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I. Report on the Dredgings in the region of St. George's Banks, in 1872.* By S. I. Smith and O. Harger.

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During the summer of 1872, a series of dredgings was carried on by the authors in the neighborhood of St. George's Banks. The work was undertaken at the instance of Professor Baird, United States Commissioner of Fish and Fisheries, and carried on, through the coöperation of the Coast Survey, from the steamer Bache, on board of which accommodations were furnished for two persons, with the necessary apparatus. On board the steamer we were received and treated throughout with the utmost courtesy by Commander J. A. Howell, and the other officers of the vessel, Lieutenants Jaques, Hagerman, Jacob and Rush; and although the dredging was carried on in connection with the special hydrographic work of the Coast Survey, all these gentlemen manifested a degree of interest in our work equal to that which they felt in their own.

On account of the lateness of the season at which operations were begun, the weather was most of the time quite unfavorable for dredging, so that the number of hauls made with the dredge was much smaller than had been expected, and no opportunities were afforded for using the large trawl or the rake dredges which had been provided, with the rest of the outfit for the natural history department of the expedition, by the United States Fish Commissioner. Still, the collections which were made from these comparatively few dredgings have proved rich and very important, giving nearly the only information which we possess of the character of the fauna of the fishing banks, and adding very largely to the knowledge of the distribution, both geographical and bathymetrical, of the marine animals of our northern coast.

* The text of this report was written and presented to the Superintendent of the Coast Survey, very nearly in its present form, in December, 1872, but its publication has been unavoidably delayed until the present. The figures illustrating some of the species mentioned have been added since the report was first prepared.

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After we were obliged, late in September, to leave the expedition, Prof. A. S. Packard, Jr., and Mr. Caleb Cooke, of Salem, Mass., went in the steamer on another trip, which was mainly devoted to dredging. On this trip a number of successful hauls were made at different localities along the northern extremity of George's Bank, in 40 to 150 fathoms. The region visited on this trip was quite distant from any of the localities examined by us, and the bottom, in the deeper dredgings, was of an entirely different character, so that the collections made by Prof. Packard and Mr. Cooke contain many species not found by us, and add very greatly to the value of the results.

We wish specially to acknowledge the assistance rendered us in the preparation of this report by Professor Verrill, who has identified all the worms mentioned, and the more difficult mollusks and radiates.

The following table will facilitate references to the localities at which the dredgings were made. The letters in the first column are the same as those used by Professor Verrill in his papers in the American Journal of Science. When more than one haul of the dredge was made at any of the localities, the number of hauls is indicated in parenthesis.

Station.	N. Lat.	W. Long.	Depth in fath.	Nature of bottom.	Temperature.		
					Air.	Surface.	Bottom.*
	° ′	° ′					
a	41 40	68 10	25	Soft sand.			
b	------	-------	30	" "			
c	41 25	66 45	28	Coarse sand.			
d	" "	66 24·8	50	Sand and shells.	66°	62°	45°
e	" "	65 58·3	60	Shells and sand.	61	58	58
f	" "	65 50·3	65	Dead shells.	64	60	55
g	" "	65 42·3	430	Sand, gravel and stones.	66	65	51
h	42 56·5	64 51·3	45	Gravel and stones.	64	61	36
i	42 44	64 36	60	Gravel, stones and sponges.	62	62	
j	------	-------	20	Mud and fine sand.			
o (3)	42 5	67 49	110	Soft mud and sand.	------	56	49
p	42 3	" "	85	" " " "	------	56	49
q (2)	42 0	67 42	45	Coarse sand.			
r	42 3	67 31	40	" "			
s (2)	42 11	67 17	150	Soft sandy mud.	------	52	52

The first dredgings were made on the evening of August 29, to the west of George's Shoal, about latitude 41° 40′ north, longitude 68° 10′

* Very little confidence can be placed in these bottom temperatures, as the Miller-Casella thermometers used did not give uniform results. Most of the temperatures are manifestly much too high.

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west. The first haul, (*a*), in 25 fathoms, soft sandy bottom, gave the following species:

CRUSTACEA.

Eupagurus Bernhardus Brandt; abundant.
Crangon vulgaris Fabricius; abundant.
Conilera polita Harger (Stimpson).
Epelys montosus Smith (Stimpson).
Balanus porcatus Costa; common.

ANNELIDA.

Cistenides granulata Malmgren.

TURBELLARIA.

Meckelia lurida (?) Verrill.

MOLLUSCA.

Bela turricula (Montagu).
B. harpularia H. and A. Adams (Couthouy).
Buccinum undatum Linné; very large and abundant.
Neptunea pygmæa H. and A. Adams (Gould); abundant.
Tritia trivittata H. and A. Adams (Say); abundant.
Lunatia heros H. and A. Adams (Say).
L. immaculata H. and A. Adams (Totten).
Crepidula plana Say (*unguiformis* Stimpson); several, living.
Ensatella Americana Verrill (Gould).
Modiolaria nigra Lovén (Gray).

RADIATA.

Echinarachnius parma Gray; very abundant.
Hydractinia polyclina Agassiz.

At the second haul (*b*), in 30 fathoms, the bottom was of the same character, but a greater variety of species was obtained.

CRUSTACEA.

Cancer irroratus Say, young; common.
Eupagurus Bernhardus Brandt; abundant.
E. pubescens Brandt; common.
Crangon vulgaris Fabricius; abundant.
Pandalus annulicornis Leach; common.
Stenothoë peltata Smith, sp. nov. [Plate III, figures 5–8.]
Photis (?) sp.
Ampelisca sp.
Xenoclea megachir Smith, sp. nov. [Plate III, figures 1–4.]
Unciola irrorata Say.
Dulichia sp.

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ANNELIDA.

Aphrodita aculeata Linné.
Clymenella torquata Verrill; tubes only.

TURBELLARIA.

Meckelia lurida (?) Verrill.
M. ingens (?) Leidy.

MOLLUSCA.

Bela turricula (Montagu).
B. harpularia H. and A. Adams (Couthouy).
Admete viridula (O. Fabricius).
Buccinum undatum Linné; large and abundant.
Neptunea pygmæa H. and A. Adams (Gould); large and common.
Lunatia heros H. and A. Adams (Say).
" " var. *triseriata* (Say).
Crepidula plana Say (*unguiformis* Stimpson).
Siliqua costata H. and A. Adams (Say).
Yoldia limatula Stimpson (Say).
Modiolaria nigra Lovén (Gray).
Farrella familiaris Smitt (Gros); abundant.
Gemellaria loricata Busk (Linné).

RADIATA.

Echinarachnius parma Gray; very abundant.
Asterias vulgaris Stimpson.
Campanularia verticillata Lamarck (Linné).
Sertularia cupressina Linné.
S. latiuscula? Stimpson.
Hydrallmania falcata Hincks (Linné); abundant.

Five successful hauls were made on the line of soundings running east from George's Bank, on the parallel of 41° 25′ north latitude, to 63° 20′ west longitude. The first of these hauls (*c*), beginning at the western end of the line, was in about longitude 66° 45′ west, from 28 fathoms, coarse sandy bottom, September 16. Here the following species occurred.

CRUSTACEA.

Cancer irroratus Say, young; abundant.
Eupagurus Bernhardus Brandt.
Crangon vulgaris Fabricius; common.
Pandalus annulicornis Leach.
Ampelisca sp.

ANNELIDA.

Nereis pelagica Linné.

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TURBELLARIA.

Meckelia lurida (?) Verrill.

MOLLUSCA.

Bela harpularia H. and A. Adams (Couthouy).
Tritia trivittata H. and A. Adams (Say); common.
Lunatia heros H. and A. Adams (Say), variety *triseriata*; common.
L. immaculata H. and A. Adams (Totten).
Crepidula fornicata Lamarck (Linné); one dead specimen.
Scalaria Grœnlandica Sowerby.
Clidiophora trilineata Carpenter.
Mactra solidissima Chemnitz (Gray).
Astarte castanea Say.
Crenella glandula H. and A. Adams (Totten).
Ostrea Virginica Lister; only dead specimens.
Glandula arenicola Verrill; abundant.

RADIATA.

Strongylocentrotus Dröbachiensis A. Agassiz.
Echinarachnius parma Gray.

At the second haul (*d*), longitude 66° 24·8′ west, 50 fathoms, sandy and shelly bottom, August 31, the following species occurred.

PYCNOGONIDA.

Nymphon grossipes Kroyer.

CRUSTACEA.

Cancer irroratus Say; young.
Hyas coarctatus Leach; abundant.
Eupagurus Bernhardus Brandt; common.
E. Kroyerii Stimpson; common.
E. pubescens Brandt; common.
Pandalus annulicornis Leach; common.
Vetumnus serratus Goës.
Melita dentata Boeck (*Gammarus purpuratus* Stimpson).
Mœra Danæ Bate (Stimpson); common.
Cerapus rubricornis Stimpson; common.
Podocerus nitidus Stimpson.
Unciola irrorata Say; common.
Balanus porcatus Costa; common.

ANNELIDA.

Aphrodita aculeata Linné.
Harmothoë imbricata Malmgren (Linné).
Phyllodoce catenula Verrill. [Plate IV, figure 3.]

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Nereis pelagica Linné; abundant.
Cistenides granulata Malmgren.
Thelepus cincinnatus Malmgren (Fabricius).
Potamilla oculifera Verrill (Leidy).
P. neglecta Malmgren.
Spirorbis nautiloides? Lamarck. [Plate IV, figure 4.]

GEPHYREA.

Phascolosoma cæmentarium Verrill.

MOLLUSCA.

Bela turricula (Montagu).
B. harpularia H. and A. Adams (Couthouy).
B. pleurotomaria H. and A. Adams (Couthouy).
B. decussata (Couthouy).
Neptunea curta Verrill (Jeffreys sp.; *Fusus Islandicus* Gould).
N. decemcostata (Say).
N. pygmæa H. and A. Adams (Gould).
Lunatia immaculata H. and A. Adams (Totten).
Natica clausa Broderip and Sowerby; common.
Amauropsis helicoides Stimpson; rare.
Crepidula plana Say (*unguiformis* Stimpson).
Acirsa borealis Mörch (Beck).
Margarita obscura Gould (Couthouy).
Hanleia mendicaria Carpenter (Mighels and Adams).
Entalis striolata Stimpson. [Plate I, figure 3.]
Æolis sp.
Thracia truncata Mighels and Adams.
Cyprina Islandica Lamarck (Linné).
Cardium pinnulatum Conrad; abundant.
Cyclocardia borealis Conrad; common.
Astarte castanea Say.
A. quadrans Gould.
Modiolaria discors Beck (Linné).
Pecten Islandicus Chemnitz (Müller).
Anomia aculeata Gmélin; abundant.
" " smooth variety.
Boltenia clavata Stimpson.
Pera crystallina Verrill (Möller); young. [Plate VIII, figure 1.]
Amaræcium sp.
Cellularia ternata Johnston (Busk).
" " var. *duplex* Smitt.
Caberea Ellisii Smitt (Fleming).

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Bugula Murrayana Busk (Bean); abundant.
Cellaria fistulosa Linné.
Discopora Skenei Smitt.
Cellepora scabra Smitt (Fabricius).
C. ramulosa Linné, var.; with the two last species abundant on hydroid stems.

RADIATA.

Strongylocentrotus Dröbachiensis A. Agassiz.
Cribrella sanguinolenta Lütken.
Leptasterias compta Verrill.
Ophiopholis aculeata Gray; common.
Amphipholis elegans Ljungman.
Ophioglypha robusta Lyman.
Hydractinia polyclina Agassiz.
Eudendrium ramosum Ehrenberg.
E. capillare Alder (?)
Tubularia indivisa Linné.
Gonothyræa Lovéni Allman.
Campanularia verticillata Lamarck (Linné); common.
C. Hincksii Alder.
C. volubilis Alder (Linné).
Lafoëa dumosa Sars (Fleming); abundant on Bryozoa.
L. gracillima G. O. Sars (Alder); with last, common.
Calycella syringa Hincks (Linné); common.
Cuspidella humilis Hincks.
Halecium Beanii Johnston.
H. tenellum Hincks.
Sertularella polyzonias Gray, var. *gigantea* Hincks; common.
S. tricuspidata Hincks (Alder); common.
Diphasia fallax Agassiz (Johnston); abundant.
Sertularia cupressina Linné; common.
S. latiuscula Stimpson.
S. abietina Linné.
Hydrallmania falcata Hincks; very abundant.
Urticina crassicornis Ehrenberg.

SPONGES.

Thecophora ibla Wyville Thompson. [Plate VII, figure 1.]
Other undetermined species.

At the third haul (*e*), longitude 65° 58·3′, 60 fathoms, shelly and sandy bottom, September 16, the following were found:

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CRUSTACEA.

Hyas coarctatus Leach; abundant.
Eupagurus Kroyerii Stimpson; abundant.
E. pubescens Brandt; abundant.
Sabinea septemcarinata Owen; one specimen.
Pandalus annulicornis Leach; common.
Paramphithoë pulchella Bruzelius (Kroyer).
Melita dentata Boeck (*Gammarus purpuratus* Stimpson).
Podocerus nitidus Stimpson.
Caprella sp.
Balanus porcatus Costa; common.

ANNELIDA.

Harmothoë imbricata Malmgren.
Rhynchobolus capitatus Verrill (Œrsted sp., *not* of Claparède).
Thelepus cincinnatus Malmgren (Fabricius).
Spirorbis nautiloides? Lamarck. [Plate IV, figure 4.]

GEPHYREA.

Phascolosoma cæmentarium Verrill.

MOLLUSCA.

Bela violacea (Mighels and Adams).
B. harpularia H. and A. Adams (Couthouy).
Neptunea curta Verrill (Jeffreys sp.; *Fusus Islandicus* Gould).
N. decemcostata (Say).
N. pygmæa H. and A. Adams (Gould); common.
Lunatia heros H. and A. Adams (Say), variety *triseriata.*
Natica clausa Broderip and Sowerby.
Amauropsis helicoides Stimpson; rare.
Stylifer Stimpsonii Verrill; parasitic on *Strongylocentrotus Drö-bachiensis* A. Agassiz. [Plate I, figure 1.]
Aporrhais occidentalis Sowerby.
Acirsa borealis Mörch (Beck).
Margarita obscura Gould (Couthouy); common.
Diodora noachina Gray (Linné).
Hanleia mendicaria Carpenter (Mighels and Adams).
Cylichna alba Lovén (Brown).
Entalis striolata Stimpson; common. [Plate I, figure 3.]
Dendronotus arborescens Alder and Hancock.
Cyprina Islandica Lamarck (Linné); very abundant.
Cardium pinnulatum Conrad; common.
Astarte castanea Say.
A. quadrans Gould.

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Astarte undata Gould. [Plate I, figures 6–9.]
Modiolaria corrugata Mörch (Stimpson).
Pecten tenuicostatus Mighels.
P. Islandicus Chemnitz (Müller).
Anomia aculeata Gmélin, smooth variety; common.
Cellularia ternata Johnston (Busk); common.
Bugula Murrayana Busk (Bean).
Cellaria fistulosa Linné.
Discopora Skenei Smitt, variety.
Cellepora scabra Smitt (Fabricius); with the last on hydroid stems.

RADIATA.

Lophothuria Fabricii Verrill.
Psolus phantapus Oken.
Strongylocentrotus Dröbachiensis A. Agassiz; abundant.
Echinarachnius parma Gray; abundant.
Crossaster papposus Müller and Troschel.
Cribrella sanguinolenta Lütken.
Hydractinia polyclina Agassiz; common.
Eudendrium capillare Alder.
E. ramosum Ehrenberg.
Tubularia indivisa Linné; common.
Campanularia verticillata Lamarck (Linné); common.
C. Hincksii Alder.
C. volubilis Alder.
Gonothyræa hyalina Hincks.
Lafoëa dumosa Sars (Fleming); common.
L. gracillima G. O. Sars (Alder).
Grammaria abietina Sars.
Coppinia arcta Hincks (Dalyell); on hydroid stems.
Sertularella tricuspidata Hincks (Alder); abundant.
Diphasia mirabilis Verrill.
D. fallax Agassiz.
Sertularia latiuscula Stimpson.
S. cupressina Linné.
Hydrallmania falcata Hincks (Linné); common.
Epizoanthus Americanus Verrill; coating shells inhabited by *Eupagurus pubescens*, and also on hydroid stems. [Plate VIII, fig. 2.]
Urticina crassicornis Ehrenberg.

At the fourth haul (*f*), longitude 65° 50·3′, 65 fathoms, the bottom composed of dead shells, September 15, midnight, the following occurred:

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CRUSTACEA.

Eupagurus Kroyerii Stimpson.
E. pubescens Brandt.
Crangon vulgaris Fabricius.

MOLLUSCA.

Bela decussata (Couthouy).
Natica clausa Broderip and Sowerby.
Stylifer Stimpsonii Verrill; parasitic on *Strongylocentrotus Dröbachiensis* A. Agassiz. [Plate I, figure 1.]
Acirsa borealis Mörch (Beck).
Margarita obscura Gould (Couthouy), variety.
Mactra solidissima Chemnitz (Gray); abundant.
Cyprina Islandica Lamarck (Linné); common.
Cyclocardia borealis Conrad.
Astarte undata Gould. [Plate I, figures 6–9.]
Crenella glandula H. and A. Adams (Totten).

RADIATA.

Strongylocentrotus Dröbachiensis A. Agassiz; common.
Echinarachnius parma Gray; abundant.
Hydractinia polyclina Agassiz.
Tubularia indivisa Linné.
Campanularia verticillata Lamarck (Linné).
Sertularella tricuspidata Hincks (Alder); common.
Sertularia cupressina Linné.
Epizoanthus Americanus Verrill; coating shells inhabited by *Eupagurus pubescens*. [Plate VIII, figure 2.]
Urticina crassicornis Ehrenberg.

The fifth haul (*g*) on this line was made on the evening of September 15, to the east of the bank, in longitude 65° 42·3′ west, at a depth of about 430 fathoms, on a bottom of sand, gravel, small and large stones. Here the following species occurred:

PYCNOGONIDA.

Pycnogonum littorale Müller (*pelagicum* Stimpson); common.

CRUSTACEA.

Eupagurus Kroyerii Stimpson; common.
Pandalus annulicornis Leach; several specimens.
Thysanopoda sp.; several specimens, perhaps not from the bottom.
Unciola irrorata Say; several specimens.
Melita dentata Boeck.
Scalpellum Stroemi Sars; on hydroid stems. [Plate III, fig. 9.]

ANNELIDA.

Nothria conchylega Malmgren (Sars); abundant. [Plate VII, fig. 3.]

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Nephthys circinata Verrill, sp. nov.
Lumbriconereis fragilis Œrsted (Müller).
Leodice vivida Verrill (*Eunice vivida* Stimpson). [Plate V, fig. 5.]
Rhynchobolus capitatus Verrill (Œrsted sp., *not* of Claparède).

GEPHYREA.

Phascolosoma tubicola Verrill.

MOLLUSCA.

Bela cancellata (Mighels and Adams).
Neptunea pygmæa H. and A. Adams (Gould).
Lunatia Grœnlandica (Möller).
L. immaculata H. and A. Adams (Totten).
Natica clausa Broderip and Sowerby.
Margarita obscura Gould (Couthouy).
Diodora noachina Gray (Linné), variety *princeps* (Mighels and Adams).
Entalis striolata Stimpson. [Plate I, figure 3.]
Astarte lens Stimpson; dwarf variety.
Cryptodon obesus Verrill. [Plate I, figure 11.]
Pecten pustulosus Verrill.
Vescicularia armata Verrill; on *Sertularia argentea.*
Several other species of Bryozoa.

RADIATA.

Lophothuria squamata Verrill.
Pentacta assimilis (Duben and Koren).
Schizaster fragilis Agassiz (Duben and Koren).
Strongylocentrotus Dröbachiensis A. Agassiz; several.
Echinarachnius parma Gray; common.
Ophioglypha Sarsii Lyman; common.
Eudendrium ramosum Ehrenberg (Linné).
Tubularia indivisa Linné.
Campanularia virticillata Lamarck (Linné).
Lafoëa gracillima G. O. Sars (Alder).
Calycella producta G. O. Sars.
Halecium robustum Verrill.
Sertularella Gayi Hincks (Lamoroux).
S. tricuspidata Hincks (Alder); with reproductive capsules.
Sertularia argentea Linné, slender variety; with reproductive capsules.
Epizoanthus Americanus Verrill; upon small stones.
Urticina crassicornis Erhenberg; young specimens.
U. nodosa Verrill (Fabricius); two large specimens.

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On the line of soundings from near Cape Sable, Nova Scotia, to latitude 41° 25′ north, longitude 63° 20′ west, two successful hauls were made September 12, on Le Have Bank. The first (*h*), latitude 42° 56·5′ north, longitude 64° 51·3′ west, 45 fathoms, gravelly and stony bottom, gave the following species:

CRUSTACEA.

Hyas coarctatus Leach; very abundant.
Eupagurus Kroyerii Stimpson; abundant.
Hippolyte spina Leach; several specimens.
H. pusiola Kroyer.
Syrrhoë crenulata Goës; a single specimen.
Tiron acanthurus Lilljeborg; one specimen.
Paramphithoë cataphracta Smith (Stimpson).
Tritropis aculeata Boeck, and several undetermined Amphipods.

ANNELIDA AND GEPHYREA.

Nychia cirrosa Malmgren (Pallas).
Eunoë nodosa Malmgren (Sars).
Harmothoë imbricata Malmgren (Linné).
Nereis pelagica Linné; abundant.
Nothria conchylega Malmgren (Sars); very abundant. [Plate VII, figure 3.]
Spiochætopterus (?); tubes only.
Cistenides granulata Malmgren.
Thelepus cincinnatus Malmgren (Fabricius).
Potamilla oculifera Verrill (Leidy).
P. neglecta Malmgren; very abundant.
Spirorbis valida Verrill, sp. nov.
S. lucidus Mörch.
Phascolosoma cæmentarium Verrill.

TURBELLARIA.

Leptoplana ellipsoides Girard.

MOLLUSCA.

Bela violacea (Mighels and Adams).
Admete viridula Stimpson (O. Fabricius).
Buccinum undatum Linné; common.
Neptunea decemcostata (Say).
N. pygmæa H. and A. Adams (Gould); common.
Trophon Gunneri Lovén; three specimens.
Lunatia Grœnlandica (Möller).
Natica clausa Broderip and Sowerby; abundant.
Crepidula plana Say (*unguiformis* Stimpson); one alive.
Trichotropis borealis Sowerby; abundant.

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Aporrhais occidentalis Sowerby.
Turritella erosa Couthouy; common.
Scalaria Grœnlandica Sowerby; abundant.
Margarita cinerea Gould; common.
M. Grœnlandica Möller (*M. undulata* Gould).
Trachydermon album Carpenter (Montagu).
Entalis striolata Stimpson; abundant. [Plate I, figure 3.]
Dendronotus arborescens Alder and Hancock.
Mya truncata Linné.
Cardium pinnulatum Conrad; common.
Cyclocardia Novangliæ Morse; common.
Astarte elliptica (Brown); very abundant. [Plate I, figure 10.]
A. Banksii Leach; common. [Plate I, figure 12.]
A. undata Gould; common. [Plate I, figures 6–9.]
Pecten Islandicus Chemnitz (Müller); abundant.
Boltenia Bolteni.
Cynthia carnea Verrill.
Terebratulina septentrionalis (Couthouy); common.
Rhynchonella psittacea (Gmélin).
Myriozoum coarctatum Smitt (Sars); common.
Eschara papposa Packard.
Escharoides rosacea Smitt.
Cellepora avicularis Hincks.

RADIATA.

Lophothuria Fabricii Verrill.
Strongylocentrotus Dröbachiensis A. Agassiz; common.
Crossaster papposus Müller and Troschel; young.
Pteraster militaris Müller and Troschel.
Ophiopholis aculeata Gray; common.
Ophioglypha Sarsii Lyman.
O. robusta Lyman.
Clytia Johnstoni Hincks (Alder).
Hydractinia polyclina Agassiz; abundant.
Eudendrium cappilare Alder.
Tubularia indivisa Linné; common.
Campanularia verticillata Lamarck (Linné).
C. Hincksii Alder.
Lafoëa gracillima G. O. Sars.
Calycella syringa Hincks (Linné).
Sertularella tricuspidata Hincks (Alder); common.
S. polyzonias Gray, variety *gigantea* Hincks.
Thuiaria articulata Fleming (Pallas).
Urticina crassicornis Ehrenberg; abundant.

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At the second haul (*i*), latitude 41° 44′ north, longitude 64° 36′ west, 60 fathoms, coarse gravel, stones, and sponges, the following occurred:

PYCNOGONIDA.

Nymphon grossipes Kroyer.

CRUSTACEA.

Hyas coarctatus Leach.
H. araneus Leach.
Eupagurus Kroyerii Stimpson; abundant.
Sabinea septemcarinata Owen; two specimens.
Tritropis aculeata Boeck.
Acanthozone cuspidata Boeck.

ANNELIDA.

Eunoë nodosa Malmgren (Sars).
Harmothoë imbricata Malmgren (Linné).
Lagisca rarispina Malmgren (Sars).
Nothria conchylega Malmgren (Sars); abundant. [Plate VII, fig. 3.]
Thelepus cincinnatus Malmgren (Fabricius).
Potamilla neglecta Malmgren; abundant.
Spirorbis valida Verrill.

MOLLUSCA.

Admete viridula Stimpson (O. Fabricius); common.
Trophon Gunneri Lovén.
Natica clausa Broderip and Sowerby.
Aporrhais occidentalis Sowerby.
Turritella reticulata Mighels and Adams.
Margarita cinerea Gould.
Hanleia mendicaria Carpenter (Mighels and Adams).
Trachydermon album Carpenter (Montagu).
Entalis striolata Stimpson; abundant.
Æolis rufibranchialis Alder and Hancock (?).
Cardium pinnulatum Conrad.
Terebratulina septentrionalis (Couthouy); common.
Amarœcium glabrum Verrill.
Discopora Skenei Smitt.

RADIATA.

Cribrella sanguinolenta Lütken.
Ophiopholis aculeata Gray; abundant.
Ophioglypha Sarsii Lyman.
Lafoëa gracillima G. O. Sars.
Sertularella tricuspidata Hincks.

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Sertularella polyzonias Gray, variety *gigantea* Hincks.
Hydrallmania falcata Hincks, var. *tenerrima* (Stimpson).
Aglaophenia myriophyllum Lamoroux (Linné).
Grammaria abietina Sars.

Many species of sponges were also obtained, but most of them are as yet undetermined. Among them is *Thecophora ibla* W. Thompson.

On leaving Halifax, N. S., September 11, one haul (*j*) was made just off Chebucto Head, in 20 fathoms, soft mud and fine sand with decaying sea weed. Here the following were found:

CRUSTACEA.

Hyas araneus Leach; common.
Eupagurus pubescens Brandt.
Diastylis quadrispinosa G. O. Sars; common.
D. sculpta G. O. Sars.
Halirages fulvocinctus Boeck (Sars).
Gammarus ornatus Edwards; perhaps from floating sea-weed.
Ampelisca sp.; common.
Monoculodes borealis Boeck.

ANNELIDA.

Harmothoë imbricata Malmgren (Linné).
Goniada maculata Œrsted.
Brada sp.
Cistenides granulata Malmgren.

MOLLUSCA.

Aporrhais occidentalis Sowerby; common.
Turritella reticulata Mighels and Adams; common.
Margarita varicosa Mighels and Adams; common.
M. obscura Gould (Couthouy), variety.
Thracia myopsis Beck.
Macoma proxima (Gray).
Astarte elliptica (Brown). [Plate I, figure 10.]
Anomia aculeata Gmélin.
Terebratulina septentrionalis (Couthouy).
Rhynchonella psittacea (Gmélin).
Crisia eburnea Lamoroux (Linné). [Plate II, figures 3–4.]
Flustra papyrea (Pallas).

RADIATA.

Ophiopholis aculeata Gray.
Ophioglypha robusta Lyman.
Manania auricula Clark (?)
Hydrallmania falcata Hincks (Linné).

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The dredging while Dr. Packard and Mr. Cooke were on board the Bache was at five localities. First (*o*), just on the northwestern border of George's Bank, latitude 42° 5′ north, longitude 67° 49′ west, in 110 fathoms, three hauls were made from a bottom of soft sandy mud with a few stones, and the following collected:

CRUSTACEA.

Caridion Gordoni Goës; one small specimen.
Harpina fusiformis (Stimpson); common.
Stegocephalus ampulla Bell; one large specimen.
Unciola irrorata Say; common.
Anthura brachiata Stimpson.

ANNELIDA.

Lænilla? mollis G. O. Sars.
Pholoë minuta Malmgren.
Nephthys ciliata Malmgren (Müller). [Plate V, figure 1.]
N. ingens Stimpson.
Phyllodoce sp.
Eteone depressa Malmgren (?).
Nereis pelagica Linné; common.
Lumbriconereis fragilis Œrsted (Müller). [Plate V, figure 2.]
Ninoë nigripes Verrill. [Plate V, figure 3.]
Leodice vivida Verrill (Stimpson). [Plate V, figure 5.]
Nothria conchylega Malmgren (Sars). [Plate VII, figure 3.]
N. opalina Verrill; common. [Plate VII, figure 4.]
Goniada maculata Œrsted.
Rhynchobolus capitatus Verrill (Œrsted sp., *not* of Claparède).
Ammotrypane fimbriata Verrill.
Eumenia crassa Œrsted.
Trophonia aspera Verrill (Stimpson).
Sternaspis fossor Stimpson.
Scolecolepis cirrata Malmgren (Sars).
Notomastus latericius Sars.
Ancistria capillaris Verrill.
Maldane Sarsii Malmgren.
Rhodine Lovéni Malmgren.
Nicomache lumbricalis Malmgren (Fabricius).
Axiothea catenata Malmgren (?).
Praxilla prætermissa Malmgren.
P. gracilis Malmgren.
P. species undetermined.
Ammochares assimilis Sars. [Plate V, figure 4.]
Ampharete arctica Malmgren.

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Ampharete Finmarchica (?) Malmgren (Sars).
A. gracilis Malmgren.
Amphicteis Gunneri Malmgren (Sars); abundant.
Samytha sexcirrata Malmgren (Sars).
Samythella elongata Verrill.
Melinna cristata Malmgren (Sars.)
Amphitrite cirrata Müller.
Pista cristata Malmgren (Müller). [Plate IV, figure 2.]
Grymæa spiralis Verrill. [Plate IV, figure 1.]
Terebellides Stroemi Sars.
Polycirrus sp.
Sabella pavonia (?) Malmgren.
Potamilla neglecta Malmgren.
Protula media Stimpson; tubes only. [Plate VI.]
P. borealis (?) Sars; tubes only.

GEPHYREA.

Phascolosoma cœmentarium Verrill.
P. boreale Keferstein (?)
P. tubicola Verrill.
Chætoderma nitidulum Lovén. [Plate VIII, figures 3–4.]

TURBELLARIA.

Meckelia lurida (?) Verrill.

MOLLUSCA.

Pleurotomella Packardii Verrill; one living.
Admete viridula Stimpson (O. Fabricius).
Neptunea pygmæa H. and A. Adams (Gould); common.
Ringicula nitida Verrill. [Plate I, figure 2.]
Lunatia Grœnlandica (Möller).
Natica clausa Broderip and Sowerby.
Velutina lævigata (Linné).
Margarita cinerea Gould.
Lepeta cæca Müller.
Trachydermon album Carpenter (Montagu).
Cylichna alba Lovén (Brown).
Philine sp.
Entalis striolata Stimpson; common.
Neæra arctica Sars.
Thracia myopsis Beck.
Macoma proxima (Gray).
Cardium pinnulatum Conrad.
Astarte lens Stimpson, dwarf var.; common.

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Cryptodon Gouldii H. and A. Adams (Philippi).
Yoldia obesa Stimpson.
Leda tenuisulcata Stimpson (Couthouy); common.
Crenella glandula H. and A. Adams (Totten).
Pecten Islandicus Chemnitz (Müller).
P. tenuicostatus Mighels.
Anomia aculeata Gmélin.
Ascidiopsis complanata Verrill (Fabricius).
Terebratulina septentrionalis (Couthouy).
Discofascigera lucernaria Sars.
Cellularia sp.
Bugula avicularia Busk, variety *fastigiata*.

RADIATA.

Lophothuria Fabricii Verrill.
Pentacta assimilis (Duben and Koren).
Thyone scabra Verrill.
Schizaster fragilis Agassiz (Duben and Koren); several.
Ctenodiscus crispatus Duben and Koren.
Ophioglypha affinis Lyman.
O. Sarsii Lyman.
Opiopholis aculeata Gray.
Ophiacantha spinulosa Müller and Troschel.
Archaster arcticus Sars.
Pennatula aculeata Danielsen.
Cerianthus borealis Verrill. [Plate II, figure 5.]

Second (*p*), a little to the southeast of the first, latitude 42° 3′, longitude 67° 49′, 85 fathoms, one haul from a bottom of the same character as at first locality:

CRUSTACEA.

Harpina fusiformis Smith (Stimpson); common.

ANNELIDA.

Antinoë Sarsii Kinberg.
Nephthys ingens Stimpson.
N. circinata Verrill, sp. nov.
Lumbriconereis fragilis Œrsted (Müller). [Plate V, figure 2.]
Nothria conchylega Malmgren (Sars). [Plate VII, figure 3.]
Ammotrypane fimbriata Verrill.
Eumenia crassa Œrsted.
Trophonia aspera Verrill (Stimpson).
Sternaspis fossor Stimpson.
Chætozone setosa Malmgren.

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Ancistria capillaris Verrill.
Maldane Sarsii Malmgren.
Rhodine Lovêni Malmgren.
Praxilla prætermissa Malmgren.
Ammochares assimilis Sars.
Ampharete arctica Malmgren.
Amphicteis Sundevalli Malmgren.
Terebellides Stroemi Sars.
Polycirrus sp.

GEPHYREA.

Phascolosoma cæmentarium Verrill.

MOLLUSCA.

Natica clausa Broderip and Sowerby.
Scalaria Grœnlandica Sowerby.
Yoldia obesa Stimpson.
Y. thraciformis Stimpson (Storer).

RADIATA.

Schizaster fragilis Agassiz (Duben and Koren).
Edwardsia sp.

Third (*q*), still farther to the southeast, latitude 42°, longitude 67° 42′, two hauls in 45 fathoms, coarse sandy bottom:

CRUSTACEA.

Hyas coarctatus Leach; very abundant.
Cancer irroratus Say; one young specimen.
Eupagurus Bernhardus Brandt.
E. Kroyeri Stimpson; common.
E. pubescens Brandt; common.
Crangon vulgaris Fabricius; abundant.
Hippolyte pusiola Kroyer.
Pandalus annulicornis Leach; common.
Vertumnus serratus Goës.
Paramphithoë cataphracta Smith (Stimpson).
P. pulchella Bruzelius.
Phoxus Kroyeri Stimpson.
Melita dentata Boeck.
Pontogeneia inermis Boeck; one specimen.
Cerapus rubricornis Stimpson.
Xenoclea megachir Smith, sp. nov. [Plate III, figures 1-4.]
Unciola irrorata Say.
Balanus porcatus Costa.

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ANNELIDA.

Harmothoë imbricata Malmgren.
Lagisca propinqua Malmgren.
Eusyllis phosphorea Verrill, sp. nov. [Plate VII, figure 2.]
Nereis pelagica Linné.
Leodice vivida Verrill (Stimpson). [Plate V, figure 5.]
Nothria conchylega Malmgren (Sars). [Plate VII, figure 3.]
Ammochares assimilis Sars. [Plate V, figure 4.]
Amphitrite Grœnlandica Malmgren.
Thelepus cincinnatus Malmgren (Fabricius).
Chone infundibuliformis Kroyer.
Spirorbis nautiloides Lamarck?

MOLLUSCA.

Bela harpularia H. and A. Adams (Couthouy).
B. pleurotomaria H. and A. Adams (Couthouy).
B. turricula (Montagu).
Buccinum undatum Linné.
Neptunea curta Verrill (Jeffreys sp., *Fusus Islandicus* Gould).
N. pygmæa H. and A. Adams (Gould).
Lunatia heros H. and A. Adams, variety *triseriata* (Say).
L. immaculata H. and A. Adams (Totten).
Turritella erosa Couthouy.
T. acicula Stimpson.
Margarita obscura Gould (Couthouy); common.
Diodora noachina Gray (Linné).
Hanleia mendicaria Carpenter; large specimens.
Mactra solidissima Chemnitz (Gray).
Cyprina Islandica Lamarck (Linné).
Cardium pinnulatum Conrad; common.
Astarte quadrans Gould.
Leda tenuisulcata Stimpson (Couthouy); common.
Crenella glandula H. and A. Adams (Totten).
Modiola modiolus Turton (Linné).
Modiolaria lævigata (Gray).
Pecten tenuicostatus Mighels.
P. Islandicus Chemnitz (Müller).
Anomia aculeata Gmélin.
Glandula arenicola Verrill.
Cellularia ternata Johnston (Busk).
Gemellaria loricata Busk (Linné).
Bugula Murrayana Busk (Bean).
Cellepora tuberosa D'Orbigny.

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RADIATA.

Lophothuria Fabricii Verrill.
Strongylocentrotus Dröbachiensis A. Agassiz; common.
Echinarachnius parma Gray; very abundant.
Solaster endeca Forbes.
Crossaster papposus Müller and Troschel.
Cribrella sanguinolenta Lütken.
Asterias vulgaris Stimpson.
Leptasterias Stimpsoni Verrill.
Ophiopholis aculeata Gray; common.
Ophioglypha robusta Lyman.
Eudendrium ramosum Ehrenberg (Linné).
E. capillare Alder.
Campanularia verticillata Lamarck; common.
C. Hincksii Alder.
Lafoëa dumosa Sars (Fleming); common.
Calycella syringa Hincks (Linné).
Grammaria abietina Sars (*G. robusta* Stimpson).
Coppinia arcta Hincks; on hydroid stems.
Halecium labrosum Alder.
Sertularella polyzonias Gray (Linné); common.
" " variety *gigantea* Hincks.
S. tricuspidata Hincks (Alder); abundant.
Sertularia abietina Linné; one, very large.
S. latiuscula Stimpson.
S. cupressina Linné; common.
S. argentea Ellis and Solander. [Plate III, figure 2.]
Hydrallmania falcata Hincks; abundant.

Third (*r*), north and a little east of the last, latitude 42° 3′, longitude 67° 31′, in 40 fathoms, coarse sandy bottom:

CRUSTACEA.

Eupagurus Bernhardus Brandt.

ANNELIDA.

Dodecaceria concharum Œrsted.
Spirorbis quadrangularis Stimpson.

MOLLUSCA.

Bela harpularia H. and A. Adams (Couthouy).
Natica clausa Broderip and Sowerby.
Scalaria Grœnlandica Sowerby.
Margarita obscura Gould (Couthouy).
Mactra solidissima Chemnitz (Gray).

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Cyprina Islandica Lamarck (Linné).
Cardium pinnulatum Conrad.
Astarte castanea Say.
Pecten tenuicostatus Mighels.
Cellularia sp.
Bugula Murrayana Busk (Bean).

RADIATA.

Echinarachnius parma Gray.
Hydractinia polyclina Agassiz.
Sertularella polyzonias Gray, var. *gigantea* Hincks.

SPONGES.

Chalina oculata Bowerbank, and a massive siliceous sponge.

Fifth (*s*) a little to northeast of the bank, latitude 42° 11′, longitude 67° 17′, two hauls in 150 fathoms, soft sandy mud with a few pebbles:

CRUSTACEA.

Hyas coarctatus Leach.
Eupagurus Bernhardus Brandt.
E. Kroyeri Stimpson.
E. pubescens Brandt.
Ptilocheirus pinguis Stimpson.
Æga psora Bate and Westwood.
Conilera polita Harger (Stimpson).
Balanus porcatus Costa.

ANNELIDA.

Lætmonice filicornis Kinberg.
Harmothoë imbricata Malmgren (Linné).
Antinoë angusta Verrill, sp. nov.
Eucranta villosa Malmgren.
Nepthys ingens Stimpson.
Lumbriconereis fragilis Œrsted (Müller). [Plate V, figure 2.]
Nothria conchylega Malmgren (Sars). [Plate VII, figure 3.]
N. opalina Verrill. [Plate VII, figure 4.]
Goniada maculata Œrsted.
Scalibregma inflatum Rathke.
Spiochætopterus (?); tubes exactly like those of this genus.
Scolecolepis cirrata Malmgren (Sars).
Ancistria capillaris Verrill.
Maldane Sarsii Malmgren; abundant.

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Cistenides granulata Malmgren.
Amphicteis Gunneri Malmgren (Sars).
Amage auricula Malmgren.
Smythella elongata Verrill.
Melinna cristata Malmgren (Sars).
Pista cristata Malmgren (Sars). [Plate IV, figure 2.]
Grymæa spiralis Verrill. [Plate IV, figure 1.]
Terebellides Stroemi Sars.
Protula borealis Sars?; tubes only.

GEPHYREA.

Phascolosoma cœmentarium Verrill.
P. boreale Keferstein (?)
P. tubicola Verrill.

MOLLUSCA.

Admete viridula Stimpson (Müller).
Neptunea pygmæa H. and A. Adams (Gould); abundant.
Ringicula nitida Verrill; one living. [Plate I, figure 2.]
Lunatia Grœnlandica (Möller).
Natica clausa Broderip and Sowerby.
Velutina zonata Gould.
Torrellia vestita Jeffreys; one specimen.
Aporrhais occidentalis Sowerby.
Margarita cinerea Gould.
M. obscura Gould (Couthouy).
Trachydermon album Carpenter.
Scaphander puncto-striatus Stimpson; one very large.
Cylichna alba Lovén (Brown).
Entalis striolata Stimpson; abundant. [Plate I, figure 3.]
Dentalium occidentale Stimpson; one specimen.
Neæra arctica Sars.
Thracia myopsis Beck.
Cardium pinnulatum Conrad.
Astarte quadrans Gould.
A. lens Stimpson, dwarf variety; common.
Yoldia obesa Stimpson.
Leda tenuisulcata Stimpson (Couthouy).
Arca pectunculoides Scacchi; several.
Pecten pustulosus Verrill; one living.
Anomia aculeata Gmélin.
Glandula arenicola Verrill; common.
Terebratulina septentrionalis (Couthouy); abundant.

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Hornerea lichenoides Smitt (Linné).
Discoporella verrucaria Smitt.
Cellularia ternata Johnston, var. *gracilis* (*arctica* Busk sp.).
C. Peachii Busk.
Caberea Ellisii Smitt (Fleming).
Bugula Murrayana Busk (Bean); abundant.
B. avicularia Busk, var. *fastigiata.*
Cellepora scabra Smitt (Fabricius).
C. ramulosa (Linné).

RADIATA.

Thyone scabra Verrill.
Schizaster fragilis Agassiz (Duben and Koren); several.
Echinarachnius parma Gray.
Solaster furcifer Duben and Koren; one specimen.
Archaster arcticus Sars; one specimen.
Ophioglypha Sarsii Lyman.
O. affinis Lyman.
Ophiacantha spinulosa Müller and Troschel.
Clytia Johnstoni Hincks.
Eudendrium capillare Alder.
Sertularella tricuspidata Hincks (Alder).
S. Gayi Hincks (Lamoroux).
Sertularia cupressina Linné.
Pennatula aculeata Danielsen.
Virgularia Lyungmanii Kölliker.
Bolocera Tuediæ Gosse; tentacles only.
Urticina crassicornis Ehrenberg.
Cerianthus borealis Verrill. [Plate II, figure 5.]

The lists of species from all the localities (*a*, *b*, *c*, *d*, *e*, *f*, *q*, *r*,) on George's Bank itself, show that the fauna of that region is almost exactly the same as in the Bay of Fundy, at the same depths and on similar bottom. To be sure, on the one hand, several arctic species, not yet found in the Bay of Fundy, occurred upon the Bank; but on the other hand, several apparently more southern forms were found, as the species of *Crepidula* and *Stylifer*. The two dredgings (*h* and *i*) upon LeHave Bank seem to indicate, as we might expect, a somewhat more arctic fauna than that upon George's Bank, since several arctic species, not known from George's Bank or the Bay of Fundy, occurred there, though *Crepidula plana* was also found.

The dredgings in deep water near the Banks indicate a fauna quite different from that upon the Banks themselves. This is undoubtedly

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partially owing to the difference in the character of the bottom as well as to the difference in depth. Of the species occurring in deep water, a much larger proportion than in the shallower waters are the same as those of northern Europe. At the greatest depth reached by the dredge, about 430 fathoms, at the locality (*g*) east of George's Bank, almost all the species which were not also found in shallow water are European. Some of these species, however, were dredged in 1872, by Prof. Verrill, in the central part of the Bay of Fundy, east of Grand Menan Island, in 95 to 106 fathoms, where the character of the bottom was quite similar to that of our deepest dredging.

At each of the three deepest of Dr. Packard's dredgings, (*o*) 110 fathoms, (*p*) 85 fathoms, and (*s*) 150 fathoms, the bottom was composed of soft sandy mud, very different in character from that at any of the localities examined by us. The fauna of the bottom at these three places was essentially the same, and, although many of the species, on account of the character of the bottom, were different from those at the locality in 430 fathoms, about the same proportion are identical with European species.

Although the dredgings in deep water were so few, the facts presented in the foregoing lists with reference to the bathymetrical distribution of species, are important and very interesting. Of the species enumerated from 430 fathoms, considerably more than half are well known shallow water forms, many of them even occurring between tides in the Bay of Fundy and at other points on the coast, while nearly all the species mentioned are also found at less than 50 fathoms depth. The same remarks will apply to the deeper dredgings of Dr. Packard and Mr. Cooke. The species from the deepest dredging belong apparently to as highly organized groups of animals as do those from shallow water. We were not able to detect any decrease in the intensity of the colors in individuals from this depth. The colors of *Pandalus annulicornis*, *Eupagurus Kroyeri*, *Unciola irrorata*, and *Urticina crassicornis*, all brightly-colored species, seemed to have lost none of their intensity at the depth of 430 fathoms.

Besides the investigation of the fauna of the bottom by means of the dredge, every opportunity was employed for collecting those animals which live in part or wholly at the surface of the water. Notwithstanding the unfavorable character of the weather during most of the time we were at sea, towing nets were used, whenever soundings were being made, and usually with very good results. Nets of small size were several times successfully used even when the

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steamer was under weigh. In this way a great number of surface species were collected, and a large proportion of them are additions to the fauna of our coast. Many of these species belong to genera previously known only from much farther south, or from the eastern or southern Atlantic, while quite a number are undescribed.

August 29, on and near Cultivator Shoal (*k*), where the surface temperature of the water was 62°, the following were taken: *Trachynema digitale* A. Agassiz, *Pleurobrachia rhododactyla* Agassiz, species of *Sagitta* and *Autolytus*, several species of Copeopod crustacea, *Calliopius læviusculus* Boeck (among floating rock-weed), the young of some Brachyuran in the zoëa and megalops stages of growth, and a species of *Motella* (?).

East of George's Bank, in latitude 41° 20′ to 30′, longitude 63° to 63° 30′, September 14, during the day, many species were taken, but as they all occurred, with many additional species, on the following day, it is not necessary to enumerate them separately.

On the evening of September 14, from nine to ten o'clock, still east of the Bank (*m*), in latitude 41° 25′, longitude 63° 55′, while the surface temperature was 65°, the following forms occurred: *Pleurobrachia* sp.; a species of *Salpa* in abundance; several species of Heteropods and Pteropods, among the latter *Spirialis Gouldii* Stimpson, and species of *Styliola;* a species of *Sagitta;* a species of *Sapphirina* and a great many other Copeopods; species of *Hyperia*, *Phrosina*, and of another allied genus; a species of *Thysanopoda*, which was beautifully phosphorescent; young Brachyura in the zoëa and megalops stages, and the young of some Macrouran.

September 15, on the same line of soundings, in latitude 41° 25′, longitude 65° 5′ to 30′, the temperature of the water varying from 66° to 70°, but most of the time at the latter point, very many species occurred, and among them the following: *Physalia pelagica* Lamarck (Portuguese man-of-war), *Cestum Veneris* Lesueur (?), *Stomolophus meleagris* Agassiz, *Charybdea periphylla* Péron and Lesueur, *Pelagia cyanella* Péron and Lesueur; species of *Salpa* and *Sagitta* in great abundance; *Lepas pectinata* Spengler and *L. fascicularis* Ellis and Solander; two species of *Sapphirina* and many other genera of Copeopoda; species of *Oxycephalus*, *Platyscelus*, *Pronoë*, *Anchylomera*, *Thyropus*, *Phronima* (?), and *Hyperia; Calliopius læviusculus* Boeck, common among floating rock-weed; species of *Lucifer* and *Mysis; Latreutes ensiferus* Stimpson, *Nautilograpsus minutus* Edwards, and *Neptunus Sayi* Stimpson among gulf-weed, and the latter frequently seen swimming at some distance from the sea-weed; three species of Heteropods and ten species of Pteropods, all new to our coast.

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Among the Pteropods are *Styliola acus* (Eschscholtz sp.), and four other species of the same genus, two of *Pleuropus*, *Spirialis Gouldii*, etc. Many of these species and genera are quite new to the fauna of the United States, and nearly all of them to the coast of New England. They are nearly all, as far as known, characteristic Gulf Stream forms.

Notes on some of the Species enumerated; by S. I. SMITH.*

CRUSTACEA.

Eupagurus Bernhardus Brandt.

Pagurus Bernhardus (Linné sp.) Fabricius, Entomologia systematica, ii, p. 469, 1793, and Supplementum, p. 411, 1798.

Pagurus (subgenus *Eupagurus*, section *Streptodactylus*) *Bernhardus* Brandt, Middendorff's Sibirische Reise, Krebse, p. 106, 1851.

Eupagurus Bernhardus Stimpson, Crust. Pacific Shores of North America, Journal Boston Soc. Nat. Hist., vi, p. 483 (separate copies, p. 43), 1857.

I have recently † wrongly given Stimpson as authority for this and the next species, not having at the time access to Brandt's work, and not being able to comprehend his absurdly complex nomenclature from the quotation of his names by other authors.

Eupagurus pubescens Brandt.

Pagurus pubescens Kroyer (in part), Grönlands Amfipoder, p. 68, 1838, and Naturhistorisk Tidsskrift, ii, p. 251, 1839.

Pagurus (subgenus *Eupagurus*, section *Orthodactylus*) *pubescens* Brandt, op. cit., p. 111, 1851.

Eupagurus pubescens Stimpson, Prodromus descriptionis Animalium evertebratorum, etc., Proceedings Acad. Nat. Sci., Philadelphia, 1858, p. 237 (separate copies, p. 75), 1859, and Notes on North American Crustacea, Annals Lyceum Nat. Hist., New York, vii, p. 89 (separate copies, p. 43), 1859.

This species is common on our eastern coast north of Cape Cod, but is not quite as abundant as the last species and is seldom found at low water. South of Cape Cod it is apparently confined to the deeper and colder waters.

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* With the exception of the portion relating to the Crustacea, these notes have had the benefit of Professor Verrill's revision, and the descriptions of all the new species have been copied from his published papers, or, in the case of those here for the first time described, have been prepared by him specially for these pages, and are marked by his initials.

† Report upon the Invertebrate Animals of Vineyard Sound, in Report of the U. S. Commissioner of Fish and Fisheries, Part I, 1873 (published in 1874).

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Eupagurus Kroyeri Stimpson.

Notes on North American Crustacea, Annals Lyceum Nat. Hist., vii, p. 89 (43)' 1859.

This species is very closely allied to the last and is very easily confounded with it, especially when young. The differences in the relative proportions of the chelipeds and ambulatory legs, given by Stimpson, will not hold for distinguishing the two species, but the differences in the amount of pubescence and especially in the form and armature of the chelipeds seem to be constant characters, sufficient for distinguishing them.

The *Kroyeri* has about the same range, on our coast, as the last species, although I have never seen it south of Cape Cod, but is apparently less abundant and more confined to the deeper waters.

Sabinea septemcarinata Owen (Sabine sp.)

This species was dredged in 68 fathoms off Casco Bay in the summer of 1873. It has also been found by Mr. Whiteaves in the Gulf of St. Lawrence and by Dr. Packard on the coast of Labrador. It is an exceedingly arctic and circumpolar species.

Caridon Gordoni Goës (Bate sp. ?)

Goës, Crustacea decapoda podophthalma marina Sueciæ (from Œfversight af Kongl. Vetenskaps-Akad. Förhandlingar, Stockholm, 1863), p. 10.

We have dredged this species in 50 fathoms in the Bay of Fundy, and Dr. Packard and Mr. Cooke obtained it on Cashe's Ledge in 1873.

Our specimens agree well with the detailed description given by Goës, except that they have a well developed epipodus ("flagellum") upon the second, third and fourth cephalothoracic legs, as in some species of *Hippolyte*, while Goës says of the second legs, "nec palpo nec (quoad viderim) flagello ullo instructis," and of the third to fifth, "flagellum basale nullum inspicere potui." From the guarded manner in which Goës mentions these wholly negative characters, I am inclined to regard them as doubtful. Our specimens agree so completely in all other respects that it seems highly improbable that they should be distinct from the European species.

Diastylis quadrispinosa G. O. Sars.

Œfversight af Kongl. Vetenskaps-Akademiens Förhandlingar, 1871, Stockholm, p. 27; and Beskrivelse af de Paa Fregatten Josephiens Expedition Fundne Cumaceer, in Kongl. Svenska Vetenskaps-Akademiens Handlingar, ix, p. 28, plates 10, 11, figs. 51–61, 1871.

This is the most abundant species of the genus from off Buzzard's Bay and Vineyard Sound to Nova Scotia. It ranges north at least as far as the Gulf of St. Lawrence.

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Diastylis sculpta G. O. Sars.

Loc. cit., Œfversight, p. 71; Handlingar, p. 24, pls. 1–9, figs. 1–49.

This species is not uncommon in Casco Bay and the Bay of Fundy.

Phoxus Kroyeri Stimpson.

Marine Invertebrates of Grand Manan, p. 58, 1853.

We have dredged this species in 10 to 29 fathoms in and off Vineyard Sound, on sandy and muddy bottoms in shallow water in Casco Bay, and have found it from low water to 20 fathoms in the Bay of Fundy. Mr. Whiteaves has dredged it in the Gulf of St. Lawrence in 200 fathoms, muddy bottom.

Our species is very closely allied to, and probably identical with, the *P. Holbollii* Kroyer which is found in Greenland, Iceland and northern Scandinavia.

Harpina fusiformis Smith.

Phoxus fusiformis Stimpson, Marine Invertebrates of Grand Manan, p. 57, 1853.

This species is very likely identical with the *H. plumosa* Boeck (*Phoxus plumosus* Kroyer), which has very nearly the same range as *Phoxus Holbollii.*

We have dredged our species in 20 to 60 fathoms, muddy bottom, in the Bay of Fundy. Mr. Whiteaves has dredged it frequently, in the Gulf of St. Lawrence.

Stenothoe peltata Smith, sp. nov.

Plate IV, figures 5 to 8.

Female. Eyes round and nearly white in alcoholic specimens. Antennulæ considerably shorter than the epimera of the fourth segment; first segment of the peduncle stout, fully as long as the head, the second shorter, and the third very short and like the segments of the flagellum; flagellum scarcely longer than the peduncle, composed of about eight segments. Antennæ slightly longer than the antennulæ; the ultimate and penultimate segments of the peduncle about equal in length; flagellum about as long as the flagellum of the antennulæ. Second epimeron (figure 5) rudely ovate, twice as high as broad; third somewhat rectangular, no wider than the second but considerably deeper; fourth (figure 6) very large, slightly deeper than the third and a third or a fourth longer than deep, being about as long as the first five segments of the thorax, the inferior margin regularly curved and the posterior convex in outline. First legs (figure 7) small and slender; merus triangular and broader distally than the carpus, which is not quite twice as long as broad and has the lateral margins parallel; propodus narrower but slightly longer than the carpus and narrowed distally; dactylus about half as long as the pro-

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podus. Second legs (figure 5) stouter; merus short triangular; carpus much broader than long and only slightly produced beneath the propodus; propodus about as long as the breadth of the epimeron, nearly twice as long as broad; palmary margin (figure 8) convex in outline, slightly oblique, with an acute lobe and a spine at the posterior angle, within which the tip of the dactylus closes. Third and fourth legs slender and nearly naked. Basal segment in the fifth legs slender, four times as long as broad, not wider than the merus. Sixth and seventh legs slightly shorter than the fifth, the basal segments posteriorly dilated and squamiform in both pairs, but broader in the seventh than in the sixth. Posterior caudal stylets with the ramus slightly longer than the peduncle.

Length of largest specimen, from front of head to tip of telson, about 6^{mm}.

The mandibles are without palpi or molar tubercles, and in all other characters the species agrees with the genus *Stenothoë* as restricted by Boeck, but it seems to be very distinct from either of the European species.

Near Cultivator Shoal (haul *b*), 30 fathoms, soft, sandy bottom, August 29.

Syrrhoe crenulata Goës.

Crustacea amphipoda maris Spetsbergiam alluentis, Œfversight af Kongl. Vetenskaps-Akad. Förhandlingar, Stockholm, 1865, p. 527, pl. xl, fig. 25; Boeck, Crustacea amphipoda borealia et arctica (Vidensk.-Selskabs Forhandlinger, Christiania, 1870), p. 67, 1870.

We have also dredged this species, in 1872, in 12 fathoms in Johnson's Bay, near Eastport, Maine, and in 90 to 100 fathoms off Grand Menan, and have examined specimens dredged, in 1873, in 30 fathoms, in Gaspé Bay, Gulf of St. Lawrence. Our specimens have all been considerably larger than the one figured by Goës, but otherwise agree perfectly. It seems to be an exceedingly arctic form, being found in Europe from Spitzbergen to the western coast of Norway.

Tiron acanthurus Lilljeborg.

Boeck, op. cit., p. 69. *Syrrhoë bicuspis* Goës, loc. cit., p. 528, pl. xl, fig. 26. ? *Thessarops hastata* Norman, Annals and Magazine Nat. Hist., IV, ii, p. 412, pl. xxii, figs. 4–7, 1868.

This species has apparently not been noticed on our coast before. It has been found in Greenland, Finmark, and on the western coast of Norway, while Norman's *Thessarops* was from the English coast.

Œdiceros lynceus Sars.

Oversigt over nordsk-arct. Krebsdyr. Forhandl. i Vidensk-Selsk. i Christiania, 1858, p. 143 (teste Boeck); Boeck, op. cit., p. 82. *Œdiceros propinquus* Goës, loc. cit.,

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p. 526, 1865, pl. xxxix, fig. 19. *Monoculodes nubilius* Packard, Memoirs Boston Soc. Nat. Hist., i, p. 398, 1867.

We dredged this species in the Bay of Fundy in 1868 and 1872, the latter year in 60 to 80 fathoms; in Casco Bay, in 27 fathoms, in 1873, and Dr. Packard and Mr. Cooke obtained it at several localities, in the "Gulf of Maine," from 50 to 90 fathoms, on the expedition of the Bache in 1873. I have also examined specimens dredged in the Gulf of St. Lawrence by Mr. Whiteaves and on the coast of Labrador by Dr. Packard. It extends to Greenland, Iceland, Spitzbergen and Finmark.

Monoculodes borealis Boeck.

Op. cit., p. 88, 1870. *Œdiceros affinis* Goës, loc. cit., p. 527, pl. xxxix, fig. 21′, 1865 (non Bruzelius).

This species is recorded from Spitzbergen and northern Norway by Goës and Boeck, but seems not to have been noticed on this side of the Atlantic before.

Paramphithoe pulchella Bruzelius (Kroyer sp.)

We have dredged this species off Casco Bay and in the Bay of Fundy, on hard bottoms, in from 40 to 90 fathoms, and it was dredged on Cashe's Ledge and Stellwagen's Bank, in 1873, by Dr. Packard and Mr. Cooke. It extends north to the Gulf of St. Lawrence, and, according to Boeck, to Greenland, Iceland, Spitzbergen, and the western coast of Norway.

Paramphithoe cataphracta Smith.

Amphithonotus cataphractus Stimpson, Synopsis of the Marine Invertebrata of Grand Manan, p. 52, 1853 (description copied in Bate, Catalogue of Amphipodus Crustacea in the British Museum, p. 152, 1862.)

This species is apparently a true *Paramphithoë*, as restricted by Boeck, and closely allied to, if not identical with, *P. panopla* Bruzelius (*Amphithoë panopla* Kroyer). Boeck places *Pleustes tuberculatus* Bate as a synonym of Kroyer's species, and if he is correct in this, our species is undoubtedly distinct. The *cataphracta* appears to be an inhabitant of hard or coarse sandy and shelly bottoms from 5 to 50 fathoms. We have dredged it sparingly in Casco Bay and the Bay of Fundy, and Dr. Packard has dredged it on the coast of Labrador.

Vertumnus serratus? Goës (Fabricius sp.)

Acanthonotus serratus Stimpson, Synopsis of the Marine Invertebrata of Grand Manan, p. 52, 1853.

Our specimens all differ from the descriptions and figures given by Boeck and Kroyer in the armature of the posterior margin of the

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third segment of the abdomen. In our specimens the upper process from this margin is armed with four or five teeth above and at the tip, while the lower process is armed with five or six teeth similarly situated, but with no teeth on the lower margin except just at the tip. In Kroyer's figure (Grönlands Ampfipoder, plate II, figure 8) the upper process is represented as terminating in a single tooth and the lower process as toothed along both sides; Boeck's description agrees with this except that he says there are two teeth at the tip of the upper process.

It is not uncommon on hard bottoms in from 5 to 50 fathoms in the Bay of Fundy. We have also dredged it in Casco Bay and have received it from the Gulf of St. Lawrence, where it was dredged by Mr. Whiteaves.

Acanthozone cuspidata Boeck.

This species is quite common on hard, and especially on spongy bottoms in 5 to 40 fathoms in the Bay of Fundy, although it is not mentioned by Stimpson in his work on Grand Menan. We have also dredged it in Casco Bay, and Mr. Whiteaves has obtained it in the Gulf of St. Lawrence. It ranges to Greenland, Spitzbergen and Finmark.

Byblis Gaimardi Boeck (Kroyer sp.)

We have frequently dredged this species in Casco Bay and the Bay of Fundy, on muddy bottoms in 10 to 60 fathoms. It extends north to the Gulf of St. Lawrence (Whiteaves), Labrador (Packard), and, according to Boeck, to Greenland, Iceland, Spitzbergen and Norway. The *Ampelisca Gaimardi* of Bate, and Bate and Westwood, is not this species but a true *Ampelisca*.

All the species of this sub-family are undoubtedly tube dwellers. Lilljeborg noticed the habit in *Haploöps;* it has been observed in species of *Ampelisca* by Professor Verrill and myself. In this species, the glands which secrete the cementing fluid are situated principally in the meral and basal segments of the third and fourth pairs of thoracic legs.

Xenoclea megachir Smith, sp. nov.

Plate IV, figures 1 to 4.

Male. Eyes large, black, very slightly elongated, and approaching closely the edges of the triangular prominence of the inferior angle of the front margin of the head. Peduncle of the antennulæ about as long as the head and the first two segments of the thorax, the second segment longest, the first and third about equal in length,

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flagellum about as long as the peduncle and composed of twelve to sixteen segments. Antennæ a little longer than the antennulæ; ultimate and penultimate segments of the peduncle sub-equal in length; flagellum slightly shorter than the peduncle and composed of eleven to fifteen segments. First epimeron (figure 1) as broad as high; second (figure 2) broader than high; third (figure 3) and fourth not broader than high and successively deeper than the first and second; fifth (figure 4) slightly deeper than the fourth and its terminal portion as broad. In the first legs (figure 1), the carpus longer and broader than the propodus, which is somewhat oval and twice as long as broad; the dactylus slender, slightly curved and fully as long as the propodus. The inferior distal margin of the propodus is regularly curved to a short distance from the extremity, where there is a small but deep emargination, beyond which and round upon the short distal margin the edge is serrate with minutely crenulated teeth; the posterior margin is furnished with numerous slender setæ and with a single stout spine at the emargination near the distal end. The inner edge of the dactylus is armed with a series of acute teeth directed obliquely toward the tip. In the second pair of legs (figure 2) the propodus is very stout, about twice as long as the epimeron and scarcely one-half longer than broad; the palmary margin oblique and armed near the middle with two stout obtuse teeth; the dactylus stout and its inner edge sinuous. Third (figure 3) and fourth pair of legs alike; ischium and carpus short, each nearly or quite as broad as long; merus fully as long as the epimeron and half as broad as long; propodus slender, not more than half as broad as the carpus but twice as long; dactylus slender, about half as long as the propodus. Basal segment in the fifth legs (figure 4) squamiform, oval, nearly as broad as long and with a marked angular emargination at the inferior posterior angle; carpus only slightly longer than the breadth of the merus; dactylus slightly curved and acute. Second and third segments of the abdomen with the inferior portion of the posterior margin sinuous, and the inferior angle prominent, but scarcely less than right-angled. The outer rami in all the caudal stylets slightly shorter than the inner, and all the rami armed with short spines above and more slender spines at the tips. Telson stout, about as broad as long and scarcely more than half as long as the peduncle of the posterior caudal stylets, the posterior margin with a few setiform hairs each side.

In the female the hands in the second pair of limbs are proportionally much smaller and more abundantly provided with hairs, while the teeth, or lobes of the palmary margin, are further apart and

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separated by a broad and deep, rounded sinus; the dactylus is not so stout, and has the inner margin evenly curved and serrated.

Length, from front of head to tip of telson, 5·5 to 7·5^{mm}.

I refer this species with some hesitation to Boeck's genus *Xenoclea*, which is known to me only from the very short diagnosis of the genus and of the single species *X. Batei*, given in his Crustacea Amphipoda Borealia et Arctica, p. 155. "Pedes 3tii et 4ti paris articulo 1mo latissimo" of the generic diagnosis would scarcely apply to our species, but in all the other generic characters it agrees perfectly, as it does also with the diagnosis of the sub-family Photinæ, except that the mandibles each bear six serrated spines instead of the usual number, four.

Near Cultivator Shoal (haul *b*), 30 fathoms, soft, sandy bottom, August 29; and on the northern side of George's Bank (haul *q*), north latitude 42°, west longitude 67° 42′, 45 fathoms, coarse sandy bottom. Also, in 18 fathoms, off Watch Hill, Rhode Island.

When first examining the alcoholic specimens of this species, I noticed a peculiar opaque glandular structure filling a large portion of the third and fourth pairs of thoracic legs, which in most, if not all, the non-tube-building Amphipoda are wholly occupied by muscles. A further examination shows that the terminal segment (dactylus) in these legs is not acute and claw-like, but truncated at the tip and apparently tubular. In this species, a large cylindrical portion of the gland lies along each side of the long basal segment, and these two portions uniting at the distal end pass through the ischial and along the posterior side of the meral and carpal segments and doubtless connect with the tubular dactylus. (See Plate III, figure 3.) There can be no doubt that these are the glands which secrete the cement with which the tubes are built, and that these two pairs of legs are specialized for that purpose. A hasty examination revealed a similar structure of the corresponding legs in *Amphithoë maculata*, *Ptilocheirus pinguis*, *Cerapus rubricornis*, *Byblis Gaimardi*, and a species of *Ampelisca*. In all these except the last two a very large proportion of the gland is in the basal segment. In the *Amphithoë* this segment is thickened and the gland is in the middle. In the *Cerapus* it is very broad and almost entirely filled by the gland, with only very slender muscles through the middle, and the orifice in the dactylus is not at the very tip but sub-terminal on the posterior side. In the *Ptilocheirus* the gland forms three longitudinal masses in the basal segment and is also largely developed in the meral and carpal segments. The dactylus is long and slender and the orifice sub-terminal. In *Ampelisca* and *Byblis* (which, like *Haploöps*, are tube-building genera) the meral segments of the specialized legs are nearly

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as large as the basal and contain a proportionally large part of the gland.

Scapellum Stroemi Sars.

Plate III, figure 9.

I am not aware that a description of this species has yet been published, although the name was used by Prof. Michael Sars in his list of animals living at great depths in the sea, published in 1869,* and the species has since been incidentally figured, without any detail, on the stems of *Mopsea borealis*, by Dr. G. O. Sars in his recent work on "Some Remarkable Forms of Animal Life from Great Depths off the Norwegian Coast" (Plate V, figure 2). Dr. G. O. Sars has, however, very kindly compared a drawing of one of our specimens, and he writes me that it agrees in every detail with the Norwegian form. It is very distinct from any of the species described in Darwin's great work, and also from the species recently described from the Challenger Expedition.

Since our specimens were obtained from 430 fathoms, Dr. Packard and Mr. Cooke have dredged in 50 to 70 fathoms near Cashe's Ledge, and in 142 fathoms, 20 miles east of Cape Race (both localities within the "Gulf of Maine"). All the specimens were attached to stems of hydroids. On the Norwegian coast the species has the same habit and has been found by Dr. G. O. Sars in from 80 to 300 fathoms.

Annelida.

Lænilla (?) mollis G. O. Sars.

Bidrag til Kundskaben om Christianiafjordens Fauna, iii, p. 7, plate xiv, figs. 1–12, 1873.

Body large, rather stout, medially convex. Head short and broad, narrowed posteriorly, prominently rounded laterally, and produced into two very small conical points anteriorly. The anterior eyes are larger than the others, situated on the outer and upper surface of the lateral prominences, and look outward and upward; the posterior pair are nearer together, on the lateral slopes of the narrowed part of the head. The median tentacle is wanting in our specimen, but its basal segment is of moderate size and cylindro-conical; the antennæ are slender, and nearly three times the length of the head, banded with brown; the palpi are rather slender and regularly tapered, smooth, or nearly so, four or five times the length of the head. The dorsal and tentacular cirri and the scales are wanting in the single specimen obtained. The lateral appendages are large and

* Forhandlinger i Videnskabs-Selskabet i Christiania, 1868, p. 259, 1869.

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prominent, with large fascicles of long, slender setæ in the lower rami, and much shorter and stouter ones in the upper rami. The appendages, including setæ, equal or exceed the breadth of the body. Breadth of body, exclusive of appendages, 7^{mm}; length of the latter, without setæ, $3{\cdot}5^{mm}$; with setæ, 10^{mm}; length of body to the 15th segment, 25^{mm}. The setæ of the upper ramus are very stout, and all of nearly the same form, the upper ones being merely smaller and stouter than the rest; they are nearly straight or slightly recurved, with rather conspicuous, moderately close transverse series of denticles, which extend nearly to the ends, leaving only stout, naked, straight tips. The setæ of the lower ramus are much longer and far more slender, with a long, slender shaft, and a slightly expanded terminal portion, which is conspicuously, but not closely, spinulated on both sides to the tips; many of these are nearly straight, but most are slightly curved; the upper ones are most slender, and mostly have the tips only very slightly bidentate, and the spinules exceed the diameter of the setæ and increase toward the end, the last ones projecting considerably beyond the tip; the middle ones are about twice as stout, having the terminal part more expanded; their spinulation is similar, but the tips are more distinctly, though slightly, bidentate, the denticles being partially obscured by the terminal spinules that project beyond them; the lower ones are more slender and like the upper ones in form and character.

Near St. George's Bank, 110 fathoms, mud. Coast of Norway, 40–200 fathoms (G. O. Sars).

Our specimen is imperfect, but the head and setæ are quite peculiar. The latter are remarkable for the length of the spinules, and for the minuteness of the denticles at the tips.—A. E. V.

Antinoe angusta Verrill, sp. nov.

Body narrow, rather slender, elongated, tapering gradually posteriorly. Head small, short, rounded, broader than long, the lateral lobes short, not prolonged into points anteriorly, but obtusely rounded; the lateral borders also well rounded. Eyes small, nearly equal; the posterior pair situated on the dorsal side of the vertex; the anterior pair farther apart on the outer and upper surface of the lateral prominences. Tentacle long and very slender, about three times the length of the head; antennæ small and short, scarcely one-third as long as the head; palpi moderately large, glabrous, considerably longer than the tentacle. Dorsal cirri slender, pretty regularly but not closely covered with slender papillæ. The lateral appendages, except anteriorly, bear large fascicles of long, fine capillary setæ, which gives a

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villous appearance to the sides. The elytra, in our specimen, are wanting. The color, in alcohol, is light brown, crossed by lighter transverse lines. Length, 15^{mm}; breadth, without appendages, 2^{mm}; breadth, including setæ, 4^{mm}.

On the middle segments the setæ of the upper ramus are quite unequal in size and length; the upper ones are stout, with the ends more or less recurved; the middle ones are still larger and more than twice as long, slightly curved, and, like the former, conspicuously transversely serrulate almost to the extreme tips; the lower ones are shorter, less stout, and slightly curved. The setæ of the lower ramus are longer and extremely slender; the upper ones are mostly but slightly expanded in the middle, with very long, flexible capillary tips, finely tapered to the end, and very minutely serrulate or nearly smooth; the median ones are stouter, more expanded in the middle, with long, acuminate, slender, sharp tips, and with conspicuous, rather distant spinules on one or both sides, which become very fine and more crowded distally; the lower ones are much shorter, and have shorter but still very slender tips, and fewer and more distant spinules. The ventral cirri are slender, tapered, with few, distantly scattered, small papillæ.—A. E. V.

Near Saint George's Bank, 150 fathoms, mud (locality *s*).

Antinoe Sarsi Kinberg.

Malmgren, Nordiska Hafs-Annulater, Œfversigt Kongl. Vetenskaps-Akad. Förhandlingar, Stockholm, 1865, p. 75, pl. 9, fig. 6; Annulata Polychæta, p. 13, 1867.

Our specimen of this species agrees very well with Malmgren's figures and description. It is much larger and stouter than the preceding, and the head is longer and quite different in form, the lateral lobes extending forward into acute conical points.

The setæ are similar to those of the former, but the median and inferior setæ of the lower ramus are relatively somewhat stouter and have the tips less attenuated and elongated, while the spinules are larger and more conspicuous, especially on the upper setæ of the lower ramus.

Near Saint George's Bank, 85 fathoms, mud. Gulf of Saint Lawrence (Whiteaves, t. McIntosh).—A. E. V.

Eucranta villosa Malmgren.

Eucranta villosa Malmgren, Nordiska Hafs-Annulater, Œfversigt af Kongl. Vetenskaps-Akad. Förhandlingar, Stockholm, 1865, p. 80, pl. 10, fig. 9; Annulata Polychæta, p. 14, 1867.

? *Eupolynoë occidentalis* McIntosh, Annals and Magazine Nat. Hist., IV, vol. xiii, p. 264, pl. 9, figs. 8–13, 1874.

This large species is easily distinguished, even when destitute of

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its scales, by the short, stout, strongly curved setæ of the upper ramus, and much longer, slender, fascicled setæ of the lower ramus, among which the upper ones have a strongly spinulose, slender, acuminate, terminal portion, with a nearly straight, split, or forceps-like, slender tip, while the middle and lower ones have a short, cuspidate terminal portion, with few large spinules, and naked acute tips.

I am unable to find anything in the figures and description of the species recently described by McIntosh to indicate that it is distinct from the present species, with which, however, he has not compared it.

Near Saint George's Bank, 150 fathoms, mud (locality *s*). Gulf of Saint Lawrence, 110 fathoms (Whiteaves, t. McIntosh).—A. E. V.

Nephthys circinata Verrill, sp. nov.

Body slender, elongated, rather depressed, tapering gradually posteriorly. Head sub-pentagonal, rather broader than long; a pair of short, tapering antennæ at the anterior angles, about one-fourth as long as the width of the anterior border of the head; another pair of longer, slender, tapering antennæ at the lateral angles; tentacular cirri long and tapering. Proboscis smooth toward the base; its distal portion with rows of slender acute papillæ, which increase rapidly in length toward the end, where they become very prominent.

The lateral appendages, including the setæ, are as long as the breadth of the body; the setæ are very numerous, long and slender. The caudal cirrus is long and slender, tapering to a slender tip. Length of body, 50^{mm}; diameter, $2{\cdot}5^{mm}$; diameter, including appendages, 5^{mm}.

The lateral appendages of the middle region are moderately long, the rami separated by a space scarcely equal to half their height. Superior ramus, with a short, broad ovate, obtuse, or slightly acuminate upper lamella, directed outward, and considerably exceeding the setigerous lobe, and a much smaller ovate median lamella; branchial cirrus long, rather slender, tapered, curved downward and inward (circinate), forming rather more than a complete whorl; the appendage at its base, on the anterior segments, is short and broad, subtruncate distally, and with a small papilliform process projecting downward from its lower angle, nearly in contact with the branchial cirrus; on the median segments it is broad and long-ovate, unequally acuminate, leaf-like. The lower ramus has a very long and wide ligulate lamella, directed obliquely upward and outward, usually more than twice as long as the setigerous lobe, and about equal to it in width; its lower edge at about the middle is sometimes incurved, and its tip is acuminate and blunt-pointed; the ventral cirrus is slender and tapered.

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The capillary setæ form large fascicles and are very long and slender, nearly smooth, and with very attenuated tips; their length is about three times that of the appendages themselves; the transversely marked setæ are scarcely one-fourth as long, and about the same in diameter, with very slender tips.—A. E. V.

East of Saint George's Bank, 430 fathoms (locality *g*); north of Saint George's Bank, 85 fathoms, mud (locality *p*).

Nephthys ingens Stimpson.

Synopsis of the Marine Invertebrata of Grand Manan, p. 33, 1853; Verrill, Report on the Invertebrate Animals of Vineyard Sound and Adjacent Waters, in Report of U. S. Commissioner of Fish and Fisheries, part I, 1873, p. 583 (separate copies, p. 289), plate xii, figs. 59, 60, 1874.

? *Nephthys incisa* Malmgren, Œfversigt af Kongl. Vet.-Akad. Förhandlingar, 1865, p. 105, plate xii, fig. 21.

This is the most common and abundant species on muddy bottoms in the deep water along the whole New England coast. It occurs at all depths from 2 to 430 fathoms.

It is easily distinguished by the stout quadrangular body, deeply incised posteriorly; by the blackish setæ, and by the remarkably elongated and widely separated rami of the posterior appendages. There is a long, odd, median papilla on the dorsal side of the proboscis, and a smaller one beneath; the papillæ in the longitudinal rows are rather small.—A. E. V.

Phyllodoce catenula Verrill.

Report on the Invertebrate Animals of Vineyard Sound, in Report of U. S. Commissioner of Fish and Fisheries, part I, 1873, p. 587, 1874; Exploration of Casco Bay by the U. S. Fish Commission, Proceedings American Association for the Advancement of Science, 1873, p. 380, pl. 3, fig. 1, 1874.

Plate IV, figure 3.

George's Bank, 50 fathoms (locality *d*). It also occurs at Watch Hill, Rhode Island, in 4 to 6 fathoms, among rocks and algæ, and in tide-pools; at Wood's Hole, at surface, evening, July 3; in Casco Bay, 8 to 30 fathoms; and is very common in the Bay of Fundy, from low-water to 50 fathoms.

This species is closely allied to *P. pulchella* Malmgren, from northern Europe, but differs somewhat in the form of the head, which is shorter and rounder in the latter; the branchiæ also differ in form.

Eusyllis phosphorea Verrill, sp. nov.

Plate VII, figure 3.

Body slender, elongated, tapering gradually posteriorly. Head, in alcoholic specimens, broader than long, well-rounded in front, the posterior margin incurved; but in living specimens the head is longer

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than broad and slightly narrowed posteriorly. Eyes small, but conspicuous, wide apart, the anterior considerably farther apart than the posterior ones. Palpi large in preserved specimens, broad ovate, and well rounded anteriorly, in contact at their bases; but in living specimens more elongated and oblong, exceeding the length of the head. Antennæ (or tentacles) long and slender, distinctly and rather regularly annulated, but not moniliform. Tentacular cirri, in preserved specimens, similar to the antennæ; the upper ones are of about the same length, but the lower are little more than half as long. Dorsal cirrus of the second segment is as long as, or even longer than, the antennæ. The dorsal cirri on the 3d, 4th, and 5th segments are shorter, about equal, longer than the lower tentacular cirrus, and about half as long as the dorsal cirri of the succeeding segments, which are alternately longer and shorter, the longer ones about half as long as the breadth of the body. While living, the alternate dorsal cirri are usually held extended and curled up over the back. The two anal cirri are long and slender; in one preserved specimen they are more than twice the breadth of the body, while in the same specimen the dorsal cirri on the second and third segments preceding the anal one are considerably longer than those on the segments farther forward.

The setæ are all compound, rather long, mostly considerably bent, with a short, acute-triangular terminal piece, which is very distinctly bidentate at the tip.

Color of body, when living, deep salmon, or light yellowish orange, with dark brown intestinal line, darker posteriorly; eyes dark brown.

Length, when living, about 25^{mm}; breadth, $1{\cdot}5^{mm}$.

Saint George's Bank, 45 fathoms, among hydroids; Bay of Fundy, off Grand Menan, 52 fathoms, among hydroids.

This species, when living, was most brilliantly phosphorescent, with a bright green light, so intense as to be distinctly visible in daylight, or close to a large kerosene lamp.—A. E. V.

Ninoe nigripes Verrill.

Report on the Invertebrate Animals of Vineyard Sound, in Report of U. S. Commissioner of Fish and Fisheries, part I, 1873, p. 595, 1874; Proceedings American Association for Advancement of Science, 1873, p. 382, pl. 3, fig. 5, 1874.

Plate V, figure 3.

Locality *o*, 110 fathoms. Also Fisher's Island Sound, Vineyard Sound, and Buzzard's Bay, and waters outside, in 8 to 29 fathoms, mud; Casco Bay, 10 to 68 fathoms; off the coast of Maine, at various depths to 107 fathoms.

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Leodice vivida Verrill.

Eunice vivida Stimpson, Marine Invertebrata of Grand Manan, p. 35, 1853.

Leodice vivida Verrill, American Journal of Science, III, vol. v, p. 9, January, 1873.

Plate V, figure 5.

Nothria conchylega Malmgren.

Onuphis conchylega Sars, Beskrivelsir og Iagttagelser, p. 61, pl. 10, fig. 28 (*teste* Malmgren), 1835.

Onuphis Eschrichti Œrsted, Grönlands Annulata Dorsibranchiata, p. 20, pl. 3, figs. 33–41, 45, 1843.

Northia conchylega Johnston, Catalogue of British Worms, p. 138, 1865.

Nothria conchylega Malmgren, Annulata Polychæta, p. 66, 1867.

Plate VII, figure 3.

This species is abundant in the deeper waters, especially upon hard bottoms, on the whole northern coast of New England, and in the Gulf of St. Lawrence. Malmgren records it, in 30 to 250 fathoms, from Greenland, Spitzbergen, Finmark, and the coast of Norway.

The name "Nothria" was substituted for *Northia* (Johnston) by Malmgren for reasons that are scarcely sufficient. The latter name was, however, previously in use for a genus of shells (Gray, 1847), and must be rejected on that account.

Nothria opalina Verrill.

American Journal of Science, III, vol. v, p. 102, 1873.

Plate VII, figure 4.

Body long and slender, narrowed anteriorly, much depressed and of nearly uniform width throughout most of its length; the five anterior segments much longer than the others. Palpi inferior, rather large, hemispherical; antennæ small, ovate, close together, on the front of head. Three central tentacles very long and slender, tapering, acute, the basal portion regularly annulated and thickened for a considerable distance, beyond which the surface is smooth, with an occasional distant annulation; the central odd one is somewhat shorter and more slender than the two adjacent ones, which reach to or beyond the 10th segment; outer pair much shorter, being less than half the length of the central ones. Tentacular cirri small and very slender. Lateral appendages or "feet" of the first six setigerous segments similar in structure but more prominent than the following ones, from which they also differ in having the ventral cirrus well developed, long and tapering, but shorter and thicker on the first segment than on the five following. Those of the first pair have a stout stalk, which terminates in a small, bluntly rounded setigerous lobe, with a long, slender, subterminal cirrus-like lobe above,

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longer than the stalk; dorsal cirrus arising from near the base, longer and more slender than the terminal cirrus; branchial filament simple, long and very slender, about equalling the dorsal cirrus and united to it above its base; ventral cirrus ovate, tapering, blunt, arising from near the base. The second pair of feet are similar to those of the first, except that in the largest specimens there are two branchial filaments, and the ventral cirrus is longer and more slender. The 3d, 4th, 5th, and 6th pairs have essentially the same structure, but the ventral cirrus becomes gradually longer to the 6th, where it is longer than the stalk and nearly equal to the terminal cirrus. The succeeding feet are much shorter; the ventral cirrus is a mere conical papilla, which soon disappears; the terminal cirriform lobe becomes smaller and disappears after the 10th pair; the branchial filament becomes larger and longer to the middle region, where it exceeds in length half the diameter of the body, while the dorsal cirrus at the same time becomes smaller and shorter, until it is less than one-fourth the length of the branchia.

The setæ of the anterior feet consist of slender, acutely pointed, curved ones, mixed with much stouter, blunt pointed compound ones; farther back there are two fascicles of more slender acute setæ, and in the lower bundles a few long, stout, bidentate hooks, with a thin, rounded, terminal expansion.

Color, in alcohol, pale yellowish white, but everywhere very brilliantly iridescent, with opaline lustre and colors.

Length, 75 to 125^{mm}; diameter, 2·5 to 4^{mm}.

Common in 110 and 150 fathoms, hauls *s* and *o*. It was also dredged in 1873, off Casco Bay, in 30 to 94 fathoms, and on Jeffrey's Bank, in 79 to 105 fathoms. It was also abundant, on muddy bottoms in deep water, at all the localities in the Gulf of Maine examined by Dr. Packard and Mr. Cooke in 1873.

Goniada maculata Œrsted.

Ann. Dan. consp., p. 33, figs. 16, 23, 91, 95, 97, 98 (t. Malmgren). *Glycera viridescens* Stimpson, Marine Invertebrata of Grand Manan, p. 33, 1853.

North of Saint George's Bank, 110 and 150 fathoms, mud (localities *o* and *s*); Saint George's Bank, 20 fathoms (locality *j*). Off Casco Bay, 30 to 90 fathoms, mud. Bay of Fundy, 20 to 70 fathoms. Common in the Gulf of Maine, 60 to 100 fathoms. Northern coasts of Europe, from Finmark to Scotland, 10 to 130 fathoms (Malmgren). —A. E. V.

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Rhynchobolus capitatus Verrill.

Glycera capitata Œrsted, Grönl. Ann. Dorsibranchiata, p. 44, plate VII, figs. 87, 88, 90–94, 96, 99; Malmgren, Annulata Polychæta, p. 70, 1867 (*non* Claparède).

This species is furnished with four well-developed jaws, and therefore belongs to the genus *Rynchobolus*, as constituted by Claparède. The species without jaws, which he refers to *Glycera*, must be distinct.

Saint George's Bank, 60 fathoms (locality *e*); 20 fathoms (locality *j*); 110 fathoms (locality *o*); east of Saint George's, 430 fathoms (locality *g*). Greenland, Iceland, Spitzbergen, and northern coasts of Europe to Great Britain.—A. E. V.

Samythella Verrill.

Body elongated, composed of about 50 segments, 15 of which bear fascicles of setæ; and posteriorly about 35 bear uncini only, but have a small conical papilla above the uncigerous lobe, as in *Melinna*; the uncini commence on the 4th setigerous ring. Branchiæ six, placed side by side in a continuous transverse row. Cephalic lobe oblique, somewhat shield-shape, with a narrowed prominent front. Buccal lobe shorter. Tentacles numerous, smooth and slender.

This genus is closely allied to *Samytha* of Malmgren, in the structure of the head and number of branchiæ, but differs in having a much larger number of segments (in this respect approaching *Melinna*), and in having only 15 setigerous segments, instead of 17.

Samythella elongata Verrill.

American Journal of Science, III, vol. v, p. 99, 1873.

Body slender, composed of 54 segments in the specimens examined, tapering regularly to the posterior end. Cephalic lobe about as broad as long, broadly rounded posteriorly, with the postero-lateral corners prominent and well rounded, the sides slightly incurved and rapidly narrowing to the front, which is about half the width of the back, and subtruncate, projecting forward; the middle region is a raised and convex oblong area as wide as the front edge, into which it runs. Buccal lobe a little shorter. Tentacles numerous, slender, tapering. Branchiæ subequal, slender, tapering, about twice the length of the cephalic lobe. Setæ numerous and long in all the fascicles except the first three, the longest nearly one-third the diameter of the body. The posterior end of the body is surrounded by about eight small papillæ, of which the two upper ones are largest.

Length of largest specimen, in alcohol, 40^{mm}; diameter, $2{\cdot}5$ to 3^{mm}.

The tubes consist of a thin and tough lining, to which a close layer of sand, in grains of moderate and nearly uniform size, is firmly cemented.

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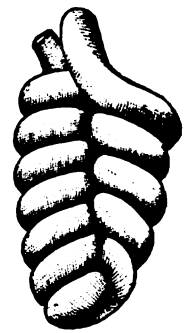
Grymæa spiralis Verrill.

Am. Journal of Science, III, vol. vii, p. 407, fig. 2, and plate V, fig. 4, April, 1874.

Plate IV, figure 1.

Fig. 1.*



Body long and slender, spirally coiled, composed of over 150 segments, of which about 120 bear fascicles of slender setæ. Branchiæ long filiform, two or three times the diameter of body, arising in three clusters on each side, easily detached and often partially absent. Setæ on the first six or seven segments a little longer than the following ones. General color dark red. Tube composed of firmly cemented mud and sand, coiled in a double spiral, the two halves revolving in opposite directions.

Also dredged, in 1872, off Grand Menan Island, Bay of Fundy, in 60 fathoms; and in 1873, off Casco Bay, in 90 fathoms, mud; and in 80 fathoms on Jeffrey's Bank.

? **Potamilla neglecta** Malmgren.

Œfversigt af Kongl. Vet.-Akad. Förhandlingar, 1865, p. 401, plate 27, fig 84. *Sabella neglecta* Sars, Reise i Lofot. og Finm., p. 83 (t. Malmgren).

This species was very abundant at localities *d*, *h*, *i*, *q*, and also occurred in 110 fathoms (locality *o*).

The tubes are long and tough, covered externally with sand. One specimen from Le Have Bank, 45 fathoms (locality *h*), had a large number of young ones within the tube, adhering to its inner surface. —A. E. V.

Spirorbis valida Verrill, sp. nov.

Tubes much larger than usual in the genus, round, strong, thick, opaque, white, transversely wrinkled, rather rapidly enlarging, sinistral, or coiling in the same direction with the hands of a watch; in some specimens, found attached to flat shells, the tubes form low, rapidly enlarging spirals of several turns, the last whorl enveloping and concealing the others externally, except near its termination, where it rises obliquely upon the preceding one, but leaving a broad, shallow umbilicus in which the previous whorls are visible; in other specimens, attached to convex univalve shells (*Turritella erosa*, etc.), the whorls rest upon the upper side of each of the preceding ones, forming an elevated and often somewhat irregular spiral, increasing in size upward, with a small umbilicus, and usually with the last part of the upper whorl slightly free from the preceding one and ascending

* Tube of *Grymæa spiralis*, natural size.

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obliquely. Diameter of the larger tubes, at end, $1{\cdot}75^{mm}$ to 2^{mm}; height of the more elevated spirals, 3^{mm} to 5^{mm}.

There are 15 large branchiæ in the adult specimens: 8 on the left side, 7 on the right, with the operculum; the pinnæ are long, slender, extending to near the ends of the branchiæ, which have slender and short, naked tips. Operculum large, white, calcareous, irregularly obconic, obliquely truncated, with the outer surface concave, the dorsal side gibbous, the margin slightly sinuous but entire, except for a small notch, or emargination, in the dorsal edge; the dorsal portion is translucent, while the ventral portion is opaque and contains small, round, ova-like bodies; the peduncle is rather short and stout, gradually expanding into the base of the operculum, but swollen in the middle, on the dorsal side. Collar, in the specimens examined, considerably mutilated, apparently with a sinuous but not revolute anterior margin, and with a long posterior dorsal point. The region covered by the collar bears, at least on the left side, three large fascicles of slender, acute, yellowish setæ, both above and below; the anterior fascicles are directed forward, and the upper anterior one is larger than the other fascicles.

Le Have Bank, 45 and 60 fathoms (localities *h* and *i*).

The size of this species is exceptionally large, and the branchiæ are unusually numerous for the genus *Spirorbis*, to which I refer it with some hesitation. When living specimens can be studied it may prove to be a new genus. It has, like *Vermilia*, a calcareous operculum, but in form and structure this organ resembles that of some species of *Spirorbis*.—A. E. V.

? **Spirorbis nautiloides** Lamarck.

Anim. sans Vert., ed. I, vol. v, p. 359, 1818. ? *Spirorbis communis* Quatrefages, Histoire naturelle des Annelés, vol. ii, p. 489.

Plate IV, figure 4.

The species figured agrees pretty well with that described by Quatrefages, but may not be the same as that of Lamarck, which is regarded by several writers as synonymous with it, and by others with *S. borealis*, the species so abundant on *Fucus* at low-water mark, on our shores.

The present species is seldom, if ever, found at low-water mark, and occurs chiefly on stones and shells in deep water. The tubes are opaque, white, cylindrical, rather closely coiled, the terminal portion not erect, and the surface is more or less conspicuously marked with lines of growth.

Abundant on the hard bottoms at Saint George's Bank; Casco

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Bay; Cashe's Ledge; and in the Bay of Fundy, 10 to 106 fathoms.—A. E. V.

Protula media Stimpson.

Marine Invertebrata of Grand Manan, p. 30, 1853.

Plate VI.

This species usually forms much contorted and irregularly bent tubes, which are cylindrical and nearly smooth, but with irregular lines of growth.

North of Saint George's Bank, 110 fathoms (locality *o*). Often brought up by fishermen on Saint George's Bank, attached to shells and stones. Abundant on Cashe's Ledge, 50 to 70 fathoms; off Grand Menan, 30 to 50 fathoms; off Casco Bay.—A. E. V.

? Protula borealis Sars.

Vidensk. Selsk. Forhandlinger, 1871, p. 417 (separate copies, p. 14).

Numerous empty tubes from the muddy bottoms in 110 and 150 fathoms (localities *o* and *s*) differ considerably in form from those of the *P. media*, ordinarily met with, and may be this species, if distinct. But they may, very likely, prove to be only a variation of the former, due to the muddy character of the bottom. The tubes are much less bent and contorted, often but slightly curved, or nearly straight, nearly smooth, but with occasional ridges or folds, indicating periods of growth.—A. E. V.

GEPHYREA.

Phascolosoma cæmentarium Verrill (Quatrefages sp.).

American Journal of Science, III, vol. v, p. 99, 1873; and Report upon the Invertebrate Animals of Vineyard Sound, in Report of U. S. Commissioner of Fish and Fisheries, part I, 1873, p. 627, pl. xviii, fig. 92, 1874.

Very common on the coast of New England, from Long Island Sound northward, in 5 to 430 fathoms, in dead univalve shells.

Phascolosoma tubicola Verrill.

American Journal of Science, III, vol. v, p. 99, 1873; Proceedings American Association for Advancement of Science, 1873, p. 388, 1874.

Body versatile in form; in contraction short, cylindrical, oval or fusiform, 12 to 25^{mm} long, 2·5 to 4^{mm} in diameter; in full extension the body is more or less fusiform, gradually tapering anteriorly into the long, slender, nearly cylindrical retractile portion, which is longer than the rest of the body, and bears, near the end, a circle of about ten to sixteen simple, slender tentacles, beyond which the terminal

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portion is often extended into a short proboscis, with the mouth at the end; below the tentacles there is sometimes a dilation, but this is without special spines or granules, and like the rest of the retractile portion in texture. The posterior end of the body is bluntly rounded, and the skin is transversely wrinkled and rough, and covered with small, round, somewhat raised verrucæ or suckers, to which dirt adheres, and at the end nearly always bears from 3 to 8 small, but prominent, peculiar bodies, having a slender pedicle and a clavate or globular head; their nature is doubtful. (They may be sense-organs, but should be examined on living specimens.) At about the posterior third of the proper body is an irregular zone of numerous, dark brown, hard chitinous hooks, arranged in several rows, broad triangular in form, with acute points directed forward; among the hooks are also a few suckers; the middle region is covered with small, round, slightly raised suckers, which become much more prominent and crowded at the anterior end toward the base of the retractile portion, and have here the form of small, subconical, elevated warts, to which dirt usually adheres firmly; the retractile portion is covered throughout with minute conical verrucæ or papillæ, most prominent toward the base.

In many respects *P. cæmentarium* agrees very closely with this, but it has the posterior end much smoother, and with less conspicuous suckers; the hooks are not so numerous, less acute, and lighter colored; the anterior part of the body has smaller and less prominent suckers or verrucæ; the skin is lighter colored, thinner, and more translucent, and there is a zone bearing several rows of minute, slender, acute, chitinous spinules, a little below the tentacles.

Hauls *p*, *o*, and *s*, 85 to 110 fathoms. It has also been dredged, in 60 to 94 fathoms, off Casco Bay.

? **Phascolosoma boreale** Keferstein.

Beiträge zur Anat. und syst. Kentniss der Sipunculiden, p. 206.

This species is rather short and thick, obtuse posteriorly, nearly smooth to the naked eye, and destitute of both hooks and distinct suckers, but the skin is minutely wrinkled transversely, and covered with almost microscopic slender papillæ, and is minutely specked with dirty yellowish brown; the retractile portion is more distinctly granulated anteriorly. The tentacles are rather numerous, small, and simple.

Dredged also off Casco Bay, 64 fathoms; Cashe's Ledge, 50 to 72 fathoms; and in the Gulf of St. Lawrence (Whiteaves).

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MOLLUSCA.

Pleurotomella Packardii Verrill.

American Journal of Science, III, vol. v, 1873, p. 15 (December, 1872).

Shell thin, fragile, translucent, pale flesh-colored, moderately stout, with an acute, somewhat turreted spire. Whorls nine; the apical whorls, for about two and one-half turns, are nearly smooth, regular, convex, chestnut-colored; below this the whorls are shouldered, strongly convex in the middle, but with a smooth concave band below the suture, corresponding to the posterior notch in the outer lip; the whorls are crossed below the sub-sutural band by about 16 strong, prominent, rounded, somewhat oblique ribs, most prominent on the middle of the whorl, but not angulated; on the last whorl these ribs become very oblique below the middle, and follow the curve of the edge of the lip, nearly fading out anteriorly; the surface between the ribs is marked by faint lines of growth and by fine, unequal, slightly raised revolving lines, which pass over the ribs without interruption. They become more evident on the lower part of the last whorl, and are very faint on the sub-sutural band, which is more decidedly marked by receding, strongly curved lines of growth. The aperture is rather broad above, elongated below, sub-oval, outer lip very thin, sharp, prominent above, separated from the preceding whorl by a wide and very deep sinus, extending back for about one-fifth of the circumference of the whorl; the anterior border of the lip is incurved near the end, and obliquely truncate, forming a short, straight canal. Columella simple, nearly straight, its inner edge toward the end sharp, and obliquely excurved. No operculum. Length, $21{\cdot}2^{mm}$; breadth, $11{\cdot}2^{mm}$; length of aperture, $12{\cdot}0^{mm}$; breadth of same, $5{\cdot}0^{mm}$. The absence of eyes and operculum, great size of the posterior sinus, and character of the apex, indicate that this shell represents a new genus.

One living specimen from (*o*) 110 fathoms.

Ringicula nitida Verrill.

American Journal of Science, III, vol. v, 1873, p. 16 (December, 1872).

Plate I, figure 2.

Shell small, white, smooth, broad oval, with five whorls, spire rapidly and regularly tapered, sub-acute, shorter than the aperture. Whorls very convex, regularly rounded, the sutures well impressed; a well marked, impressed, revolving line just below the suture; the surface otherwise nearly smooth, but with more or less distinct, distant, microscopic revolving lines, most distinct anteriorly. Aperture somewhat crescent-shaped. Outer lip evenly rounded, forming

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the segment of a circle, the border regularly thickened, receding a little posteriorly, near the suture. Callus on the body whorl narrow, nearly even, but a little swollen in the middle and slightly raised. Columella stout, recurved at the end, with two strong, very prominent, equal, spiral folds—the anterior one projecting beyond the canal, with the end rounded. Length, $4{\cdot}2^{mm}$; breadth, $3{\cdot}1^{mm}$; length of aperture, $2{\cdot}5^{mm}$; breadth of aperture, $\cdot 11^{mm}$.

From 110 and 150 fathoms (localities *s* and *o*).

Torellia vestita Jeffreys.

This shell in form and size somewhat resembles large specimens of *Margarita helicina*, but it has a ciliated epidermis resembling that of *Velutina lævigata*. The spire is small and low; whorls four, the last large, well rounded, forming the bulk of the shell. Suture deep. Umbilicus small and deep, somewhat concealed by the reflected outer edge of the columella, which recedes in front and joins the outer lip at an obtuse angle, forming a broad, shallow, anterior emargination; inner border of the columella a little excavated near the body whorl, slightly swollen in the middle. Outer lip sharp, regularly rounded. Epidermis thick, greenish, with conspicuous lines of growth, finely reticulated by raised revolving lines, along which arise numerous slender, but short, hair-like processes. Shell beneath the epidermis white, nearly smooth. Length, $7{\cdot}5^{mm}$; breadth, 10^{mm}; length of aperture, 6^{mm}; breadth, $4{\cdot}5^{mm}$.

Only one specimen, dead and inhabited by a *Phascolosoma*, was found in 1872. Since this, however, during the explorations of 1873, it was dredged by Dr. Packard and Mr. Cooke, in 52 to 90 fathoms, on Cashe's Ledge, off the coast of Maine.

Stylifer Stimpsonii Verrill.

American Journal of Science, III, vol. iii, p. 283, 1872.

Plate I, figure 1.

Shell white, short, swollen, broad oval; spire short, rapidly enlarging. Whorls four or five, the last one forming a large part of the shell; convex, rounded, with the suture impressed; surface smooth, or with faint striæ of growth. Color, when living, pale orange yellow. Length, about 4^{mm}; breadth, 3^{mm}.

Parasitic on *Strongylocentrotus Dröbachiensis*. In 32 fathoms off the coast of New Jersey (Capt. Gedney); 60 and 65 fathoms (*e* and *f*), George's Banks; 8 fathoms off Fisher's Island, mouth of Long Island Sound.

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Astarte undata Gould.

Verrill, American Journal of Science, III, vol. iii, pp. 213, 287, 1872; and Report on the Invertebrate Animals of Vineyard Sound, in Report of U. S. Commissioner of Fish and Fisheries, part I, 1873, p. 384, pl. 29, fig. 203, 1874.

Plate I, figures 6 to 9.

The figures given in Gould's works are scarcely characteristic of this, the most abundant species of the northern coast of New England, and we here publish several figures, prepared by Professor Verrill, which more fully illustrate the different forms of the species. The name *undata* was proposed by Gould for a form of his *Astarte sulcata.*

Astarte lens Stimpson.

Astarte crebricostata Gould, Invertebrata of Massachusetts, 2d edition, edited by Binney, p. 126, fig. 440, 1870 (not of Forbes, *teste* Verrill).

Astarte lens Stimpson, MS., Gould, op. cit., p. 127; Verrill, American Journal of Science, III, vol. iii, pp. 213, 287, 1872.

Plate I, figures 4 and 5.

This species seems to be more exclusively a deep-water form than the last, although the specimens dredged by us at the localities (*g*, *o*, and *s*) mentioned are all much smaller than the common form of the species in the Bay of Fundy, and may well be regarded as a dwarf variety.

Pecten pustulosus Verrill.

American Journal of Science, III, vol. v, 1873, p. 14 (December, 1872).

Upper valve more convex than the lower, a little swollen toward the umbo; length and breadth nearly equal, the margin diverging nearly at right angles from the beak to the middle of the anterior and posterior borders, on each of which there is an obtuse angle, from which the outline of the ventral margin forms a regular curve, nearly semicircular, but a little produced ventrally; the surface with about 14 radiating rows of relatively large, prominent, round, hollow vesicles, those in the middle rows nearly hemispherical, while part of those of the lateral ones are subconical and smaller; seven or eight of the rows are first developed, at a short distance from the apex of the shell, the other ones afterward coming in between the primary ones; the rows are distant in the middle and more crowded together toward the borders; between the rows of vesicles the surface is marked by distant, fine, impressed grooves, which pass between and separate the vesicles; on the umbos, above the origin of the vesicles, the border of the groove rises into a thin, slightly elevated lamella. Lower valve with fine, close, slightly raised, concentric lamellæ, be-

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coming faint toward the beak. Auricles unequal, that of the upper valve small, and a little projecting posteriorly, much longer and more prominent, with a deep, curved emargination anteriorly, its surface with concentric lamellæ and radiating rows of small, conical vesicles; that of the lower valve with a deep, angular byssal notch anteriorly, its surface with concentric lamellæ and faint radiating ridges. Color yellowish white. Length, $7{\cdot}5^{mm}$; height, $8{\cdot}0^{mm}$; thickness, $2{\cdot}5^{mm}$.

East of St. George's Banks (*g*), in 430 fathoms, dead but fresh valves; and north of the Banks, locality (*s*), 150 fathoms, living.

Pera crystallina Verrill.

Clavelina crystallina Möller, Naturhistorisk Tidsskrift, vol. iv, p. 95, 1842.

Pera pellucida Stimpson, Proceedings Boston Soc. Nat. Hist., vol. iv, p. 232, 1852.

Pera crystallina Verrill, American Journal of Science, III, vol. iii, p. 213, pl. 8, fig. 9, 1872.

Plate VIII, figure 1.

This species was described by Stimpson from specimens, adhering to stems *Sertularella polyzonias*, variety *gigantea*, taken in 30 fathoms on St. George's Banks. Professor Verrill records it from Murray Bay, Gulf of St. Lawrence.

Glandula arenicola Verrill.

American Journal of Science, III, vol. iii, pp. 211, 288, 1872; Report on the Invertebrate Animals of Vineyard Sound, in Report of U. S. Commissioner of Fish and Fisheries, 1873, p. 701, 1874.

This species, which was dredged by us in immense numbers in 28 fathoms (haul *c*), has also been dredged, by Dr. Dawson, at Murray Bay, Gulf of St. Lawrence, by Mr. T. M. Prudden, in Buzzard's Bay, and off New London, Conn., by A. E. Verrill.

Thyone scabra Verrill.

American Journal of Science, III, vol. v, p. 100, 1873.

Thyone fusus? Verrill, American Journal of Science, III, vol. v, p. 14, 1873 (*non* Koren).

Body fusiform, gradually tapered behind, with a long, slender, posterior portion, covered throughout with very numerous, rather rigid, slender, scabrous papillæ; skin rather rigid, scabrous with small, rough points, which project from the plates. Tentacles ten; eight large ones much elongated and arborescently divided from near the base; the two small ones are very short, nearly sessile, subdivided from the base. The calcareous plates of the skin are very flat, somewhat imbricated, irregularly oval, triangular, or subpolygonal, with an undulated or crenulated margin, pierced by about 20 to 24 unequal round openings, two or three central ones larger than the rest, the

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interspaces mostly as wide as the pores; from the center of the upper side arises an open, slender, flat, acute spinous process, composed of two anastomosing pieces. The plates of the papillæ or suckers are narrow, elongated, bent into a bow-shape, the middle expanded and usually pierced by about four pores, two of which are larger; the ends are also usually dilated and pierced with small pores; from the middle arises a flat, spinous process, similar to that of the skin-plates, but smaller.

Length, in alcohol, about 50^{mm}; greatest diameter, 6 to 9^{mm}; length of longest tentacles, $7{\cdot}5^{mm}$. Color of preserved specimens, yellowish brown.

Localities *o* and *s*, 110 and 150 fathoms. Also dredged, in 1878, off Casco Bay.

This species resembles *T. raphanus* Duben and Koren (Troschel sp.) in form, but the latter has long-stalked tentacles, branching only near the ends, and the plates of the skin are different in form, and in the perforations, and lack the spinous processes which give the species its rough, scabrous surface.

? **Charybdea periphylla** Péron and Lesueur.

Verrill, Report upon the Invertebrate Animals of Vineyard Sound, p. 724, 1874.

This species, originally described and figured by Péron and Lesueur from mutilated specimens taken under the equator in the Atlantic Ocean, is doubtfully identified by Professor Verrill with a specimen obtained by us east of George's Banks.

The body in the alcoholic specimen is elevated, bell-shaped, rounded above, with a marked constriction toward the border; transparent, the inner cavity showing through as a large, conical, dark reddish brown spot, with the apex slightly truncated. Border deeply divided into sixteen long, flat lobes, which are of nearly uniform breadth throughout, and slightly rounded, or sub-truncate, at the end; the edges and end thin and more or less frilled; the inner side with two sub-marginal carinæ. Eyes inconspicuous, but small bright red specks are scattered over the marginal lobes. The intervals between the lobes are narrow and generally smoothly rounded, without distinct evidence of the existence of tentacles, except that, in one of these intervals, there is a small and short papilliform process, with brown pigment at the base. The ovaries are mostly wanting, but portions are to be seen as slightly convoluted organs in the marginal region, opposite the intervals between the lobes.

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Lafoea gracillima G. O. Sars (Alder sp.).

Lafoëa fruticosa Hincks, History of British Hydroid Zoophytes, p. 202, pl. 41, fig. 2, 1868; and Annals and Magazine Nat. Hist., IV, vol. xiii, pp. 132, 148, pl. 6, figs. 6–10, pl. 7, fig. 16. 1874.

Lafoëa gracillima G. O. Sars (Alder sp.), Bidrag til Kundskaben om Norges Hydroider, in Vidensk.-Selskabs Forhandlinger, Christiania, for 1873, p. 115 (27), pl. 4, figs. 19–21.

Hincks reports this species from 100 fathoms off the coast of Iceland, and G. O. Sars from a depth of 150 fathoms off the Norwegian coast. It has been dredged by Professor Verrill in the Bay of Fundy and in Casco Bay.

Halecium robustum Verrill.

American Journal of Science, III, vol. v, 1873, p. 9, December, 1872.

Stem stout and coarse, composed of many tubes; branches stout, tapering, compound except at tips, pinnately or bipinnately branched, the branchlets spreading at an angle of about 45°; yellowish white and translucent, about ·5 of an inch long, divided by simple distant constrictions, the long internodes usually bearing from two to four hydroids. Hydrothecæ alternate, large, deep, somewhat vase-shaped, with an even, slightly everted rim, below which there is a slight constriction; the middle region is slightly smaller, gradually narrowed toward the base, with a simple diaphragm near the base within. The hydrothecæ are articulated upon slightly prominent projections from the stem, in an oblique and excentric position, so as to produce a decidedly geniculated appearance. Most of the hydrothecæ are simple, but some have one or two slightly prominent secondary rims near the margin. Height about 100^{mm}.

East of St. George's Bank, 430 fathoms (haul *g*).

Sertularella polyzonias Gray, var. **gigantea** Hincks.

Annals and Magazine Nat. Hist., IV, vol. xiii, p. 151, pl. 7, figs. 11, 12, 1874.

Diphasia mirabilis Verrill.

American Journal of Science, III, vol. v, 1873, p. 9, December, 1872.

Stem stout, rather rigid, narrowed at base, pinnately branched, somewhat flexuous between the branches, which are alternate, stout, rigid, straight, constricted at base, spreading at an angle of about 45°. Hydrothecæ on the main stem in two rows, nearly opposite; on the branches mostly in six regular rows, occupying all sides of the branches, those in the adjacent rows alternating. The hydrothecæ have large, appressed, somewhat swollen bases, but the upper portion is rapidly narrowed, prominent and curved outward; aperture strongly bilabiate, operculated. Reproductive capsules not observed.

Le Have Bank, 60 fathoms (haul *e*).

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Pennatula aculeata Danielsen.

Pennatula aculeata Danielsen, Forhandlinger i Vedenskabs-Selskabet i Christiania, 1858, p. 25 (*teste* Kölliker); Verrill, loc. cit., p. 100, 1873.

Pennatula phosphorea, var. *aculeata*, Kölliker, Anatomisch-systematische Beschreibung der Alcyonarien, 1 Abtheilung, 1 Hälfte, p. 134, pl. 9, fig. 73, 1870 (from Ahandlungen d. Senckenberg. Naturf. Gesellschaft, Frankfort, Bd. vii).

Pennatula Canadensis Whiteaves, Annals and Magazine of Natural History, IV, vol. x, p. 346, November, 1872.

Pennatula, near *P. phosphorea* Verrill, Am. Journal of Science, III, vol. v, p. 5, 1873.

Localities *o* and *s*, 110 and 150 fathoms. Also dredged by Mr. Whiteaves in 200 fathoms in the Gulf of St. Lawrence.

Virgularia Lyungmanii Kölliker.

Op. cit., 2 Hälfte, 1 Heft, p. 196, pl. 13, figs. 133, 134, 1871; Verrill, American Journal of Science, III, vol. v, p. 100, 1873; Whiteaves, Report on a Second Deep-sea Dredging Expedition to the Gulf of St. Lawrence, p. 13, 1873.

This species was described by Kölliker from specimens obtained in 30 to 80 fathoms, among the Azores, by the Josephine Expedition sent out by the Swedish government. It was also dredged in 1872, in the Gulf of St. Lawrence, at a depth of 200 fathoms, by Mr. Whiteaves.

Urticina nodosa Verrill.

Actinia nodosa Fabricius, Fauna Grœnlandica, p. 350, 1780.

Urticina digitata Verrill, Am. Jour. of Science, III, vol. v, p. 5, 1873 (not of Müller?).

This species has been dredged also in deep water off Casco Bay (Professor Verrill), and in the Gulf of St. Lawrence (Mr. Whiteaves).

Cerianthus borealis Verrill.

American Journal of Science, III, vol. v, 1873, p. 5, December, 1872.

Plate II, figure 5.

Body much elongated, tapering gradually to the abactinal opening, the surface smooth but more or less sulcated longitudinally. Marginal tentacles very numerous and unequal, the inner ones longest, in the largest specimens 56^{mm} long, and 3^{mm} in diameter at base, gradually tapering, acute; the outer ones 25^{mm} and less in length. Oral tentacles numerous, crowded in several rows, in the largest specimens about 25^{mm} long, slender, acute. Color of body olive-brown or dark chestnut-brown, sometimes pale bluish or purplish just below the tentacles; disk pale yellowish-brown; space within the oral tentacles, around the mouth, deep brown, with lighter radiating lines; oral tentacles pale chestnut-brown; marginal ones deep salmon or yellowish-brown, the longest usually barred transversely with six to eight dark reddish-brown spots, each spot partially divided along the median line into two lateral ones; part of the tentacles often have flake white spots on each side, at the base.

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The two largest specimens, dredged in 1872, in 28 fathoms, east of Grand Menan, by Professor Verrill, measured 125^{mm} across the disk and tentacles, but their bodies were mutilated. Entire ones of much smaller size were dredged by Dr. Packard and Mr. Cook in 110 and 150 fathoms, soft muddy bottom, hauls *s* and *o*. The largest of these was 200^{mm} long, and like other species of the genus, inhabited a thick, tough, felt-like, muddy tube. It was also dredged, in 1873, in Casco Bay, from 7 to 94 fathoms. One of these specimens, dredged off Seguin Island, in 70 fathoms, was 450^{mm} long, 40^{mm} in diameter, and 175^{mm} across the tentacles. A small specimen has been dredged in 18 fathoms off Watch Hill, R. I.

Epizoanthus Americanus Verrill.

Plate VIII, figure 2.

This species lives upon stones as well as upon shells inhabited by *Eupagurus*. The specimens from 430 fathoms (*g*) were on stones, while those from 60 and 65 fathoms (*s* and *f*) were on shells. It ranges from off the coast of New Jersey to the Gulf of St. Lawrence.

Sponges.

Most of the sponges obtained have not yet been sufficiently studied to be reported upon, but the two following species are of special interest.

Hyalonema longissimum Sars.

G. O. Sars, on some Remarkable Forms of Animal Life from the Great Depths off the Norwegian Coast, p. 70, pl. 6, figs. 35–45, 1872.

Only a single and somewhat abnormal specimen of this remarkable species was dredged by us in 430 fathoms, but it has since been dredged in considerable abundance by Professor Verrill, in 95 fathoms, off Casco Bay, and by Dr. Packard and Mr. Cooke on Cashe's Ledge.* Mr. Whiteaves reports it also from deep water in the Gulf of St. Lawrence.

Thecophora ibla Wyville Thompson.

Depths of the Sea, p. 147, fig. 24, 1873; Verrill, American Journal of Science, III, vol. vii, p. 500, pl. 8, fig. 8, 1874.

Plate VII, figure 1.

This species, first described by Wyville Thompson, from specimens dredged in 344 fathoms, off the Shetland Islands, by the Porcupine expedition, and dredged by us in 50 and 60 fathoms (hauls *e* and *d*), has since been dredged by Dr. Packard and Mr. Cook on Cashe's Ledge and Jeffrey's Ledge in the Gulf of Maine.

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* American Journal of Science, III, vol. vi, p. 440, 1873.

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EXPLANATION OF PLATES.

Plate I.

Figure 1.—*Stylifer Stimpsonii* Verrill; specimen from 60 fathoms, George's Bank (haul *i*); enlarged 10 diameters.

Figure 2.—*Ringicula nitida* Verrill; specimen from 110 fathoms; enlarged 14 diameters.

Figure 3.—*Entalis striolata* Stimpson; several views of animal, with the foot in different states of expansion; enlarged about 1½ diameters.

Figure 4.—*Astarte lens* Stimpson; adult; natural size.

Figure 5.—The same; young specimen; natural size.

Figure 6.—*Astarte undata* Gould; inside of valves, showing the hinge; natural size.

Figure 7.—The same; young specimen; natural size.

Figure 8.—The same; adult specimen; natural size.

Figure 9.—Variety of the same; adult specimen; natural size.

Figure 10.—*Astarte elliptica* (Brown); natural size.

Figure 11.—*Cryptodon obesus* Verrill; inside of valve; enlarged 3 diameters.

Figure 12.—*Astarte Banksii* Leach; natural size.

Figure 1 was drawn from nature by S. I. Smith; 2, 5, 6, 7, 8, 9, 11, by Professor Verrill; 3, by J. H. Emerton; the rest from Binney's Gould.

Plate II.

Figure 1.—*Sertularia argentea* Ellis and Solander; a branch bearing reproductive capsules (gonothecæ) with the soft parts removed; much enlarged.

Figure 2.—*Alcyonium carneum* Agassiz; three of the polyps fully expanded; much enlarged.

Figure 3.—*Crisia eburnea* Lamouroux; a cluster of branches, enlarged.

Figure 4.—The same; a branch bearing ovicells, more highly magnified.

Figure 5.—*Cerianthus borealis* Verrill; entire animal removed from its tube and fully expanded; about one-third natural size.

Figures 1 and 2 were drawn from nature by Professor Verrill; 3 and 4 by Professor A. Hyatt; 5 by J. H. Emerton.

Plate III.

Figure 1.—*Xenoclea megachir* Smith, male; one of the first pair of legs with its epimeron, seen from the outside; enlarged 20 diameters.

Figure 2.—The same; one of the second pair of legs, seen in the same position and enlarged the same amount.

Figure 3.—The same; one of the third pair of legs, with its epimeron and gill, seen from the outside, and showing the glandular organ within; enlarged 20 diameters; *a*, the tip of the dactylus, showing the perforation; enlarged 100 diameters.

Figure 4.—The same; one of the fifth pair of legs, with its epimeron and gill, seen from the outside; enlarged 20 diameters.

Figure 5.—*Stenothoë peltata* Smith, female; one of the second pair of legs, with its epimeron, seen from the outside; enlarged 16 diameters.

Figure 6.—The same; one of the fourth pair of legs, with its epimeron, seen from the outside; enlarged 16 diameters.

Figure 7.—The same; one of the first pair of legs, seen from the outside; enlarged 50 diameters.

Figure 8.—The same; distal portion of the propodus, with the dactylus, of one of the second pair of legs, seen from the outside; enlarged 125 diameters.

Figure 9.—*Scalpellum Stroemi* Sars; side view; enlarged 5 diameters.

All the figures were drawn on wood, from alcoholic specimens, by S. I. Smith.

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PLATE IV.

Figure 1.—*Grymæa spiralis* Verrill; head and anterior part of body; enlarged.

Figure 2.—*Pista cristata* Malmgren; head and anterior part of body; enlarged.

Figure 3.—*Phyllodoce catenula* Verrill; dorsal view of anterior part of body and head, and extended proboscis: enlarged about 4 diameters.

Figure 4.—? *Spirorbis nautiloides* Lamarck; entire animal; much enlarged.

All the figures were drawn from life by J. H. Emerton.

PLATE V.

Figure 1.—*Nephthys ciliata* Rathke; one of the lateral appendages; enlarged 10 diameters.

Figure 2.—*Lumbriconereis fragilis* Œrsted; anterior part of body and head, dorsal

ERRATA.

Page 1, line 13, for Hagerman, read Hagemann.
" 13, " 34, " *cappilare*, read *capillare*.
" 28, " 19, " Caridon, read Caridion.
" 35, " 3, " Scapellum, read Sealpellum.
" 58, " 14, " branches, read branchiæ.
" 60, " 12, " Plate X, read Plate IX.

Bay of Fundy, by Professor Verrill, in 1872.

PLATE VII.

Figure 1.—*Thecophora ibla* W. Thompson; specimen from 60 fathoms, Le Have Bank (haul 5); natural size.

Figure 2.—*Eusyllis phosphorea* Verrill; anterior and posterior portions of the animal; dorsal view; much enlarged.

Figure 3.—*Nothria conchylega* Malmgren; anterior portion; enlarged.

Figure 4.—*Nothria opalina* Verrill; anterior portion; enlarged.

Figure 1 was drawn from nature by Sherman; the others from life by J. H. Emerton.

PLATE VIII.

Figure 1.—*Pera crystallina* Verrill; enlarged 3 diameters.

Figure 2.—*Epizoanthus Americanus* Verrill; a single polyp expanded; enlarged about 6 diameters.

Figure 3.—*Chætoderma nitidulum* Lovén; entire animal; enlarged 4 diameters.

Figure 4.—The same; posterior portion with the gills expanded; enlarged 24 diameters.

Figure 1 was drawn from nature by Professor Verrill; the others were drawn from life by J. H. Emerton.

ERRATA.

Page 5, line 30, for *Vetumnus*, read *Vertumnus*.
" 9, last line, for 1873, read 1874.
" 11, line 31, for *virticillata*, read *verticillata*.

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II.—Descriptions of New and Rare Species of Hydroids from the New England Coast. By S. F. Clark.

The material for this paper was gathered while at work on the Hydroids in the Museum of Yale College. This collection is mostly from the New England Coast, and is very large and complete.

Obelia bicuspidata, sp. nov.

Plate IX, fig. 1.

The stem is erect, slender, straight or nearly so, compound, consisting of many united tubes which gradually diminish in number toward the top, varying in color from a light horn, to a light whitish brown, sparingly branched, and with three or four annulations just above the origin of each branch; branches short, ascending, slender and irregularly arranged, sometimes one, and often two branches starting from a node; branches few, very short, slender and ascending.

Hydrothecæ very deeply campanulate, narrow, tapering slightly toward the base, very hyaline, and with eight to ten longitudinal lines extending from the distal extremity nearly to the base; the rim is armed with very acute teeth, varying in number from sixteen to twenty-two, and arranged in pairs, the spaces in which the longitudinal lines terminate being a trifle wider and deeper than the alternate spaces; the pedicels supporting the hydrothecæ are long and tapering, consisting of about fifteen annulations. Gonothecæ unknown.

Height of largest specimens, about three inches (80^{mm}).

The specimens from which this species is described were taken in 3–5 fathoms, on the reefs near Thimble Islands, Long Island Sound, September 23, 1874.

This species is closely allied to *O. bidentata*, but is readily distinguished from the latter by its entirely different habit, the narrower and deeper calycles, and by the long tapering pedicels upon which the calycles are supported.

Obelia bidentata, sp. nov.

Plate IX, fig. 2.

Stems clustered, straight or slightly flexuous, compound, composed, at the base, of eight or ten slender, united tubes, varying in color from a light horn to a dingy white, densely branched, and with three

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or four annulations just above the origin of each branch; mode of branching irregular, two branches often starting from a node, sometimes an alternate arrangement of branches on opposite sides of the stem, one branch at each node. A few of the lowest branches sometimes attain a considerable length and resemble the main stem, the upper ones are short, sparingly branched and with the pinnæ diverging at a slight angle; the branchlets and ends of the branches are simple, slender, translucent, and very graceful. Hydrothecæ very deeply campanulate, tapering slightly toward the base, and with nine to twelve longitudinal lines extending from the distal extremity nearly to the base; the rim is ornamented with from eighteen to twenty-four very acute teeth, arranged in pairs, the spaces in which the longitudinal markings terminate being a trifle wider and deeper than the alternate spaces; the pedicels supporting the hydrothecæ are usually short and stout, consisting of three to six strong rings, but some of the hydrothecæ near the base of the stem have the pedicel slightly tapering, and composed of from ten to twelve annulations. Gonothecæ unknown.

The largest specimen has a height of about 6 inches (150^{mm}).

We have had this species from but one locality, Greenport, Long Island, where it was collected August 5th, 1874, in considerable abundance, on the piles of the wharves at low water,—U. S. Fish Commission.

O. bidentata resembles *O. gelatinosa* in the delicacy and grace of its habit, in the flexibility of the compound stem and branches, and in the pellucid whiteness of the upper portion of its branches and branchlets.

Campanularia pygmæa, sp. nov.

Plate IX, fig. 9.

Stem often creeping, with short, stout, coarsely annulated, upright pedicels, sometimes with one or two short, annulated branches, each bearing a single calycle. Hydrothecæ large, deep campanulate, tapering slightly toward the base, and with longitudinal lines at regular intervals, extending down about one-fourth the length of the calycle; the rim is ornamented with from ten to fourteen square-cut denticles, which are more or less hollowed out above, and separated by rather shallow evenly rounded notches, of about the same breadth. Gonothecæ unknown. Height about 1^{mm}.

Found growing on a specimen of *Sertularia latiuscula*, from Casco Bay, Maine,—U. S. Fish Commission.

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Campanularia noliformis McCrady.

Plate X, fig. 5.

A few specimens of this species were collected at low-water, near Savin Rock, in the latter part of September, 1874, attached to *Zostera marina.*

The hydrarium agrees very well with McCrady's description. We were not fortunate enough to find the gonothecæ.

This is the first time this species has been recorded since McCrady described it from the harbor of Charleston, S. Carolina, in 1857.

Campanularia calceolifera Hincks.

Ann. and Mag. of Nat. Hist., vol. viii, Aug., 1871, page 78, pl. vi.

Plate X, figs. 7, 8.

Stem filiform, slender, flexuous, sometimes slightly branched, ringed at the base and above the origin of the branches, light horn-color, with the upper portions pellucid white; branches short, curving outward, undivided, and bearing but two or three calycles. Hydrothecæ alternate, broadly campanulate, deep, with a slightly everted entire rim, and borne on annulated pedicels of variable length, those on the upper portion of the stem consisting of five to eight rings, those near the base, of twelve to twenty. At each bend of the stem a single hydrotheca is given off, and these all curving outward give to this species a very graceful habit. Gonothecæ axillary, borne on pedicels consisting of three or four rings, largest at the distal extremity and tapering gradually toward the base, with a peculiar incurved coil or twist at the distal end near the opening; the aperture is shield-shaped and placed in a depression on one side of the distal end. An internal membrane extends inward from the shield-shaped opening and terminates in a circular orifice near the distal extremity. Height about one inch (25^{mm}).

Noank, Conn., from the bottom of an old scow, Sept. 9, 1874, with gonothecæ; piles of wharves at Woods Hole, Mass., Aug., 1871, with gonothecæ—U. S. Fish Commission.

Hincks' figures represent the hydrothecæ as being more everted than they are in the American specimens; otherwise they exactly correspond. This is the first time this species has been recorded from the American coast.

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Gonothyræa tenuis, sp. nov.

Plate X, fig. 8.

Stem simple, somewhat flexuous, slender, and annulated above the origin of each of the numerous branches, which are arranged alternately, some simple and some compound, the latter bearing numerous branchlets, the lower branches sometimes half the length of the main stem; base of the stem and of the lower branches light horn-color, the upper portions of the same and all of the branchlets pellucid white; branches and branchlets spreading, giving quite a bushy appearance to a well developed colony. Hydrothecæ variable in size, deeply campanulate, tapering quite rapidly from a little below the middle to the base; the rim is ornamented with teeth which show considerable variation, both in number and in shape; in some cases they are quite sharp and shallow, while on other calycles upon the same stem they are of a castellated form and sometimes slightly emarginate; in number they vary from ten to sixteen; the pedicels which support the hydrothecæ also vary greatly, some being composed of but three or four annulations, others of as many as fourteen.

Gonothecæ axillary, very much elongated, narrow, obconic, tapering gradually from the distal to the proximal end, borne on short pedicels of but three to five rings; the number of medusæ holding planulæ contained in each reproductive capsule varies from two to five and the number of planulæ in each medusa varies to the same extent; the tentacles of the medusæ vary considerably in length and in number, some of them being over half as long as the diameter of the medusa, while others are not more than one-third that length, in number they vary from eight to fourteen. The planulæ at the time of liberation are regularly cylindrical, and their length is equal to nearly four times the width. Height usually 1 to 1·25 inches (25 to 38^{mm}).

New Haven, Conn., on piles of Long Wharf, June 2nd, 1875,—S. F. Clark. Found in considerable abundance at low-water, loaded with reproductive capsules. The large size of the latter, together with the clusters of extracapsular medusæ, make this quite a showy species for one of such humble growth.

Opercularella pumila, sp. nov.

Plate IX, figs. 3, 4, 5.

Stem rather stout, erect or creeping, slightly flexuous, annulated throughout, sparingly branched; branches erect, undivided, some-

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times attaining a considerable length. Hydrothecæ largest in the middle, tapering very slightly toward the base, rapidly converging at the distal end, and supported on short annulated pedicels, consisting of three to five rings. Gonothecæ fusiform, with the tapering neck often somewhat elongated; length about twice that of the hydrothecæ. They contain one to three small globular or ovate immature medusoids. The pedicels consist of three to six annulations. In the creeping form the hydrothecæ appear at intervals borne on short ringed stalks consisting of about three to six rings.

Portland, Maine, August, 1873, with gonothecæ, on piles of wharves; off Montauk Pt., Long Island, 5–15 fathoms, August, 1874,—U. S. Fish Commission.

This species closely resembles *O. lacerata* of Hincks, from which it is distinguished by the forms both of the hydrothecæ and gonothecæ. The hydrothecæ are also smaller than those of *O. lacerata*. As the reproduction has not been traced in this species, it can only be referred to the genus provisionally. I am inclined to think, from the shape of the gonothecæ and from the fact that they often contain two or three distinct reproductive bodies, that it may not belong under *Opercularella*.

Opercularella lacerata Hincks.

Campanularia lacerata Johnston, Brit. Zoöph., p. 111, Pl. XXVIII, fig. 3.
Opercularella lacerata Hincks, Brit. Hydr. Zoöph., p. 194, Pl. XXXIX, fig. 1.

Plate IX, fig. 6.

Stem erect, simple, slightly flexuous, more or less annulated throughout, sparingly branched; branches short undivided. Hydrothecæ ovato-fusiform and borne on short pedicels of but two or three annulations; operculum composed of six to eight segments. Gonothecæ, of the female colonies, a trifle wider at the distal end, and tapering very slightly toward the base, supported on short ringed pedicels; the medusoid (sporosac) containing the planulæ is quite large, the diameter being about equal to the length of the gonotheca; from two to five planulæ in each medusoid.

New Haven, Conn., on piles of Long Wharf, May 13th, 1875, with extracapsular medusæ.—S. F. Clark.

The hydrothecæ in this species average about one third larger than those of *O. pumila*; the segments of the opercula are more deeply cleft; and there are differences in the form of the gonothecæ.

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Calycella syringa Hincks. Peculiar variety.

Plate X, figs. 1, 2, 3.

Sertularia syringa Linn., Syst. 1311.

Calicella syringa Hincks, Cat. Devon Zooph., 23; Ann. N. H. (3d ser.), VIII, 294.

Calycella syringa Hincks, British Hydroid Zoophytes, Vol. I, p. 206, Plate XXXIX, figs. 2, 2*a*.

Stem simple, creeping, nearly smooth. Hydrothecæ hyaline, colorless or tinged with a light horn-color, cylindrical, rounded off below, with an everted rim, to which is attached an operculum consisting of from five to eight segments and supported on twisted pedicels of considerable length, with eight to fifteen twists; some of the hydrothecæ have an addition in the shape of a wide ring, ornamented with from ten to fourteen longitudinal markings, which rises for some distance above the rim and on the summit of which there is borne either the operculum or another ring; in some cases there are as many as four of these rings with an operculum at the summit. The opercula usually point upward, but are occasionally deflected into the calycle.

From Casco Bay, Me., 9 fathoms,—U. S. Fish Commission.

Halecium articulosum, sp. nov.

Plate X, fig. 6.

Stem dark brown and tapering gradually, very stout, sparingly branched, compound, consisting of many, slender, anastomosing, serpentine tubes; branches short and irregularly arranged on all sides of the main stem; branchlets few and very short; both branches and branchlets are divided into very short, stout internodes by distinct joints placed at right angles to the stem; branches and branchlets simple, whitish, delicate, becoming more numerous toward the top of the stem. The internodes become shorter very gradually toward the ends of the branches and branchlets. Hydrothecæ alternate, short and wide, one to each segment; some of them have a cup within a cup, as is so often the case in the species of *Halecium*. Gonothecæ borne in rows on the upper side of the pinnæ. The female gonothecæ are large, obovate, and have the opening on one side and nearer the distal than the proximal end; the male gonothecæ are oblong, subcylindrical, and, like the female, are sessile. Height of the largest specimens, 5 inches (125^{mm}).

Eastern end of Long Island Sound, 8–12 fathoms; Coxe's Ledge, S. E. of Block Island, 17–21 fathoms; Casco Bay, Maine; Eastport, Me.,—U. S. Fish Commission.

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H. articulosum resembles *H. plumosum*, but has a stouter habit; the internodes are shorter and stouter; and it may also be distinguished by the direction of the joints, at right angles to the stem, and by the very wide-mouthed calycles. The female gonothecæ somewhat resemble those of *H. Beanii*, but are of a stouter build; they are relatively larger at the distal extremity, the orifice is differently shaped and is differently situated, being nearer the distal extremity.

Sertularia argentea Ellis and Solander, var. **divaricata** nov.

Plate X, fig. 7.

Stem simple, stout, erect, straight or slightly flexuous, of a deep horn color, regularly jointed, each joint having two or three branches; branches alternate, sparingly branched, diverging at right angles from the main stem and all in the same plane, divided quite regularly by joints, each bearing two pairs of hydrothecæ, much resembling a young shoot of the usual form of *S. argentea*. Hydrothecæ nearly opposite, curving strongly outwards, with a bilabiate mouth, the upper lip being considerably smaller than the lower; hydrothecæ are also scattered along the stem in pairs. Gonothecæ unknown.

Collected at Casco Bay, Me., 1873,—U. S. Fish Commission.

I at first thought that this was a distinct species, but I have since had intermediate forms which prove quite conclusively that it is only a variety of *S. argentea*. Considerable variation is shown in the hydrothecæ of this variety; some of them, on the same stem, are more directly opposite and curve outwards more than others.

Plumularia Verrillii, sp. nov.

Plate X, fig. 9.

Stems erect, simple, straight or slightly curved, slender, two to four inches high, of a bright horn-color, branched and regularly jointed by transverse divisions; the branches have their origin near the base of the stem, are erect and resemble the main stem in all particulars; pinnæ occasionally branched, regularly arranged on two sides of the main stem and branches, separated by an angle of ninety degrees, composed of long similar joints, each bearing a hydrotheca and a number of nematophores; occasionally there is an odd, intermediate joint bearing only one or two nematophores and no hydrothecæ; a single pinna to each joint. Nematophores sessile, compound, large, tapering to the base, with a round cup-like opening: there are four to six on each hydrotheca-bearing joint, one on each side of the upper

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edge of the hydrotheca, two or three between the hydrotheca and the proximal end of the joint, and occasionally one at the distal end; on the main stem there are usually two in each axil and two or three on each joint. Hydrothecæ small, shallow, about equal in depth to the length of the nematophores, attached to the stem by their entire length; rim entire. Gonothecæ borne in the axils of the stem and of the branched pinnæ, sessile, tapering at the base, the remaining portion either regularly cylindrical or slightly swollen in the middle; aperture large, terminal. Height, 2·5 inches (64^{mm}).

Eastport, Maine, 10–20 fathoms, 1868,—A. E. Verrill and S. I. Smith.

P. Verrillii is a beautiful little, pellucid, white species, with a delicate, graceful habit which readily distinguishes it from any of the forms now known upon our coast. It is the second genuine *Plumularia* from the New England coast, both having been discovered by Professor Verrill. The previously described species (*P. tenella* Verrill) was dredged in 1871, off Gay Head, Martha's Vineyard, in 10 fathoms. It has since been dredged in 4–5 fathoms off the Thimble Islands, near New Haven, Conn., and it was also found on the piles at Greenport, Long Island, August 5th, 1874, with gonothecæ. It differs greatly from the present species in the form of the gonothecæ, which are in the shape of an elegantly curved cornucopia, slender at the base and gradually enlarged to the end, and with a cluster of nematophores at the base. The hydrothecæ are also different in form.

EXPLANATION OF PLATES.

Plate IX.

Figure 1. *Obelia bicuspidata;* from Thimble Islands.

Figure 2. *Obelia bidentata;* from Greenport, Long Island.

Figure 3. *Opercularella pumila;* creeping form.

Figure 4. The same, young, with stem erect; from off Montauk.

Figure 5. The same, from Portland, Me., showing a more luxuriant growth; *a* and *c*, the hydrarium; *b*, gonotheca, enlarged 32 diameters.

Figure 6. *Opercularella lacerata;* *a*, hydrarium; *b*, gonotheca; *c*, medusoid; *d*, undeveloped planulæ.

Figure 7. *Campanularia calceolifera;* from Noank, Conn.

Figure 8. The same; *a*, hydrotheca; *b*, gonotheca; from Noank, Conn.

Figure 9. *Campanularia pygmæa;* from Casco Bay.

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PLATE X.

Figure 1. *Calycella syringa;* from Casco Bay, showing a peculiar variation in the operculum.

Figure 2. The same, showing the variation in the size and shape of the hydrothecæ and in the length of the pedicel.

Figure 3. The same, with one secondary ring.

Figure 4. *Clytia Johnstoni*, from Noank, Conn.

Figure 5. *Campanularia noliformis*, showing variations in the pedicels.

Figure 6. *Halecium articulosum;* from Coxe's Ledge. A, a branch bearing both hydrothecæ and gonothecæ; *a*, gonothecæ; *b*, hydrothecæ. B, a branch with hydrothecæ only; *b*, a branchlet; *c*, hydrothecæ.

Figure 7. *Sertularia argentea*, var. *divaricata;* from Casco Bay.

Figure 8. *Gonothyræa tenuis;* *a*, branch with hydrothecæ; *b*, gonotheca with extracapsular medusæ; *c*, medusa with radiating tubes and tentacles; *d*, planulæ.

Figure 9. *Plumularia Verrillii;* *a*, branch showing hydrothecæ and nematophores and the arrangement of the joints; *b*, gonotheca; *c*, a single joint.

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III. On the Chondrodite from the Tilly-Foster Iron Mine, Brewster, New York. By Edward S. Dana.

WITH THREE PLATES.

The interesting discovery by Scacchi,* of the existence of three types in the crystals of the Vesuvian humite, gives especial interest to the study of chondrodite—a mineral identical with humite in chemical composition, and yet very different in appearance, as well as in origin and method of occurrence. The same subject of humite has since been more exhaustively investigated by vom Rath,† with the entire confirmation of Scacchi's views. These authors have shown that the crystals of humite are to be divided into three groups, all bearing the same relation to each other in respect to their lateral axes, while the vertical axis has a distinct value for each type. In other words, the planes occurring upon a given crystal bear simple relations to each other, whereas only very complex symbols result when the planes of one type are referred to the axes of another. For a full explanation of this subject reference must be made to the valuable memoirs above alluded to. It will be sufficient to give as an example the symbols of the occurring pyramids of the r series on the second and third types of chondrodite (see beyond); (1) as referred to their own axes; (2) as referred to the axes of the second type.

(1.)		(2.)	
II.	III.	II.	III.
$\frac{4}{7}\text{-}\bar{2}$ (r^1)	$\frac{8}{11}\text{-}\bar{2}$ (ρ^3)	$\frac{4}{7}\text{-}\bar{2}$	$\frac{36}{55}\text{-}\bar{2}$
$\frac{4}{5}\text{-}\bar{2}$ (r^2)	$\frac{8}{9}\text{-}\bar{2}$ (ρ^4)	$\frac{4}{5}\text{-}\bar{2}$	$\frac{36}{45}\text{-}\bar{2}$
$\frac{4}{3}\text{-}\bar{2}$ (r^3)	$\frac{8}{7}\text{-}\bar{2}$ (ρ^5)	$\frac{4}{3}\text{-}\bar{2}$	$\frac{36}{35}\text{-}\bar{2}$
$4\text{-}\bar{2}$ (r^4)	$\frac{8}{5}\text{-}\bar{2}$ (ρ^6)	$4\text{-}\bar{2}$	$\frac{36}{25}\text{-}\bar{2}$
	$\frac{8}{3}\text{-}\bar{2}$ (ρ^7)		$\frac{36}{15}\text{-}\bar{2}$
	$8\text{-}\bar{2}$ (ρ^8)		$\frac{36}{5}\text{-}\bar{2}$

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* Pogg. Ann., Erg. Bd. iii, 161, 1851.

† Pogg. Ann., Erg. Bd. v, 321, 1871; vi, 385, 1873.

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The chemical composition of the three types of humite has been most recently investigated by vom Rath,* and although analyses lead to somewhat different results in the three cases, he concludes that in composition they are still essentially the same, and that the cause of the variation in crystalline form is *not* to be found in the relative amount of fluorine present, as has been often assumed.

A further remarkable peculiarity true of two of the three types is their hemihedral character, which is clearly set forth in the memoirs referred to. These points are alluded to here because of their direct bearing on the crystallization of chondrodite, which forms the subject of this paper.

Chondrodite was first shown by Rammelsberg to be identical with humite in chemical composition, but its crystallographic relation to it was not brought out until the investigations of Kokscharow. He showed, in his "Materialien zur Mineralogie Russlands," vol. vi, p. 73, 1870, that the crystals from Pargas, Finland, were identical in form and angles with type II of humite. Vom Rath has followed with the description of crystals from Nya-Kopparberg, Sweden, and proved that the same fact is true of them.

The study of the chondrodite from the Tilly-Foster iron mine, Brewster, Putnam Co., New York, which I have been able to make during the past season, has shown that it, too, is for the most part identical in crystalline form with type II of humite, but that at the same time crystals exist belonging to *type I*, and others which belong to *type III*. Further than this, the chemical composition of the second type crystals, as shown by an analysis by Mr. G. W. Hawes (p. 21), agrees with great exactness with that of the Swedish mineral analyzed by vom Rath. Moreover, the detailed study of these crystals has shown that while they agree with humite in the character of their hemihedrism, as well as in angles, they surpass it in the multiplicity of secondary planes. Thus a single solid angle has been observed which was modified by fifteen distinct and well-defined, though very minute, planes. This, as will be seen when the facts are described in detail, implies a delicacy in the action of the crystallogenic forces at work which is unparalleled, and sustains the opinion that chondrodite, or humite, is unique among mineral species.

The method of occurrence at the Tilly-Foster iron mine has been fully described by Prof. Dana in a memoir entitled, "Serpentine pseudomorphs and other kinds, etc.," Journal of Science, viii, pp. 371,

* Pogg. Ann., cxlvii, 246, 1872.

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447, 1874. It may be of interest, however, to review the subject again so far as the chondrodite itself is immediately concerned.

The chondrodite forms the gangue of the magnetite, being everywhere disseminated through it in varying proportions. In the parts of the mine where the ore is purest and perfectly firm and solid,—the so-called "blue ore,"—the associated chondrodite is sparsely sprinkled through it in small yellow grains, showing no trace of crystalline form. Occasionally, however, the firmer ore contains the chondrodite in very large but imperfect crystals, or crystalline masses, associated directly with enstatite and enveloped with dolomite, which have a dark, rich brown color, and a brilliant luster on the fracture. A distinctly laminated structure is uniformly present, which is perhaps due to cleavage (?) (See page 21.) Isolated grains imbedded in dolomite often show traces of crystalline faces, though nothing that admits of even approximate determination. An analysis of this variety of the mineral gave Mr. Breidenbaugh (Am. J. Sci., III, vi, 209),

$\dot{S}i$ 35·42, $\dot{F}e$ 5·72, $\dot{M}g$ 54·22, Fl 9·00=104·36; equivalent of oxygen replaced by fluorine, 3·79.

In the larger portion of the mine as now opened the soft "yellow ore" predominates: the chondrodite is present in it in much larger quantities, and, like the other minerals present, it has almost universally suffered extensive alteration. A long list of these products of alteration has been fully described by Prof. Dana in the memoir already alluded to. The chondrodite forms the main portion of the material taken out, and many tons of this refuse matter are yearly thrown away. It varies much in color, but is generally of a light yellow; it usually has more or less of a soapy feel and shows a variety of transition-products between the semi-altered material and serpentine. The chondrodite in this "yellow ore" is generally massive; but occasionally fragments of large coarse crystals have been found, some of which measure five or six inches in length. These are always more or less altered; moreover, the material of which they are formed is far from homogeneous, masses of magnetite, and also chlorite, being often enclosed. Dolomite is the most constantly associated mineral and occurs in rhombohedrons of considerable size; these, as well as the crystals of chondrodite, are often coated with magnetite.

Better crystals of chondrodite than those just mentioned are sometimes found in what were once cavities in this massive material. Unfortunately these have all suffered from the general alteration and now have little or no luster, and often are not even smooth. These cavities are almost invariably filled with a soft mealy serpentine,

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which can be cut out with a knife. These crystals vary in size, being sometimes an inch or two in length. A crystallographic examination of them is seldom possible, but a few of the crystals found allow of it, and the results are described beyond. The form is usually very simple, and the color varies from a deep red to a light yellow. This may be said to be the common method of occurrence at the locality.

Fortunately, material much better adapted for crystallographic study also occurs, though this is very rarely true. Narrow veins are sometimes met with, two or three inches across, which were originally lined with more or less perfectly crystallized chondrodite and also with dodecahedrons of magnetite, crystals of ripidolite, and rarely apatite, and then subsequently filled in with dolomite. Where this has been the case and the dolomite has remained intact the chondrodite has been protected and the crystals have retained perfectly their luster and color. Only in a few instances were the crystals polished when covered simply by a soft serpentine. The chemical composition of this chondrodite is given beyond, after the description of the crystals. It has a deep, garnet-red color, and a luster equal to that of the finest Binnenthal blende.

1. *Description of Crystals belonging to Type II.*

The remark of vom Rath in regard to the irregularity of form of the Swedish chondrodite is eminently true of the Brewster crystals. For in the same little group no two are alike; so that each one deserves and requires an especial study.

The first point to be determined was the values of the fundamental angles. Some difficulty was found in obtaining these from the fact that many crystals, though faultless in luster, yet gave uncertain measurements. This was due to the fractured condition of many of the planes, which, though often not very apparent at first sight, yet gave rise to a variety of reflected images in the goniometer, of which no one could be accepted as trustworthy. All the larger crystals show a multitude of internal fractures; and, where such crystals have been subjected to altering influences, this circumstance has hastened their destruction, and in all cases the external condition of the planes has been more or less affected. The direction of the fracture lines was in most cases entirely irregular, though in a number of cases they were distinctly parallel to $e^2(2\text{-}\bar{2})$. The presence of these cracks gave the crystals the appearance of having suffered sudden contraction, by which the planes had been irregularly drawn inward, forming re-entrant angles; in fact, in this respect, as in general appearance, this chondrodite might be aptly compared to a resin.

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The smallest crystals proved to be free from this cause of irregularity, and one of them, on which the faces were exceptionably brilliant, was chosen for careful measurement. It may be added that all the measurements were made with an Oertling goniometer, provided with two telescopes.

The mean of 30 measurements of A (O=001) on r^1 $\left(\frac{4}{7}\text{-}\bar{2}=\bar{2}47\right)$ gave: 135° 18′ 50″. The maximum variation from the mean given was ±45″. The mean of 30 measurements of A on e^a $\left(\frac{2}{5}\text{-}\bar{1}=205\right)$ gave: 149° 55′ 48″. Maximum variation ±45″.

These were accepted as the fundamental angles, and as the agreement between the other angles measured and the calculations made

TABLE I.

			Chondrodite.									*Humite.*		
			$C=i\text{-}\bar{1}$ (010) Calculated.			A = O (001) Measured.			Calculated.			A (v. Rath) Calculated.		
i	1-$\bar{1}$	011	147°	32′	39″	122°	44′	(ap.)	122°	27′	21″	122°	27′	49″
e^a	$\frac{2}{5}$-$\bar{1}$	205	90			*149	55	48	149	58	48			
e^1	$\frac{2}{3}$-$\bar{1}$	203	90			135	59		136	1	17	135	52	15
e^2	2-$\bar{1}$	201	90			109	4		109	3	24	108	57	50
r^1	$\frac{4}{7}$-$\bar{2}$	$\bar{2}47$	129	42	9	*135	18	50	135	18	50	135	17	40
r^2	$\frac{4}{5}$-$\bar{2}$	245	137	25	45	125	52		125	50	6	125	49	0
r^3	$\frac{4}{3}$-$\bar{2}$	$\bar{2}43$	146	27	42	113	25½		113	25	36	113	24	45
r^4	4-$\bar{2}$	241	154	2	9	98	14		98	13	6	98	12	47
m^3	6-$\frac{3}{2}$	$\bar{6}41$	125	43	56	95	22		95	19	40	95	17	59
n^1	$\frac{2}{3}$	$\bar{2}23$	127	1	31				125	3	49	125	2	47
n^2	2	221	135	45	24	103	11		103	10	4	103	9	35

upon the above basis proved to be as close as could be desired, having regard to the nature of the plane in each individual case, no

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attempt to correct them was deemed desirable. Calculated from these angles the parameters are:

$$a \text{ (vert.)} = 1{\cdot}57236\,;\ b = 1;\ c = 1{\cdot}08630\,;$$

and the angle for the fundamental prism is

$$I \wedge I\,(110 \wedge 1\bar{1}0) = 85^\circ\ 15'\ 46'' \text{ or } 94^\circ\ 44'\ 14''.$$

It may be added that the angle of A on r' behind ($\bar{2}\bar{4}7$) was measured with equal care, and found to be 135° 18′ 40″.

The preceding table* (I) includes the principal angles measured on the same crystal, and also those calculated from the above parameters; in addition, the corresponding angles for humite, type II, are also given, as calculated by vom Rath. The angles of the macrodomes agree very closely, it will be observed, in chondrodite and humite; in the brachydomes, on the other hand, there is a divergence of 6 or 7 minutes.

The angles given in table I, and also in tables XII, XIII, XIV, for types I and III, are the actual angles. In all the other tables, however, the supplement (normal) angles, as measured and calculated, are uniformly given. The reason for this was that these angles having special reference to the sphere of projection, and being chiefly of value in calculating with it, it did not seem worth while to change them from the form in which they had been used.

It is necessary to explain, also, the system of symbols and letters here and subsequently employed. The fundamental form adopted is the same as that used in Dana's "Mineralogy," and first suggested by the author of that work in Am. J. Sci. II, xiv, 175, 185. It is to be remembered that Scacchi made the prism of the r series (i. e., i-$\bar{2}$) the fundamental prism, and gave to the vertical axes lengths, in the ratio of 7 : 5 : 9 to each other, for types I, II and III respectively. Vom Rath followed him in this latter respect, but for the vertical prism took that of the n series (i. e., I, or 110 of Dana). Prof. Dana took a modified view of the relations of the three forms, and chose for the fundamental macrodome in each type the plane making with $A(O)$ an angle of 122° to 125°: thus on humite (vom Rath), type I, 124° 17′, II, 122° 28′, III, 125° 15′. In this view the vertical axes have one-third (I), one-half (II) and one-quarter (III) of the lengths assumed by Scacchi; in other words, their relation to each other are

* Both the symbols of Naumann (in the form used in Dana's "Mineralogy") and also of Miller are given; the signs belonging to each plane omitted here, as the relations of the planes are shown with sufficient clearness on the spherical projection, Plate xiii.

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as $\frac{7}{3}:\frac{5}{2}:\frac{9}{4}$ or 28 : 30 : 27. The method adopted by Scacchi and vom Rath has the advantage of expressing the simplest possible numerical ratios between the three types. It is the view of the author above referred to, however, that the variation in the angle of $A \wedge 1\text{-}\bar{1}(011)$ is to be regarded in the same light as the variation in the vertical axes of the rhombohedral carbonates, or in the orthorhombic sulphates; or in other words, the three types form an isomorphous series, and the variation observed is no greater than is constantly seen in analogous isomorphous groups. This view seems to find confirmation in the crystallographic relations of humite and *chrysolite*, a subject already discussed by Scacchi, Rammelsberg and vom Rath. Taking the fundamental form, as in Dana's "Mineralogy" (here $i\text{-}\bar{2}=I$ of other authors), the lateral axes are nearly identical with those of humite, while the vertical axis (1·25928 Kokscharow) has exactly the ratio of $\frac{8}{10}$ to that of humite, type II, and $\frac{8}{9}$ to that of humite, type III; in other words, we have the ratios:

	Humite.			
Chrysolite.	III.	I.	II.	
24 :	27 :	28 :	30.	

If we adopt the vertical axes of Scacchi and quadruple that of chrysolite, we obtain

II.	I.	Chrysolite.	III.
5 :	7 :	8 :	9

These relations were in effect brought out by Scacchi when he showed that what he called the common fundamental form of humite, obtained by dividing the vertical axes by 7 (I), 5 (II), and 9 (III) respectively, was nearly identical with that accepted by him for chrysolite. This fact seemed to Rammelsberg of so much importance that he proposed to refer all the planes of humite to this common fundamental form; and in this he has been followed by Kokscharow. The result of this will be seen in the following table, which gives the symbols thus obtained for the two common macrodomes in each type.

I.	II.	III.
$\frac{7}{10}\left(\frac{3}{5}\text{-}\bar{i}\right)$	$\frac{5}{8}\left(\frac{1}{2}\text{-}\bar{i}\right)$	$\frac{3}{4}\left(\frac{2}{3}\text{-}\bar{i}\right)$
$\frac{7}{6}\ (1\text{-}\bar{i})$	$\frac{5}{4}\ (1\text{-}\bar{i})$	$\frac{9}{8}\ (1\text{-}\bar{i})$
$\frac{7}{2}\ (3\text{-}\bar{i})$		$\frac{9}{4}\ (2\text{-}\bar{i})$

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It will be seen from these few examples that the plan proposed introduces a set of common molecular axes at the expense of all crystallographic simplicity in the relations of the several planes of each type. Moreover, the view of Rammelsberg loses some of its plausibility, if, as shown by vom Rath, the vertical axes do *not* stand in direct relation to the amount of fluorine present. The view of Prof. Dana here advocated seems to have the advantage of presenting all the relations in their most natural light.

It may be added, as completing the history of the subject, that DesCloizeaux refers all the humite planes to one, and that the second, type.

In regard to the letters employed, it seemed to offer the simplest solution of an obvious difficulty to retain all the letters of Scacchi for the second type, and for the third to use simply the corresponding Greek letters in the same order, and for the first type to use the corresponding capital letters. It was not deemed advisable to use the same letter for two planes, on different types, which bore no immediate relation to each other.

TABLE II.

$A = O(001)$.

			Calculated.	II.	III.	IV.
i^1	$1\text{-}\breve{\imath}$	011	57° 33′	57° 28′		
e^a	$\frac{2}{5}\text{-}\breve{\imath}$	205	30 4			
e^1	$\frac{2}{3}\text{-}\breve{\imath}$	203	43 58½			
e^2	$2\text{-}\breve{\imath}$	201	70 57			
r^1	$\frac{4}{7}\text{-}\bar{2}$	$\bar{2}47$	44 41	44 40½ 44 40	44° 37′	44° 41′
r^2	$\frac{4}{5}\text{-}\bar{2}$	245	54 10		54 12	
r^3	$\frac{4}{3}\text{-}\bar{2}$	$\bar{2}43$	66 34	66 39 66 32	66 25	66 35
r^4	$4\text{-}\bar{2}$	241	81 47	81 52		
n^1	$\frac{2}{3}$	$\bar{2}23$	54 56			
n^2	2	221	76 50			
m^2	$6\text{-}\frac{3}{2}$	$\bar{6}41$	84 40			

TABLE III.

$C = \breve{\imath}\text{-}\breve{\imath}(010)$.

Calculated.	II.	III.	IV.
32° 27′	32° 28′ 32 32		
90			
90			
90			
50 18	50 7 50 30	50° 13′ 50 30	50° 12′
42 34	42 39	42 32	42 35
33 32	33 31 33 32	33 30	33 27
25 58		26 0 25 57½	
44 14½		44 11	
52 58			
54 16		54 18	

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In the tables, II, III, IV, are given the angles as measured in a series of crystals (each crystal is numbered). They are important as showing how far the angles are constant. Some considerable variations from calculated angles in a few instances are to be explained by the cause of irregularity already mentioned—the occurrence of irregular fractures across the planes. In table V are given the supplement angles calculated for all the planes on e^2 and also the angles measured on the several crystals (as before numbered). The angles are in pairs corresponding to 201 and $\bar{2}01$, or $\bar{2}01$ and 201 in each case. (Compare the spherical projection, plate xiii.)

TABLE IV.

Angles measured on $C = i\text{-}\bar{i}$ (010).

			V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
r^1	$\frac{4}{7}\text{-}\bar{2}$	$\bar{2}47$			50° 11′	50° 16′ 50 16	50° 17′ 50 17	50° 12′		50° 24′
r^2	$\frac{4}{5}\text{-}\bar{2}$	245	42° 34′	42° 20′ 42 12	42 30 42 23	42 28	42 36 42 29		42° 34′	42 38 42 36
r^3	$\frac{4}{3}\text{-}\bar{2}$	$\bar{2}43$			33 26	33 32½	33 33 33 34	33 27		33 32
r^4	$4\text{-}\bar{2}$	241	25 59	26 1 26 0	25 55		25 57	25 56½	26 0	
n^1	$\frac{2}{3}$	$\bar{2}23$	44 20 44 10	44 26 44 14	44 14		44 15			
n^2	2	221	52 57	52 55			52 52			
m^2	$6\text{-}\frac{3}{2}$	$\bar{6}41$			54 31					

Of the planes which occur, according to Scacchi and vom Rath, on type II of humite, I have identified all but m of Scacchi ($\frac{6}{5}\text{-}\frac{3}{2}$) and $\frac{3}{5}\text{-}e$ of vom Rath ($\frac{6}{5}\text{-}\bar{i}$). Of new planes I have found the following, which fall in the old vertical or horizontal zones, and many others to be described later; o ($i\text{-}\breve{2}=210$), i^3 ($2\text{-}\bar{i}=021$), i^a ($\frac{4}{7}\text{-}\bar{i}=047$), $i\beta=(\frac{2}{5}\text{-}\bar{i}=025$), $e^a(\frac{2}{5}\text{-}\breve{i}=205)$, $r^a(\frac{8}{9}\text{-}\breve{2}=\bar{4}89)$; of these the most interesting is the prism $i\text{-}\breve{2}$, as hitherto no prism has been found on either the 2d or 3d types.

Hemihedrism—The peculiar hemihedral character of humite has been alluded to, and it is a little striking that the crystals of chondrodite should show so entire a correspondence to it. Taking the same position for the crystals as vom Rath, r^2 and r^4 appear uniformly in the positive (or upper) quadrants, r^1 and r^3 always in the

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TABLE V.

Angles measured on $e^2 = 2\text{-}\breve{\imath}$ (201 and $\bar{2}01$).

			Calculated.	V.	VII.	IX.	I.	X.	VI.
e^1	$\frac{2}{3}\text{-}\breve{\imath}$	203	26° 57′ 53″	26° 55′			26° 51′		26° 59′
r^1	$\frac{4}{7}\text{-}\bar{2}$	$\bar{2}47$	59 19 58 87 22 38				59 20	59° 20′	
r^2	$\frac{4}{5}\text{-}\bar{2}$	245	59 14 0 82 34 28	59 14	59° 33′ (approx)	59° 30′ 59 30	82 34		
r^3	$\frac{4}{3}\text{-}\bar{2}$	$\bar{2}43$	60 30 0 76 32 14		76 26	76 15		76 35	XI.
r^4	$4\text{-}\bar{2}$	241	64 2 9 69 51 3		64 0	63 57			64° 0′
n^1	$\frac{2}{3}$	$\bar{2}23$	44 38 20 70 20 31	44 34					
n^2	2	221	45 45 24 56 42 18	45 40 45 50	45 46 56 42	56 26	45 42 56 42		
m	$6\text{-}\frac{3}{2}\breve{\imath}$	$\bar{6}41$	37 34 34 42 57 12			42 43 42 32	42 51		

negative (or lower) and n^2 is both + and −, but where occurring alone is generally negative; n^1 is generally, and m^2 always, negative. Of the brachydomes it may be said that they are often holohedral, but this is not always the case. The various figures on the two plates will show the true relations better than words. It is to be said, however, that when the brachydomes are ± they are still distinguished from each other physically. Thus the + series may be largely developed and rough, destitute of any semblance to polish, when the negative series is as lustrous as the pyramidal planes. When e^2 ($2\text{-}\breve{\imath}=201$) is only once present it is uniformly positive. The macrodomes are always holohedral on humite; here this is sometimes the case, but there is a good deal of irregularity (as will be seen in the figures), and this is conspicuous in figure 7 where i and $i\frac{1}{2}$ occur together and also i^α, $i\beta$, and i.

Habit.—With regard to the general habit of the crystals, it is interesting to note the wide variation which is shown. Figures 1, 2, 6, 9, 10, are intended to give some idea of the crystals as drawn symmetrically, and figures 7, 10, 14, 15, 16, 17, 18, 19, , of their actual appearance. As will be seen, the figures are drawn with $C(\dot{\imath}\text{-}\bar{\imath}, 010)$ in front: this was necessary in order to give a true idea of their real appearance. The prism $\dot{\imath}\text{-}2$ is so acute ($49\frac{1}{2}°$) that when directed

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toward the eye the projection gives it but little width. As a matter of fact the crystals have *C* uniformly well developed and are generally attached approximately by the extremity of the brachydiagonal axis. This having been explained, it will be clear that while fig. 3 is an almost exact reproduction of an actual crystal, fig. 5, by the other method of projection, gives an entirely false idea of its appearance. It is certainly true that the latter method shows the hemihedrism in its true light, but this should not weigh against the other more important consideration.

The crystals from which the partial figures, 7, 10, 16, 17, were drawn were united, along with others quite as diverse, in one small fragment only half an inch in length. It is to be noted that figures 14 and 15 are really more different than would appear at first glance. The crystals drawn in figures 16 and 19 also occurred closely conjoined in the same group; and other examples might be mentioned. One crystal of a very prismatic appearance (when placed in an inverted position) is shown in figure 19.

Presence of minute planes.—The most remarkable feature of the mineral from this locality is yet to be mentioned. I refer to the multitude of minute planes which modify many of the solid angles. One single case will be discussed in detail, as the planes admitted of more than usually exact determination; it serves well to illustrate the subject. A horizontal projection of a portion of the crystal is shown in fig. 14. The crystal itself was small, and unfortunately so imbedded in dolomite that it was for the most part rough and beyond even approximate measurements. The part available showed *C* (i-$\bar{i}$) faultless; also r^3 good; and less satisfactory r^1, r^2 and r^4. On the solid angle between *C*, r^3 and r^2 a large number of minute planes were observed; they were so extremely small (all covering a surface not ·03 of an inch in breadth) that any exact measurements seemed at first hopeless. They were sharply defined, however, and brilliant, and when the attempt was made it was found that they gave perfectly distinct though faint reflections. It may be remarked here that measurements in this case were only made possible by the substitution of a cross, cut in tin foil and illuminated very brilliantly by a gas burner, for the ordinary spider lines in the second telescope—a device for which I am indebted to Prof. Schrauf of Vienna.*

The measurements were all made with the greatest care, though, as will be readily understood, the exact adjustment of planes so small

* Ber. Ak. Wien, lxvii, 1873.

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was not an easy task. The symbols were calculated from the angles thus obtained. But as even then some doubt remained as to the degree of dependence which was to be placed in them, the measurements were all repeated with the same care as before. The result was perfectly satisfactory, as the variation in no case exceeded the probable error of observation given to each angle when measured for the first time. This variation in most cases did not exceed $\pm 2'$. The following supplement angles were obtained for the more prominent planes: $C \wedge r^3$ gave 33° 33′ (required 33° 32¼′); and $C \wedge r^4$ gave 25° 53′ (required 25° 58′); and $r^3 \wedge r^4$ gave 31° 31′ (required 31° 38′). The following table contains the angles for each of these minute planes as measured on C, and r^3 and also on i^2, itself one of this group.

TABLE VI.

			$C = i\text{-}\bar{i}(010)$. Meas.		Calc.		$r^3 = \bar{2}43$. Meas.		Calc.		$i^2 = 201$. Meas.		Calc.	
i^2	$2\text{-}\bar{i}$	021	17°	37′	17°	38½	23°	58′	23°	49′				
$+x^1$	$\frac{12}{7}\text{-}6$	2·12·7	22	9	21	53	31	4	31	12	8°	21′	8°	37′
$+x^2$	$\frac{26}{9}\text{-}13$	2·26·9	12	55	12	58	29	32	29	27	6	36	6	35
$+x^3$	$\frac{26}{7}\text{-}13$	2·26·7	10	34	10	30	30	37	30	42	8	50	8	54
$+x^4$	$\frac{34}{7}\text{-}\frac{17}{2}$	4·34·7	9	42	9	39	33	41	33	41	12	0	11	53
$-x^6$	$\frac{12}{7}\text{-}24$	$\bar{1}$·24·14	20	21	20	28	21	11	21	6	3	25	3	24
$-x^5$	$\frac{13}{12}\text{-}\frac{13}{5}$	$\bar{5}$·13·12	34	32	34	27	7	32	7	15	21	15	21	8
$-x^7$	$\frac{13}{7}\text{-}13$	$\bar{1}$·13·7	18	50	19	17	19	40	19	47	4	5	4	3
$-x^8$	$\frac{24}{13}\text{-}4$	$\bar{6}$·24·13	22	24	22	31	12	15	11	59	12	22	12	22
$-x^9$	$\frac{5}{2}\text{-}\frac{15}{2}$	$\bar{2}$·15·6	16	5	15	47	18	51	19	7	7	39	7	34
$-x^{10}$	$\frac{13}{7}\text{-}\frac{13}{4}$	$\bar{4}$·13·7	24	22	23	59	9	26	9	47	15	25	15	3
$-y^2$	$\frac{25}{9}\text{-}\frac{25}{7}$	$\bar{7}$·25·9	18	54	19	2	14	45	14	38	14	40	14	52
$-y^1$	$3\text{-}\frac{8}{3}$	$\bar{9}$·24·8	21	55	22	4	13	18	13	14	19	32	19	29
$-y^4$	$9\text{-}\frac{9}{2}$	$\bar{2}$91	11	43	12	13	23	31	23	14				
$-y^3$	$24\text{-}\frac{24}{9}$	$\bar{9}$·24·1	18	45	19	5	25	34	25	27				

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The calculated symbols are also given with the angles which belong to them. It will be noticed that $i^2(2\bar{i}=021)$ is itself one of the minute planes of the same character as those surrounding it, and its presence gives a reality to them which they would not otherwise have, and shows what degree of reliance is to be placed on the angles. Cases of a similar character will be noted hereafter.

The symbols* calculated for this series of planes are certainly not simple; and yet a moment's consideration will show that this was exactly what was to be expected. Crowded together so closely, they would be abnormal if occurring on crystals of any species, while this becomes still more true for a mineral like chondrodite. The constantly recurring, common planes have ratios which in any other species would be considered next to impossible: thus, in type II, $1:\frac{1}{3}:\frac{1}{5}:\frac{1}{7}$; and in type III, $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}$. It is to be noticed that these are the true ratios of the r series of pyramids, which exist no matter what change be made in the assumed axes. It is not surprising, then, that these secondary planes should themselves have symbols totally at variance with the accepted law of simplicity of the indices. Many cases of planes with what may be called abnormal indices have been described, but frequently they are to be explained as has been done by Brezina in the case of the $\frac{25}{24}\cdot\bar{i}$ (25·0·24), which he proved to exist in wiserine, as a tendency to a plane with a simple index ($1\text{-}\bar{i}=101$), which has resulted in a plane with approximately the given index. The case in hand, however, is quite different.

It will be noticed, however, that, lawless as they appear at first, there is an attempt at system in the symbols given. Thus in the ratio of the brachydiagonal to the vertical axis, we have:

x^1	12 : 7	x^7	13 : 7
x^6	12 : 7	x^{10}	13 : 7
y^2	24 : 13	x^3	26 : 7
y^1	24 : 8	x^2	26 : 9
y^3	24 : 1	x^5	13 : 12

and so on. The ratios for the other axes might be drawn out in the same way, but they are already contained in the symbols given in the table. A little surprise is felt at first that the uniformity in ratio is not greater, that, for instance, x^1 is not 2·13·7, instead of 2·12·7; but the measurements are too good and reliable to allow of such a

* In the symbols given in the tables (i. e., those according to Naumann) the dash over the second figure, or fraction, has been omitted (in order to simplify the work of the printer). This has also been done in all the following tables, being made possible by the fact that all these planes, with one or two exceptions, belong to the *macrodiagonal* series.

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supposition. It is to be noticed that when ratios of this character are allowed, a slight change in the measured angle will alter entirely the calculated index; the liberty in this respect is not, however, quite so great as it would seem at first sight to be. For example, the ratios $\frac{5}{9}(\frac{35}{63})$ and $\frac{4}{7}(\frac{36}{63})$ approach pretty closely to each other, and it might be a question which was to be accepted as the true ratio of the two axes for a certain plane; and yet if the ratio of one of these axes with the third be unquestionably expressed in sevenths, e. g., $\frac{8}{7}$, then there seems little doubt but that the ratio $\frac{4}{7}$ is to be accepted, for that would give 8·4·7 or $\frac{8}{7}$-2, while the other supposition would give 35·72·63 or $\frac{8}{9}$·$\frac{72}{35}$. This principle has been accepted in obtaining all the indices given in the following tables.

A remarkable fact connected with these planes,—in fact, implied in what has already been said,—is that there is so little tendency among them to lie in zones. For example, x^1, x^9, y^2 and y^1 lie very nearly in a zone with each other and i^2, and yet the reflections in the goniometer deny that this is exactly true, while no satisfactory indices can be obtained on this supposition. (x^1, i^2 and y^2 are, however, in a zone.)

In regard to these planes two points are to be noticed. In the first place, the question suggests itself whether, if referred to a common fundamental form (see above), or to that of either of the other types, the relations of the planes would be at all more simple. This is answered in the negative, as will be seen to be necessary if the trial is made, and also evidently because planes whose normals make angles of a few degrees only with one another can never bear simple relations to each other, no matter what axes be assumed.

In the second place, it might be urged that such ratios as those above given being accepted, there is no reason why we should attempt to express the relations of the prominent planes—those of humite, type II, for example, with simple numbers (see above, page 7). But, as has just been stated, the attempt to refer these planes themselves to other axes leads to disastrous results, while further, as has been shown, these planes are truly secondary and subordinate and bear no relations to other types of the species.

This case has been dwelt upon at considerable length, because it was believed that theoretically the existence of such planes was a matter of some interest and importance, and because this single crystal offered opportunities for exact determination which did not exist to the same degree in any other case. Almost all of the twenty and more smaller crystals examined showed some of these secondary planes. In some cases, however, there was a tendency to rounded edges

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without the formation of distinct planes, as is so common in many species; and then nothing of course could be done.

The following tables, VII, VIII, IX, X, include the measured angles, with the symbols obtained, and the corresponding angles belonging to them, for a considerable number of these minute planes. Those occurring on each individual crystal are arranged together, being expressed by the same letter, and where the crystal has been figured, this is also indicated. Upwards of one-hundred of these planes were measured, and an attempt was made in every case to obtain a satisfactory index. It was concluded, however, to discard the

TABLE VII.

				$C = i\text{-}\bar{\imath}$ (010). Meas.		Calc.		$+e^3 = 2\text{-}\bar{\imath}$ ($20\bar{1}$). Meas.		Calc.	
Crystal VII (figure 16).	$-a$	8-4	$\bar{2}81$	14°	6′	13°	41′	79°	2′	79°	16′
	$-v_1$	$\frac{11}{4}$-$\frac{11}{5}$	$\bar{5}\cdot11\cdot4$	25	14	25	34	73	29	73	7
	$-v_2$	$\frac{8}{3}$-$\frac{16}{11}$	$\overline{11}\cdot16\cdot6$	33	50	34	5	64	40	64	28
	$-v_3$	$\frac{13}{4}$-$\frac{5}{4}$	$\overline{52}\cdot65\cdot20$	37	7	37	19	60	0	59	49
	$-s_1$	$\frac{3}{2}$-$\frac{15}{16}$	$\overline{16}\cdot15\cdot10$	46	43	46	56	57	38	57	22
	$-t_1$	$\frac{5}{7}$-$\frac{5}{3}$	$\bar{3}57$	46	4	46	21	81	35	80	49
	$-t_2$	$\frac{9}{14}$-$\frac{9}{4}$	$\bar{4}\cdot9\cdot14$	46	57	46	58	87	7	87	31
	$-s_2$	$\frac{7}{9}$-$\frac{7}{8}$	$\bar{8}79$	53	6	53	7	64	9	64	7
	$-t_3$	$\frac{1}{2}$-$\frac{3}{2}$	$\bar{2}36$	54	31	54	42	85	0	84	33
	$+t_4$	$\frac{7}{11}$-$\frac{7}{2}$	$2\cdot7\cdot11$	45	47	45	57	92	46	93	6
	$+t_5$	$\frac{9}{10}$-$\frac{9}{2}$	$2\cdot9\cdot10$	36	16	36	21	91	20	91	44
Crystal X (figure 15).	$-n^2$	2	$\bar{2}21$	44	13	44	14½	56	45	56	42
	$-\zeta$	$\frac{9}{5}$-$\frac{9}{7}$	$\bar{7}95$	38	45	38	37	63	44	63	58
	$-\vartheta$	$\frac{15}{4}$-$\frac{3}{4}$	$\overline{20}\cdot15\cdot4$	50	58	51	6	46	13	46	4
	ξ^1	1-ī	011	32	34	32	27	79	55	79	54
	$+p^1$	$\frac{5}{7}$-5	157	42	0	42	17	109	36	110	7
	$-p^2$	$\frac{20}{17}$-5	$\bar{4}\cdot20\cdot17$	29	41	29	44½	90	0	89	58

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larger portion as untrustworthy, retaining those which had given the best angles. It is not pretended that the symbols deduced are, even in the majority of cases, correct beyond question; for the angles, while perfectly reliable in some cases, are in others somewhat uncertain, and for reasons already explained this throws still greater doubt over the indices which calculation may produce.

TABLE VIII.

				$C = i\text{-}\bar{\imath}$ (010). Meas.		Calc.		$A = O$ (001). Meas.		Calc.	
Crystal III (figure 17).	$-z^1$	$\frac{7}{6}\text{-}\frac{7}{5}$	$\bar{6}75$	40°	0′	39°	52′	65°	25′	65°	51′
	$-z^2$	$\frac{11}{6}\text{-}\frac{11}{5}$	$\bar{5}\cdot11\cdot6$	28	44	28	32	72	13	72	15
	$-z^3$	$\frac{15}{4}\text{-}\frac{15}{14}$	$\overline{14}\cdot15\cdot4$	41	0	41	14	82	45	82	40
	$-z^4$	$\frac{17}{3}\text{-}\frac{17}{4}$	$\bar{4}\cdot17\cdot3$	14	6	13	43	83	55	83	44
	$-z^5$	$\frac{10}{11}\text{-}5$	$\bar{2}\cdot10\cdot11$	35	42	35	54	55	44	55	28
	$-m$	$6\text{-}\frac{3}{2}$	$\bar{6}41$	54	18	54	17	84	37	84	40
Crystal IV (figure 10).	$-\beta$	$\frac{5}{9}\text{-}\frac{5}{2}$	$\bar{2}59$	50	32	50	16	42	45	42	56
	$-\xi_1$	$\frac{7}{9}\text{-}\frac{7}{5}$	$\bar{5}79$	46	45	46	10	55	48	55	29
	$-\xi^2$	$\frac{8}{9}\text{-}\frac{4}{3}$	$\bar{6}89$	45	18	44	51	59		59	30

The group of planes clustered about i^2 has already been described. It is interesting to note that in two instances analogous groups were observed, of which $i(1\text{-}\bar{\imath} = 011)$ was a member (see fig. 15), and in two other cases the common and prominent planes n^2 (fig. 15) and m^2 (fig. 17) were found in the same relations. The angles obtained for these planes show conclusively the degree of dependence to be placed on those measured for the other planes. It will be remembered that in all cases these planes were exceedingly minute.

The fact already mentioned, that all of these planes belong to the macrodiagonal series, may possibly be explained in part by the fact that it is uniformly that portion of the crystal (i. e., near C, $i\ \bar{\imath}$) which is exposed and well developed. There still remains the fact, which will be noticed by a glance at the spherical projection (plate xiii) and which does not allow of an analogous explanation, that

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TABLE IX.

				$e^2 = 2\bar{1}$ (201). Meas.		Calc.		$A = O$ (001). Meas.		Calc.	
Crystal II (figure 7).	$i^{\frac{1}{2}}$	$\frac{1}{2}$-$\bar{1}$	012	51°	50′	51°	50′				
	i^{α}	$\frac{4}{7}$-$\bar{1}$	047	47½	48	48	4				
	i^{β}	$\frac{2}{5}$-$\bar{1}$	025	58	18	57	51				
	$-r^{\alpha}$	$\frac{8}{9}$-$\bar{2}$	$\bar{4}89$	40	6	40	24	56°	42′	56°	59′
	$-r$	$\frac{1}{2}$-$\frac{5}{4}$	$\bar{4}\cdot5\cdot10$	55	45	55	47	45	7	44	18
	$-q$	$\frac{11}{10}$-$\frac{11}{7}$	$\bar{7}\cdot11\cdot10$	39	33	39	32	63	23	63	26
	$+\pi$	$\frac{1}{2}$-$\frac{5}{3}$	$3\cdot5\cdot10$	54	2	54	13	41	58	41	55
Crystal I.	$-n^{y}$	$\frac{5}{2}$	$\bar{5}52$	43	40	43	42	79	33	79	22
	$-n^{x}$	30	$\bar{3}\bar{0}\cdot30\cdot1$	42	40	42	39	89	7	89	6

these minute planes almost always lie in the negative (lower) quadrants; this has necessitated the drawing of some of the crystals in an inverted position. The figures show, in addition to planes mentioned in the tables, some others for which no symbols were obtained.

TABLE X.

				$C = i\text{-}\bar{1}$ (010).				$-e^1 = \frac{2}{3}\text{-}\bar{1}$ ($\bar{2}03$).			
Crystal VIII.	i	1-$\bar{1}$	011	32°	20′	32°	27′	67°	15′	67°	16′
	$-w_1$	1-$\frac{15}{4}$	$\bar{4}\cdot15\cdot15$	34	41	34	18	58	21	58	43
	$-w_2$	$\frac{17}{10}$-$\frac{17}{6}$	$\bar{6}\cdot17\cdot10$	26	33	26	25	63	29	63	37
	$-w_3$	$\frac{3}{4}$-3	$\bar{1}34$	41	34	42	3	52	45	52	18
	$-r$	$\frac{1}{2}$-$\frac{5}{4}$	$\bar{4}\cdot5\cdot10$	55	27	55	47	36	40	36	37
	$-\delta$	$\frac{8}{9}$-4	$\bar{2}89$	37	10	36	57	56	27	57	21
	$-u_1$	$\frac{15}{13}$-5	$\bar{3}\cdot15\cdot13$	29	22	30	10	63	45	63	2
	$-u_2$	$\frac{12}{11}$-6	$\bar{2}\cdot12\cdot11$	31	8	31	6	62	55	63	13

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Crystals of simpler habit.—The crystals thus far described have all been of a more or less complicated character. But allusion has been made to some very simple crystals, which also deserve description. The distinction is in most cases probably unimportant, though believed to be of interest at first. The simple crystals are uniformly large; they are so generally altered, and appear so differently from their small brilliant relatives, that it was supposed that they differed from them at least in the purity of the original material, if not more essentially. One brilliant exception, however, to the general rule in regard to the altered condition of these crystals, in the form of an isolated crystal of faultless luster, and deep red color, about $\frac{3}{4}$ of an inch in length, as well as numerous examples of transition products between the altered and unaltered material, made it probable that all the crystals in question were originally of the same character. Some examples are given in figures 3, 4 and 5. The angles could be measured approximately only with a hand goniometer, but there is no question that they, as well as others, belong to type II. On one of them $\pm r$ was observed.

In what has been said exception must be made in regard to the large coarse crystals, and crystalline masses, mentioned in the early part of this article, and which are made up of a more or less heterogeneous mass of chondrodite, magnetite and sometimes ripidolite; some, at least, of these last, belong to type I. (See p. 25.)

Twins.—The humite crystals of Vesuvius, as well as the Swedish chondrodite, has been shown by vom Rath to possess so great a tendency to twinning that it is a little remarkable that the contrary should be true of the mineral from Brewster. Figs. 20, 21, show the only method of twinning which has been found, as well as the only distinct twin-crystal. The axis of revolution here is the vertical axis of the crystal, and the composition-face the basal plane A. Unfortunately the crystal in question was quite imperfect, and all that was available is shown in the figure. The plane B ($i\text{-}\check{i}=\bar{1}00$) gave no reflections, so that all measurements were made on e^2 (that is 201 and $20\bar{1}$); in this case these planes were similar in luster as a result of the twinning. A revolution of the kind mentioned (in a perfectly symmetrical crystal) would, so far as this half of the crystal goes, have the effect only of making it holohedral, giving no re-entrant angles; but, in case of any irregularity, it might give, as here, a re-entrant angle in the planes which are hemihedral in their occurrence.

The measurement of the re-entrant angle here observed gave for $m^2 \wedge \overline{m}^2$, 10° 38′ and 10° 40′; required 10° 39′. The other angles measured on the same crystal are given in the following table.

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TABLE XI.

		$e^2 = 2\text{-}\breve{\imath}$ (201). Meas.		Calc.		$e^2 = 2\text{-}\breve{\imath}$ ($20\bar{1}$). Meas.		Calc.	
$\bar{m}^2$	641	37°	39′	37°	34½′	42°	54′	42°	57 ′
m^2	$64\bar{1}$	42	57	42	57	37	28	37	34½
$\bar{m}^2$	$6\bar{4}1$	37	36	37	34½	42	55	42	57
m^2	$6\bar{4}\bar{1}$	42	57	42	57	37	33	37	34½
n^2	221	45	45	45	45	56	42	56	42
n^2	$22\bar{1}$	56	44	56	42	45	45	45	45
n^2	$2\bar{2}1$	45	34	45	45	56	41	56	42
n^2	$2\bar{2}\bar{1}$	56	15	56	42	45	34	45	45
n^1	223	44	45	44	38	70	16	70	20
	$2\bar{2}3$	44	48	44	38				
r^2	$2\bar{4}5$	59	13	59	14	82	34	82	34½
r^1	241	64	3	64	2				
o	$2\bar{1}0$	33	52	33	50½	33	48	33	50½
ϕ	60·38·1	38	14	38	17	39	24	39	27½
		641				$6\bar{4}1$			
"	"	4	31	4	25	6	31	6	36¾

Besides the interest of the twin, a noteworthy fact in the crystal is the occurrence of the prism ($i\text{-}\breve{2}=210$) the first time that any one of the vertical prisms has been observed in the 2d type of either humite or chondrodite. It lies in both the zones $e^2(201)$, $n^2(2\bar{2}\bar{1})$, and $e^2(22\bar{1})$, $n^2(2\bar{2}1)$ which answers sufficiently to determine what it is; and the result thus obtained is fully sustained by good measurements, as will be seen in the table. This plane is distinctly present on one side only of B; on the other side its presence is barely indicated. Its place here is taken by a well polished and conspicuous plane Φ, which is another striking instance of the peculiar nature of this species; its position is indicated in the sphere of projection,* and the angles on e^2 are given in the table. The index was calculated for each pair of measurements 38° 14′, 39° 24 and 4° 31, 6° 31 (see in the table), and the results obtained were identical. From the first pair of measured angles $\Phi \wedge B$ was found to be 34° 30½′ and from the second $\Phi \wedge B=$ 34° 31′ (required 34° 32′ 38″). The index obtained was 60-$\frac{38}{19}$ or 60·38·1, and abnormal as it certainly is, it expresses the exact posi-

* In the projection Φ is placed incorrectly in the negative quadrants; it should be in the positive with $+n$, $+n^2$, $+r$, etc.

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tion of the plane. It will be noticed that the four planes upon which the inclination of Φ was measured are so situated, that any variation from the true position in the index would show itself very conspicuously. The fundamental angles for Φ are as follows:

$$\Phi \wedge A(001) = 88^\circ\ 51'\ 41''$$
$$\Phi \wedge B(100) = 34\ \ 32\ \ 38$$
$$\Phi \wedge C(010) = 55\ \ 28\ \ 49$$

In figure 16 a crystal, or portion of one, is exhibited which is holohedral. It is irregular in this respect, however, that r^2 forms a reentrant angle with r^3. This is not a point of special importance, as an irregularity such as this is often observed; but, in view of the crystal which has just been described, it is possible that here also there has been a semi-revolution parallel to the basal plane. A more interesting crystal, already once alluded to, is shown in fig. 4. It is conspicuously hemimorphic, as far as the form goes. It is large, and admits only of approximate measurements, but there is no doubt but that the planes as given have been determined correctly. In view of the fact that a revolution parallel to *O* would produce just the effect we have here, and as such a twinning law has been shown to exist in another conspicuous case, it is altogether probable that this forms an ample explanation of what is observed. Another exactly parallel case is noted under the description of two crystals of the 3d type. The above described crystal was somewhat altered, and so far imbedded in the matrix that any experiments as to its pyro-electrical character were out of the question.

Chemical composition.—I am glad to be able to add here the results of a chemical examination of the chondrodite of the 2d type from this locality, by Mr. G. W. Hawes of the Sheffield Scientific School. It obviously increases much the value of this memoir. An analysis by Mr. Breidenbaugh has already been quoted (p. 3).

The material analyzed by Mr. Hawes consisted of fragments of crystals of the 2d type, selected with great care to avoid the presence of any altered material. It had a deep garnet-red color and a brilliant vitreous luster. Its specific gravity as determined by Mr. Hawes was 3·22.

	Analysis I.	Analysis II.
Silica	34·10	34·05
Magnesia	53·17	53·72
Ferrous oxide	7·17	7·28
Alumina	·48	·41
Fluorine	4·14	3·88
	99·06	99·34

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Following the view of Rammelsberg, that the higher values of each constituent are nearest to the truth, Mr. Hawes's analysis becomes as follows. For comparison the results obtained by vom Rath for 2d type crystals from Vesuvius and from Sweden are added.

	Chondrodite.		Humite.
	Brewster, N. Y., *Hawes.*	Sweden, *v. Rath.*	Vesuvius, *v. Rath.*
Silica	34·10	33·96	34·02
Magnesia	53·72	53·51	59·23
Ferrous oxide	7·28	6·83	1·78
Alumina	0·48	0·72	0·99
Fluorine	4·14	4·24	2·74
	99·72	99·26	98·76
Silicon	15·91	15·85	15·88
Magnesium	32·23	32·11	35·54
Iron	5·66	5·31	1·38
Aluminum	0·26	0·38	0·53
Fluorine	4·14	4·24	2·74
Oxygen	39·78	39·58	41·54
	97·98	97·47	97·61

Transforming the iron into an equivalent of magnesium, as also the alumina (2Al=3Mg), Mr. Hawes obtains further:

Silicon 15·91, Magnesium 35·00, Fluorine 4·14, Oxygen 39·78.

From these values a formula is deduced, which is essentially that of the Swedish mineral according to v. Rath, $20(Mg_5Si_2O_9)+Mg_5Si_2Fl_{18}$. The close correspondence between the three analyses in the above table is certainly very remarkable. It would have been extremely interesting to have added an analysis also of crystals of the 1st and 3d types; but, as will be apparent from what follows, the material was not to be obtained.

In completing the description of this variety of the mineral, in general it may be repeated that it occurs usually in narrow veins, and when free from alteration has uniformly a deep *garnet-red* color. A *cleavage* such as exists in humite (parallel to the basal plane) and has been observed by Kokscharow on chondrodite from Pargas, could in no case be discovered. The fracture is always conchoidal, and the only thing that suggested cleavage was the laminated structure of the massive specimens described, and the fractures parallel to e^2 which were quite conspicuous on two or three specimens. The material in hand was not well adapted for optical determinations;

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but some interesting results have been obtained which are described in the closing pages of this paper.

2. *Description of Crystals of Type III.*

Since it was shown by Kokscharow that the Pargas chondrodite was identical with the second type of humite, it was natural to expect that further investigation would prove the existence also of the two remaining types. Up to the present time that expectation has been unfulfilled, and it has been reserved for the Brewster locality to give this confirmation of Scacchi's interesting discovery. The crystals of the 3d type are exceedingly rare, three or four specimens being all that have thus far been found, and from these only two individual crystals could be obtained which allowed of measurement. Fortunately these two crystals are very satisfactory, being small and brilliant, and establish the fact as well as a hundred could do. Figures 11 and 12 show one of the crystals, and figure 13 the other. The appearance of the first crystal is best shown in the second of these figures. As will be seen, the planes are the same as in humite, and they are for the most part hemihedral and situated in the same way; i.e., they include $+\rho^4, +\rho^6, +\rho^8$, and $-\rho^3, -\rho^5, -\rho^7$, and $-\nu^2, -\nu^3, -\nu^3, -\nu^4$ as also $\iota^2, \iota^3, \iota^4$. In the $n(\nu)$ series the planes are distinct in the negative half, but the crystal is incomplete, and it is a little uncertain whether the + series should not in part be added in the symmetrical drawing fig. 11; on humite, III, these planes are both positive and negative. No brachydomes are visible, the edge being rounded and rough. m^2 of Scacchi may also be present, but that is a little doubtful.

The second crystal is of very different form, and while the first was affixed to the rock so that only one-half was developed, this one was imbedded in brucite, and entirely free in it. It was perfectly formed on all sides, being almost as perfect as the projected drawing, with the exception, however, of the acute (brachydiagonal) edge, which was mostly broken. When only the upper part of the crystal is considered, it will be seen that the hemihedrism is like that in the other case, except that ρ^8 is holohedral. For macrodomes there are $\iota^1(\frac{2}{3}\text{-}\bar{\iota} = 023)$, $\iota^2(1\text{-}\bar{\iota}=011)$, $\iota^3(2\text{-}\bar{\iota}=021)$, $\iota^4(4\text{-}\bar{\iota}=041)$; the last has not been observed on humite. On measuring the planes below it was found that they were not distributed as was expected in accordance with the monoclinic character of the crystal; instead, either extremity of the brachydiagonal axis was differently developed. This is clear in the figure, it being but a more complicated repetition of what was observed in one

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of the very simple crystals of the 2d type (see fig. 4 and page 18). There are present also at one extremity $\pm\varepsilon'(\frac{4}{7}\text{-}\bar{1}=407)$, though the plane could only be approximately measured. This is probably also to be explained as having resulted from a revolution parallel to the basal plane. The crystal was very small and not at all adapted to experiments having in view the discovery of any proper hemimorphic development. The angles measured on both these crystals are contained in the following tables.

TABLE XII.

			Chondrodite.								Humite.		
			$A = O$ (001). Calc.			Meas (XX).		Meas (XXI)			v. Rath. Calculated.		
ε	$\frac{4}{7}\text{-}\bar{1}$	407	143°	20′	9″						143°	11′	29″
ι^4	$4\text{-}\bar{1}$	041	100	1	7								
ι^3	$2\text{-}\bar{1}$	021	109	27	35						109	27	54
ι^2	$1\text{-}\bar{1}$	011	125	14	49						125	15	18
ι^1	$\frac{2}{3}\text{-}\bar{1}$	023	136	40	4						136	40	34
ρ^3	$\frac{8}{11}\text{-}\bar{2}$	$\bar{2}811$	131	25	57	131°	46′	131°	24′		131	24	49
ρ^4	$\frac{8}{9}\text{-}\bar{2}$	289	125	50	6	125 125	37 47	125	48		125	49	0
ρ^5	$\frac{8}{7}\text{-}\bar{2}$	$\bar{2}87$	119	19	18	119 119	35 15	118	36		119	18	19
ρ^6	$\frac{8}{5}\text{-}2$	285	111	51	38	111 111	44 49	112	0		111	50	50
ρ^7	$\frac{8}{3}\text{-}\bar{2}$	$\bar{2}83$	103	32	4	103	41	103	38		103	31	33
ρ^8	$8\text{-}\bar{2}$	281	94	35	15	94 94	31 13	94	48		94	35	4
ν^1	$\frac{4}{7}$	447	132	17	48			132	16		132	16	43
ν^2	$\frac{4}{5}$	445	123	1	8			122	32		123	0	8
ν^3	$\frac{4}{3}$	443	111	18	7			111	5		111	17	23
ν^4	4	441	97	24	20			97	29		97	24	3

Unfortunately the inclination to C on no one of the pyramidal planes could be measured with perfect accuracy; the measurements are good, yet not entirely trustworthy. These planes, though brilliant, are uniformly fractured in the manner already explained, and this made all

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TABLE XIII.

Chondrodite.

			$C = i\text{-}\bar{i}$ (010). Calc.			Meas (XX).		Meas (XXI)	
ι^4	$4\text{-}\bar{\imath}$	041	169°	58′	53″	170°	2′		
ι^3	$2\text{-}\bar{\imath}$	021	160	32	25	160	35		
ι^2	$1\text{-}\bar{\imath}$	011	144	45	11	144	47		
ι^1	$\frac{2}{3}\text{-}\bar{\imath}$	023	133	19	56	133	(ap.)		
ρ^3	$\frac{8}{11}\text{-}\bar{2}$	$\bar{2}811$	132	56	12	133 132	13 38	132°	46′
ρ^4	$\frac{8}{9}\text{-}\bar{2}$	289	137	26	13	137 137	36 27	137	21
ρ^5	$\frac{8}{7}\text{-}\bar{2}$	$\bar{2}87$	142	22	50	142 142	35 20	142	38
ρ^6	$\frac{8}{5}\text{-}\bar{2}$	285	147	28	24			147	26
ρ^7	$\frac{8}{3}\text{-}\bar{2}$	$\bar{2}83$	152	1	44			152	2
ρ^8	$8\text{-}\bar{2}$	281	154	53	20	155	0	154	58
ν^1	$\frac{4}{7}$	447	123	31	12				
ν^2	$\frac{4}{5}$	445	128	5	52				
ν^3	$\frac{4}{3}$	443	133	16	51			133	57
ν^4	4	441	136	51	9			136	42

these angles a little uncertain. The macrodomes in one case gave good measurements; and making use of the best of them, $C \wedge \iota^2(1\text{-}\bar{\imath})$ $=144° 47'$, and also the same prism as in type II, after the analogy of humite, the angles were calculated throughout. It was found, however, that wherever trustworthy they corresponded so closely to what was required to satisfy the ratio of 10 : 9 for the vertical axes (asserted as approximately true by Scacchi, i. e., 5 : 9 for his axes, and finally proved rigidly by vom Rath), that the calculations were made on this supposition. The calculated angles, as now given, consequently have as their basis the prismatic angle $I \wedge I = 94° 44' 14''$ and the macrodome angle $C \wedge \iota^2 = 144° 45' 11''$.

The corresponding parameters are:

$$a \text{ (vert.)} = 1{\cdot}41512;\ b = 1;\ c = 1{\cdot}08630.$$

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Very little further can be said in regard to the crystals of the 3d type. Those observed had a somewhat different color from those of type II; that is, the color was more yellowish, less of a pure garnet-red—though this may be accidental. No analysis was possible of course; and even the specific gravity was out of the question also, for the one loose crystal, in addition to its small size, had imbedded in it a still smaller crystal of ripidolite, making any gravity determinations obviously unreliable.

The method of occurrence was much like that of the brilliant crystals of the second type; that is, they were found implanted on the massive rock adjoining small veins. The associated minerals were magnetite, ripidolite in clear transparent crystals, and, probably as a later formation, brucite.

3. *Description of Crystals of Type I.*

The occurrence of large, coarse crystals of quite impure chondrodite, imbedded in the massive material, has already been described; these belong, at least in part, to the *first* of Scacchi's types. As has been remarked, the crystals of this character do not often admit of exact determination, but in two cases they were so good as to allow of their crystallographic relations being accurately made out. The accompanying wood-cuts, figures 22 and 23, give faithful representations of their appearance and size.

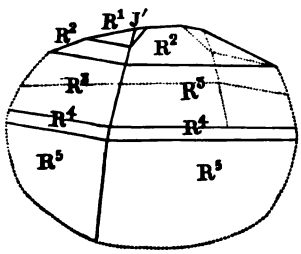

Fig. 22.

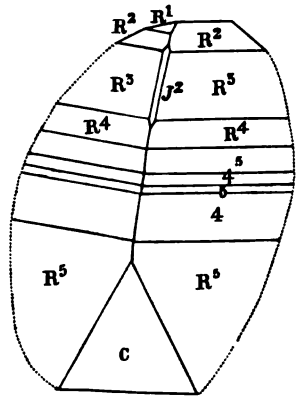

Fig. 23.

It will be seen that they are both quite imperfect, and it was on this account that no attempt was made to make a symmetrical drawing of either of them. In each case the crystals were so distorted as to give a sharp edge between the diagonally situated pyramidal planes; this furnished an opportunity for reliable measurements. The planes

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were, of course, destitute of all luster, but they were mostly smooth and large enough to allow of the convenient use of the hand goniometer.

R^2 on R^2 (behind) gave measurements varying, in a series of trials, from 78° to 79°, required 79° 4′.

R^2 on R^2 (behind) gave 62½°, required 63° 1′.
R^2 on R^2 (behind) gave 72 , required 71 17½.
R^2 on R^2 (behind) gave 72 , required 71 17½.

These angles on both crystals were identical within the allowed error of observation (say 30′). The above are the best angles afforded by any of the planes.

It is entirely impossible to refer these angles to any of the forms of the second type. When compared with the third type, it is seen that on making the supposition that R^2 and R^3 (front) are ρ^3 and ρ^4 respectively, and R^2 and R^3 (behind) are ρ^4 and ρ^5, we obtain for:

$\rho^3 \wedge \rho^4$ (behind)=77° 12′; $\rho^4 \wedge \rho^5$ (behind)=65° 6′;
$\rho^3 \wedge \rho^5$ (behind)=70° 32′; $\rho^4 \wedge \rho^4$ (behind)=71° 37′.

It will be seen, by comparing these with the previously given angles, that the measured angles correspond much better with the first type, and my confidence in them is so great that this would alone be regarded as sufficient to establish the point; and that, without reference to the fact that the supposed method of occurrence of the third type planes is contrary to all the laws of the species.

The decisive proof is derived from the fact that both crystals are certainly holohedral, the planes on both sides being similar with the exception of R^1, and there is nothing of the obliquity which is observed in the hemihedral forms.

The measured angles of C on R^5, right and left, were identical, though not obtainable with exactness; the measurements gave 152½° –154°: this is also true for C on R^3, right and left,=140½°–142½°.

In the first crystal $J^1=\frac{3}{5}\text{-}i$ (035) occurs, and in the other J^3 ($1\text{-}\bar{i}$=011). The occurrence of C is also to be noticed, as it is rare on humite; in fig. 23, the oscillatory combination of R^4 and R^5 will be also observed.

The following table includes the most important angles for the occurring planes, calculated from the fundamental form of the second type on the assumption that the lateral axes are equal, and the vertical axes have the ratio of 14 : 15. The measured angles are also added though only approximate; in the form given they were obtained immediately from the measurements over the top of the crystals (see above).

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TABLE XIV.

			Chondrodite.							Humite.		
			$C = \bar{i}\text{-}\bar{i}$ (010). Calculated.			$A = O$ (001). Meas.	Calc.			v. Rath. Calculated.		
J^2	$1\text{-}\bar{i}$	011	145°	43′	44″		124°	16′	16″	124°	16′	45″
J^1	$\frac{3}{5}\text{-}\bar{i}$	035								138	38	38
R^1	$\frac{3}{5}\text{-}\bar{2}$	3·6·10	129	12	57	135°	135	53	35	135	52	23
R^2	$\frac{3}{4}\text{-}\bar{2}$	368	134	28	38	129½	129	32	3	129	30	52
R^3	$1\text{-}\bar{2}$	122	140	30	10	121½	121	45	28	121	44	23
R^4	$\frac{3}{2}\text{-}\bar{2}$	364	147	6	34		112	25	28	112	24	37
R^5	$3\text{-}\bar{2}$	362	152	49	49	101	101	39	30	101	39	2

The two crystals described are the only ones which could be positively identified. It is very probable, however, that of those found others also belong here, as they have much the same appearance and habit. These crystals are all considerably altered, being generally soft enough to be cut with a knife, and for this reason a chemical analysis would be of little value. The color of the crystals is gray to grayish-yellow, and the material of which they are composed is never pure, and often quite heterogeneous. In this respect they recall the brown crystals described by v. Rath as occurring at Nya-Kopparberg.

Whether brilliant crystals of the first type exist, as they do of the other types, must be left for the present undecided.

4. *On the Optical Properties of Chondrodite.*

In the preceding pages the question of the orthorhombic or clinorhombic crystallization of the chondrodite has not been discussed. In fact, nothing was detected by the measurements sustaining any other conclusion than that of Scacchi and vom Rath, that the crystals were fundamentally orthometric. Still the hemihedral character of the second and third types seem to point to a clinometric form, and this is apparently supported by the optical characters obtained. The material available for optical investigations was very scanty, and, with the exception of one crystal, poorly adapted for the purpose.

The crystal referred to was, properly, but the fragment of what was originally a specimen of considerable size and beauty;

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when unbroken it must have been nearly an inch in length. In the condition in which it was found it showed only the brachydomes e^1 and e^2, with the pyramids n^1, n^2, and m^2; it had the deep garnet-red color of crystals of the second type, and with the exception of the universally present fractures was perfectly clear and transparent.

On the optical properties of the mineral in question, we have, as far as I have been able to find, no information except what has been given by DesCloizeaux, Manuel de Mineralogie, 1862, p. 141. He says: "Double refraction energetic; positive bisectrix normal to g^1; optic-axes situated in the plane parallel to the base; divergence in oil for red and yellow rays, 82° 14′. Dispersion almost nothing; $\rho < v$?"

Guided by the above, a section was cut from the crystal described, which was parallel to C, i. e., perpendicular to the brachydomes present. The examination of this section showed: 1st, that the acute bisectrix is normal to C (i-$\bar{i}$, 010); 2d, that this bisectrix is positive; 3d, that the optic-axial angle is large, the axes being seen only when oil is used; but 4th, that *the axes do not lie in the basal plane*, but in a plane making an angle of about 154° with it. This last point was so unexpected and anomalous that every effort was made to explain the measurements in some other way, but with no success. The planes on the crystal had been carefully measured, before the slicing, and the angles agreed perfectly with those of type II for the planes mentioned, so that it was impossible to assume that the crystal had not been correctly put into position. By means of a stauroscope, made by Fuess in Berlin, after the excellent pattern of Groth,* the position of the two axes of polarization, as referred to e^1, and also to e^2 in plane C, were carefully determined. The measurements were repeated twenty times, the error arising from an imperfect adjustment of the Nicols being eliminated in the usual manner. The result was as follows:

Supplement angle made by the plane of the axes—

with e^1 ($\frac{2}{3}$-$\bar{i}$=203), 18° 9′; hence with the basal plane, C, 25° 50′.
with e^2 (2-$\bar{i}$=201), 45° 9′; " " " " 25° 46′.

In order to confirm these results, other crystals were sought, which would admit of like determinations. None could be found which would serve for measuring the axial angle; but two small ones, on which the plane C was naturally developed, proved to be clear enough to allow of measurements with the stauroscope. The first

* See Pogg. Ann., cxliv, 34, 1870.

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alone gave accurate results; on it the angle of the same plane with $e^a(\frac{2}{3}\breve{i}=205)$ was determined with equal care. The results were:

4° 55′ for the angle with e^a; and hence 25° 59′ with *C*.

The agreement with the angles given above is as close as could be desired. In the other case, the rather rare plane *B* ($\dot{i}\text{-}\breve{i}=100$) was present; the crystal was minute, however, and the determination only approximate. It was found that the normal to the axial plane made with *B* an angle of 65°–70°, and hence with the normal to the basal plane 20°-25°.

With so ample confirmation the point made cannot be even questioned, and it remains to reconcile it with the crystallographic properties of the species. It will be seen at once that the position of the optic axes is totally at variance with the accepted orthorhombic character of the crystals; but it conforms to the rule for monoclinic crystals, as one axis of polarization is normal to the plane of symmetry *C*, and the others lie in it, or in other words, the optic-axes lie in a plane perpendicular to the axis of symmetry. The angles measured and calculated, given in the various tables, show that the variation from the rectangular type, if it really exist, must be very slight, as the agreement between the angles measured and those calculated on the assumed prismatic basis is very close—it being remarked that some considerable variations in the angles given in the tables are amply explained by the imperfection of the crystals. Note the angles measured for $m^2 \wedge \bar{m}^2$ on the twin crystal described on page 18. It was not to be expected that the variation in the optical character of the crystals would be so decided in view of the slight divergence which is possible in the crystalline form. I reserve for the future the careful revision of the angles of this species, when I shall hope to be able to command a more abundant supply of satisfactory material. It may be added that the hemihedral character of the second and third types of humite long ago suggested the idea that they were oblique in form; but all the crystallographic investigations thus far have seemed to deny this. In the Mineralogy of Brook and Miller, the form is made oblique, but this seems to be due to a misunderstanding of the planes occurring on the crystals.

It would have been interesting to extend these observations to the two remaining types, but the material did not allow of it. It was also desired to investigate the same subject for humite, but, though some good specimens are to be found in the Yale cabinet, there were no satisfactory crystals to be had, and the matter is left for others, who have a larger choice of specimens. The axes as already mentioned do not appear distinctly except in oil; in the first-mentioned

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section they admitted of good measurements. The mean of thirty determinations of the angle for red rays gave—

$$2H\alpha = 88^\circ\ 48' : \text{ the extremes being } 88^\circ\ 36' \text{ and } 89^\circ\ 0'.$$

With a yellow light (sodium) the angle was essentially the same, but the mean was 10′ or 15′ smaller, which would indicate that the dispersion is $\rho > v$, but the matter cannot be considered to be beyond doubt. No other dispersion was observed, that is, none parallel or perpendicular to the plane of polarization.

The index of refraction of the oil employed, as determined by Professor Wright and myself, was 1·466.

In conclusion, I have to express my very great obligations to Prof. Allen for his kindness in giving me free use of all the specimens in his valuable cabinet. Both of the crystals of the third type, as well as several others mentioned, came from his collection; in fact it was Prof. Allen who first made known the special interest connected with the locality. To Mr. Cosgriff, the superintendent of the Tilly-Foster Iron Mine, I am also much indebted for his uniform kindness and courtesy to me at the several occasions when I have visited the mine; as also for the gift of several fine specimens.

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IV.—On the Transcendental Curves $\sin y \sin my = a \sin x \sin nx + b$. With Plates XIV—XXXVII. By H. A. Newton and A. W. Phillips.

1. Algebraic curves have been studied hitherto more than transcendental. A few of the latter have been given in the text books, but attempts to classify the numerous varieties of transcendental curves have been rare.

From the form of a transcendental curve it is not easy to state an equation that can represent it. The simpler inverse problem of describing the curve from the equation is naturally the first to be undertaken. The forms that result may, when compared, suggest the solution of the direct problem. We have thought it worth while, therefore, to select for study a single one of the numberless transcendental equations, and to exhibit a few of the very many plane curves which that one equation furnishes. The equation selected is,

$$\sin y \sin my = a \sin x \sin nx + b, \qquad (1)$$

in which there are four arbitrary constants a, b, m, and n, with two coördinates, x and y.

2. We assume that m and n are each less than unity. If either, for example m, is greater than unity, we may change the unit for y in the ratio of $1 : m$ by writing $y' = my$. The first member of Eq. (1) then becomes $\sin y' \sin \frac{1}{m} y'$, where the coefficient of y' is $\frac{1}{m}$, which is less than unity.

In the equation thus changed, we have assumed in our figures the units for x and y equal, and the axes rectangular. The effect of a different supposition in either particular can be readily understood.

3. *Curves whose equations are* $y = a \sin x \sin mx$. It was found convenient to draw several auxiliary curves whose equations are of the form,

$$y = a \sin x \sin mx. \qquad (2)$$

A convenient arbitrary value being assumed for a, to m was given in turn all the values of the proper fractions, which, reduced to their lowest terms, have denominators less than 12. The forms of these curves are shown on plates XIV and XV, excepting a few in which m has 11 for denominator. In fig. 37 is shown the beginning of the curve when m has the irrational value $\sqrt{\frac{1}{3}}$. The axis of x is drawn

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in the figures. The origin is the point at the left of each figure where the curve touches the axis of x.

It will be convenient at times to put $m=\frac{p}{q}$, and $n=\frac{p'}{q'}$, where p, q, p', and q', are integers, and the fractions are reduced to their lowest terms.

4. *Properties of the curves of Eq.* (2). By inspection of the curves on plates XIV and XV, and of their equations, we readily deduce the following properties:

a. The value of y is not greater than a.
b. When either x or mx is a multiple of π, $y=0$.
c. There are maxima or minima values of y when $m \tan x = -\tan mx$.
d. When m is rational the values of y repeat; after $q\pi$ if p and q are both *odd;* after $2q\pi$ if either p or q is even.
e. When m is irrational the curve does not repeat its form.
f. The curve is symmetrical about the axis of y, and about an axis through the middle point of each cycle.
g. If p or q is *even*, the curve is symmetrical about the point $y=0$, $x=\frac{q\pi}{2}$.
h. There are, in each distance $2q\pi$ along x, $p+q$ maxima, and an equal number of minima values of y.

5. The value of y in Eq. (2) may be regarded as made up of two parts, since $y=a \sin x \sin mx = \frac{a}{2}\cos(1-m)x - \frac{a}{2}\cos(1+m)x$. In fig. 22, where $m=\frac{7}{8}$, these parts are separately shown. The continuous line represents the curve $y=\frac{a}{2}\cos(1-\frac{7}{8})x$, having one complete oscillation in a distance of 16π along x. By laying off below and above this curve the second part of y, that is $-\frac{a}{2}\cos(1+\frac{7}{8})x$, we have the curve $y=a \sin x \sin \frac{7}{8}x$.

6. *Use of the auxiliary curves, Eq.* (2). To draw the curves from equation (1), even after all the usual devices for saving labor have been employed, requires the frequent solution of equations of the form $\sin x \sin mx = c$. This equation gives a set of values of x for each cycle of the curve. To find each value of x requires a solution by trial and error, a very simple process, but when often repeated quite tedious. By the curves figs. 1–37 carefully traced on cross-section paper we may by merely running the eye along the line $y=c$

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obtain by inspection all the values of x to a sufficient degree of accuracy.

7. *Equation* (1) *when* $a=b=0$. The equation

$$\sin y \sin my=0, \qquad (3)$$

consists of the two equations $\sin y=0$, and $\sin my=0$, and is satisfied by the values $y=l\pi$, and $my=l\pi$, where l is 0, or any integer. In fig. 66 the horizontal lines belong to the equation $\sin y \sin \frac{4}{5} y=0$. They consist of two series, one at intervals of π, the other at intervals of $2\frac{1}{4}\pi$. If through the intersections of the curve in fig. 25 with the axis of x there be drawn lines perpendicular to that axis, the lines for $\sin x \sin \frac{4}{5}x=0$ would be obtained. The heavy lines of fig. 66 represent double lines, corresponding to points of tangency in fig. 25.

8. *Equation* (1) *where* $a=0$. The equation (1) becomes by making $a=0$, and for convenience changing the axes,

$$\sin x \sin mx=b. \qquad (4)$$

This does not contain y, and therefore represents straight lines parallel to the axis of y. If the straight line $y=b$ be drawn parallel to the axis of x to cut the curve $y=\sin x \sin mx$, and through the several points of intersection straight lines be drawn parallel to the axis of y, these lines will evidently be those represented by the equation $\sin x \sin mx=b$.

In fig. 66 the vertical lines represent the equation $\sin x \sin \frac{4}{5}x=\frac{2}{5}$. If the curve in fig. 26 be cut by a line parallel to the axis of x and distant from it two-fifths of the largest ordinate, the intersections will correspond with the intersections of any horizontal line in fig. 66 by the several vertical lines.*

9. *Equation* (1) *where* $a=1$, $b=0$, $m=n=1$. The equation

$$\sin y \sin y=\sin x \sin x \qquad (5)$$

becomes $\sin y=\pm\sin x$, or $y=l\pi \pm x$, l being 0, or an integer. The curve consists of two series of parallel equidistant straight lines, the one parallel to $y=x$, the other to $y=-x$, and both cutting the axes at intervals of π. The locus is represented in fig. 38, where the origin is any point of intersection.

10. *Equation* (1) *where* $a=1, b=0, m=n=\frac{1}{q}$. The equation

$$\sin y \sin \frac{1}{q} y=\sin x \sin \frac{1}{q} x \qquad (6)$$

is one of the simpler examples of equation (1).

* The unit of abscissas in plates XIV and XV is smaller than in the other plates.

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a. It is satisfied if $y=x$, or if $y=-x$. Hence the two straight lines $y=\pm x$ form part of the locus of equation (6).

b. If $2lq\pi+x$ be put for x, l being an integer, the equation is unchanged, whether q be *odd* or *even.*

c. If q be *odd* the equation will be unchanged if $lq\pi+x$ be put for x.

d. The curve repeats itself to the right and left, and also above and below, at intervals of $q\pi$ if q is *odd*, and at intervals of $2q\pi$ if q is *even.*

e. Straight lines parallel to $y=\pm x$, and cutting the axes at intervals of $q\pi$, or $2q\pi$, according as q is *odd* or *even*, belong to the locus of equation (6).

f. These straight lines divide the infinite plane of coördinates into equal squares for a given value of q. Each square contains a similar and equal portion of the locus. If q is odd, that portion is not always similarly placed, for it may have two positions with respect to an axis.

g. If q is even, isolated points at the centers of the squares (*f*) belong to the locus.

h. The equation (6) is satisfied if $\sin x=0$, and $\sin y=0$. Hence the locus of (6) passes through the angular points of all the squares formed by the lines $\sin x=0$, and $\sin y=0$ (Art. 7.)

i. A few curves representing equation (6) are shown in figs. 40–47. The axes are not drawn. Any point of intersection of straight lines that is surrounded by an oval may be taken for the origin. The several propositions of this article will be more easily understood by inspection of the curves.

11. *Equation* (1) *when* $a=1$, $b=0$, $m=n=\frac{p}{q}$. In this case the equation becomes,

$$\sin y \sin\frac{p}{q}y=\sin x \sin\frac{p}{q}x. \qquad (7)$$

The properties of the curves of equation (7) are in many respects like those of equation (6).

a. The two straight lines $y=\pm x$ belong to the locus.

b. If p and q are both *odd*, the equation is unchanged, if y or x is increased or diminished by multiples of $q\pi$.

c. If either p or q is *even*, the equation is unchanged if y or x is increased or diminished by multiples of $2q\pi$.

d. The curve repeats in the direction of either axis; at intervals of $q\pi$ if p and q are both *odd*, at intervals of $2q\pi$ if either p or q is even.

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e. Straight lines parallel to $y=\pm x$ and cutting the axes at intervals of $q\pi$, or $2q\pi$, according as $p+q$ is *even* or *odd*, belong to the locus of equation (7).

f. These straight lines divide the plane of coördinates into equal squares for any given value of m. Each square contains a similar and equal portion of the locus, though not always similarly placed.

g. Equation (7) is satisfied if $\sin x \sin mx=0$, and $\sin y \sin my=0$. Hence the locus passes through all the angular points of the rectangles formed by these two series of parallel straight lines (Art. 7).

h. If $p+q$ is *odd* isolated points appear, belonging to the locus, at the centers of the squares.

i. The maxima and minima values of y are determined by the equation $\frac{p}{q}\tan x=-\tan\frac{p}{q}x$ (Art. 4, *c*). This equation represents straight lines parallel to the axis of y. There are $2(p+q)$ of the lines (Art. 4, *h*) in an interval of $2q\pi$.

j. The same equation in y gives the maxima and minima values of x.

k. These equations are also the conditions of the isolated and double points. Hence there can be isolated or double points only at the intersections of the lines $\frac{p}{q}\tan x=-\tan\frac{p}{q}x$ with the lines $\frac{p}{q}\tan y=-\tan\frac{p}{q}y$.

l. The propositions (*i*), (*j*), and (*k*) hold equally true for any values of a and b in equation (1), and there are similar properties if m is not equal to n.

m. The figs. 48–65, 68, and 70, represent curves belonging to equation (7). Any point where two straight lines meet, and that is surrounded by an oval may be the origin.

n. If through the double points on the line $y=x$ vertical and horizontal lines be drawn, these lines will pass through all the points of maxima and minima ordinates and abscissas. By their intersections they will mark all the possible positions of double points for any values of a and b.

12. *Equation* (1) *when* $a=1$, $b=0$, $m=n=$*an irrational number.* The equation

$$\sin y \sin \sqrt{\tfrac{1}{5}}\, y=\sin x \sin \sqrt{\tfrac{1}{5}}\, x, \qquad (8)$$

represents a class of curves that do not repeat their forms but change

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continually throughout the plane of coördinates. The curve is symmetrical about either axis, and also about either line $y=\pm x$. These two lines belong to the curve.

The origin and a portion of the curve, principally in the first quadrant, are given in fig. 67, plate XXII.

13. *Equation* (1) *when* $a=-1$, $b=0$, $m=n=\frac{1}{q}$. If q is *even* the equation

$$\sin y \sin\frac{1}{q}y = -\sin x \sin\frac{1}{q}x \qquad (9)$$

merely changes the sign of the second member if we substitute $q\pi+x$ for x. Hence the curves in figures 40, 42, 44, and 47 represent equation (9), when q is *even*, the origin being at an isolated point.

But if q is *odd* we obtain new forms which have these properties.

a. The origin is an isolated point.
b. If $q=1$, the locus consists solely of points (fig. 39).
c. If $q=3$, each point is surrounded by one closed curve (fig. 93).
d. If $q=5$, each point is surrounded by two closed curves (fig. 74).
e. The resemblance of these figures to parts of figs. 40, 42, and 44, and the law of their formation makes it unnecessary to give further examples.
f. A dot and four surrounding closed curves in fig. 47, would fairly represent the element for equation (9), when $q=9$.

14. *Equation* (1) *when* $a=-1$, $b=0$, $m=n=\frac{p}{q}$. Curves whose equations are of the form

$$\sin y \sin\frac{p}{q}y = -\sin x \sin\frac{p}{q}x \qquad (10)$$

are shown in figures 69, 71, 99, and 108. There are no straight lines belonging to the locus. The origin is at any one of the isolated points. The first two are placed beside figures 68 and 70 for ease in comparing.

The following propositions of Art. 11, for equation (7) apply also to equation (10), without change of terms, viz: *b*, *c*, *d*, *g*, *i*, *j*, and *k*.

15. *Equation* (1) *when* $a=1$, $b=0$, $m=1$, and $n=\frac{p'}{q'}$. The figures 76–79, and 81, represent curves whose equations are

$$\sin y \sin y = \sin x \sin\frac{p'}{q'}x. \qquad (11)$$

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In the direction of y they repeat at intervals of π. In the direction of x they repeat at intervals of $q'\pi$, or $2q'\pi$, according as $p'+q'$ is *even* or *odd.*

Fig. 80 gives a similar curve except that $a=-1$.

16. *Equation* (1) *when* $a=1$, $b=0$, $m=\frac{p}{q}$, and $n=\frac{p'}{q'}$. The equation (11) is a special case of the equation

$$\sin y \sin \frac{p}{q} y = \sin x \sin \frac{p'}{q'} x. \qquad (12)$$

Examples of curves from equation (12) are given in figures 82–91, 123, and 141. The number of different curves that this equation gives us is quite large, even if q and q' are limited to small numbers. If 11 is the maximum value of q and q', the number of independent curves belonging to the equation is nearly a thousand. Equations (5), (6), (7) and (11) are special cases of (12).

17. *Further consideration of the curves of equation* (12).

a. If the parallel straight lines $\sin x \sin \frac{p'}{q'} x=0$ be drawn (Art. 7) the plane of coördinates is divided by those lines into portions. When two lines coincide the portion between them may be regarded as real but infinitessimal. In crossing any of these lines the sign of the second member of (12) changes from plus to minus, or *vice-versa.*

b. In like manner in crossing any of the parallel lines $\sin y \sin \frac{p}{q} y=0$, the sign of the first member changes.

c. The lines $\sin y \sin \frac{p}{q} y=0$, and $\sin x \sin \frac{p'}{q'} x=0$, divide the plane into rectangles (some of which are infinitessimal). The curve of equation (12) passes through each of the angular points of these rectangles.

d. Since the signs of the two members of (12) must be alike the curve passes at any angle of a rectangle into the rectangle vertically opposite. It passes from a rectangle only at the angles.

e. If, however, any rectangle is of infinitessimal breadth and finite length, the curve at its extremity becomes tangent to the line that limits the infinitessimal parallelogram.

f. If a rectangle becomes infinitessimal in both directions, the curve has at that point an isolated or a double point.

g. The horizontal and vertical lines of fig. 148, and the rectangles formed by them, illustrate the above propositions. The con-

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tinuous closed line represents the curve of equation (12) in this case. The axis of y is the heavy vertical line, and the axis of x the upper heavy horizontal line. These heavy lines are double lines.

h. Several of the propositions of Art. 11 apply to equation (12) with evident modifications. If $\frac{p}{q}$ is not equal to $\frac{p'}{q'}$, there are no straight lines belonging to the locus.

i. We may regard the plane of the curve as divided into equal rectangles by lines parallel to the axes, the altitudes of the rectangles being $q\pi$, or $2q\pi$, according as $p+q$ is *even* or *odd*, and whose bases are $q'\pi$, or $2q'\pi$, according as $p'+q'$ is *even* or *odd*. The curve (12) repeats itself in each of these rectangles without any variation, through the whole extent of the plane.

j. The origin of (12) is a real double point.

18. *Effect of a change of the value of a in equation* (1), *when* $b=0$. The effect of a change in the value of the coefficient of the second member may be observed by comparing some of the figures: for example, figs. 38 with 39; figs. 41 with 93; figs. 45 with 72 and 73; figs. 77 with 80; figs. 43 with 74 and 75; figs. 123 with 131–135; figs. 136 with 141 and 145.

19. The effect of the change of this factor can be better observed in the simpler equation

$$\sin y = k \sin x, \qquad (13)$$

where k represents a as assuming several values. Figure 130 represents a faisceau of curves for equation (13). The origin is the nodal point near the lower left hand corner of the figure. Let k change from $-\infty$ to $+\infty$.

a. If $k=\infty$, we have the vertical equidistant straight lines.

b. If $k=-2$, we have the curved lines represented by uniform fine dots. At the origin it is tangent to $y=-2x$.

c. If $k=-1$, we have the straight lines in which dots and strokes alternate.

d. If $k=-\frac{1}{2}$, we have the continuous curved lines.

e. If $k=0$, we have horizontal straight lines.

f. If $k=\frac{1}{2}$, we have the heavy dotted curved lines.

g. If $k=1$, we have straight lines of which $y=x$ is one, and the others are similarly marked.

h. If $k=2$, we have the curved lines consisting of a stroke and three dots alternating.

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i. If $k=+\infty$, we have vertical lines again. The curve is at the origin always tangent to $y=kx$. The faisceau has nodal points wherever x and y are both multiples of π.

20. If we consider in like manner the faisceau of curves

$$\sin y \sin \tfrac{3}{5}y = k \sin x \sin \tfrac{3}{7}x, \qquad (14)$$

for various values of k (fig. 148), we shall find similar but more complicated changes. The origin is the intersection of the heavy lines near the top of the figure. The figure represents the loci for six values of k, viz: ∞, -1, $-\frac{1}{2}$, 0, $+1$, and $+2$. Each of the six loci passes through each nodal point, if isolated points be counted as branches of a locus.

a. For $k=\infty$, we have the vertical straight lines. The heavy line is a double line.

b. For $k=-1$, we have the uniformly dotted curves.

c. For $k=-\frac{1}{2}$, we have the curves represented by strokes and four dots alternating.

d. For $k=0$, we have horizontal straight lines, the heavy lines being double.

e. For $k=1$, we have the continuous curves (compare fig. 147).

f. For $k=2$, we have the curved lines consisting of a long stroke and a short stroke alternating.

By removal upward or downward a distance of 5π, the curve (*b*) coincides with (*e*). In general any one of the curves by such change coincides with that one for which k has an equal value with opposite sign.

21. We may in like manner obtain a faisceau of curves from the equation

$$\sin y \sin my = k \sin x \sin nx + b, \qquad (15)$$

by giving to k different values.

The curve will be the horizontal lines $\sin y \sin my = b$ (Art. 8), if $k=0$. It will be the vertical lines $\sin x \sin mx = 0$, if $k=0$. For other values of k, the curve will pass through all the points of intersections of these series of straight lines. Figure 66 represents (with the axes interchanged) the vertical and horizontal lines in a special case.

The lines of maxima and minima values of x and y, and the possible positions of double points (Art. 11, *i*, *j*, *k*,), are independent of k and b. The origin is not upon the curve if k and b are finite.

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22. *Change of value of b in equation* (1). It remains to consider the effect of a change in the constant b in equation (1). That it may change entirely the appearance of the locus will be seen by a glance at figures 92, 93, and 94. The same curves are superposed in fig. 95. Though each locus may have its own double points, they cannot when superposed cut each other.

23. In the figures 96–103, the curves of the equation

$$\sin y \sin \tfrac{3}{7}y = -\sin x \sin \tfrac{3}{7}x + k \qquad (16)$$

are shown for certain specified values of k. The origin is the place of the isolated point in fig. 99. The several curves if superposed will not intersect. The values of k were selected so as to furnish curves with double points.

24. A series of twelve curves from the equation

$$\sin y \sin \tfrac{3}{11}y = -\sin x \sin \tfrac{3}{11}x + k \qquad (17)$$

is given in the figures 104–115. By tracing any selected portions of the figure through the series the effect of the change in k will be seen. As in equation (16) values of k were chosen which give (except fig. 108) real double points. In each case other curves of the series with real double points might have been given.

25. Another series of fourteen curves is given in figures 116–129 from the equation

$$\sin y \sin \tfrac{1}{2}y = \sin x \sin \tfrac{3}{7}x + k. \qquad (18)$$

The complete series would give 18 curves with double or isolated points. The omitted curves are those having isolated points, one at the beginning and one at the end of the series, one between figs. 127 and 128, and one between figs. 129 and 130.

26. Similar partial series can be seen in figs. 136–138, in figs. 139–143, and in figs. 144–146.

27. The superposition of the several curves of a series is shown in figure 147 where the curves represent the equation

$$\sin y \sin \tfrac{2}{5}y = \sin x \sin \tfrac{3}{7}x + k.$$

A little more than one complete figure of the curves is shown. The origin is at the double point near the top of the figure. The value of k varies from curve to curve by intervals of $\frac{1}{5}$, and it cannot numerically exceed 2. The full line corresponds to $k=0$.

The multiple that k is of $+\frac{1}{5}$ is denoted by the number of *dots* between the long strokes of the lines.

The multiple that k is of $-\frac{1}{5}$ is denoted by the number of *short strokes* between the long strokes in the lines.

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28. The resemblance of fig. 147 to a series of contour lines in surveying, suggests a corresponding treatment of the equation. Let

$$z = \sin y \sin my - a \sin x \sin nx - b \qquad (19)$$

be the equation of a surface, and let it be intersected by planes parallel to the plane of xy, and we may obtain the groups of curves described in Arts. 22–27.

The surface of equation (19) may be described by continuous motion, as follows: Let $z = \sin y \sin my$ be a plane curve (figs. 1–37), and let it move parallel to itself so that each point of it shall describe a straight line parallel to the axis of x. The curve shall then describe a cylindrical surface whose equation is

$$z = \sin y \sin my. \qquad (20)$$

Let $z = -a \sin x \sin nx - b$ be the equation of a second plane curve, and let this curve move parallel to the plane xz, in such a manner that the axis of x of the curve shall always lie in the cylindrical surface (20). The curve will describe by its motion the surface of equation (19).

The surface will consist of one continuous sheet lying between the two parallel planes $z = \pm (1 + a + b)$, the positive numerical values of a and b being here taken.

29. By means of the two arbitrary constants, a and b, in equation (1) the curve may be made to pass through any two points of the plane.

In a rectangle whose base is $2q'\pi$, and altitude $2q\pi$, there are $4(p+q)(p'+q')$ possible positions of double points (Art. 11, *k*.) If the curve passes through such a point it must have there two branches real or imaginary.

Hence we may assign to a and b such values that the curve will have double points, in general, at any two of the $4(p+q)(p'+q')$ possible positions.

ERRATUM IN PLATE XVI.

In figure 40, plate XVI, there is a series of ovals about one-half of the real double points. There should be added to the curve, as represented, a like series of ovals around each of the remaining real double points.

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V. On the Equilibrium of Heterogeneous Substances.

By J. Willard Gibbs.

"Die Energie der Welt ist constant.
Die Entropie der Welt strebt einem Maximum zu."
Clausius.*

The comprehension of the laws which govern any material system is greatly facilitated by considering the energy and entropy of the system in the various states of which it is capable. As the difference of the values of the energy for any two states represents the combined amount of work and heat received or yielded by the system when it is brought from one state to the other, and the difference of entropy is the limit of all the possible values of the integral $\int \frac{dQ}{t}$, (dQ denoting the element of the heat received from external sources, and t the temperature of the part of the system receiving it,) the varying values of the energy and entropy characterize in all that is essential the effects producible by the system in passing from one state to another. For by mechanical and thermodynamic contrivances, supposed theoretically perfect, any supply of work and heat may be transformed into any other which does not differ from it either in the amount of work and heat taken together or in the value of the integral $\int \frac{dQ}{t}$. But it is not only in respect to the external relations of a system that its energy and entropy are of predominant importance. As in the case of simply mechanical systems, (such as are discussed in theoretical mechanics,) which are capable of only one kind of action upon external systems, viz., the performance of mechanical work, the function which expresses the capability of the system for this kind of action also plays the leading part in the theory of equilibrium, the condition of equilibrium being that the variation of this function shall vanish, so in a thermodynamic system, (such as all material systems actually are,) which is capable of two different kinds of action upon external systems, the two functions which express the twofold capabilities of the system afford an almost equally simple criterion of equilibrium.

*Pogg. Ann. Bd. cxxv (1865), S. 400; or Mechanische Wärmetheorie, Abhand. ix., S. 44.

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CRITERIA OF EQUILIBRIUM AND STABILITY.

The criterion of equilibrium for a material system which is isolated from all external influences may be expressed in either of the following entirely equivalent forms:

I. *For the equilibrium of any isolated system it is necessary and sufficient that in all possible variations of the state of the system which do not alter its energy, the variation of its entropy shall either vanish or be negative.* If ε denote the energy, and η the entropy of the system, and we use a subscript letter after a variation to indicate a quantity of which the value is not to be varied, the condition of equilibrium may be written

$$(\delta\eta)_\varepsilon \leqq 0. \tag{1}$$

II. *For the equilibrium of any isolated system it is necessary and sufficient that in all possible variations in the state of the system which do not alter its entropy, the variation of its energy shall either vanish or be positive.* This condition may be written

$$(\delta\varepsilon)_\eta \geqq 0. \tag{2}$$

That these two theorems are equivalent will appear from the consideration that it is always possible to increase both the energy and the entropy of the system, or to decrease both together, viz., by imparting heat to any part of the system or by taking it away. For, if condition (1) is not satisfied, there must be some variation in the state of the system for which

$$\delta\eta > 0 \text{ and } \delta\varepsilon = 0;$$

therefore, by diminishing both the energy and the entropy of the system *in its varied state*, we shall obtain a state for which (considered as a variation from the original state)

$$\delta\eta = 0 \text{ and } \delta\varepsilon < 0;$$

therefore condition (2) is not satisfied. Conversely, if condition (2) is not satisfied, there must be a variation in the state of the system for which

$$\delta\varepsilon < 0 \text{ and } \delta\eta = 0;$$

hence there must also be one for which

$$\delta\varepsilon = 0 \text{ and } \delta\eta > 0;$$

therefore condition (1) is not satisfied.

The equations which express the condition of equilibrium, as also its statement in words, are to be interpreted in accordance with the general usage in respect to differential equations, that is, infinitesimals

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of higher orders than the first relatively to those which express the amount of change of the system are to be neglected. But to distinguish the different kinds of equilibrium in respect to stability, we must have regard to the absolute values of the variations. We will use Δ as the sign of variation in those equations which are to be construed *strictly*, i. e., in which infinitesimals of the higher orders are not to be neglected. With this understanding, we may express the necessary and sufficient conditions of the different kinds of equilibrium as follows;—for stable equilibrium

$$(\Delta\eta)_\varepsilon < 0, \text{ i. e., } (\Delta\varepsilon)_\eta > 0; \tag{3}$$

for neutral equilibrium there must be some variations in the state of the system for which

$$(\Delta\eta)_\varepsilon = 0, \text{ i. e., } (\Delta\varepsilon)_\eta = 0, \tag{4}$$

while in general

$$(\Delta\eta)_\varepsilon \leqq 0, \text{ i. e., } (\Delta\varepsilon)_\eta \geqq 0; \tag{5}$$

and for unstable equilibrium there must be some variations for which

$$(\Delta\eta)_\varepsilon > 0, \tag{6}$$

i. e., there must be some for which

$$(\Delta\varepsilon)_\eta < 0, \tag{7}$$

while in general

$$(\delta\eta)_\varepsilon \leqq 0, \text{ i. e., } (\delta\varepsilon)_\eta \geqq 0. \tag{8}$$

In these criteria of equilibrium and stability, account is taken only of *possible* variations. It is necessary to explain in what sense this is to be understood. In the first place, all variations in the state of the system which involve the transportation of any matter through any finite distance are of course to be excluded from consideration, although they may be capable of expression by infinitesimal variations of quantities which perfectly determine the state of the system. For example, if the system contains two masses of the same substance, not in contact, nor connected by other masses consisting of or containing the same substance or its components, an infinitesimal increase of the one mass with an equal decrease of the other is not to be considered as a possible variation in the state of the system. In addition to such cases of essential impossibility, if heat can pass by conduction or radiation from every part of the system to every other, only those variations are to be rejected as impossible, which involve changes which are prevented by passive forces or analogous resistances to change. But, if the system consist of parts between which there is supposed to be no thermal communication, it will be necessary to regard as impossible any diminution of the entropy of any of

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these parts, as such a change can not take place without the passage of heat. This limitation may most conveniently be applied to the second of the above forms of the condition of equilibrium, which will then become

$$(\delta\varepsilon)_{\eta', \eta'', \text{etc.}} \geqq 0, \tag{9}$$

η', η'', etc., denoting the entropies of the various parts between which there is no communication of heat. When the condition of equilibrium is thus expressed, the limitation in respect to the conduction of heat will need no farther consideration.

In order to apply to any system the criteria of equilibrium which have been given, a knowledge is requisite of its passive forces or resistances to change, in so far, at least, as they are capable of *preventing* change. (Those passive forces which only retard change, like viscosity, need not be considered.) Such properties of a system are in general easily recognized upon the most superficial knowledge of its nature. As examples, we may instance the passive force of friction which prevents sliding when two surfaces of solids are pressed together,—that which prevents the different components of a solid, and sometimes of a fluid, from having different motions one from another,—that resistance to change which sometimes prevents either of two forms of the same substance (simple or compound), which are capable of existing, from passing into the other,—that which prevents the changes in solids which imply plasticity, (in other words, changes of the form to which the solid tends to return,) when the deformation does not exceed certain limits.

It is a characteristic of all these passive resistances that they prevent a certain kind of motion or change, however the initial state of the system may be modified, and to whatever external agencies of force and heat it may be subjected, within limits, it may be, but yet within limits which allow finite variations in the values of all the quantities which express the initial state of the system or the mechanical or thermal influences acting on it, without producing the change in question. The equilibrium which is due to such passive properties is thus widely distinguished from that caused by the balance of the active tendencies of the system, where an external influence, or a change in the initial state, infinitesimal in amount, is sufficient to produce change either in the positive or negative direction. Hence the ease with which these passive resistances are recognized. Only in the case that the state of the system lies so near the limit at which the resistances cease to be operative to prevent change, as to create a

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doubt whether the case falls within or without the limit, will a more accurate knowledge of these resistances be necessary.

To establish the validity of the criterion of equilibrium, we will consider first the sufficiency, and afterwards the necessity, of the condition as expressed in either of the two equivalent forms.

In the first place, if the system is in a state in which its entropy is greater than in any other state of the same energy, it is evidently in equilibrium, as any change of state must involve either a decrease of entropy or an increase of energy, which are alike impossible for an isolated system. We may add that this is a case of *stable* equilibrium, as no infinitely small cause (whether relating to a variation of the initial state or to the action of any external bodies) can produce a finite change of state, as this would involve a finite decrease of entropy or increase of energy.

We will next suppose that the system has the greatest entropy consistent with its energy, and therefore the least energy consistent with its entropy, but that there are other states of the same energy and entropy as its actual state. In this case, it is impossible that any motion of masses should take place; for if any of the energy of the system should come to consist of *vis viva* (of sensible motions), a state of the system identical in other respects but without the motion would have less energy and not less entropy, which would be contrary to the supposition. (But we cannot apply this reasoning to the motion within any mass of its different components in different directions, as in diffusion, when the momenta of the components balance one another.) Nor, in the case supposed, can any conduction of heat take place, for this involves an increase of entropy, as heat is only conducted from bodies of higher to those of lower temperature. It is equally impossible that any changes should be produced by the transfer of heat by radiation. The condition which we have supposed is therefore sufficient for equilibrium, so far as the motion of masses and the transfer of heat are concerned, but to show that the same is true in regard to the motions of diffusion and chemical or molecular changes, when these can occur without being accompanied or followed by the motions of masses or the transfer of heat, we must have recourse to considerations of a more general nature. The following considerations seem to justify the belief that the condition is sufficient for equilibrium in every respect.

Let us suppose, in order to test the tenability of such a hypothesis, that a system may have the greatest entropy consistent with its energy without being in equilibrium. In such a case, changes in the

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state of the system must take place, but these will necessarily be such that the energy and the entropy will remain unchanged and the system will continue to satisfy the same condition, as initially, of having the greatest entropy consistent with its energy. Let us consider the change which takes place in any time so short that the change may be regarded as uniform in nature throughout that time. This time must be so chosen that the change does not take place in it infinitely slowly, which is always easy, as the change which we suppose to take place cannot be infinitely slow except at particular moments. Now no change whatever in the state of the system, which does not alter the value of the energy, and which commences with the same state in which the system was supposed at the commencement of the short time considered, will cause an increase of entropy. Hence, it will generally be possible by some slight variation in the circumstances of the case to make all changes in the state of the system like or nearly like that which is supposed actually to occur, and not involving a change of energy, to involve a necessary decrease of entropy, which would render any such change impossible. This variation may be in the values of the variables which determine the state of the system, or in the values of the constants which determine the nature of the system, or in the form of the functions which express its laws,—only there must be nothing in the system as modified which is thermodynamically impossible. For example, we might suppose temperature or pressure to be varied, or the composition of the different bodies in the system, or, if no small variations which could be actually realized would produce the required result, we might suppose the properties themselves of the substances to undergo variation, subject to the general laws of matter. If, then, there is any tendency toward change in the system as first supposed, it is a tendency which can be entirely checked by an infinitesimal variation in the circumstances of the case. As this supposition cannot be allowed, we must believe that a system is always in equilibrium when it has the greatest entropy consistent with its energy, or, in other words, when it has the least energy consistent with its entropy.

The same considerations will evidently apply to any case in which a system is in such a state that $\Delta\eta \leqq 0$ for any possible infinitesimal variation of the state for which $\Delta\varepsilon = 0$, even if the entropy is not the least of which the system is capable with the same energy. (The term *possible* has here the meaning previously defined, and the character Δ is used, as before, to denote that the equations are to be

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construed strictly, i. e., without neglect of the infinitesimals of the higher orders.)

The only case in which the sufficiency of the condition of equilibrium which has been given remains to be proved is that in which in our notation $\delta\eta \leqq 0$ for all possible variations not affecting the energy, but for some of these variations $\Delta\eta > 0$, that is, when the entropy has in some respects the characteristics of a minimum. In this case the considerations adduced in the last paragraph will not apply without modification, as the change of state may be infinitely slow at first, and it is only in the initial state that the condition $\delta\eta_\varepsilon \leqq 0$ holds true. But the differential coefficients of all orders of the quantities which determine the state of the system, taken with respect of the time, must be functions of these same quantities. None of these differential coefficients can have any value other than 0, for the state of the system for which $\delta\eta_\varepsilon \leqq 0$. For otherwise, as it would generally be possible, as before, by some infinitely small modification of the case, to render impossible any change like or nearly like that which might be supposed to occur, this infinitely small modification of the case would make a finite difference in the value of the differential coefficients which had before the finite values, or in some of lower orders, which is contrary to that continuity which we have reason to expect. Such considerations seem to justify us in regarding such a state as we are discussing as one of theoretical equilibrium; although as the equilibrium is evidently unstable, it cannot be realized.

We have still to prove that the condition enunciated is in every case necessary for equilibrium. It is evidently so in all cases in which the active tendencies of the system are so balanced that changes of every kind, except those excluded in the statement of the condition of equilibrium, can take place *reversibly*, (i. e., both in the positive and the negative direction,) in states of the system differing infinitely little from the state in question. In this case, we may omit the sign of inequality and write as the condition of such a state of equilibrium

$$(\delta\eta)_\varepsilon = 0, \quad \text{i. e.,} \quad (\delta\varepsilon)_\eta = 0 \tag{10}$$

But to prove that the condition previously enunciated is in every case necessary, it must be shown that whenever an isolated system remains without change, if there is any infinitesimal variation in its state, not involving a finite change of position of any (even an infinitesimal part) of its matter, which would diminish its energy by a

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quantity which is not infinitely small relatively to the variations of the quantities which determine the state of the system, without altering its entropy,—or, if the system has thermally isolated parts, without altering the entropy of any such part,—this variation involves changes in the system which are prevented by its passive forces or analogous resistances to change. Now, as the described variation in the state of the system diminishes its energy without altering its entropy, it must be regarded as theoretically possible to produce that variation by some process, perhaps a very indirect one, so as to gain a certain amount of work (above all expended on the system). Hence we may conclude that the active forces or tendencies of the system favor the variation in question, and that equilibrium cannot subsist unless the variation is prevented by passive forces.

The preceding considerations will suffice, it is believed, to establish the validity of the criterion of equilibrium which has been given. The criteria of stability may readily be deduced from that of equilibrium. We will now proceed to apply these principles to systems consisting of heterogeneous substances and deduce the special laws which apply to different classes of phenomena. For this purpose we shall use the second form of the criterion of equilibrium, both because it admits more readily the introduction of the condition that there shall be no thermal communication between the different parts of the system, and because it is more convenient, as respects the form of the general equations relating to equilibrium, to make the entropy one of the independent variables which determine the state of the system, than to make the energy one of these variables.

THE CONDITIONS OF EQUILIBRIUM FOR HETEROGENEOUS MASSES IN CONTACT WHEN UNINFLUENCED BY GRAVITY, ELECTRICITY, DISTORTION OF THE SOLID MASSES, OR CAPILLARY TENSIONS.

In order to arrive as directly as possible at the most characteristic and essential laws of chemical equilibrium, we will first give our attention to a case of the simplest kind. We will examine the conditions of equilibrium of a mass of matter of various kinds enclosed in a rigid and fixed envelop, which is impermeable to and unalterable by any of the substances enclosed, and perfectly non-conducting to heat. We will suppose that the case is not complicated by the action of gravity, or by any electrical influences, and that in the solid portions of the mass the pressure is the same in every direction.

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We will farther simplify the problem by supposing that the variations of the parts of the energy and entropy which depend upon the surfaces separating heterogeneous masses are so small in comparison with the variations of the parts of the energy and entropy which depend upon the quantities of these masses, that the former may be neglected by the side of the latter; in other words, we will exclude the considerations which belong to the theory of capillarity.

It will be observed that the supposition of a rigid and non-conducting envelop enclosing the mass under discussion involves no real loss of generality, for if any mass of matter is in equilibrium, it would also be so, if the whole or any part of it were enclosed in an envelop as supposed; therefore the conditions of equilibrium for a mass thus enclosed are the general conditions which must always be satisfied in case of equilibrium. As for the other suppositions which have been made, all the circumstances and considerations which are here excluded will afterward be made the subject of special discussion.

Conditions relating to the Equilibrium between the initially existing Homogeneous Parts of the given Mass.

Let us first consider the energy of any homogeneous part of the given mass, and its variation for any possible variation in the composition and state of this part. (By *homogeneous* is meant that the part in question is uniform throughout, not only in chemical composition, but also in physical state.) If we consider the amount and kind of matter in this homogeneous mass as fixed, its energy ε is a function of its entropy η, and its volume v, and the differentials of these quantities are subject to the relation

$$d\varepsilon = t\,d\eta - p\,dv, \tag{11}$$

t denoting the (absolute) temperature of the mass, and p its pressure. For $t\,d\eta$ is the heat received, and $p\,dv$ the work done, by the mass during its change of state. But if we consider the matter in the mass as variable, and write $m_1, m_2, \ldots m_n$ for the quantities of the various substances $S_1, S_2, \ldots S_n$ of which the mass is composed, ε will evidently be a function of $\eta, v, m_1, m_2, \ldots m_n$, and we shall have for the complete value of the differential of ε

$$d\varepsilon = t\,d\eta - p\,dv + \mu_1\,dm_1 + \mu_2\,dm_2 \ldots + \mu_n\,dm_n, \tag{12}$$

$\mu_1, \mu_2, \ldots \mu_n$ denoting the differential coefficients of ε taken with respect to $m_1, m_2, \ldots m_n$.

The substances $S_1, S_2, \ldots S_n$, of which we consider the mass composed, must of course be such that the values of the differen-

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tials dm_1, dm_2, . . . dm_n shall be independent, and shall express every possible variation in the composition of the homogeneous mass considered, including those produced by the absorption of substances different from any initially present. It may therefore be necessary to have terms in the equation relating to component substances which do not initially occur in the homogeneous mass considered, provided, of course, that these substances, or their components, are to be found in some part of the whole given mass.

If the conditions mentioned are satisfied, the choice of the substances which we are to regard as the components of the mass considered, may be determined entirely by convenience, and independently of any theory in regard to the internal constitution of the mass. The number of components will sometimes be greater, and sometimes less, than the number of chemical elements present. For example, in considering the equilibrium in a vessel containing water and free hydrogen and oxygen, we should be obliged to recognize three components in the gaseous part. But in considering the equilibrium of dilute sulphuric acid with the vapor which it yields, we should have only two components to consider in the liquid mass, sulphuric acid (anhydrous, or of any particular degree of concentration) and (additional) water. If, however, we are considering sulphuric acid in a state of maximum concentration in connection with substances which might possibly afford water to the acid, it must be noticed that the condition of the independence of the differentials will require that we consider the acid in the state of maximum concentration as one of the components. The quantity of this component will then be capable of variation both in the positive and in the negative sense, while the quantity of the other component can increase but cannot decrease below the value 0.

For brevity's sake, we may call a substance S_a an *actual component* of any homogeneous mass, to denote that the quantity m_a of that substance in the given mass may be either increased or diminished (although we may have so chosen the other component substances that $m_a = 0$); and we may call a substance S_b a *possible component* to denote that it may be combined with, but cannot be substracted from the homogeneous mass in question. In this case, as we have seen in the above example, we must so choose the component substances that $m_b = 0$.

The units by which we measure the substances of which we regard the given mass as composed may each be chosen independently. To fix our ideas for the purpose of a general discussion, we may suppose

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all substances measured by weight or mass. Yet in special cases, it may be more convenient to adopt chemical equivalents as the units of the component substances.

It may be observed that it is not necessary for the validity of equation (12) that the variations of nature and state of the mass to which the equation refers should be such as do not disturb its homogeneity, provided that in all parts of the mass the variations of nature and state are infinitely small. For, if this last condition be not violated, an equation like (12) is certainly valid for all the infinitesimal parts of the (initially) homogeneous mass; i. e., if we write $D\varepsilon$, $D\eta$, etc., for the energy, entropy, etc., of any infinitesimal part,

$$dD\varepsilon = t\,dD\eta - p\,dDv + \mu_1\,dDm_1 + \mu_2\,dDm_2 \ldots + \mu_n\,dDm_n, \quad (13)$$

whence we may derive equation (12) by integrating for the whole initially homogeneous mass.

We will now suppose that the whole mass is divided into parts so that each part is homogeneous, and consider such variations in the energy of the system as are due to variations in the composition and state of the several parts remaining (at least approximately) homogeneous, and together occupying the whole space within the envelop. We will at first suppose the case to be such that the component substances are the same for each of the parts, each of the substances $S_1, S_2, \ldots S_n$ being an actual component of each part. If we distinguish the letters referring to the different parts by accents, the variation in the energy of the system may be expressed by $\delta\varepsilon' + \delta\varepsilon'' +$ etc., and the general condition of equilibrium requires that

$$\delta\varepsilon' + \delta\varepsilon'' + \text{ etc.} \geqq 0 \quad (14)$$

for all variations which do not conflict with the *equations of condition*. These equations must express that the entropy of the whole given mass does not vary, nor its volume, nor the total quantities of any of the substances $S_1, S_2, \ldots S_n$. We will suppose that there are no other equations of condition. It will then be necessary for equilibrium that

$$\begin{aligned} &t'\,\delta\eta' - p'\,\delta v' + \mu_1'\,\delta m_1' + \mu_2'\,\delta m_2' \ldots + \mu_n'\,\delta m_n' \\ &+ t''\,\delta\eta'' - p''\,\delta v'' + \mu_1''\,\delta m_1'' + \mu_2''\,\delta m_2'' \ldots + \mu_n''\,\delta m_n'' \\ &+ \text{ etc.} \geqq 0 \end{aligned} \quad (15)$$

for any values of the variations for which

$$\delta\eta' + \delta\eta'' + \delta\eta''' + \text{ etc.} = 0, \quad (16)$$

$$\delta v' + \delta v'' + \delta v''' + \text{ etc.} = 0, \quad (17)$$

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$$\left.\begin{array}{l} \delta m_1' + \delta m_1'' + \delta m_1''' + \text{etc.} = 0, \\ \delta m_2' + \delta m_2'' + \delta m_2''' + \text{etc.} = 0, \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ \text{and} \quad \delta m_n' + \delta m_n'' + \delta m_n''' + \text{etc.} = 0. \end{array}\right\} \tag{18}$$

For this it is evidently necessary and sufficient that

$$t' = t'' = t''' = \text{etc.} \tag{19}$$

$$p' = p'' = p''' = \text{etc.} \tag{20}$$

$$\left.\begin{array}{l} \mu_1' = \mu_1'' = \mu_1''' = \text{etc.} \\ \mu_2' = \mu_2'' = \mu_2''' = \text{etc.} \\ \quad . \quad . \quad . \quad . \\ \mu_n' = \mu_n'' = \mu_n''' = \text{etc.} \end{array}\right\} \tag{21}$$

Equations (19) and (20) express the conditions of thermal and mechanical equilibrium, viz., that the temperature and the pressure must be constant throughout the whole mass. In equations (21) we have the conditions characteristic of chemical equilibrium. If we call a quantity μ_s, as defined by such an equation as (12), the *potential* for the substance S_s in the homogeneous mass considered, these conditions may be expressed as follows:

The potential for each component substance must be constant throughout the whole mass.

It will be remembered that we have supposed that there is no restriction upon the freedom of motion or combination of the component substances, and that each is an actual component of all parts of the given mass.

The state of the whole mass will be completely determined (if we regard as immaterial the position and form of the various homogeneous parts of which it is composed), when the values are determined of the quantities of which the variations occur in (15). The number of these quantities, which we may call the independent variables, is evidently $(n+2)\nu$, ν denoting the number of homogeneous parts into which the whole mass is divided. All the quantities which occur in (19), (20), (21), are functions of these variables, and may be regarded as known functions, if the energy of each part is known as a function of its entropy, volume, and the quantities of its components. (See eq. (12).) Therefore, equations (19), (20), (21), may be regarded as $(\nu - 1)(n+2)$ independent equations between the independent variables. The volume of the whole mass and the total quantities of the various substances being known afford $n+1$ additional equations. If we also know the total energy of the given mass, or its total entropy, we will have as many equations as there are independent variables.

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But if any of the substances $S_1, S_2 \ldots S_n$ are only possible components of some parts of the given mass, the variation δm of the quantity of such a substance in such a part cannot have a negative value, so that the general condition of equilibrium (15) does not require that the potential for that substance in that part should be equal to the potential for the same substance in the parts of which it is an actual component, but only that it shall not be less. In this case instead of (21) we may write

$$\left.\begin{array}{c}
\mu_1 = M_1 \\
\text{for all parts of which } S_1 \text{ is an actual component, and} \\
\mu_1 \geqq M_1 \\
\text{for all parts of which } S_1 \text{ is a possible (but not actual) component,} \\
\mu_2 = M_2 \\
\text{for all parts of which } S_2 \text{ is an actual component, and} \\
\mu_2 \geqq M_2 \\
\text{for all parts of which } S_2 \text{ is a possible (but not actual) component,} \\
\text{etc.,}
\end{array}\right\} \quad (22)$$

M_1, M_2, etc., denoting constants of which the value is only determined by these equations.

If we now suppose that the components (actual or possible) of the various homogeneous parts of the given mass are not the same, the result will be of the same character as before, provided that all the different components are *independent*, (i. e., that no one can be made out of the others,) so that the total quantity of each component is fixed. The general condition of equilibrium (15) and the equations of condition (16), (17), (18) will require no change, except that, if any of the substances $S_1, S_2 \ldots S_n$ is not a component (actual or possible) of any part, the term $\mu\,\delta m$ for that substance and part will be wanting in the former, and the δm in the latter. This will require no change in the form of the particular conditions of equilibrium as expressed by (19), (20), (22); but the number of single conditions contained in (22) is of course less than if all the component substances were components of all the parts Whenever, therefore, each of the different homogeneous parts of the given mass may be regarded as composed of some or of all of the same set of substances, no one of which can be formed out of the others, the condition which (with equality of temperature and pressure) is necessary and sufficient for equilibrium between the different parts of the given mass may be expressed as follows:

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The potential for each of the component substances must have a constant value in all parts of the given mass of which that substance is an actual component, and have a value not less than this in all parts of which it is a possible component.

The number of *equations* afforded by these conditions, after elimination of $M_1, M_2, \ldots M_n$, will be less than $(n+2)(\nu-1)$ by the number of terms in (15) in which the variation of the form δm is either necessarily nothing or incapable of a negative value. The number of variables to be determined is diminished by the same number, or, if we choose, we may write an equation of the form $m=0$ for each of these terms. But when the substance is a possible component of the part concerned, there will also be a condition (expressed by $\geqq$) to show whether the supposition that the substance is not an actual component is consistent with equilibrium.

We will now suppose that the substances $S_1, S_2, \ldots S_n$ are not all independent of each other, i. e., that some of them can be formed out of others. We will first consider a very simple case. Let S_3 be composed of S_1 and S_2 combined in the ratio of a to b, S_1 and S_2 occurring as actual components in some parts of the given mass, and S_3 in other parts, which do not contain S_1 and S_2 as separately variable components. The general condition of equilibrium will still have the form of (15) with certain of the terms of the form $\mu\,\delta m$ omitted. It may be written more briefly [(23)

$$\Sigma(t\,\delta\eta) - \Sigma(p\,\delta v) + \Sigma(\mu_1\delta m_1) + \Sigma(\mu_2\,\delta m_2)\ldots + \Sigma(\mu_n\,\delta m_n) \geqq 0,$$

the sign Σ denoting summation in regard to the different parts of the given mass. But instead of the three equations of condition,

$$\Sigma\,\delta m_1 = 0, \quad \Sigma\,\delta m_2 = 0, \quad \Sigma\,\delta m_3 = 0, \tag{24}$$

we shall have the two,

$$\left.\begin{aligned} \Sigma\,\delta m_1 + \frac{a}{a+b}\Sigma\,\delta m_3 = 0, \\ \Sigma\,\delta m_2 + \frac{b}{a+b}\Sigma\,\delta m_3 = 0. \end{aligned}\right\} \tag{25}$$

The other equations of condition,

$$\Sigma\,\delta\eta = 0, \quad \Sigma\,\delta v = 0, \quad \Sigma\,\delta m_4 = 0, \text{ etc.}, \tag{26}$$

will remain unchanged. Now as all values of the variations which satisfy equations (24) will also satisfy equations (25), it is evident that all the particular conditions of equilibrium which we have already deduced, (19), (20), (22), are necessary in this case also. When these are satisfied, the general condition (23) reduces to

$$M_1\,\Sigma\,\delta m_1 + M_2\,\Sigma\,\delta m_2 + M_3\,\Sigma\,\delta m_3 \geqq 0. \tag{27}$$

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For, although it may be that μ_1', for example, is greater than M_1, yet it can only be so when the following $\delta m_1'$ is incapable of a negative value. Hence, if (27) is satisfied, (23) must also be. Again, if (23) is satisfied, (27) must also be satisfied, so long as the variation of the quantity of every substance has the value 0 in all the parts of which it is not an actual component. But as this limitation does not affect the range of the possible values of $\Sigma\,\delta m_1$, $\Sigma\,\delta m_2$, and $\Sigma\,\delta m_3$, it may be disregarded. Therefore the conditions (23) and (27) are entirely equivalent, when (19), (20), (22) are satisfied. Now, by means of the equations of condition (25), we may eliminate $\Sigma\,\delta m_1$ and $\Sigma\,\delta m_2$ from (27), which becomes

$$-\,a\,M_1\,\Sigma\,\delta m_3 - b\,M_2\,\Sigma\,\delta m_3 + (a+b)\,M_3\,\Sigma\,\delta m_3 \gtreqless 0, \qquad (28)$$

i. e., as the value of $\Sigma\,\delta m_3$ may be either positive or negative,

$$a\,M_1 + b\,M_2 = (a+b)\,M_3, \qquad (29)$$

which is the additional condition of equilibrium which is necessary in this case.

The relations between the component substances may be less simple than in this case, but in any case they will only affect the equations of condition, and these may always be found without difficulty, and will enable us to eliminate from the general condition of equilibrium as many variations as there are equations of condition, after which the coefficients of the remaining variations may be set equal to zero, except the coefficients of variations which are incapable of negative values, which coefficients must be equal to or greater than zero. It will be easy to perform these operations in each particular case, but it may be interesting to see the form of the resultant equations in general.

We will suppose that the various homogeneous parts are considered as having in all n components, $S_1, S_2, \ldots S_n$, and that there is no restriction upon their freedom of motion and combination. But we will so far limit the generality of the problem as to suppose that each of these components is an actual component of some part of the given mass.* If some of these components can be formed out of others, all such relations can be expressed by equations such as

$$\alpha\,\mathfrak{S}_a + \beta\,\mathfrak{S}_b + \text{etc.} = \varkappa\,\mathfrak{S}_k + \lambda\,\mathfrak{S}_l + \text{etc.} \qquad (30)$$

where $\mathfrak{S}_a$, $\mathfrak{S}_b$, $\mathfrak{S}_k$, etc. denote the units of the substances S_a, S_b, S_k, etc.,

* When we come to seek the conditions of equilibrium relating to the formation of masses unlike any previously existing, we shall take up *de novo* the whole problem of the equilibrium of heterogeneous masses enclosed in a non-conducting envelop, and give it a more general treatment, which will be free from this limitation.

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(that is, of certain of the substances $S_1, S_2, \ldots S_n$,) and $\alpha, \beta, \varkappa$, etc. denote numbers. These are not, it will be observed, equations between abstract quantities, but the sign $=$ denotes qualitative as well as quantitative equivalence. We will suppose that there are r independent equations of this character. The equations of condition relating to the component substances may easily be derived from these equations, but it will not be necessary to consider them particularly. It is evident that they will be satisfied by any values of the variations which satisfy equations (18); hence, the particular conditions of equilibrium (19), (20), (22) must be necessary in this case, and, if these are satisfied, the general equation of equilibrium (15) or (23) will reduce to

$$M_1 \Sigma \delta m_1 + M_2 \Sigma \delta m_2 \ldots + M_n \Sigma \delta m_n \geqq 0. \tag{31}$$

This will appear from the same considerations which were used in regard to equations (23) and (27). Now it is evidently possible to give to $\Sigma \delta m_a$, $\Sigma \delta m_b$, $\Sigma \delta m_k$, etc. values proportional to $\alpha, \beta, -\varkappa$, etc. in equation (30), and also the same values taken negatively, making $\Sigma \delta m = 0$ in each of the other terms; therefore

$$\alpha M_a + \beta M_b + \text{etc.} \ldots - \varkappa M_k - \lambda M_l - \text{etc.} = 0, \tag{32}$$

or,

$$\alpha M_a + \beta M_b + \text{etc.} = \varkappa M_k + \lambda M_l + \text{etc.} \tag{33}$$

It will be observed that this equation has the same form and coefficients as equation (30), M taking the place of $\mathfrak{S}$. It is evident that there must be a similar condition of equilibrium for every one of the r equations of which (30) is an example, which may be obtained simply by changing $\mathfrak{S}$ in these equations into M. When these conditions are satisfied, (31) will be satisfied with any possible values of $\Sigma \delta m_1$, $\Sigma \delta m_2, \ldots \Sigma \delta m_n$. For no values of these quantities are possible, except such that the equation

$$(\Sigma \delta m_1) \mathfrak{S}_1 + (\Sigma \delta m_2) \mathfrak{S}_2 \ldots + (\Sigma \delta m_n) \mathfrak{S}_n = 0 \tag{34}$$

after the substitution of these values, can be derived from the r equations like (30), by the ordinary processes of the reduction of linear equations. Therefore, on account of the correspondence between (31) and (34), and between the r equations like (33) and the r equations like (30), the conditions obtained by giving any possible values to the variations in (31) may also be derived from the r equations like (33); that is, the condition (31) is satisfied, if the r equations like (33) are satisfied. Therefore the r equations like (33) are with (19), (20), and (22) the equivalent of the general condition (15) or (23).

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For determining the state of a given mass when in equilibrium and having a given volume and given energy or entropy, the condition of equilibrium affords an additional equation corresponding to each of the r independent relations between the n component substances. But the equations which express our knowledge of the matter in the given mass will be correspondingly diminished, being $n - r$ in number, like the equations of condition relating to the quantities of the component substances, which may be derived from the former by differentiation.

Conditions relating to the possible Formation of Masses Unlike any Previously Existing.

The variations which we have hitherto considered do not embrace every possible infinitesimal variation in the state of the given mass, so that the particular conditions already formed, although always necessary for equilibrium (when there are no other equations of condition than such as we have supposed), are not always sufficient. For, besides the infinitesimal variations in the state and composition of different parts of the given mass, infinitesimal masses may be formed entirely different in state and composition from any initially existing. Such parts of the whole mass in its varied state as cannot be regarded as parts of the initially existing mass which have been infinitesimally varied in state and composition, we will call *new parts*. These will necessarily be infinitely small. As it is more convenient to regard a vacuum as a limiting case of extreme rarefaction than to give a special consideration to the possible formation of empty spaces within the given mass, the term *new parts* will be used to include any empty spaces which may be formed, when such have not existed initially. We will use $D\varepsilon$, $D\eta$, Dv, Dm_1, Dm_2, . . . Dm_n to denote the infinitesimal energy, entropy, and volume of any one of these new parts, and the infinitesimal quantities of its components. The component substances S_1, S_2, . . . S_n must now be taken to include not only the independently variable components (actual or possible) of all parts of the given mass as initially existing, but also the components of all the new parts, the possible formation of which we have to consider. The character δ will be used as before to express the infinitesimal variations of the quantities relating to those parts which are only infinitesimally varied in state and composition, and which for distinction we will call *original parts*, including under this term the empty spaces, if such exist initially, within the envelop bounding the system. As we may divide the given mass into as many parts as we choose, and as not only the

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initial boundaries, but also the movements of these boundaries during any variation in the state of the system are arbitrary, we may so define the parts which we have called original, that we may consider them as initially homogeneous and remaining so, and as initially constituting the whole system.

The most general value of the energy of the whole system is evidently

$$\Sigma\,\delta\varepsilon + \Sigma\,D\varepsilon, \tag{35}$$

the first summation relating to all the original parts, and the second to all the new parts. (Throughout the discussion of this problem, the letter δ or D following Σ will sufficiently indicate whether the summation relates to the original or to the new parts.) Therefore the general condition of equilibrium is

$$\Sigma\,\delta\varepsilon + \Sigma\,\delta\varepsilon \geqq 0, \tag{36}$$

or, if we substitute the value of $\delta\varepsilon$ taken from equation (12), [(37)

$$\Sigma D\varepsilon + \Sigma(t\delta\eta) - \Sigma(p\delta v) + \Sigma(\mu_1\delta m_1) + \Sigma(\mu_2\delta m_2)\,.\,.\,+\Sigma(\mu_n\delta m_n) \geqq 0.$$

If any of the substances $S_1, S_2, \ldots S_n$ can be formed out of others, we will suppose, as before (see page 122), that such relations are expressed by equations between the units of the different substances. Let these be

$$\left.\begin{array}{l} a_1\,\mathfrak{S}_1 + a_2\,\mathfrak{S}_2 \ldots + a_n\,\mathfrak{S}_n = 0 \\ b_1\,\mathfrak{S}_1 + b_2\,\mathfrak{S}_2 \ldots + b_n\,\mathfrak{S}_n = 0 \\ \quad\text{etc.} \end{array}\right\} r \text{ equations.} \tag{38}$$

The equations of condition will be (if there is no restriction upon the freedom of motion and composition of the components)

$$\Sigma\,\delta\eta + \Sigma\,D\eta = 0, \tag{39}$$

$$\Sigma\,\delta v + \Sigma\,Dv = 0, \tag{40}$$

and $n - r$ equations of the form

$$\left.\begin{array}{l} h_1\,(\Sigma\,\delta m_1 + \Sigma\,Dm_1) + h_2\,(\Sigma\,\delta m_2 + \Sigma\,Dm_2)\;.\;.\;. \\ \qquad\qquad + h_n\,(\Sigma\,\delta m_n + \Sigma\,Dm_n) = 0 \\ i_1\,(\Sigma\,\delta m_1 + \Sigma\,Dm_1) + i_2\,(\Sigma\,\delta m_2 + \Sigma\,Dm_2)\;.\;. \\ \qquad\qquad + i_n\,(\Sigma\,\delta m_n + \Sigma\,Dm_n) = 0 \\ \quad\text{etc.} \end{array}\right\} \tag{41}^*$$

* In regard to the relation between the coefficients in (41) and those in (38), the reader will easily convince himself that the coefficients of any one of equations (41) are such as would satisfy all the equations (38) if substituted for $S_1, S_2, \ldots S_n$; and that this is the only condition which these coefficients must satisfy, except that the $n - r$ sets of coefficients shall be independent, i. e., shall be such as to form independent equations; and that this relation between the coefficients of the two sets of equations is a reciprocal one.

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Now, using Lagrange's "method of multipliers,"* we will subtract $T\ (\Sigma\,\delta\eta + \Sigma\,D\eta) - P\,(\Sigma\,\delta v + \Sigma\,Dv)$ from the first member of the general condition of equilibrium (37), T and P being constants of which the value is as yet arbitrary. We might proceed in the same way with the remaining equations of condition, but we may obtain the same result more simply in another way. We will first observe that

$$(\Sigma\,\delta m_1 + \Sigma\,Dm_1)\,\mathfrak{S}_1 + (\Sigma\,\delta m_2 + \Sigma\,Dm_2)\,\mathfrak{S}_2 \ldots + (\Sigma\,\delta m_n + \Sigma\,Dm_n)\,\mathfrak{S}_n = 0, \quad (42)$$

which equation would hold identically for any possible values of the quantities in the parentheses, if for r of the letters $\mathfrak{S}_1, \mathfrak{S}_2, \ldots \mathfrak{S}_n$ were substituted their values in terms of the others as derived from equations (38). (Although $\mathfrak{S}_1, \mathfrak{S}_2, \ldots \mathfrak{S}_n$ do not represent abstract quantities, yet the operations necessary for the reduction of linear equations are evidently applicable to equations (38).) Therefore, equation (42) will hold true if for $\mathfrak{S}_1, \mathfrak{S}_2, \ldots \mathfrak{S}_n$ we substitute n numbers which satisfy equations (38). Let $M_1, M_2, \ldots M_n$ be such numbers, i. e., let

$$\left.\begin{array}{l} a_1\,M_1 + a_2\,M_2 \ldots + a_n\,M_n = 0, \\ b_1\,M_1 + b_2\,M_2 \ldots + b_n\,M_n = 0, \\ \quad \text{etc.} \end{array}\right\} r \text{ equations}, \quad (43)$$

then

$$M_1\,(\Sigma\,\delta m_1 + \Sigma\,Dm_1) + M_2\,(\Sigma\,\delta m_2 + \Sigma\,Dm_2) \ldots + M_n\,(\Sigma\,\delta m_n + \Sigma\,Dm_n) = 0. \quad (44)$$

This expression, in which the values of $n - r$ of the constants $M_1, M_2, \ldots M_n$ are still arbitrary, we will also subtract from the first member of the general condition of equilibrium (37), which will then become

$$\begin{aligned} &\Sigma\,D\varepsilon + \Sigma\,(t\,\delta\eta) - \Sigma\,(p\,\delta v) + \Sigma\,(\mu_1\,\delta m_1) \ldots + \Sigma\,(\mu_n\,\delta m_n) \\ &\quad - T\,\Sigma\,\delta\eta + P\,\Sigma\,\delta v - M_1\,\Sigma\,\delta m_1 \ldots + M_n\,\Sigma\,\delta m_n \\ &\quad - T\,\Sigma\,D\eta + P\,\Sigma\,Dv - M_1\,\Sigma\,Dm_1 \ldots - M_n\,\Sigma\,Dm_n \geqq 0. \quad (45) \end{aligned}$$

That is, having assigned to $T, P, M_1, M_2, \ldots M_n$ any values consistent with (43), we may assert that it is necessary and sufficient for equilibrium that (45) shall hold true for any variations in the state of the system consistent with the equations of condition (39), (40), (41). But it will always be possible, in case of equilibrium, to assign such values to $T, P, M_1, M_2, \ldots M_n$, without violating equations (43),

* On account of the sign $\geqq$ in (37), and because some of the variations are incapable of negative values, the successive steps in the reasoning will be developed at greater length than would be otherwise necessary.

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that (45) shall hold true for all variations in the state of the system and in the quantities of the various substances composing it, even though these variations are not consistent with the equations of condition (39), (40), (41). For, when it is not possible to do this, it must be possible by applying (45) to variations in the system not necessarily restricted by the equations of condition (39), (40), (41) to obtain conditions in regard to T, P, M_1, M_2, . . . M_n, some of which will be inconsistent with others or with equations (43). These conditions we will represent by

$$A \geqq 0, \quad B \geqq 0, \quad \text{etc.}, \tag{46}$$

A, B, etc. being linear functions of T, P, M_1, M_2, . . M_n. Then it will be possible to deduce from these conditions a single condition of the form

$$\alpha A + \beta B + \text{etc.} \geqq 0, \tag{47}$$

α, β, etc. being positive constants, which cannot hold true consistently with equations (43). But it is evident from the form of (47) that, like any of the conditions (46), it could have been obtained directly from (45) by applying this formula to a certain change in the system (perhaps not restricted by the equations of condition (39), (40), (41)). Now as (47) cannot hold true consistently with eqs. (43), it is evident, in the first place, that it cannot contain T or P, therefore in the change in the system just mentioned (for which (45) reduces to (47))

$$\Sigma\, \delta\eta + \Sigma\, D\eta = 0, \quad \text{and} \quad \Sigma\, \delta v + \Sigma\, Dv = 0,$$

so that the equations of condition (39) and (40) are satisfied. Again, for the same reason, the homogeneous function of the first degree of M_1, M_2, . . . M_n in (47) must be one of which the value is fixed by eqs. (43). But the value thus fixed can only be zero, as is evident from the form of these equations. Therefore

$$(\Sigma\, \delta m_1 + \Sigma\, Dm_1)\, M_1 + (\Sigma\, \delta m_2 + \Sigma\, Dm_2)\, M_2 \;\; . \; . \; . \\ + (\Sigma\, \delta m_n + \Sigma\, Dm_n)\, M_n = 0 \tag{48}$$

for any values of M_1, M_2 . . . M_n which satisfy eqs. (43), and therefore

$$(\Sigma\, \delta m_1 + \Sigma\, Dm_1)\, \mathfrak{S}_1 + (\Sigma\, \delta m_2 + \Sigma\, Dm_2)\, \mathfrak{S}_2 \;\; . \; . \; . \\ + (\Sigma\, \delta m_n + \Sigma\, Dm_n)\, \mathfrak{S}_n = 0 \tag{49}$$

for any numerical values of $\mathfrak{S}_1$, $\mathfrak{S}_2$, . . . $\mathfrak{S}_n$ which satisfy eqs. (38). This equation (49) will therefore hold true, if for r of the letters $\mathfrak{S}$, $\mathfrak{S}_2$, . . . $\mathfrak{S}_n$ we substitute their values in terms of the others taken from eqs. (38), and therefore it will hold true when we use

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$\mathfrak{S}_1, \mathfrak{S}_2, \ldots \mathfrak{S}_n$, as before, to denote the units of the various components. Thus understood, the equation expresses that the values of the quantities in the parentheses are such as are consistent with the equations of condition (41). The change in the system, therefore, which we are considering, is not one which violates any of the equations of condition, and as (45) does not hold true for this change, and for all values of $T, P, M_1, M_2, \ldots M_n$ which are consistent with eqs. (43), the state of the system cannot be one of equilibrium. Therefore it is necessary, and it is evidently sufficient for equilibrium, that it shall be possible to assign to $T, P, M_1, M_2, \ldots M_n$ such values, consistent with eqs. (43), that the condition (45) shall hold true for any change in the system irrespective of the equations of condition (39), (40), (41).

For this it is necessary and sufficient that

$$t = T, \qquad p = P, \tag{50}$$

$$\mu_1\, \delta m_1 \geqq M_1\, \delta m_1, \quad \mu_2\, \delta m_2 \geqq M_2\, \delta m_2, \ \ldots \quad \mu_n\, \delta m_n \geqq M_n\, \delta m_n \tag{51}$$

for each of the *original parts* as previously defined, and that

$$D\varepsilon - T\,D\eta + P\,Dv - M_1\,Dm_1 - M_2\,Dm_2 \ldots - M_n\,Dm_n \geqq 0, \tag{52}$$

for each of the *new parts* as previously defined. If to these conditions we add equations (43), we may treat $T, P, M_1, M_2, \ldots M_n$ simply as unknown quantities to be eliminated.

In regard to conditions (51), it will be observed that if a substance, S_1, is an actual component of the part of the given mass distinguished by a single accent, $\delta m_1'$ may be either positive or negative, and we shall have $\mu_1' = M_1$; but if S_1 is only a possible component of that part, $\delta m_1'$ will be incapable of a negative value, and we will have $\mu_1' \geqq M_1$.

The formulæ (50), (51), and (43) express the same particular conditions of equilibrium which we have before obtained by a less general process. It remains to discuss (52). This formula must hold true of any infinitesimal mass in the system in its varied state which is not approximately homogeneous with any of the surrounding masses, the expressions $D\varepsilon, D\eta, Dv, Dm_1, Dm_2, \ldots Dm_n$ denoting the energy, entropy, and volume of this infinitesimal mass, and the quantities of the substances $S_1, S_2, \ldots S_n$ which we regard as composing it, (not necessarily as *independently* variable components). If there is more than one way in which this mass may be considered as composed of these substances, we may choose whichever is most convenient. Indeed it follows directly from the relations existing between $M_1, M_2, \ldots$ and M_n that the result would be the same in

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any case. Now, if we assume that the values of $D\varepsilon$, $D\eta$, Dv, Dm_1, $Dm_2, \ldots Dm_n$ are proportional to the values of ε, η, v, m_1, $m_2, \ldots m_n$ for any large homogeneous mass of similar composition, and of the same temperature and pressure, the condition is equivalent to this, that

$$\varepsilon - T\eta + Pv - M_1 m_1 - M_2 m_2 \ldots - M_n m_n \geqq 0 \qquad (53)$$

for any large homogeneous body which can be formed out of the substances $S_1, S_2 \ldots S_n$.

But the validity of this last transformation cannot be admitted without considerable limitation. It is assumed that the relation between the energy, entropy, volume, and the quantities of the different components of a very small mass surrounded by substances of different composition and state is the same as if the mass in question formed a part of a large homogeneous body. We started, indeed, with the assumption that we might neglect the part of the energy, etc., depending upon the surfaces separating heterogeneous masses. Now, in many cases, and for many purposes, as, in general, when the masses are large, such an assumption is quite legitimate, but in the case of these masses which are formed within or among substances of different nature or state, and which at their first formation must be infinitely small, the same assumption is evidently entirely inadmissible, as the surfaces must be regarded as infinitely large in proportion to the masses. We shall see hereafter what modifications are necessary in our formulæ in order to include the parts of the energy, etc., which are due to the surfaces, but this will be on the assumption, which is usual in the theory of capillarity, that the radius of curvature of the surfaces is large in proportion to the radius of sensible molecular action, and also to the thickness of the lamina of matter at the surface which is not (sensibly) homogeneous in all respects with either of the masses which it separates. But although the formulæ thus modified will apply with sensible accuracy to masses (occurring within masses of a different nature) much smaller than if the terms relating to the surfaces were omitted, yet their failure when applied to masses infinitely small in all their dimensions is not less absolute.

Considerations like the foregoing might render doubtful the validity even of (52) as the necessary and sufficient condition of equilibrium in regard to the formation of masses not approximately homogeneous with those previously existing, when the conditions of equilibrium between the latter are satisfied, unless it is shown that in establishing this formula there have been no quantities neglected relating to the

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mutual action of the new and the original parts, which can affect the result. It will be easy to give such a meaning to the expressions $D\varepsilon$, $D\eta$, Dv, Dm_1, Dm_2, . . . Dm_n that this shall be evidently the case. It will be observed that the quantities represented by these expressions have not been perfectly defined. In the first place, we have no right to assume the existence of any surface of absolute discontinuity to divide the new parts from the original, so that the position given to the dividing surface is to a certain extent arbitrary. Even if the surface separating the masses were determined, the energy to be attributed to the masses separated would be partly arbitrary, since a part of the total energy depends upon the mutual action of the two masses. We ought perhaps to consider the case the same in regard to the entropy, although the entropy of a system never depends upon the mutual relations of parts at sensible distances from one another. Now the condition (52) will be valid if the quantities $D\varepsilon$, $D\eta$, Dv, Dm_1, Dm_2 . . . Dm_n are so defined that none of the assumptions which have been made, tacitly or otherwise, relating to the formation of these new parts, shall be violated. These assumptions are the following:—that the relation between the variations of the energy, entropy, volume, etc., of any of the original parts is not affected by the vicinity of the new parts; and that the energy, entropy, volume, etc., of the system in its varied state are correctly represented by the sums of the energies, entropies, volumes, etc., of the various parts (original and new), so far at least as any of these quantities are determined or affected by the formation of the new parts. We will suppose $D\varepsilon$, $D\eta$, Dv, Dm_1, Dm_2 . . . Dm_n to be so defined that these conditions shall not be violated. This may be done in various ways. We may suppose that the position of the surfaces separating the new and the original parts has been fixed in any suitable way. This will determine the space and the matter belonging to the parts separated. If this does not determine the division of the entropy, we may suppose this determined in any suitable arbitrary way. Thus we may suppose the total energy in and about any new part to be so distributed that equation (12) as applied to the original parts shall not be violated by the formation of the new parts. Or, it may seem more simple to suppose that the imaginary surface which divides any new part from the original is so placed as to include all the matter which is affected by the vicinity of the new formation, so that the part or parts which we regard as original may be left homogeneous in the strictest sense, including uniform *densities of energy and entropy*, up to the very

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bounding surface. The homogeneity of the new parts is of no consequence, as we have made no assumption in that respect. It may be doubtful whether we can consider the new parts, *as thus bounded*, to be infinitely small even in their earliest stages of development. But if they are not infinitely small, the only way in which this can affect the validity of our formulæ will be that in virtue of the equations of condition, i. e., in virtue of the evident necessities of the case, finite variations of the energy, entropy, volume, etc., of the original parts will be caused, to which it might seem that equation (12) would not apply. But if the nature and state of the mass be not varied, equation (12) will hold true of finite differences. (This appears at once, if we integrate the equation under the above limitation.) Hence, the equation will hold true for finite differences, provided that the nature and state of the mass be infinitely little varied. For the differences may be considered as made up of two parts, of which the first are for a constant nature and state of the mass, and the second are infinitely small. We may therefore regard the new parts to be bounded as supposed without prejudice to the validity of any of our results.

The condition (52) understood in either of these ways (or in others which will suggest themselves to the reader) will have a perfectly definite meaning, and will be valid as the necessary and sufficient condition of equilibrium in regard to the formation of new parts, when the conditions of equilibrium in regard to the original parts, (50), (51), and (43), are satisfied.

In regard to the condition (53), it may be shown that with (50), (51), and (43) it is always sufficient for equilibrium. To prove this, it is only necessary to show that when (50), (51), and (43) are satisfied, and (52) is not, (53) will also not be satisfied.

We will first observe that an expression of the form

$$-\varepsilon + T\eta - Pv + M_1 m_1 + M_2 m_2 \ldots + M_n m_n \qquad (54)$$

denotes the work obtainable by the formation (by a reversible process) of a body of which ε, η, v, m_1, m_2, . . . m_n are the energy, entropy, volume, and the quantities of the components, within a medium having the pressure P, the temperature T, and the potentials M_1, M_2, . . . M_n. (The medium is supposed so large that its properties are not sensibly altered in any part by the formation of the body.) For ε is the energy of the body formed, and the remaining terms represent (as may be seen by applying equation (12) to the medium) the decrease of the energy of the medium, if, after the

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formation of the body, the joint entropy of the medium and the body, their joint volumes and joint quantities of matter, were the same as the entropy, etc., of the medium before the formation of the body. This consideration may convince us that for any given finite values of v and of T, P, M_1, etc. this expression cannot be infinite when ε, η, m_1, etc. are determined by any real body, whether homogeneous or not, (but of the given volume), even when T, P, M_1, etc. do not represent the values of the temperature, pressure, and potentials of any real substance. (If the substances S_1, S_2, . . . S_n are all actual components of any homogeneous part of the system of which the equilibrium is discussed, that part will afford an example of a body having the temperature, pressure, and potentials of the medium supposed.)

Now by integrating equation (12) on the supposition that the nature and state of the mass considered remain unchanged, we obtain the equation

$$\varepsilon = t\eta - pv + \mu_1 m_1 + \mu_2 m_2 \ . \ . \ . \ + \mu_n m_n \tag{55}$$

which will hold true of any homogeneous mass whatever. Therefore for any one of the original parts, by (50) and (51),

$$\varepsilon - T\eta + Pv - M_1 m_1 - M_2 m_2 \ . \ . \ . \ - M_n m_n = 0. \tag{56}$$

If the condition (52) is not satisfied in regard to all possible new parts, let N be a new part occurring in an original part O, for which the condition is not satisfied. It is evident that the value of the expression

$$\varepsilon - T\eta + Pv - M_1 m_1 - M_2 m_2 \ . \ . \ . \ - M_n m_n \tag{57}$$

applied to a mass like O including some very small masses like N, will be negative, and will decrease if the number of these masses like N is increased, until there remains within the whole mass no portion of any sensible size without these masses like N, which, it will be remembered, have no sensible size. But it cannot decrease without limit, as the value of (54) cannot become infinite. Now we need not inquire whether the least value of (57) (for constant values of T, P, M_1, M_2, . . . M_n) would be obtained by excluding entirely the mass like O, and filling the whole space considered with masses like N, or whether a certain mixture would give a smaller value,—it is certain that the least possible value of (57) per unit of volume, and that a negative value, will be realized by a mass having a certain homogeneity. If the new part N for which the condition (52) is not satisfied occurs between two different original parts O' and O'', the argument need not be essentially varied. We may consider the

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value of (57) for a body consisting of masses like O' and O'' separated by a lamina N. This value may be decreased by increasing the extent of this lamina, which may be done within a given volume by giving it a convoluted form; and it will be evident, as before, that the least possible value of (57) will be for a homogeneous mass, and that the value will be negative. And such a mass will be not merely an ideal combination, but a body capable of existing, for as the expression (57) has for this mass in the state considered its least possible value per unit of volume, the energy of the mass included in a unit of volume is the least possible for the same matter with the same entropy and volume,—hence, if confined in a non-conducting vessel, it will be in a state of not unstable equilibrium. Therefore when (50), (51), and (43) are satisfied, if the condition (52) is not satisfied in regard to all possible new parts, there will be some homogeneous body which can be formed out of the substances $S_1, S_2, \ldots S_n$ which will not satisfy condition (53).

Therefore, if the initially existing masses satisfy the conditions (50), (51), and (43), and condition (53) is satisfied by every homogeneous body which can be formed out of the given matter, there will be equilibrium.

On the other hand, (53) is not a necessary condition of equilibrium. For we may easily conceive that the condition (52) shall hold true (for any very small formations within or between any of the given masses), while the condition (53) is not satisfied (for all large masses formed of the given matter), and experience shows that this is very often the case. Supersaturated solutions, superheated water, etc., are familiar examples. Such an equilibrium will, however, be *practically* unstable. By this is meant that, although, strictly speaking, an infinitely small disturbance or change may not be sufficient to destroy the equilibrium, yet a very small change in the initial state, perhaps a circumstance which entirely escapes our powers of perception, will be sufficient to do so. The presence of a small portion of the substance for which the condition (53) does not hold true, is sufficient to produce this result, when this substance forms a variable component of the original homogeneous masses. In other cases, when, if the new substances are formed at all, different kinds must be formed simultaneously, the initial presence of the different kinds, and that in immediate proximity, may be necessary.

It will be observed, that from (56) and (53) we can at once obtain (50) and (51), viz., by applying (53) to bodies differing infinitely little from the various homogeneous parts of the given mass. There-

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fore, the condition (56) (relating to the various homogeneous parts of the given mass) and (53) (relating to any bodies which can be formed of the given matter) with (43) are always sufficient for equilibrium, and always necessary for an equilibrium which shall be practically stable. And, if we choose, we may get rid of limitation in regard to equations (43). For, if we compare these equations with (38), it is easy to see that it is always immaterial, in applying the tests (56) and (53) to any body, how we consider it to be composed. Hence, in applying these tests, we may consider all bodies to be composed of the *ultimate* components of the given mass. Then the terms in (56) and (53) which relate to other components than these will vanish, and we need not regard the equations (43). Such of the constants M_1, M_2 . . . M_n as relate to the ultimate compo ponents, may be regarded, like T and P, as unknown quantities subject only to the conditions (56) and (53).

These two conditions, which are sufficient for equilibrium and necessary for a practically stable equilibrium, may be united in one, viz., (if we choose the ultimate components of the given mass for the component substances to which m_1, m_2, . . . m_n relate) that it shall be possible to give such values to the constants T, P, M_1, M_2, . . . M_n in the expression (57) that the value of the expression for each of the homogeneous parts of the mass in question shall be as small as for any body whatever made of the same components.

Effect of Solidity of any Part of the given Mass.

If any of the homogeneous masses of which the equilibrium is in question are solid, it will evidently be proper to treat the proportion of their components as invariable in the application of the criterion of equilibrium, even in the case of *compounds of variable proportions*, i. e., even when bodies can exist which are compounded in proportions infinitesimally varied from those of the solids considered. (Those solids which are capable of absorbing fluids form of course an exception, so far as their fluid components are concerned.) It is true that a solid may be increased by the formation of new solid matter on the surface where it meets a fluid, which is not homogeneous with the previously existing solid, but such a deposit will properly be treated as a distinct part of the system, (viz., as one of the parts which we have called *new*). Yet it is worthy of notice that if a homogeneous solid which is a compound of variable proportions is in contact and equilibrium with a fluid, and the actual components of the solid (considered as of variable composition) are also actual com-

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ponents of the fluid, and the condition (53) is satisfied in regard to all bodies which can be formed out of the actual components of the fluid, (which will always be the case unless the fluid is practically unstable,) all the conditions will hold true of the solid, which would be necessary for equilibrium if it were fluid.

This follows directly from the principles stated on the preceding pages. For in this case the value of (57) will be zero as determined either for the solid or for the fluid considered with reference to their ultimate components, and will not be negative for any body whatever which can be formed of these components; and these conditions are sufficient for equilibrium independently of the solidity of one of the masses. Yet the point is perhaps of sufficient importance to demand a more detailed consideration.

Let $S_a, \ldots S_g$ be the actual components of the solid, and $S_h, \ldots S_k$ its possible components (which occur as actual components in the fluid); then, considering the proportion of the components of the solid as variable, we shall have for this body by equation (12)

$$d\varepsilon' = t\,d\eta' - p'\,dv' + \mu_a'\,dm_a' \ldots + \mu_g'\,dm_g' + \mu_h'\,dm_h' \ldots + \mu_k'\,dm_k'. \tag{58}$$

By this equation the potentials $\mu_a' \ldots \mu_k'$ are perfectly defined. But the differentials $dm_a' \ldots dm_k'$, considered as independent, evidently express variations which are not *possible* in the sense required in the criterion of equilibrium. We might, however, introduce them into the general condition of equilibrium, if we should express the dependence between them by the proper equations of condition. But it will be more in accordance with our method hitherto, if we consider the solid to have only a single independently variable component S_z, of which the nature is represented by the solid itself. We may then write

$$\delta\varepsilon' = t'\,\delta\eta' - p'\,\delta v' + \mu_z'\,\delta m_z'. \tag{59}$$

In regard to the relation of the potential μ_z' to the potentials occurring in equation (58) it will be observed, that as we have by integration of (58) and (59)

$$\varepsilon' = t'\,\eta' - p'\,v' + \mu_a'\,m_a' \ldots + \mu_g'\,m_g', \tag{60}$$

and

$$\varepsilon' = t'\,\eta' - p'\,v' + \mu_z'\,m_z'; \tag{61}$$

therefore

$$\mu_z'\,m_z' = \mu_a'\,m_a' \ldots + \mu_g'\,m_g'. \tag{62}$$

Now, if the fluid has besides $S_a \ldots S_g$ and $S_h \ldots S_k$ the actual components $S_l \ldots S_n$, we may write for the fluid

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$$\delta\varepsilon'' = t''\,\delta\eta'' - p''\,\delta v'' + \mu_a''\,\delta m_a'' \ldots + \mu_g''\,\delta m_g'' + \mu_h''\,\delta m_h'' \ldots + \mu_k''\,\delta m_k'' + \mu_l''\,\delta m_l'' \ldots + \mu_n''\,\delta m_n'', \quad (63)$$

and as by supposition

$$m_x'\,\mathfrak{S}_x = m_a'\,\mathfrak{S}_a \ldots + m_g'\,\mathfrak{S}_g \quad (64)$$

equations (43), (50), and (51) will give in this case on elimination of the constants T, P, etc.,

$$t' = t'', \quad p' = p'', \quad (65)$$

and

$$m_x'\,\mu_x' = m_a'\,\mu_a'' \ldots + m_g'\,\mu_g''. \quad (66)$$

Equations (65) and (66) may be regarded as expressing the conditions of equilibrium between the solid and the fluid. The last condition may also, in virtue of (62), be expressed by the equation

$$m_a'\,\mu_a' \ldots + m_g'\,\mu_g' = m_a'\,\mu_a'' \ldots + m_g'\,\mu_g''. \quad (67)$$

But if condition (53) holds true of all bodies which can be formed of $S_a \ldots S_g$, $S_h, \ldots S_k$, $S_l \ldots S_n$, we may write for all such bodies

$$\varepsilon - t''\,\eta + p''\,v - \mu_a''\,m_a \ldots - \mu_g''\,m_g - \mu_h''\,m_h \ldots - \mu_k''\,m_k - \mu_l''\,m_l \ldots \mu_n''\,m_n \geqq 0. \quad (68)$$

(In applying this formula to various bodies, it is to be observed that only the values of the unaccented letters are to be determined by the different bodies to which it is applied, the values of the accented letters being already determined by the given fluid.) Now, by (60), (65), and (67), the value of the first member of this condition is zero when applied to the solid in its given state. As the condition must hold true of a body differing infinitesimally from the solid, we shall have

$$d\varepsilon' - t''\,d\eta' + p''\,dv' - \mu_a''\,dm_a' \ldots \mu_g''\,dm_g' - \mu_h''\,dm_h' \ldots - \mu_k''\,dm_k' \geqq 0, \quad (69)$$

or, by equations (58) and (65),

$$(\mu_a' - \mu_a'')\,dm_a' \ldots + (\mu_g' - \mu_g'')\,dm_g' + (\mu_h' - \mu_h'')\,dm_h' \ldots + (\mu_k' - \mu_k'')\,dm_k' \geqq 0. \quad (70)$$

Therefore, as these differentials are all independent,

$$\mu_a' = \mu_a'', \ldots \mu_g' = \mu_g'', \qquad \mu_h' \geqq \mu_h'', \ldots \mu_k' \geqq \mu_k''; \quad (71)$$

which with (65) are evidently the same conditions which we would have obtained if we had neglected the fact of the solidity of one of the masses.

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We have supposed the solid to be homogeneous. But it is evident that in any case the above conditions must hold for every separate point where the solid meets the fluid. Hence, the temperature and pressure and the potentials for all the actual components of the solid must have a constant value in the solid at the surface where it meets the fluid. Now, these quantities are determined by the nature and state of the solid, and exceed in number the independent variations of which its nature and state are capable. Hence, if we reject as improbable the supposition that the nature or state of a body can vary without affecting the value of any of these quantities, we may conclude that a solid which varies (continuously) in nature or state at its surface cannot be in equilibrium with a stable fluid which contains, as independently variable components, the variable components of the solid. (There may be, however, in equilibrium with the same stable fluid, a *finite* number of different solid bodies, composed of the variable components of the fluid, and having their nature and state completely determined by the fluid.)*

Effect of Additional Equations of Condition.

As the equations of condition, of which we have made use, are such as always apply to matter enclosed in a rigid, impermeable, and non-conducting envelop, the particular conditions of equilibrium which we have found will always be sufficient for equilibrium. But the number of conditions necessary for equilibrium, will be diminished, in a case otherwise the same, as the number of equations of condition is increased. Yet the problem of equilibrium which has been treated will sufficiently indicate the method to be pursued in all cases and the general nature of the results.

It will be observed that the position of the various homogeneous parts of the given mass, which is otherwise immaterial, may determine the existence of certain equations of condition. Thus, when different parts of the system in which a certain substance is a variable component are entirely separated from one another by parts of which this substance is not a component, the quantity of this substance will be invariable for each of the parts of the system which are thus separated, which will be easily expressed by equations of condition. Other equations of condition may arise from the passive forces (or resistances to change) inherent in the given masses. In the prob-

* The solid has been considered as subject only to isotropic stresses. The effect of other stresses will be considered hereafter.

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lem which we are next to consider there are equations of condition due to a cause of a different nature.

Effect of a Diaphragm (Equilibrium of Osmotic Forces).

If the given mass, enclosed as before, is divided into two parts, each of which is homogeneous and fluid, by a diaphragm which is capable of supporting an excess of pressure on either side, and is permeable to some of the components and impermeable to others, we shall have the equations of condition

$$\delta\eta' + \delta\eta'' = 0, \tag{72}$$

$$\delta v' = 0, \quad \delta v'' = 0, \tag{73}$$

and for the components which cannot pass the diaphragm

$$\delta m_a' = 0, \quad \delta m_a'' = 0, \quad \delta m_b' = 0, \quad \delta m_b'' = 0, \quad \text{etc.}, \tag{74}$$

and for those which can

$$\delta m_h' + \delta m_h'' = 0, \quad \delta m_i' + \delta m_i'' = 0, \quad \text{etc.} \tag{75}$$

With these equations of condition, the general condition of equilibrium (see (15)) will give the following particular conditions:

$$t' = t'', \tag{76}$$

and for the components which can pass the diaphragm, if actual components of both masses,

$$\mu_h' = \mu_h'', \quad \mu_i' = \mu_i'', \quad \text{etc.}, \tag{77}$$

but not $$p' = p'',$$

nor $$\mu_a' = \mu_a'', \quad \mu_b' = \mu_b'', \quad \text{etc.}$$

Again, if the diaphragm is permeable to the components in certain proportions only, or in proportions not entirely determined yet subject to certain conditions, these conditions may be expressed by equations of condition, which will be linear equations between $\delta m_1'$, $\delta m_2'$, etc., and if these be known the deduction of the particular conditions of equilibrium will present no difficulties. We will however observe that if the components S_1, S_2, etc. (being actual components on each side) can pass the diaphragm simultaneously in the proportions a_1, a_2, etc. (without other resistances than such as vanish with the velocity of the current), values proportional to a_1, a_2, etc. are possible for $\delta m_1'$, $\delta m_2'$, etc. in the general condition of equilibrium, $\delta m_1''$, $\delta m_2''$, etc. having the same values taken negatively, so that we shall have for one particular condition of equilibrium

$$a_1\,\mu_1' + a_2\,\mu_2' + \text{etc.} = a_1\,\mu_1'' + a_2\,\mu_2'' + \text{etc.} \tag{78}$$

There will evidently be as many independent equations of this form

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as there are independent combinations of the elements which can pass the diaphragm.

These conditions of equilibrium do not of course depend in any way upon the supposition that the volume of each fluid mass is kept constant, if the diaphragm is in any case supposed immovable. In fact, we may easily obtain the same conditions of equilibrium, if we suppose the volumes variable. In this case, as the equilibrium must be preserved by forces acting upon the external surfaces of the fluids, the variation of the energy of the sources of these forces must appear in the general condition of equilibrium, which will be

$$\delta\varepsilon' + \delta\varepsilon'' + P'\,\delta v' + P''\,\delta v'' \geqq 0, \tag{79}$$

P' and P'' denoting the external forces per unit of area. (Compare (14).) From this condition we may evidently derive the same internal conditions of equilibrium as before, and in addition the external conditions

$$p' = P', \quad p'' = P''. \tag{80}$$

In the preceding paragraphs it is assumed that the permeability of the diaphragm is perfect, and its impermeability absolute, i. e., that it offers no resistance to the passage of the components of the fluids in certain proportions, except such as vanishes with the velocity, and that in other proportions the components cannot pass at all. How far these conditions are satisfied in any particular case is of course to be determined by experiment.

If the diaphragm is permeable to all the n components without restriction, the temperature and the potentials for all the components must be the same on both sides. Now, as one may easily convince himself, a mass having n components is capable of only $n+1$ independent variations in nature and state. Hence, if the fluid on one side of the diaphragm remains without change, that on the other side cannot (in general) vary in nature or state. Yet the pressure will not necessarily be the same on both sides. For, although the pressure is a function of the temperature and the n potentials, it may be a many-valued function (or any one of several functions) of these variables. But when the pressures are different on the two sides, the fluid which has the less pressure will be *practically unstable*, in the sense in which the term has been used on page 133. For

$$\varepsilon'' - t''\eta'' + p''v'' - \mu_1''m_1'' - \mu_2''m_2'' \ldots - \mu_n''m_n'' = 0, \tag{81}$$

as appears from equation (12) if integrated on the supposition that the nature and state of the mass remain unchanged. Therefore, if $p' < p''$ while $t' = t''$, $\mu_1' = \mu_1''$, etc.,

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$$\varepsilon'' - t'\eta'' + p'v'' - \mu_1' m_1'' - \mu_2' m_2'' \ldots - \mu_n' m_n'' < 0. \quad (82)$$

This relation indicates the instability of the fluid to which the single accents refer. (See page 133.)

But independently of any assumption in regard to the permeability of the diaphragm, the following relation will hold true in any case in which each of the two fluid masses may be regarded as uniform throughout in nature and state. Let the character D be used with the variables which express the nature, state, and quantity of the fluids to denote the increments of the values of these quantities actually occurring in a time either finite or infinitesimal. Then, as the heat received by the two masses cannot exceed $t' \mathrm{D}\eta' + t'' \mathrm{D}\eta''$, and as the increase of their energy is equal to the difference of the heat they receive and the work they do,

$$\mathrm{D}\varepsilon' + \mathrm{D}\varepsilon'' \leqq t' \mathrm{D}\eta' + t'' \mathrm{D}\eta'' - p' \mathrm{D}v' - p'' \mathrm{D}v'', \quad (83)$$

i. e., by (12),

$$\mu_1' \mathrm{D}m_1' + \mu_1'' \mathrm{D}m_1'' + \mu_2' \mathrm{D}m_2' + \mu_2'' \mathrm{D}m_2'' + \text{etc.} \leqq 0, \quad (84)$$

or

$$(\mu_1'' - \mu_1') \mathrm{D}m_1'' + (\mu_2'' - \mu_2') \mathrm{D}m_2'' + \text{etc.} \leqq 0. \quad (85)$$

It is evident that the sign $=$ holds true only in the limiting case in which no motion takes place.

DEFINITION AND PROPERTIES OF FUNDAMENTAL EQUATIONS.

The solution of the problems of equilibrium which we have been considering has been made to depend upon the equations which express the relations between the energy, entropy, volume, and the quantities of the various components, for homogeneous combinations of the substances which are found in the given mass. The nature of such equations must be determined by experiment. As, however, it is only *differences* of energy and of entropy that can be measured, or indeed, that have a physical meaning, the values of these quantities are so far arbitrary, that we may choose independently for each simple substance the state in which its energy and its entropy are both zero. The values of the energy and the entropy of any compound body in any particular state will then be fixed. Its energy will be the sum of the work and heat expended in bringing its components from the states in which their energies and their entropies are zero into combination and to the state in question; and its entropy is the value of the integral $\int \frac{dQ}{t}$ for any *reversible* process

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by which that change is effected (dQ denoting an element of the heat communicated to the matter thus treated, and t the temperature of the matter receiving it). In the determination both of the energy and of the entropy, it is understood that at the close of the process, all bodies which have been used, other than those to which the determinations relate, have been restored to their original state, with the exception of the sources of the work and heat expended, which must be used only as such sources.

We know, however, *a priori*, that if the quantity of any homogeneous mass containing n independently variable components varies and not its nature or state, the quantities $\varepsilon, \eta, v, m_1, m_2, \ldots m_n$ will all vary in the same proportion; therefore it is sufficient if we learn from experiment the relation between all but any one of these quantities for a given constant value of that one. Or, we may consider that we have to learn from experiment the relation subsisting between the $n+2$ ratios of the $n+3$ quantities $\varepsilon, \eta, v, m_1, m_2, \ldots m_n$. To fix our ideas we may take for these ratios $\frac{\varepsilon}{v}, \frac{\eta}{v}, \frac{m_1}{v}, \frac{m_2}{v}$, etc., that is, the separate densities of the components, and the ratios $\frac{\varepsilon}{v}$ and $\frac{\eta}{v}$, which may be called the *densities of energy and entropy.* But when there is but one component, it may be more convenient to choose $\frac{\varepsilon}{m}, \frac{\eta}{m}, \frac{v}{m}$ as the three variables. In any case, it is only a function of $n+1$ independent variables, of which the form is to be determined by experiment.

Now if ε is a known function of $\eta, v, m_1, m_2, \ldots m_n$, as by equation (12)

$$d\varepsilon = t\,d\eta - p\,dv + \mu_1\,dm_1 + \mu_2\,dm_2 \ldots + \mu_n\,dm_n, \qquad (86)$$

$t, p, \mu_1, \mu_2, \ldots \mu_n$ are functions of the same variables, which may be derived from the original function by differentiation, and may therefore be considered as known functions. This will make $n+3$ independent known relations between the $2n+5$ variables, $\varepsilon, \eta, v, m_1, m_2, \ldots m_n, t, p, \mu_1, \mu_2, \ldots \mu_n$. These are all that exist, for of these variables, $n+2$ are evidently independent. Now upon these relations depend a very large class of the properties of the compound considered,—we may say in general, all its thermal, mechanical, and chemical properties, so far as *active tendencies* are concerned, in cases in which the form of the mass does not require consideration. A single equation from which all these relations may

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be deduced we will call a *fundamental equation* for the substance in question. We shall hereafter consider a more general form of the fundamental equation for solids, in which the pressure at any point is not supposed to be the same in all directions. But for masses subject only to isotropic stresses an equation between $\varepsilon, \eta, v, m_1, m_2, \ldots m_n$ is a fundamental equation. There are other equations which possess this same property.*

Let

$$\psi = \varepsilon - t\eta, \tag{87}$$

then by differentiation and comparison with (86) we obtain

$$d\psi = -\eta\, dt - p\, dv + \mu_1\, dm_1 + \mu_2\, dm_2 \ldots + \mu_n\, dm_n. \tag{88}$$

If, then, ψ is known as a function of $t, v, m_1, m_2, \ldots m_n$, we can find $\eta, p, \mu_1, \mu_2, \ldots \mu_n$ in terms of the same variables. If we then substitute for ψ in our original equation its value taken from eq. (87), we shall have again $n+3$ independent relations between the same $2n+5$ variables as before.

Let

$$\chi = \varepsilon + p\, v, \tag{89}$$

then by (86),

$$d\chi = t\, d\eta + v\, dp + \mu_1\, dm_1 + \mu_2\, dm_2 \ldots + \mu_n\, dm_n. \tag{90}$$

If, then, χ be known as a function of $\eta, p, m_1, m_2, \ldots m_n$, we can find $t, v, \mu_1, \mu_2, \ldots \mu_n$ in terms of the same variables. By eliminating χ, we may obtain again $n+3$ independent relations between the same $2n+5$ variables as at first.

Let

$$\zeta = \varepsilon - t\eta + p\, v, \tag{91}$$

then, by (86)

$$d\zeta = -\eta\, dt + v\, dp + \mu_1\, dm_1 + \mu_2\, dm_2 \ldots + \mu_n\, dm_n. \tag{92}$$

If, then, ζ is known as a function of $t, p, m_1, m_2, \ldots m_n$, we can

* M. Massieu (Comptes Rendus, T. lxix, 1869, p. 858 and p. 1057) has shown how all the properties of a fluid "which are considered in thermodynamics" may be deduced from a single function, which he calls a characteristic function of the fluid considered. In the papers cited, he introduces two different functions of this kind; viz., a function of the temperature and volume, which he denotes by ψ, the value of which in our notation would be $\frac{-\varepsilon + t\eta}{t}$ or $\frac{-\psi}{t}$; and a function of the temperature and pressure, which he denotes by ψ', the value of which in our notation would be $\frac{-\varepsilon + t\eta - pv}{t}$ or $\frac{-\zeta}{t}$. In both cases he considers a constant quantity (one kilogram) of the fluid, which is regarded as invariable in composition.

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find η, v, μ_1, μ_2, . . . μ_n in terms of the same variables. By eliminating ζ, we may obtain again $n + 3$ independent relations between the same $2n + 5$ variables as at first.

If we integrate (86), supposing the quantity of the compound substance considered to vary from zero to any finite value, its nature and state remaining unchanged, we obtain

$$\varepsilon = t\eta - pv + \mu_1 m_1 + \mu_2 m_2 \ldots + \mu_n m_n, \tag{93}$$

and by (87), (89), (91)

$$\psi = -pv + \mu_1 m_1 + \mu_2 m_2 \ldots + \mu_n m_n, \tag{94}$$

$$\chi = t\eta + \mu_1 m_1 + \mu_2 m_2 \ldots + \mu_n m_n, \tag{95}$$

$$\zeta = \mu_1 m_1 + \mu_2 m_2 \ldots + \mu_n m_n. \tag{96}$$

The last three equations may also be obtained directly by integrating (88), (90), and (92).

If we differentiate (93) in the most general manner, and compare the result with (86), we obtain

$$-v\,dp + \eta\,dt + m_1\,d\mu_1 + m_2\,d\mu_2 \ldots + m_n\,d\mu_n = 0, \tag{97}$$

or

$$dp = \frac{\eta}{v}\,dt + \frac{m_1}{v}\,d\mu_1 + \frac{m_2}{v}\,d\mu_2 \ldots + \frac{m_n}{v}\,d\mu_n = 0. \tag{98}$$

Hence, there is a relation between the $n + 2$ quantities t, p, μ_1, μ_2, . . . μ_n, which, if known, will enable us to find in terms of these quantities all the ratios of the $n + 2$ quantities η, v, m_1, m_2 . . . m_n. With (93), this will make $n + 3$ independent relations between the same $2n + 5$ variables as at first.

Any equation, therefore, between the quantities

$$\varepsilon,\quad \eta,\quad v,\quad m_1,\quad m_2, \ldots m_n, \tag{99}$$

or

$$\psi,\quad t,\quad v,\quad m_1,\quad m_2, \ldots m_n, \tag{100}$$

or

$$\chi,\quad \eta,\quad p,\quad m_1,\quad m_2, \ldots m_n, \tag{101}$$

or

$$\zeta,\quad t,\quad p,\quad m_1,\quad m_2, \ldots m_n, \tag{102}$$

or

$$t,\quad p,\quad \mu_1,\quad \mu_2, \ldots \mu_n, \tag{103}$$

is a fundamental equation, and any such is entirely equivalent to any other.* For any homogeneous mass whatever, considered (in general) as variable in composition, in quantity, and in thermodynamic state, and having n independently variable components, to which

* The distinction between equations which are, and which are not, *fundamental*, in the sense in which the word is here used, may be illustrated by comparing an equation

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the subscript numerals refer, (but not excluding the case in which $n = 1$ and the composition of the body is invariable,) there is a relation between the quantities enumerated in any one of the above sets, from which, if known, with the aid only of *general* principles and relations, we may deduce all the relations subsisting for such a mass between the quantities $\varepsilon, \psi, \chi, \zeta, \eta, v, m_1, m_2, \ldots m_n, t, p, \mu_1, \mu_2, \ldots \mu_n$. It will be observed that, besides the equations which define ψ, χ, and ζ, there is one finite equation, (93), which subsists between these quantities independently of the form of the fundamental equation.

Other sets of quantities might of course be added which possess the same property. The sets (100), (101), (102) are mentioned on account of the important properties of the quantities ψ, χ, ζ, and because the equations (88), (90), (92), like (86), afford convenient definitions of the potentials, viz.,

$$\mu_1 = \left(\frac{d\varepsilon}{dm_1}\right)_{\eta, v, m} = \left(\frac{d\psi}{dm_1}\right)_{t, v, m} = \left(\frac{d\chi}{dm_1}\right)_{\eta, p, m} = \left(\frac{d\zeta}{dm_1}\right)_{t, p, m} \tag{104}$$

etc., where the subscript letters denote the quantities which remain constant in the differentiation, m being written for brevity for all the letters $m_1, m_2, \ldots m_n$ except the one occurring in the denominator. It will be observed that the quantities in (103) are all independent of the quantity of the mass considered, and are those which must, in general, have the same value in contiguous masses in equilibrium.

On the quantities ψ, χ, ζ.

The quantity ψ has been defined for any homogeneous mass by the equation

$$\psi = \varepsilon - t\eta. \tag{105}$$

---

between

$$\varepsilon, \eta, v, m_1, m_2, \ldots m_n$$

with one between

$$\varepsilon, t, v, m_1, m_2, \ldots m_n.$$

As, by (86),

$$t = \left(\frac{d\varepsilon}{d\eta}\right)_{vm},$$

the second equation may evidently be derived from the first. But the first equation cannot be derived from the second; for an equation between

$$\varepsilon, \left(\frac{d\varepsilon}{d\eta}\right)_{vm}, v, m_1, m_2, \ldots m_n$$

is equivalent to one between

$$\left(\frac{d\eta}{d\varepsilon}\right)_{vm}, \varepsilon, v, m_1, m_2, \ldots m_n.$$

which is evidently not sufficient to determine the value of η in terms of the other variables.

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We may extend this definition to any material system whatever which has a uniform temperature throughout.

If we compare two states of the system of the same temperature, we have

$$\psi' - \psi'' = \varepsilon' - \varepsilon'' - t(\eta' - \eta''). \tag{106}$$

If we suppose the system brought from the first to the second of these states without change of temperature and by a reversible process in which W is the work done and Q the heat received by the system, then

$$\varepsilon' - \varepsilon'' = W - Q, \tag{107}$$

and

$$t(\eta'' - \eta') = Q. \tag{108}$$

Hence

$$\psi' - \psi'' = W; \tag{109}$$

and for an infinitely small reversible change in the state of the system, in which the temperature remains constant, we may write

$$-d\psi = dW. \tag{110}$$

Therefore, $-\psi$ is the force function of the system for constant temperature, just as $-\varepsilon$ is the force function for constant entropy. That is, if we consider ψ as a function of the temperature and the variables which express the distribution of the matter in space, for every different value of the temperature $-\psi$ is the different force function required by the system if maintained at that special temperature.

From this we may conclude that when a system has a uniform temperature throughout, the additional conditions which are necessary and sufficient for equilibrium may be expressed by

$$(\delta\psi)_t \geqq 0.^* \tag{111}$$

* This general condition of equilibrium might be used instead of (2) in such problems of equilibrium as we have considered and others which we shall consider hereafter with evident advantage in respect to the brevity of the formulæ, as the limitation expressed by the subscript t in (111) applies to every part of the system taken separately, and diminishes by one the number of independent variations in the state of these parts which we have to consider. The more cumbersome course adopted in this paper has been chosen, among other reasons, for the sake of deducing *all* the particular conditions of equilibrium from one general condition, and of having the quantities mentioned in this general condition such as are most generally used and most simply defined; and because in the longer formulæ as given, the reader will easily see in each case the form which they would take if we should adopt (111) as the general condition of equilibrium, which would be in effect to take the thermal condition of equilibrium for granted, and to seek only the remaining conditions. For example, in the problem treated on pages 116 ff., we would obtain from (111) by (88) a condition precisely like (15), except that the terms $t\,\delta\eta'$, $t\,\delta\eta''$, etc. would be wanting.

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When it is not possible to bring the system from one to the other of the states to which ψ' and ψ'' relate by a reversible process without altering the temperature, it will be observed that it is not necessary for the validity of (107)–(109) that the temperature of the system should remain constant during the reversible process to which W and Q relate, provided that the only source of heat or cold used has the same temperature as the system in its initial or final state. Any external bodies may be used in the process in any way not affecting the condition of reversibility, if restored to their original condition at the close of the process; nor does the limitation in regard to the use of heat apply to such heat as may be restored to the source from which it has been taken.

It may be interesting to show directly the equivalence of the conditions (111) and (2) when applied to a system of which the temperature in the given state is uniform throughout.

If there are any variations in the state of such a system which do not satisfy (2), then for these variations

$$\delta\varepsilon < 0 \quad \text{and} \quad \delta\eta = 0.$$

If the temperature of the system in its varied state is not uniform, we may evidently increase its entropy without altering its energy by supposing heat to pass from the warmer to the cooler parts. And the state having the greatest entropy for the energy $\varepsilon + \delta\varepsilon$ will necessarily be a state of uniform temperature. For this state (regarded as a variation from the original state)

$$\delta\varepsilon < 0 \quad \text{and} \quad \delta\eta > 0.$$

Hence, as we may diminish both the energy and the entropy by cooling the system, there must be a state of uniform temperature for which (regarded as a variation of the original state)

$$\delta\varepsilon < 0 \quad \text{and} \quad \delta\eta = 0.$$

From this we may conclude that for systems of initially uniform temperature condition (2) will not be altered if we limit the variations to such as do not disturb the uniformity of temperature.

Confining our attention, then, to states of uniform temperature, we have by differentiation of (105)

$$\delta\varepsilon - t\,\delta\eta = \delta\psi + \eta\,\delta t. \tag{112}$$

Now there are evidently changes in the system (produced by heating or cooling) for which

$$\delta\varepsilon - t\,\delta\eta = 0 \quad \text{and therefore} \quad \delta\psi + \eta\,\delta t = 0, \tag{113}$$

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neither $\delta\eta$ nor δt having the value zero. This consideration is sufficient to show that the condition (2) is equivalent to

$$\delta\varepsilon - t\,\delta\eta \geqq 0. \tag{114}$$

and that the condition (111) is equivalent to

$$\delta\psi + \eta\,\delta t \geqq 0 \tag{115}$$

and by (112) the two last conditions are equivalent.

In such cases as we have considered on pages 115–137, in which the form and position of the masses of which the system is composed is immaterial, uniformity of temperature and pressure are always necessary for equilibrium, and the remaining conditions, when these are satisfied, may be conveniently expressed by means of the function ζ, which has been defined for a homogeneous mass on page 142, and which we will here define for any mass of uniform temperature and pressure by the same equation

$$\zeta = \varepsilon - t\,\eta + p\,v. \tag{116}$$

For such a mass, the condition of (internal) equilibrium is

$$(\delta\zeta)_{t,p} \geqq 0. \tag{117}$$

That this condition is equivalent to (2) will easily appear from considerations like those used in respect to (111).

Hence, it is necessary for the equilibrium of two contiguous masses identical in composition that the values of ζ as determined for equal quantities of the two masses should be equal. Or, when one of three contiguous masses can be formed out of the other two, it is necessary for equilibrium that the value of ζ for any quantity of the first mass should be equal to the sum of the values of ζ for such quantities of the second and third masses as together contain the same matter. Thus, for the equilibrium of a solution composed of a parts of water and b parts of a salt which is in contact with vapor of water and crystals of the salt, it is necessary that the value of ζ for the quantity $a+b$ of the solution should be equal to the sum of the values of ζ for the quantities a of the vapor and b of the salt. Similar propositions will hold true in more complicated cases. The reader will easily deduce these conditions from the particular conditions of equilibrium given on page 128.

In like manner we may extend the definition of χ to any mass or combination of masses in which the pressure is everywhere the same, using ε for the energy and v for the volume of the whole and setting as before

$$\chi = \varepsilon + p\,v. \tag{118}$$

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If we denote by Q the heat received by the combined masses from external sources in any process in which the pressure is not varied, and distinguish the initial and final states of the system by accents we have

$$\chi'' - \chi' = \varepsilon'' - \varepsilon' + p(v'' - v') = Q. \tag{119}$$

This function may therefore be called the *heat function for constant pressure* (just as the energy might be called the heat function for constant volume), the diminution of the function representing in all cases in which the pressure is not varied the heat given out by the system. In all cases of chemical action in which no heat is allowed to escape the value of χ remains unchanged.

POTENTIALS.

In the definition of the potentials μ_1, μ_2, etc., the energy of a homogeneous mass was considered as a function of its entropy, its volume, and the quantities of the various substances composing it. Then the potential for one of these substances was defined as the differential coefficient of the energy taken with respect to the variable expressing the quantity of that substance. Now, as the manner in which we consider the given mass as composed of various substances is in some degree arbitrary, so that the energy may be considered as a function of various different sets of variables expressing quantities of component substances, it might seem that the above definition does not fix the value of the potential of any substance in the given mass, until we have fixed the manner in which the mass is to be considered as composed. For example, if we have a solution obtained by dissolving in water a certain salt containing water of crystallization, we may consider the liquid as composed of m_S weight-units of the hydrate and m_W of water, or as composed of m_s of the anhydrous salt and m_w of water. It will be observed that the values of m_S and m_s are not the same, nor those of m_W and m_w, and hence it might seem that the potential for water in the given liquid considered as composed of the hydrate and water, viz.,

$$\left(\frac{d\varepsilon}{dm_W}\right)_{\eta,\, v,\, m_S}$$

would be different from the potential for water in the same liquid considered as composed of anhydrous salt and water, viz.,

$$\left(\frac{d\varepsilon}{dm_w}\right)_{\eta,\, v,\, m_s}$$

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The value of the two expressions is, however, the same, for, although m_W is not equal to m_w, we may of course suppose dm_W to be equal to dm_w, and then the numerators in the two fractions will also be equal, as they each denote the increase of energy of the liquid, when the quantity dm_W or dm_w of water is added without altering the entropy and volume of the liquid. Precisely the same considerations will apply to any other case.

In fact, we may give a definition of a potential which shall not presuppose any choice of a particular set of substances as the components of the homogeneous mass considered.

Definition.—If to any homogeneous mass we suppose an infinitesimal quantity of any substance to be added, the mass remaining homogeneous and its entropy and volume remaining unchanged, the increase of the energy of the mass divided by the quantity of the substance added is the *potential* for that substance in the mass considered. (For the purposes of this definition, any chemical element or combination of elements in given proportions may be considered a substance, whether capable or not of existing by itself as a homogeneous body.)

In the above definition we may evidently substitute for entropy, volume, and energy, respectively, either temperature, volume, and the function ψ; or entropy, pressure, and the function χ; or temperature, pressure, and the function ζ. (Compare equation (104).)

In the same homogeneous mass, therefore, we may distinguish the potentials for an indefinite number of substances, each of which has a perfectly determined value.

Between the potentials for different substances in the same homogeneous mass the same equations will subsist as between the units of these substances. That is, if the substances, S_a, S_b, etc., S_k, S_l, etc., are components of any given homogeneous mass, and are such that

$$\alpha\,\mathfrak{S}_a + \beta\,\mathfrak{S}_b + \text{etc.} = \varkappa\,\mathfrak{S}_k + \lambda\,\mathfrak{S}_l + \text{etc.}, \tag{120}$$

$\mathfrak{S}_a$, $\mathfrak{S}_b$, etc., $\mathfrak{S}_k$, $\mathfrak{S}_l$, etc. denoting the units of the several substances, and α, β, etc., $\varkappa$, λ, etc. denoting numbers, then if μ_a, μ_b, etc., μ_k, μ_l, etc. denote the potentials for these substances in the homogeneous mass,

$$\alpha\,\mu_a + \beta\,\mu_b + \text{etc.} = \varkappa\,\mu_k + \lambda\,\mu_l + \text{etc.} \tag{121}$$

To show this, we will suppose the mass considered to be very large. Then, the first number of (121) denotes the increase of the energy of the mass produced by the addition of the matter represented by the first member of (120), and the second member of (121) denotes the

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increase of energy of the same mass produced by the addition of the matter represented by the second member of (120), the entropy and volume of the mass remaining in each case unchanged. Therefore, as the two members of (120) represent the same matter in kind and quantity, the two members of (121) must be equal.

But it must be understood that equation (120) is intended to denote equivalence of the substances represented *in the mass considered*, and not merely chemical identity; in other words, it is supposed that there are no passive resistances to change in the mass considered which prevent the substances represented by one member of (120) from passing into those represented by the other. For example, in respect to a mixture of vapor of water and free hydrogen and oxygen (at ordinary temperatures), we may not write

$$9\,\mathfrak{S}_{Aq} = 1\,\mathfrak{S}_H + 8\,\mathfrak{S}_O,$$

but water is to be treated as an independent substance, and no necessary relation will subsist between the potential for water and the potentials for hydrogen and oxygen.

The reader will observe that the relations expressed by equations (43) and (51) (which are essentially relations between the potentials for actual components in different parts of a mass in a state of equilibrium) are simply those which by (121) would necessary subsist between the same potentials in any homogeneous mass containing as variable components all the substances to which the potentials relate.

In the case of a body of invariable composition, the potential for the single component is equal to the value of ζ for one unit of the body, as appears from the equation

$$\zeta = \mu m \tag{122}$$

to which (96) reduces in this case. Therefore, when $n = 1$, the fundamental equation between the quantities in the set (102) (see page 143) and that between the quantities in (103) may be derived either from the other by simple substitution. But, with this single exception, an equation between the quantities in one of the sets (99)–(103) cannot be derived from the equation between the quantities in another of these sets without differentiation.

Also in the case of a body of variable composition, when all the quantities of the components except one vanish, the potential for that one will be equal to the value of ζ for one unit of the body. We may make this occur for any given composition of the body by

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choosing as one of the components the matter constituting the body itself, so that the value of ζ for one unit of a body may always be considered as a potential. Hence the relations between the values of ζ for contiguous masses given on page 147 may be regarded as relations between potentials.

The two following propositions afford definitions of a potential which may sometimes be convenient.

The potential for any substance in any homogeneous mass is equal to the amount of mechanical work required to bring a unit of the substance by a reversible process from the state in which its energy and entropy are both zero into combination with the homogeneous mass, which at the close of the process must have its original volume, and which is supposed so large as not to be sensibly altered in any part. All other bodies used in the process must by its close be restored to their original state, except those used to supply the work, which must be used only as the source of the work. For, in a reversible process, when the entropies of other bodies are not altered, the entropy of the substance and mass taken together will not be altered. But the original entropy of the substance is zero; therefore the entropy of the mass is not altered by the addition of the substance. Again, the work expended will be equal to the increment of the energy of the mass and substance taken together, and therefore equal, as the original energy of the substance is zero, to the increment of energy of the mass due to the addition of the substance, which by the definition on page 149 is equal to the potential in question.

The potential for any substance in any homogeneous mass is equal to the work required to bring a unit of the substance by a reversible process from a state in which $\psi = 0$ and the temperature is the same as that of the given mass into combination with this mass, which at the close of the process must have the same volume and temperature as at first, and which is supposed so large as not to be sensibly altered in any part. A source of heat or cold of the temperature of the given mass is allowed, with this exception, other bodies are to be used only on the same conditions as before. This may be shown by applying equation (109) to the mass and substance taken together.

The last proposition enables us to see very easily, how the value of the potential is affected by the arbitrary constants involved in the definition of the energy and the entropy of each elementary substance. For we may imagine the substance brought from the state in which $\psi = 0$ and the temperature is the same as that of the given

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mass, first to any specified state of the same temperature, and then into combination with the given mass. In the first part of the process the work expended is evidently represented by the value of ψ for the unit of the substance in the state specified. Let this be denoted by ψ', and let μ denote the potential in question, and W the work expended in bringing a unit of the substance from the specified state into combination with the given mass as aforesaid; then

$$\mu = \psi' + W. \tag{123}$$

Now as the state of the substance for which $\varepsilon = 0$ and $\eta = 0$ is arbitrary, we may simultaneously increase the energies of the unit of the substance in all possible states by any constant C, and the entropies of the substance in all possible states by any constant K. The value of ψ, or $\varepsilon - t\eta$, for any state would then be increased by $C - tK$, t denoting the temperature of the state. Applying this to ψ' in (123) and observing that the last term in this equation is independent of the values of these constants, we see that the potential would be increased by the same quantity $C - tK$, t being the temperature of the mass in which the potential is to be determined.

ON COEXISTENT PHASES OF MATTER.

In considering the different homogeneous bodies which can be formed out of any set of component substances, it will be convenient to have a term which shall refer solely to the composition and thermodynamic state of any such body without regard to its quantity or form. We may call such bodies as differ in composition or state different *phases* of the matter considered, regarding all bodies which differ only in quantity and form as different examples of the same phase. Phases which can exist together, the dividing surfaces being plane, in an equilibrium which does not depend upon passive resistances to change, we shall call *coexistent.*

If a homogeneous body has n independently variable components, the phase of the body is evidently capable of $n + 1$ independent variations. A system of r coexistent phases, each of which has the same n independently variable components is capable of $n + 2 - r$ variations of phase. For the temperature, the pressure, and the potentials for the actual components have the same values in the different phases, and the variations of these quantities are by (97) subject to as many conditions as there are different phases. Therefore, the number of independent variations in the values of these quantities, i. e., the number of independent variations of phase of the system, will be $n + 2 - r$.

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Or, when the r bodies considered have not the same independently variable components, if we still denote by n the number of independently variable components of the r bodies taken as a whole, the number of independent variations of phase of which the system is capable will still be $n+2-r$. In this case, it will be necessary to consider the potentials for more than n component substances. Let the number of these potentials be $n+h$. We shall have by (97), as before, r relations between the variations of the temperature, of the pressure, and of these $n+h$ potentials, and we shall also have by (43) and (51) h relations between these potentials, of the same form as the relations which subsist between the units of the different component substances.

Hence, if $r=n+2$, no variation in the phases (remaining coexistent) is possible. It does not seem probable that r can ever exceed $n+2$. An example of $n=1$ and $r=3$ is seen in the coexistent solid, liquid, and gaseous forms of any substance of invariable composition. It seems not improbable that in the case of sulphur and some other simple substances there is more than one triad of coexistent phases; but it is entirely improbable that there are four coexistent phases of any simple substance. An example of $n=2$ and $r=4$ is seen in a solution of a salt in water in contact with vapor of water and two different kinds of crystals of the salt.

Concerning $n+1$ Coexistent Phases.

We will now seek the differential equation which expresses the relation between the variations of the temperature and the pressure in a system of $n+1$ coexistent phases (n denoting, as before, the number of independently variable components in the system taken as a whole).

In this case we have $n+1$ equations of the general form of (97) (one for each of the coexistent phases), in which we may distinguish the quantities η, v, m_1, m_2, etc. relating to the different phases by accents. But t and p will each have the same value throughout, and the same is true of μ_1, μ_2, etc., so far as each of these occurs in the different equations. If the total number of these potentials is $n+h$, there will be h independent relations between them, corresponding to the h independent relations between the units of the component substances to which the potentials relate, by means of which we may eliminate the variations of h of the potentials from the equations of the form of (97) in which they occur.

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Let one of these equations be

$$v'\,dp = \eta'\,dt + m_a'\,d\mu_a + m_b'\,d\mu_b + \text{etc.}, \tag{124}$$

and by the proposed elimination let it become

$$v'\,dp = \eta'\,dt + A_1'\,d\mu_1 + A_2'\,d\mu_2 \;.\;.\;.\; + A_n'\,d\mu_n. \tag{125}$$

It will be observed that μ_a, for example, in (124) denotes the potential in the mass considered for a substance S_a which may or may not be identical with any of the substances S_1, S_2, etc. to which the potentials in (125) relate. Now as the equations between the potentials by means of which the elimination is performed are similar to those which subsist between the units of the corresponding substances, (compare equations (38), (43), and (51),) if we denote these units by $\mathfrak{S}_a$, $\mathfrak{S}_b$, etc., $\mathfrak{S}_1$, $\mathfrak{S}_2$, etc., we must also have

$$m_a'\,\mathfrak{S}_a + m_b'\,\mathfrak{S}_b + \text{etc.} = A_1'\,\mathfrak{S}_1 + A_2'\,\mathfrak{S}_2 \;.\;.\;.\; + A_n'\,\mathfrak{S}_n. \tag{126}$$

But the first member of this equation denotes (in kind and quantity) the matter in the body to which equations (124) and (125) relate. As the same must be true of the second member, we may regard this same body as composed of the quantity A_1' of the substance S_1, with the quantity A_2' of the substance S_2, etc. We will therefore, in accordance with our general usage, write m_1' m_2', etc. for A_1', A_2', etc. in (125), which will then become

$$v'\,dp = \eta'\,dt + m_1'\,d\mu_1 + m_2'\,d\mu_2 \;.\;.\;.\; + m_n'\,d\mu_n. \tag{127}$$

But we must remember that the components to which the m_1', m_2', etc. of this equation relate are not necessarily independently variable, as are the components to which the similar expressions in (97) and (124) relate. The rest of the $n+1$ equations may be reduced to a similar form, viz.,

$$v''\,dp = \eta''\,dt + m_1''\,d\mu_1 + m_2''\,d\mu_2 \;.\;.\;.\; + m_n''\,d\mu_n, \tag{128}$$

etc.

By elimination of $d\mu_1, d\mu_2, \ldots d\mu_n$ from these equations we obtain

$$\begin{vmatrix} v' & m_1' & m_2' & \ldots & m_n' \\ v'' & m_1'' & m_2'' & \ldots & m_n'' \\ v''' & m_1''' & m_2''' & \ldots & m_n''' \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \cdot & \ldots & \cdot \end{vmatrix} dp = \begin{vmatrix} \eta' & m_1' & m_2' & \ldots & m_n' \\ \eta'' & m_1'' & m_2'' & \ldots & m_n'' \\ \eta''' & m_1''' & m_2''' & \ldots & m_n''' \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \cdot & \ldots & \cdot \end{vmatrix} dt. \tag{129}$$

In this equation we may make v', v'', etc. equal to unity. Then m_1', m_2', m_1'', etc. will denote the separate densities of the components in the different phases, and η', η'', etc. the densities of entropy.

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When $n = 1$,

$$(m'' v' - m' v'') \, dp = (m'' \eta' - m' \eta'') \, dt, \tag{130}$$

or, if we make $m' = 1$ and $m'' = 1$, we have the usual formula

$$\frac{dp}{dt} = \frac{\eta' - \eta''}{v' - v''} = \frac{Q}{t\,(v'' - v')}, \tag{131}$$

in which Q denotes the heat absorbed by a unit of the substance in passing from one state to the other without change of temperature or pressure.

Concerning Cases in which the Number of Coexistent Phases is less than $n + 1$.

When $n > 1$, if the quantities of all the components $S_1, S_2, \ldots S_n$ are proportional in two coexistent phases, the two equations of the form of (127) and (128) relating to these phases will be sufficient for the elimination of the variations of all the potentials. In fact, the condition of the coexistence of the two phases together with the condition of the equality of the $n - 1$ ratios of $m_1', m_2', \ldots m_n'$ with the $n - 1$ ratios of $m_1'', m_2'', \ldots m_n''$ is sufficient to determine p as a function of t if the fundamental equation is known for each of the phases. The differential equation in this case may be expressed in the form of (130), m' and m'' denoting either the quantities of any one of the components or the total quantities of matter in the bodies to which they relate. Equation (131) will also hold true in this case, if the total quantity of matter in each of the bodies is unity. But this case differs from the preceding in that the matter which absorbs the heat Q in passing from one state to another, and to which the other letters in the formula relate, although the same in quantity, is not in general the same in kind at different temperatures and pressures. Yet the case will often occur that one of the phases is essentially invariable in composition, especially when it is a crystalline body, and in this case the matter to which the letters in (131) relate will not vary with the temperature and pressure.

When $n = 2$, two coexistent phases are capable, when the temperature is constant, of a single variation in phase. But as (130) will hold true in this case when $m_1' : m_2' :: m_1'' : m_2''$, it follows that for constant temperature the pressure is in general a maximum or a minimum when the composition of the two phases is identical. In like manner, the temperature of the two coexistent phases is in general a maximum or a minimum, for constant pressure, when the composition of the two phases is identical. Hence, the series of simultaneous values of t and p for which the composition of two coexistent phases

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is identical separates those simultaneous values of t and p for which no coexistent phases are possible from those for which there are two pair of coexistent phases. This may be applied to a liquid having two independently variable components in connection with the vapor which it yields, or in connection with any solid which may be formed in it.

When $n = 3$, we have for three coexistent phases three equations of the form of (127), from which we may obtain the following,

$$\begin{vmatrix} v' & m_1' & m_2' \\ v'' & m_1'' & m_2'' \\ v''' & m_1''' & m_2''' \end{vmatrix} dp = \begin{vmatrix} \eta' & m_1' & m_2' \\ \eta'' & m_1'' & m_2'' \\ \eta''' & m_1''' & m_2''' \end{vmatrix} dt + \begin{vmatrix} m_1' & m_2' & m_3' \\ m_1'' & m_2'' & m_3'' \\ m_1''' & m_2''' & m_3''' \end{vmatrix} d\mu_3. \quad (132)$$

Now the value of the last of these determinants will be zero, when the composition of one of the three phases is such as can be produced by combining the other two. Hence, the pressure of three coexistent phases will in general be a maximum or minimum for constant temperature, and the temperature a maximum or minimum for constant pressure, when the above condition in regard to the composition of the coexistent phases is satisfied. The series of simultaneous values of t and p for which the condition is satisfied separates those simultaneous values of t and p for which three coexistent phases are not possible, from those for which there are two triads of coexistent phases. These propositions may be extended to higher values of n, and illustrated by the boiling temperatures and pressures of saturated solutions of $n - 2$ different solids in solvents having two independently variable components.

INTERNAL STABILITY OF HOMOGENEOUS FLUIDS AS INDICATED BY FUNDAMENTAL EQUATIONS.

We will now consider the stability of a fluid enclosed in a rigid envelop which is non-conducting to heat and impermeable to all the components of the fluid. The fluid is supposed initially homogeneous in the sense in which we have before used the word, i. e., uniform in every respect throughout its whole extent. Let $S_1, S_2, \ldots S_n$ be the *ultimate* components of the fluid; we may then consider every body which can be formed out of the fluid to be composed of $S_1, S_2, \ldots S_n$, and that in only one way. Let $m_1, m_2, \ldots m_n$ denote the quantities of these substances in any such body, and let ε, η, v, denote its energy, entropy, and volume. The fundamental equation for compounds of $S_1, S_2, \ldots S_n$, if completely determined, will give us all possible sets of simultaneous values of these variables for homogeneous bodies.

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Now, if it is possible to assign such values to the constants T, P, M_1, M_2, . . . M_n that the value of the expression

$$\varepsilon - T\eta + Pv - M_1 m_1 - M_2 m_2 \ldots - M_n m_n \qquad (133)$$

shall be zero for the given fluid, and shall be positive for every other phase of the same components, i. e., for every homogeneous body* not identical in nature and state with the given fluid (but composed entirely of S_1, S_2, . . . S_n), the condition of the given fluid will be stable.

For, in any condition whatever of the given mass, whether or not homogeneous, or fluid, if the value of the expression (133) is not negative for any homogeneous part of the mass, its value for the whole mass cannot be negative; and if its value cannot be zero for any homogeneous part which is not identical in phase with the mass in its given condition, its value cannot be zero for the whole except when the whole is in the given condition. Therefore, in the case supposed, the value of this expression for any other than the given condition of the mass is positive. (That this conclusion cannot be invalidated by the fact that it is not entirely correct to regard a composite mass as made up of homogeneous parts having the same properties in respect to energy, entropy, etc., as if they were parts of larger homogeneous masses, will easily appear from considerations similar to those adduced on pages 131–133.) If, then, the value of the expression (133) for the mass considered is less when it is in the given condition than when it is in any other, the energy of the mass in its given condition must be less than in any other condition in which it has the same entropy and volume. The given condition is therefore stable. (See page 110.)

Again, if it is possible to assign such values to the constants in (133) that the value of the expression shall be zero for the given fluid mass, and shall not be negative for any phase of the same components, the given condition will be evidently not unstable. (See page 110.) It will be stable unless it is possible for the given matter in the given volume and with the given entropy to consist of homogeneous parts for all of which the value of the expression (133) is zero, but which are not all identical in phase with the mass in its given condition. (A mass consisting of such parts would be in equilibrium, as we have already seen on pages 133, 134.) In this case, if we disregard the quantities connected with the surfaces which divide the

* A vacuum is throughout this discussion to be regarded as a limiting case of an extremely rarified body. We may thus avoid the necessity of the specific mention of a vacuum in propositions of this kind.

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homogeneous parts, we must regard the given condition as one of neutral equilibrium. But in regard to these homogeneous parts, which we may evidently consider to be all different phases, the following conditions must be satisfied. (The accents distinguish the letters referring to the different parts, and the unaccented letters refer to the whole mass.)

$$\left.\begin{array}{l} \eta' + \eta'' + \text{etc.} = \eta, \\ v' + v'' + \text{etc.} = v, \\ m_1' + m_1'' + \text{etc.} = m_1, \\ m_2' + m_2'' + \text{etc.} = m_2, \\ \quad \text{etc.} \end{array}\right\} \tag{134}$$

Now the values of η, v, m_1, m_2, etc. are determined by the whole fluid mass in its given state, and the values of $\frac{\eta'}{v'}$, $\frac{\eta''}{v''}$, etc., $\frac{m_1'}{v'}$, $\frac{m_1''}{v''}$, etc., $\frac{m_2'}{v'}$, $\frac{m_2''}{v''}$, etc., etc., are determined by the phases of the various parts. But the phases of these parts are evidently determined by the phase of the fluid as given. They form, in fact, the whole set of coexistent phases of which the latter is one. Hence, we may regard (134) as $n + 2$ linear equations between v', v'', etc. (The values of v', v'', etc. are also subject to the condition that none of them can be negative.) Now one solution of these equations must give us the given condition of the fluid; and it is not to be expected that they will be capable of any other solution, unless the number of different homogeneous parts, that is, the number of different coexistent phases, is greater than $n + 2$. We have already seen (page 153) that it is not probable that this is ever the case.

We may, however, remark that in a certain sense an infinitely large fluid mass will be in neutral equilibrium in regard to the formation of the substances, if such there are, other than the given fluid, for which the value of (133) is zero (when the constants are so determined that the value of the expression is zero for the given fluid, and not negative for any substance); for the tendency of such a formation to be reabsorbed will diminish indefinitely as the mass out of which it is formed increases.

When the substances S_1, S_2, . . . S_n are all independently variable components of the given mass, it is evident from (86) that the conditions that the value of (133) shall be zero for the mass as given, and shall not be negative for any phase of the same components, can only be fulfilled when the constants T, P, M_1, M_2, . . . M_n are equal to the temperature, the pressure, and the several potentials in the given

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mass. If we give these values to the constants, the expression (133) will necessarily have the value zero for the given mass and we shall only have to inquire whether its value is positive for all other phases. But when $S_1, S_2, \ldots S_n$ are not all independently variable components of the given mass, the values which it will be necessary to give to the constants in (133) cannot be determined entirely from the properties of the given mass; but T and P must be equal to its temperature and pressure, and it will be easy to obtain as many equations connecting $M_1, M_2, \ldots M_n$ with the potentials in the given mass as it contains independently variable components.

When it is not possible to assign such values to the constants in (133) that the value of the expression shall be zero for the given fluid, and either zero or positive for any phase of the same components, we have already seen (pages 129–134) that if equilibrium subsists without passive resistances to change, it must be in virtue of properties which are peculiar to small masses surrounded by masses of different nature, and which are not indicated by fundamental equations. In this case, the fluid will necessarily be unstable, if we extend this term to embrace all cases in which an initial disturbance confined to a small part of an indefinitely large fluid mass will cause an ultimate change of state not indefinitely small in degree throughout the whole mass. In the discussion of stability as indicated by fundamental equations it will be convenient to use the term in this sense.*

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* If we wish to know the stability of the given fluid when exposed to a constant temperature, or to a constant pressure, or to both, we have only to suppose that there is enclosed in the same envelop with the given fluid another body (which cannot combine with the fluid) of which the fundamental equation is $\varepsilon = T\eta$, or $\varepsilon = -Pv$, or $\varepsilon = T\eta - Pv$, as the case may be, (T and P denoting the constant temperature and pressure, which of course must be those of the given fluid,) and to apply the criteria of page 110 to the whole system. When it is possible to assign such values to the constants in (133) that the value of the expression shall be zero for the given fluid and positive for every other phase of the same components, the value of (133) for the whole system will be less when the system is in its given condition than when it is in any other. (Changes of form and position of the given fluid are of course regarded as immaterial.) Hence the fluid is stable. When it is not possible to assign such values to the constants that the value of (133) shall be zero for the given fluid and zero or positive for any other phase, the fluid is of course unstable. In the remaining case, when it is possible to assign such values to the constants that the value of (133) shall be zero for the given fluid and zero or positive for every other phase, but not without the value zero for some other phase, the state of equilibrium of the fluid as stable or neutral will be determined by the possibility of satisfying, for any other than the given condition of the fluid, equations like (134), in which, however, the first or the second or both are to be stricken out, according as we are considering the

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In determining for any given positive values of T and P and any given values whatever of $M_1, M_2, \ldots M_n$ whether the expression (133) is capable of a negative value for any phase of the components $S_1, S_2, \ldots S_n$, and if not, whether it is capable of the value zero for any other phase than that of which the stability is in question, it is only necessary to consider phases having the temperature T and pressure P. For we may assume that a mass of matter represented by any values of $m_1, m_2, \ldots m_n$ is capable of at least one state of not unstable equilibrium (which may or may not be a homogeneous state) at this temperature and pressure. It may easily be shown that for such a state the value of $\varepsilon - T\eta + Pv$ must be as small as for any other state of the same matter. The same will therefore be true of the value of (133). Therefore if this expression is capable of a negative value for any mass whatever, it will have a negative value for that mass at the temperature T and pressure P. And if this mass is not homogeneous, the value of (133) must be negative for at least one of its homogeneous parts. So also, if the expression (133) is not capable of a negative value for any phase of the components, any phase for which it has the value zero must have the temperature T and the pressure P.

It may easily be shown that the same must be true in the limiting cases in which $T = 0$ and $P = 0$. For negative values of P, (133) is always capable of negative values, as its value for a vacuum is Pv.

For any body of the temperature T and pressure P, the expression (133) may by (91) be reduced to the form

$$\zeta - M_1 m_1 - M_2 m_2 \ldots - M_n m_n. \tag{135}$$

We have already seen (pages 131, 132) that an expression like (133), when $T, P, M_1, M_2, \ldots M_n$ and v have any given finite values, cannot have an infinite negative value as applied to any real body. Hence, in determining whether (133) is capable of a negative value for any phase of the components $S_1, S_2, \ldots S_n$, and if not, whether it is capable of the value zero for any other phase than that of which the stability is in question, we have only to consider the least value of which it is capable for a constant value of v. Any body giving this value must satisfy the condition that for constant volume

$$d\varepsilon - T\,d\eta - M_1\,dm_1 - M_2\,dm_2 \ldots - M_n\,dm_n \geqq 0, \tag{136}$$

stability of the fluid for constant temperature, or for constant pressure, or for both. The number of coexistent phases will sometimes exceed by one or two the number of the remaining equations, and then the equilibrium of the fluid will be neutral in respect to one or two independent changes.

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or, if we substitute the value of $d\varepsilon$ taken from equation (86), using subscript $a \ldots g$ for the quantities relating to the actual components of the body, and subscript $h \ldots k$ for those relating to the possible,

$$t\,d\eta + \mu_a\,dm_a \ldots + \mu_g\,dm_g + \mu_h\,dm_h \ldots + \mu_k\,dm_k$$
$$- T\,d\eta - M_1\,dm_1 - M_2\,dm_2 \ldots - M_n\,dm_n \geqq 0. \quad (137)$$

That is, the temperature of the body must be equal to T, and the potentials of its components must satisfy the same conditions as if it were in contact and in equilibrium with a body having potentials $M_1, M_2, \ldots M_n$. Therefore the same relations must subsist between $\mu_a \ldots \mu_g$, and $M_1 \ldots M_n$ as between the units of the corresponding substances, so that

$$m_a\,\mu_a \ldots + m_g\,\mu_g = m_1\,M_1 \ldots + m_n\,M_n; \quad (138)$$

and as we have by (93)

$$\varepsilon = t\,\eta - p\,v + \mu_a\,m_a \ldots + \mu_g\,m_g, \quad (139)$$

the expression (133) will reduce (for the body or bodies for which it has the least value per unit of volume) to

$$(P - p)\,v, \quad (140)$$

the value of which will be positive, null, or negative, according as the value of

$$P - p \quad (141)$$

is positive, null, or negative.

Hence, the conditions in regard to the stability of a fluid of which all the ultimate components are independently variable admit a very simple expression. If the pressure of the fluid is greater than that of any other phase of the same components which has the same temperature and the same values of the potentials for its actual components, the fluid is stable without coexistent phases; if its pressure is not as great as that of some other such phase, it will be unstable; if its pressure is as great as that of any other such phase, but not greater than that of every other, the fluid will certainly not be unstable, and in all probability it will be stable (when enclosed in a rigid envelop which is impermeable to heat and to all kinds of matter), but it will be one of a set of coexistent phases of which the others are the phases which have the same pressure.

The considerations of the last two pages, by which the tests relating to the stability of a fluid are simplified, apply to such bodies as actually exist. But if we should form arbitrarily any equation as a fundamental equation, and ask whether a fluid of which the proper-

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ties were given by that equation would be stable, the tests of stability last given would be insufficient, as some of our assumptions might not be fulfilled by the equation. The test, however, as first given (pages 156–159) would in all cases be sufficient.

Stability in respect to Continuous Changes of Phase.

In considering the changes which may take place in any mass, we have already had occasion to distinguish between infinitesimal changes in existing phases, and the formation of entirely new phases. A phase of a fluid may be stable in regard to the former kind of change, and unstable in regard to the latter. In this case it may be capable of continued existence in virtue of properties which prevent the commencement of discontinuous changes. But a phase which is unstable in regard to continuous changes is evidently incapable of permanent existence on a large scale except in consequence of passive resistances to change. We will now consider the conditions of stability in respect to continuous changes of phase, or, as it may also be called, stability in respect to adjacent phases. We may use the same general test as before, except that the expression (133) is to be applied only to phases which differ infinitely little from the phase of which the stability is in question. In this case the component substances to be considered will be limited to the independently variable components of the fluid, and the constants M_1, M_2, etc. must have the values of the potentials for these components in the given fluid. The constants in (133) are thus entirely determined and the value of the expression for the given phase is necessarily zero. If for any infinitely small variation of the phase, the value of (133) can become negative, the fluid will be unstable; but if for every infinitely small variation of the phase the value of (133) becomes positive, the fluid will be stable. The only remaining case, in which the phase can be varied without altering the value of (133) can hardly be expected to occur. The phase concerned would in such a case have coexistent adjacent phases. It will be sufficient to discuss the condition of stability (in respect to continuous changes) without coexistent adjacent phases.

This condition, which for brevity's sake we will call the condition of stability, may be written in the form

$$\varepsilon'' - t'\eta'' + p'v'' - \mu_1' m_1'' \ldots - \mu_n' m_n'' > 0, \qquad (142)$$

in which the quantities relating to the phase of which the stability is in question are distinguished by single accents, and those relating to

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the other phase by double accents. This condition is by (93) equivalent to

$$\varepsilon'' - t'\eta'' + p'v'' - \mu_1' m_1'' \ldots - \mu_n' m_n''$$
$$- \varepsilon' + t'\eta' - p'v' + \mu_1' m_1' \ldots + \mu_n' m_n' > 0, \qquad (143)$$

and to

$$- t'\eta'' + p'v'' - \mu_1' m_1'' \ldots - \mu_n' m_n''$$
$$+ t''\eta'' - p''v'' + \mu_1'' m_1'' \ldots + \mu_n'' m_n'' > 0. \qquad (144)$$

The condition (143) may be expressed more briefly in the form

$$\Delta\varepsilon > t\,\Delta\eta - p\,\Delta v + \mu_1\,\Delta m_1 \ldots + \mu_n\,\Delta m_n, \qquad (145)$$

if we use the character Δ to signify that the condition, although relating to infinitesimal differences, is not to be interpreted in accordance with the usual convention in respect to differential equations with neglect of infinitesimals of higher orders than the first, but is to be interpreted *strictly*, like an equation between finite differences. In fact, when a condition like (145) (interpreted strictly) is satisfied for infinitesimal differences, it must be possible to assign limits within which it shall hold true of finite differences. But it is to be remembered that the condition is not to be applied to any arbitrary values of $\Delta\eta$, Δv, Δm_1, . . . Δm_n, but only to such as are determined by a change of phase. (If only the quantity of the body which determines the value of the variables should vary and not its phase, the value of the first member of (145) would evidently be zero.) We may free ourselves from this limitation by making v constant, which will cause the term $-p\,\Delta v$ to disappear. If we then divide by the constant v, the condition will become

$$\Delta\frac{\varepsilon}{v} > t\,\Delta\frac{\eta}{v} + \mu_1\,\Delta\frac{m_1}{v} \ldots + \mu_n\,\Delta\frac{m_n}{v}, \qquad (146)$$

in which form it will not be necessary to regard v as constant. As we may obtain from (86)

$$d\frac{\varepsilon}{v} = t\,d\frac{\eta}{v} + \mu_1\,d\frac{m_1}{v} \ldots + \mu_n\,d\frac{m_n}{v}, \qquad (147)$$

we see that *the stability of any phase in regard to continuous changes depends upon the same conditions in regard to the second and higher differential coefficients of the density of energy regarded as a function of the density of entropy and the densities of the several components, which would make the density of energy a minimum, if the necessary conditions in regard to the first differential coefficients were fulfilled.* When $n = 1$, it may be more convenient to regard m as constant

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in (145) than v. Regarding m a constant, it appears that the stability of a phase depends upon the same conditions in regard to the second and higher differential coefficients of the energy of a unit of mass regarded as a function of its entropy and volume, which would make the energy a minimum, if the necessary conditions in regard to the first differential coefficients were fulfilled.

The formula (144) expresses the condition of stability for the phase to which t', p', etc. relate. But it is evidently the necessary and sufficient condition of the stability of all phases of certain kinds of matter, or of all phases within given limits, that (144) shall hold true of any two infinitesimally differing phases within the same limits, or, as the case may be, in general. For the purpose, therefore, of such *collective* determinations of stability, we may neglect the distinction between the two states compared, and write the condition in the form

$$-\eta\,\Delta t + v\,\Delta p - m_1\,\Delta\mu_1 \;.\;.\;.\; - m_n\,\Delta\mu_n > 0, \tag{148}$$

or

$$\Delta p > \frac{\eta}{v}\,\Delta t + \frac{m_1}{v}\,\Delta\mu_1 \;.\;.\;.\; + \frac{m_n}{v}\,\Delta\mu_n. \tag{149}$$

Comparing (98), we see that it is necessary and sufficient for the stability in regard to continuous changes of all the phases within any given limits, that within those limits the same conditions should be fulfilled in respect to the second and higher differential coefficients of the pressure regarded as a function of the temperature and the several potentials, which would make the pressure a minimum, if the necessary conditions with respect to the first differential coefficients were fulfilled.

By equations (87) and (94), the condition (142) may be brought to the form

$$\begin{aligned} &\psi'' + t''\,\eta'' + p'\,v'' - \mu_1'\,m_1'' \;.\;.\;.\; - \mu_n'\,m_n'' \\ &-\psi' - t'\,\eta'' - p'\,v' + \mu_1'\,m_1' \;.\;.\;.\; + \mu_n'\,m_n' > 0. \end{aligned} \tag{150}$$

For the stability of all phases within any given limits it is necessary and sufficient that within the same limits this condition shall hold true of any two phases which differ infinitely little. This evidently requires that when $v' = v''$, $m_1' = m_1''$, . . . $m_n' = m_n''$,

$$\psi'' - \psi' + (t'' - t')\,\eta'' > 0; \tag{151}$$

and that when $t' = t''$

$$\begin{aligned} &\psi'' + p'\,v'' - \mu_1'\,m_1'' \;.\;.\;.\; + \mu_n'\,m_n'' \\ &-\psi' - p'\,v' + \mu_1'\,m_1' \;.\;.\;.\; + \mu_n'\,m_n' > 0. \end{aligned} \tag{152}$$

These conditions may be written in the form

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$$[\Delta\psi + \eta\,\Delta t]_{v,m} < 0, \tag{153}$$

$$[\Delta\psi + p\,\Delta v - \mu_1\,\Delta m_1 \;.\;.\;.\; - \mu_n\,\Delta m_n]_t > 0, \tag{154}$$

in which the subscript letters indicate the quantities which are to be regarded as constant, m standing for all the quantities $m_1 \;.\;.\;.\; m_n$. If these conditions hold true within any given limits, (150) will also hold true of any two infinitesimally differing phases within the same limits. To prove this, we will consider a third phase, determined by the equations

$$t''' = t', \tag{155}$$

and

$$v''' = v'', \quad m_1''' = m_1'', \quad .\;.\;.\; m_n''' = m_n''. \tag{156}$$

Now by (153),

$$\psi''' - \psi'' + (t''' - t'')\,\eta'' < 0; \tag{157}$$

and by (154),

$$\begin{aligned} &\psi''' + p'v''' - \mu_1' m_1''' \;.\;.\;.\; - \mu_n' m_n''' \\ &- \psi' \;\; - p'v' \;\; + \mu_1' m_1' \;.\;.\;.\; + \mu_n' m_n' > 0. \end{aligned} \tag{158}$$

Hence,

$$\begin{aligned} &\psi'' + t''\,\eta'' + p'v''' - \mu_1' m_1''' \;.\;.\;.\; - \mu_n' m_n''' \\ &- \psi' \;\; - t'''\eta'' - p'v' \;\; + \mu_1' m_1' \;.\;.\;.\; + \mu_n' m_n' > 0, \end{aligned} \tag{159}$$

which by (155) and (156) is equivalent to (150). Therefore, the conditions (153) and (154) in respect to the phases within any given limits are necessary and sufficient for the stability of all the phases within those limits. It will be observed that in (153) we have the condition of thermal stability of a body considered as unchangeable in composition and in volume, and in (154), the condition of mechanical and chemical stability of the body considered as maintained at a constant temperature. Comparing equation (88), we see that the condition (153) will be satisfied, if $\frac{d^2\psi}{dt^2} < 0$, i. e., if $\frac{d\eta}{dt}$ or $t\frac{d\eta}{dt}$ (the specific heat for constant volume) is positive. When $n = 1$, i. e., when the composition of the body is invariable, the condition (154) will evidently not be altered, if we regard m as constant, by which the condition will be reduced to

$$[\Delta\psi + p\,\Delta v]_{t,m} > 0. \tag{160}$$

This condition will evidently be satisfied if $\frac{d^2\psi}{dv^2} > 0$, i. e., if $-\frac{dp}{dv}$ or $-v\frac{dp}{dv}$ (the elasticity for constant temperature) is positive. But when $n > 1$, (154) may be abbreviated more symmetrically by making v constant.

Again, by (91) and (96), the condition (142) may be brought to the form

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$$\zeta'' + t''\eta'' - p''v'' - \mu_1'' m_1'' \ldots - \mu_n'' m_n''$$
$$- \zeta' - t'\eta'' + p'v'' + \mu_1' m_1' \ldots + \mu_n' m_n' > 0. \qquad (161)$$

Therefore, for the stability of all phases within any given limits it is necessary and sufficient that within the same limits

$$[\Delta\zeta + \eta\,\Delta t - v\,\Delta p]_m < 0, \qquad (162)$$

and

$$[\Delta\zeta - \mu_1\,\Delta m_1 \ldots - \mu_n\,\Delta m_n]_{t,p} > 0, \qquad (163)$$

as may easily be proved by the method used with (153) and (154). The first of these formulæ expresses the thermal and mechanical conditions of stability for a body considered as unchangeable in composition, and the second the conditions of chemical stability for a body considered as maintained at a constant temperature and pressure. If $n = 1$, the second condition falls away, and as in this case $\zeta = m\mu$, condition (162) becomes identical with (148).

The foregoing discussion will serve to illustrate the relation of the general condition of stability in regard to continuous changes to some of the principal forms of fundamental equations. It is evident that each of the conditions (146), (149), (154), (162), (163) involve in general several particular conditions of stability. We will now give our attention to the latter. Let

$$\Phi = \varepsilon - t'\eta + p'v - \mu_1' m_1 \ldots - \mu_n' m_n, \qquad (164)$$

the accented letters referring to one phase and the unaccented to another. It is by (142) the necessary and sufficient condition of the stability of the first phase that, for constant values of the quantities relating to that phase and of v, the value of Φ shall be a minimum when the second phase is identical with the first. Differentiating (164), we have by (86)

$$d\Phi = (t - t')\,d\eta - (p - p')\,dv + (\mu_1 - \mu_1')\,dm_1$$
$$\ldots - (\mu_n - \mu_n')\,dm_n. \qquad (165)$$

Therefore, the above condition requires that if we regard $v, m_1, \ldots m_n$ as having the constant values indicated by accenting these letters, t shall be an increasing function of η, when the variable phase differs sufficiently little from the fixed. But as the fixed phase may be any one within the limits of stability, t must be an increasing function of η (within these limits) for any constant values of $v, m_1, \ldots m_n$. This condition may be written

$$\left(\frac{\Delta t}{\Delta \eta}\right)_{v,\, m_1, \ldots m_n} > 0. \qquad (166)$$

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When this condition is satisfied, the value of Φ, for any given values of $v, m_1, \ldots m_n$, will be a minimum when $t = t'$. And therefore, in applying the general condition of stability relating to the value of Φ, we need only consider the phases for which $t = t'$.

We see again by (165) that the general condition requires that if we regard $t, v, m_2, \ldots m_n$ as having the constant values indicated by accenting these letters, μ_1 shall be an increasing function of m_1, when the variable phase differs sufficiently little from the fixed. But as the fixed phase may be any one within the limits of stability, μ_1 must be an increasing function of m_1 (within these limits) for any constant values of $t, v, m_2, \ldots m_n$. That is,

$$\left(\frac{\Delta \mu_1}{\Delta m_1}\right)_{t,\, v,\, m_2 \ldots m_n} > 0. \tag{167}$$

When this condition is satisfied, as well as (166), Φ will have a minimum value, for any constant values of $v, m_2, \ldots m_n$, when $t = t'$ and $\mu_1 = \mu_1'$; so that in applying the general condition of stability we need only consider the phases for which $t = t'$ and $\mu_1 = \mu_1'$.

In this way we may also obtain the following particular conditions of stability:

$$\left(\frac{\Delta \mu_2}{\Delta m_2}\right)_{t,\, v,\, m_1,\, m_3, \ldots m_n} > 0, \tag{168}$$

. . . . .

$$\left(\frac{\Delta \mu_n}{\Delta m_n}\right)_{t,\, v,\, m_1, \ldots m_{n-1}} > 0. \tag{169}$$

When the $n + 1$ conditions (166)–(169) are all satisfied, the value of Φ, for any constant value of v, will be a minimum when the temperature and the potentials of the variable phase are equal to those of the fixed. The pressures will then also be equal and the phases will be entirely identical. Hence, the general condition of stability will be completely satisfied, when the above particular conditions are satisfied.

From the manner in which these particular conditions have been derived, it is evident that we may interchange in them $\eta, m_1, \ldots m_n$ in any way, provided that we also interchange in the same way $t, \mu_1, \ldots \mu_n$. In this way we may obtain different sets of $n + 1$ conditions which are necessary and sufficient for stability. The quantity v might be included in the first of these lists, and $-p$ in the second, except in cases when, in some of the phases considered, the entropy or the quantity of one of the components has the value zero.

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Then the condition that that quantity shall be constant would create a restriction upon the variations of the phase, and cannot be substituted for the condition that the volume shall be constant in the statement of the general condition of stability relative to the minimum value of Φ.

To indicate more distinctly all these particular conditions at once, we observe that the condition (144), and therefore also the condition obtained by interchanging the single and double accents, must hold true of any two infinitesimally differing phases within the limits of stability. Combining these two conditions we have

$$(t'' - t')(\eta'' - \eta') - (p'' - p')(v'' - v') + (\mu_1'' - \mu_1')(m_1'' - m_1') \ldots (\mu_n'' - \mu_n')(m_n'' - m_n') > 0, \quad (170)$$

which may be written more briefly

$$\Delta t \, \Delta\eta - \Delta p \, \Delta v + \Delta\mu_1 \, \Delta m_1 \ldots + \Delta\mu_n \, \Delta m_n > 0. \quad (171)$$

This must hold true of any two infinitesimally differing phases within the limits of stability. If, then, we give the value zero to one of the differences in every term except one, but not so as to make the phases completely identical, the values of the two differences in the remaining term will have the same sign, except in the case of Δp and Δv, which will have opposite signs. (If both states are stable this will hold true even on the limits of stability.) Therefore, within the limits of stability, either of the two quantities occurring (after the sign Δ) in any term of (171) in an increasing function of the other, —except p and v, of which the opposite is true,—when we regard as constant one of the quantities occurring in each of the other terms, but not such as to make the phases identical.

If we write d for Δ in (166)–(169), we obtain conditions which are always sufficient for stability. If we also substitute $\geqq$ for $>$, we obtain conditions which are necessary for stability. Let us consider the form which these conditions will take when η, v, m_1, . . . m_n are regarded as independent variables. When $dv = 0$, we shall have

$$\left.\begin{aligned} dt &= \frac{dt}{d\eta} d\eta + \frac{dt}{dm_1} dm_1 \ldots + \frac{dt}{dm_n} dm_n \\ d\mu_1 &= \frac{d\mu_1}{d\eta} d\eta + \frac{d\mu_1}{dm_1} dm_1 \ldots + \frac{d\mu_n}{dm_n} dm_n \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ d\mu_n &= \frac{d\mu_n}{d\eta} d\eta + \frac{d\mu_n}{dm_1} dm_1 \ldots + \frac{d\mu_n}{dm_n} dm_n \end{aligned}\right\} \quad (172)$$

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Let us write R_{n+1} for the determinant of the order $n+1$

$$\begin{vmatrix} \frac{d^2\varepsilon}{d\eta^2} & \frac{d^2\varepsilon}{dm_1\,d\eta} & \cdots & \frac{d^2\varepsilon}{dm_n\,d\eta} \\ \frac{d^2\varepsilon}{d\eta\,dm_1} & \frac{d^2\varepsilon}{dm_1{}^2} & \cdots & \frac{d^2\varepsilon}{dm_n\,dm_1} \\ \cdot & \cdot & \cdots & \cdot \\ \frac{d^2\varepsilon}{d\eta\,dm_n} & \frac{d^2\varepsilon}{dm_1\,dm_n} & \cdots & \frac{d^2\varepsilon}{dm_n{}^2} \end{vmatrix} \tag{173}$$

of which the constituents are by (86) the same as the coefficients in equations (172), and R_n, R_{n-1}, etc. for the minors obtained by erasing the last column and row in the original determinant and in the minors successively obtained, and R_1 for the last remaining constituent. Then if dt, $d\mu_1$, . . . $d\mu_{n-1}$, and dv all have the value zero, we have by (172)

$$R_n\,d\mu_n = R_{n+1}\,dm_n, \tag{174}$$

that is,

$$\left(\frac{d\mu_n}{dm_n}\right)_{t,\,v,\,\mu_1,\,\ldots\,\mu_{n-1}} = \frac{R_{n+1}}{R_n}. \tag{175}$$

In like manner we obtain

$$\left.\begin{aligned} &\left(\frac{d\mu_{n-1}}{dm_{n-1}}\right)_{t,\,v,\,\mu_1,\,\ldots\,\mu_{n-2},\,m_n} = \frac{R_n}{R_{n-1}}, \\ &\text{etc.} \end{aligned}\right\} \tag{176}$$

Therefore, the conditions obtained by writing d for Δ in (166)–(169) are equivalent to this, that the determinant given above with the n minors obtained from it as above mentioned and the last remaining constituent $\frac{d^2\varepsilon}{d\eta^2}$ shall all be positive. Any phase for which this condition is satisfied will be stable, and no phase will be stable for which any of these quantities has a negative value. But the conditions (166)–(169) will remain valid, if we interchange in any way η, m_1, . . . m_n (with corresponding interchange of t, μ_1, . . . μ_n). Hence the order in which we erase successive columns with the corresponding rows in the determinant is immaterial. Therefore none of the minors of the determinant (173) which are formed by erasing corresponding rows and columns, and none of the constituents of the principal diagonal, can be negative for a stable phase.

We will now consider the conditions which characterize the *limits of stability* (i. e., the limits which divide stable from unstable phases)

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with respect to continuous changes.* Here, evidently, one of the conditions (166)–(169) must cease to hold true. Therefore, one of the differential coefficients formed by changing Δ into d in the first members of these conditions must have the value zero. (That it is the numerator and not the denominator in the differential coefficient which vanishes at the limit appears from the consideration that the denominator is in each case the differential of a quantity which is necessarily capable of progressive variation, so long at least as the phase is capable of variation at all under the conditions expressed by the subscript letters.) The same will hold true of the set of differential coefficients obtained from these by interchanging in any way $\eta, m_1, \ldots m_n$, and simultaneously interchanging $t, \mu_1, \ldots \mu_n$ in the same way. But we may obtain a more definite result than this.

Let us give to η or t, to m_1 or $\mu_1, \ldots$ to m_{n-1} or μ_{n-1}, and to v, the constant values indicated by these letters when accented. Then by (165)

$$d\Phi = (\mu_n - \mu_n')\, dm_n. \tag{177}$$

Now

$$\mu_n - \mu_n' = \left(\frac{d\mu_n}{dm_n}\right)'(m_n - m_n') \tag{178}$$

approximately, the differential coefficient being interpreted in accordance with the above assignment of constant values to certain variables, and its value being determined for the phase to which the accented letters refer. Therefore,

$$d\Phi = \left(\frac{d\mu_n}{dm_n}\right)'(m_n - m_n')\, dm_n, \tag{179}$$

and

$$\Phi = \tfrac{1}{2}\left(\frac{d\mu_n}{dm_n}\right)'(m_n - m_n')^2. \tag{180}$$

The quantities neglected in the last equation are evidently of the same order as $(m_n - m_n')^3$. Now this value of Φ will of course be different (the differential coefficient having a different meaning) according as we have made η or t constant, and according as we have made m_1 or μ_1 constant, etc.; but since, within the limits of stability, the value of Φ, for any constant values of m_n and v, will be the least when $t, p, \mu_1 \ldots \mu_{n-1}$ have the values indicated by accenting these letters, the value of the differential coefficient will be at least as small

* The limits of stability with respect to discontinuous changes are formed by phases which are coexistent with other phases. Some of the properties of such phases have already been considered. See pages 152–156.

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when we give these variables these constant values, as when we adopt any other of the suppositions mentioned above in regard to the quantities remaining constant. And in all these relations we may interchange in any way $\eta, m_1, \ldots m_n$, if we interchange in the same way $t, \mu_1, \ldots \mu_n$. It follows that, within the limits of stability, when we choose for any one of the differential coefficients

$$\frac{dt}{d\eta}, \quad \frac{d\mu_1}{dm_1}, \ldots \frac{d\mu_n}{dm_n} \tag{181}$$

the quantities following the sign d in the numerators of the others together with v as those which are to remain constant in differentiation, the value of the differential coefficient as thus determined will be at least as small as when one or more of the constants in differentiation are taken from the denominators, one being still taken from each fraction, and v as before being constant.

Now we have seen that none of these differential coefficients, as determined in any of these ways, can have a negative value within the limit of stability, and that some of them must have the value zero at that limit. Therefore, in virtue of the relations just established, one at least of these differential coefficients determined by considering constant the quantities occurring in the numerators of the others together with v, will have the value zero. But if one such has the value zero, all such will in general have the same value. For if

$$\left(\frac{d\mu_n}{dm_n}\right)_{t,\, v,\, \mu_1, \ldots \mu_{n-1}} \tag{182}$$

for example, has the value zero, we may change the density of the component S_n without altering (if we disregard infinitesimals of higher orders than the first) the temperature or the potentials, and therefore, by (98), without altering the pressure. That is, we may change the phase without altering any of the quantities $t, p, \mu_1, \ldots \mu_n$. (In other words, the phases adjacent to the limits of stability exhibit *approximately* the relations characteristic of neutral equilibrium.) Now this change of phase, which changes the density of one of the components, will in general change the density of the others and the density of entropy. Therefore, all the other differential coefficients formed after the analogy of (182), i. e., formed from the fractions in (181) by taking as constants for each the quantities in the numerators of the others together with v, will in general have the value zero at the limit of stability. And the relation which characterizes the limit of stability may be expressed, in general, by setting any one of these differential coefficients equal to zero. Such

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an equation, when the fundamental equation is known, may be reduced to the form of an equation between the independent variables of the fundamental equation.

Again, as the determinant (173) is equal to the product of the differential coefficients obtained by writing d for Δ in the first members of (166)–(169), the equation of the limit of stability may be expressed by setting this determinant equal to zero. The form of the differential equation as thus expressed will not be altered by the interchange of the expressions η, m_1, . . . m_n, but it will be altered by the substitution of v for any one of these expressions, which will be allowable whenever the quantity for which it is substituted has not the value zero in any of the phases to which the formula is to be applied.

The condition formed by setting the expression (182) equal to zero is evidently equivalent to this, that

$$\left(\frac{d\mu_n}{d\frac{m_n}{v}}\right)_{t,\,\mu_1,\,\ldots\,\mu_{n-1}} = 0, \tag{183}$$

that is, that

$$\left(\frac{d\frac{m_n}{v}}{d\mu_n}\right)_{t,\,\mu_1,\,\ldots\,\mu_{n-1}} = \infty, \tag{184}$$

or by (98), if we regard t, μ_1, . . . μ_n as the independent variables,

$$\left(\frac{d^2p}{d\mu_n{}^2}\right) = \infty. \tag{185}$$

In like manner we may obtain

$$\frac{d^2p}{dt^2} = \infty, \quad \frac{d^2p}{d\mu_1{}^2} = \infty, \ \ldots \ \frac{d^2p}{d\mu_{n-1}{}^2} = \infty. \tag{186}$$

Any one of these equations, (185), (186), may be regarded, in general, as the equation of the limit of stability. We may be certain that at every phase at that limit one at least of these equations will hold true.

GEOMETRICAL ILLUSTRATIONS.

Surfaces in which the Composition of the Body represented is Constant.

In vol. ii, p. 382, of the Trans. Conn. Acad., a method is described of representing the thermodynamic properties of substances of invariable composition by means of surfaces. The volume, entropy, and energy

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of a constant quantity of a substance are represented by rectangular co-ordinates. This method corresponds to the first kind of fundamental equation described on pages 140–144. Any other kind of fundamental equation for a substance of invariable composition will suggest an analogous geometrical method. Thus, if we make m constant, the variables in any one of the sets (99)–(103) are reduced to three, which may be represented by rectangular co-ordinates. This will, however, afford but four different methods, for, as has already (page 150) been observed, the two last sets are essentially equivalent when $n = 1$.

The method described in the preceding volume has certain advantages, especially for the purposes of theoretical discussion, but it may often be more advantageous to select a method in which the properties represented by two of the co-ordinates shall be such as best serve to identify and describe the different states of the substance. This condition is satisfied by temperature and pressure as well, perhaps, as by any other properties. We may represent these by two of the co-ordinates and the potential by the third. (See page 143.) It will not be overlooked that there is the closest analogy between these three quantities in respect to their parts in the general theory of equilibrium. (A similar analogy exists between volume, entropy, and energy.) If we give m the constant value unity, the third co-ordinate will also represent ζ, which then becomes equal to μ.

Comparing the two methods, we observe that in one

$$v = x, \quad \eta = y, \quad \varepsilon = z, \tag{187}$$

$$p = -\frac{dz}{dx}, \quad t = \frac{dz}{dy}, \quad \mu = \zeta = z - \frac{dz}{dx}x - \frac{dz}{dy}y; \tag{188}$$

and in the other

$$t = x, \quad p = y, \quad \mu = \zeta = z, \tag{189}$$

$$\eta = -\frac{dz}{dx}, \quad v = \frac{dz}{dy}, \quad \varepsilon = z - \frac{dz}{dx}x - \frac{dz}{dy}y. \tag{190}$$

Now $\frac{dz}{dx}$ and $\frac{dz}{dy}$ are evidently determined by the inclination of the tangent plane, and $z - \frac{dz}{dx}x - \frac{dz}{dy}y$ is the segment which it cuts off on the axis of Z. The two methods, therefore, have this reciprocal relation, that the quantities represented in one by the position of a point in a surface are represented in the other by the position of a tangent plane.

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The surfaces defined by equations (187) and (189) may be distinguished as the v-η-ε surface, and the t-p-ζ surface, of the substance to which they relate.

In the t-p-ζ surface a line in which one part of the surface cuts another represents a series of pairs of coexistent states. A point through which pass three different parts of the surface represents a triad of coexistent states. Through such a point will evidently pass the three lines formed by the intersection of these sheets taken two by two. The perpendicular projection of these lines upon the p-t plane will give the curves which have recently been discussed by Professor J. Thomson.* These curves divide the space about the projection of the triple point into six parts which may be distinguished as follows: Let $\zeta^{(V)}$, $\zeta^{(L)}$, $\zeta^{(S)}$ denote the three ordinates determined for the same values of p and t by the three sheets passing through the triple point, then in one of the six spaces

$$\zeta^{(V)} < \zeta^{(L)} < \zeta^{(S)}, \tag{191}$$

in the next space, separated from the former by the line for which $\zeta^{(L)} = \zeta^{(S)}$,

$$\zeta^{(V)} < \zeta^{(S)} < \zeta^{(L)}, \tag{192}$$

in the third space, separated from the last by the line for which $\zeta^{(V)} = \zeta^{(S)}$,

$$\zeta^{(S)} < \zeta^{(V)} < \zeta^{(L)}, \tag{193}$$

in the fourth $$\zeta^{(S)} < \zeta^{(L)} < \zeta^{(V)}, \tag{194}$$

in the fifth $$\zeta^{(L)} < \zeta^{(S)} < \zeta^{(V)}, \tag{195}$$

in the sixth $$\zeta^{(L)} < \zeta^{(V)} < \zeta^{(S)}. \tag{196}$$

The sheet which gives the least values of ζ is in each case that which represents the stable states of the substance. From this it is evident that in passing around the projection of the triple point we pass through lines representing alternately coexistent stable and coexistent unstable states. But the states represented by the intermediate values of ζ may be called stable *relatively* to the states represented by the highest. The differences $\zeta^{(L)} - \zeta^{(V)}$, etc. represent the amount of work obtained in bringing the substance by a reversible process from one to the other of the states to which these quantities relate, in a medium having the temperature and pressure common to the two states. To illustrate such a process, we may suppose a plane perpendicular to the axis of temperature to pass through the points

* See the Reports of the British Association for 1871 and 1872; and Philosophical Magazine, vol. xlvii. (1874), p. 447.

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representing the two states. This will in general cut the double line formed by the two sheets to which the symbols (L) and (V) refer. The intersections of the plane with the two sheets will connect the double point thus determined with the points representing the initial and final states of the process, and thus form a *reversible path* for the body between those states.

The geometrical relations which indicate the stability of any state may be easily obtained by applying the principles stated on pp. 156 ff. to the case in which there is but a single component. The expression (133) as a test of stability will reduce to

$$\varepsilon - t'\eta + p'v - \mu' m, \tag{197}$$

the accented letters referring to the state of which the stability is in question, and the unaccented letters to any other state. If we consider the quantity of matter in each state to be unity, this expression may be reduced by equations (91) and (96) to the form

$$\zeta - \zeta' + (t - t')\eta - (p - p')v, \tag{198}$$

which evidently denotes the distance of the point (t', p', ζ') below the tangent plane for the point (t, p, ζ), measured parallel to the axis of ζ. Hence if the tangent plane for every other state passes above the point representing any given state, the latter will be stable. If any of the tangent planes pass below the point representing the given state, that state will be unstable. Yet it is not always necessary to consider these tangent planes. For, as has been observed on page 160, we may assume that (in the case of any real substance) there will be at least one not unstable state for any given temperature and pressure, except when the latter is negative. Therefore the state represented by a point in the surface on the positive side of the plane $p = 0$ will be unstable only when there is a point in the surface for which t and p have the same values and ζ a less value. It follows from what has been stated, that where the surface is doubly convex upwards (in the direction in which ζ is measured) the states represented will be stable in respect to adjacent states. This also appears directly from (162). But where the surface is concave upwards in either of its principal curvatures the states represented will be unstable in respect to adjacent states.

When the number of component substances is greater than unity, it is not possible to represent the fundamental equation by a single surface. We have therefore to consider how it may be represented by an infinite number of surfaces. A natural extension of either of the methods already described will give us a series of surfaces in

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which every one is the v-η-ε surface, or every one the t-p-ζ surface for a body of constant composition, the proportion of the components varying as we pass from one surface to another. But for a simultaneous view of the properties which are exhibited by compounds of two or three components without change of temperature or pressure, we may more advantageously make one or both of the quantities t or p constant in each surface.

Surfaces and Curves in which the Composition of the Body represented is Variable and its Temperature and Pressure are Constant.

When there are three components, the position of a point in the X-Y plane may indicate the composition of a body most simply, perhaps, as follows. The body is supposed to be composed of the quantities m_1, m_2, m_3 of the substances S_1, S_2, S_3, the value of $m_1 + m_2 + m_3$ being unity. Let P_1, P_2, P_3 be any three points in the plane, which are not in the same straight line. If we suppose masses equal to m_1, m_2, m_3 to be placed at these three points, the center of gravity of these masses will determine a point which will indicate the value of these quantities. If the triangle is equiangular and has the height unity, the distances of the point from the three sides will be equal numerically to m_1, m_2, m_3. Now if for every possible phase of the components, of a given temperature and pressure, we lay off from the point in the X-Y plane which represents the composition of the phase a distance measured parallel to the axis of Z and representing the value of ζ (when $m_1 + m_2 + m_3 = 1$), the points thus determined will form a surface, which may be designated as the m_1-m_2-m_3-ζ surface of the substances considered, or simply as their m-ζ surface, for the given temperature and pressure. In like manner, when there are but two component substances, we may obtain a curve, which we will suppose in the X-Z plane. The coordinate y may then represent temperature or pressure. But we will limit ourselves to the consideration of the properties of the m-ζ surface for $n = 3$, or the m-ζ curve for $n = 2$, regarded as a surface, or curve, which varies with the temperature and pressure.

As by (96) and (92)

$$\zeta = \mu_1 m_1 + \mu_2 m_2 + \mu_3 m_3,$$

and (for constant temperature and pressure)

$$d\zeta = \mu_1 dm_1 + \mu_2 dm_2 + \mu_3 dm_3,$$

if we imagine a tangent plane for the point to which these letters relate, and denote by ζ' the ordinate for any point in the plane,

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and by m_1', m_2', m_3', the distances of the foot of this ordinate from the three sides of the triangle $P_1 P_2 P_3$, we may easily obtain

$$\zeta' = \mu_1 m_1' + \mu_2 m_2' + \mu_3 m_3', \tag{199}$$

which we may regard as the equation of the tangent plane. Therefore the ordinates for this plane at P_1, P_2, and P_3 are equal respectively to the potentials μ_1, μ_2, and μ_3. And in general, the ordinate for any point in the tangent plane is equal to the potential (in the phase represented by the point of contact) for a substance of which the composition is indicated by the position of the ordinate. (See page 149.) Among the bodies which may be formed of S_1, S_2, and S_3, there may be some which are incapable of variation in composition, or which are capable only of a single kind of variation. These will be represented by single points and curves in vertical planes. Of the tangent plane to one of these curves only a single line will be fixed, which will determine a series of potentials of which only two will be independent. The phase represented by a separate point will determine only a single potential, viz., the potential for the substance of the body itself, which will be equal to ζ.

The points representing a set of coexistent phases have in general a common tangent plane. But when one of these points is situated on the edge where a sheet of the surface terminates, it is sufficient if the plane is tangent to the edge and passes below the surface. Or, when the point is at the end of a separate line belonging to the surface, or at an angle in the edge of a sheet, it is sufficient if the plane pass through the point and below the line or sheet. If no part of the surface lies below the tangent plane, the points where it meets the plane will represent a stable (or at least not unstable) set of coexistent phases.

The surface which we have considered represents the relation between ζ and m_1, m_2, m_3 for homogeneous bodies when t and p have any constant values and $m_1 + m_2 + m_3 = 1$. It will often be useful to consider the surface which represents the relation between the same variables for bodies which consist of parts in different but coexistent phases. We may suppose that these are stable, at least in regard to adjacent phases, as otherwise the case would be devoid of interest. The point which represents the state of the composite body will evidently be at the center of gravity of masses equal to the parts of the body placed at the points representing the phases of these parts. Hence from the surface representing the properties of homogeneous bodies, which may be called the primitive surface, we

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may easily construct the surface representing the properties of bodies which are in equilibrium but not homogeneous. This may be called the secondary or derived surface. It will consist, in general, of various portions or sheets. The sheets which represent a combination of two phases may be formed by rolling a double tangent plane upon the primitive surface: the part of the envelop of its successive positions which lies between the curves traced by the points of contact will belong to the derived surface. When the primitive surface has a triple tangent plane or one of higher order, the triangle in the tangent plane formed by joining the points of contact, or the smallest polygon without re-entrant angles which includes all the points of contact, will belong to the derived surface, and will represent masses consisting in general of three or more phases.

Of the whole thermodynamic surface as thus constructed for any temperature and any positive pressure, that part is especially important which gives the least value of ζ for any given values of m_1, m_2, m_3. The state of a mass represented by a point in this part of the surface is one in which no dissipation of energy would be possible if the mass were enclosed in a rigid envelop impermeable both to matter and to heat; and the state of any mass composed of S_1, S_1, S_3 in any proportions, in which the dissipation of energy has been completed, so far as internal processes are concerned, (i. e., under the limitations imposed by such an envelop as above supposed,) would be represented by a point in the part which we are considering of the m-ζ surface for the temperature and pressure of the mass. We may therefore briefly distinguish this part of the surface as the *surface of dissipated energy.* It is evident that it forms a continuous sheet, the projection of which upon the X-Y plane coincides with the triangle $P_1 P_2 P_3$, (except when the pressure for which the m-ζ surface is constructed is negative, in which case there is no surface of dissipated energy,) that it nowhere has any convexity upward, and that the states which it represents are in no case unstable.

The general properties of the m-ζ lines for two component substances are so similar as not to require separate consideration. We now proceed to illustrate the use of both the surfaces and the lines by the discussion of several particular cases.

Three coexistent phases of two component substances may be represented by the points A, B, and C, in figure 1, in which ζ is measured toward the top of the page from P_1P_2, m_1 toward the left from P_2Q_2, and m_2 toward the right from P_1Q_1. It is supposed that $P_1P_2 = 1$. Portions of the curves to which these points belong

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are seen in the figure, and will be denoted by the symbols (A), (B), (C). We may, for convenience, speak of these as separate curves, without implying anything in regard to their possible continuity in parts of the diagram remote from their common tangent AC. The *line of dissipated energy* includes the straight line AC and portions of the primitive curves (A) and (C). Let us first consider how the diagram will be altered, if the temperature is varied while the pressure remains constant. If the temperature receives the increment dt, an ordinate of which the position is fixed will receive the increment $\left(\frac{d\zeta}{dt}\right)_{p,\,m} dt$, or $-\eta\, dt$. (The reader will easily convince himself that this is true of the ordinates for the secondary line AC, as well as of the ordinates for the primitive curves.) Now if we denote by η' the entropy of the phase represented by the point B considered as belonging to the curve (B), and by η'' the entropy of the composite state of the same matter represented by the point B considered as belonging to the tangent to the curves (A) and (C), $t(\eta' - \eta'')$ will denote the heat yielded by a unit of matter in passing from the first to the second of these states. If this quantity is positive, an elevation of temperature will evidently cause a part of the curve (B) to protrude below the tangent to (A) and (C), which will no longer form a part of the line of dissipated energy. This line will then include portions of the three curves (A), (B), and (C), and of the tangents to (A) and (B) and to (B) and (C). On the other hand, a lowering of the temperature will cause the curve (B) to lie entirely above the tangent to (A) and (C), so that all the phases of the sort represented by (B) will be unstable. If $t(\eta' - \eta'')$ is negative, these effects will be produced by the opposite changes of temperature.

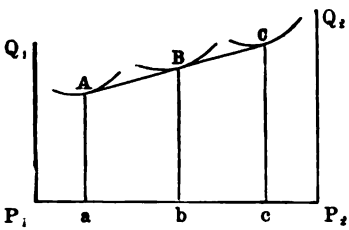

FIG. 1.

The effect of a change of pressure while the temperature remains constant may be found in a manner entirely analogous. The variation of any ordinate will be $\left(\frac{d\zeta}{dp}\right)_{t,\,m} dp$ or $v\, dp$. Therefore, if the volume of the homogeneous phase represented by the point B is a greater than the volume of the same matter divided between the the phases represented by A and C, an increase of pressure will give diagram indicating that all phases of the sort represented by curve (B) are unstable, and a decrease of pressure will give a diagram indi-

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cating two stable pairs of coexistent phases, in each of which one of the phases is of the sort represented by the curve (B). When the relation of the volumes is the reverse of that supposed, these results will be produced by the opposite changes of pressure.

When we have four coexistent phases of three component substances, there are two cases which must be distinguished. In the first, one of the points of contact of the primitive surface with the quadruple tangent plane lies within the triangle formed by joining the other three; in the second, the four points may be joined so as to form a quadrilateral without re-entrant angles. Figure 2 represents the projection upon the X-Y plane (in which m_1, m_2, m_3 are measured) of a part of the surface of dissipated energy, when one of the points of contact D falls within the triangle formed by the other three A, B, C. This surface includes the triangle ABC in the quadruple tangent plane, portions of the three sheets of the primitive surface which touch the triangle at its vertices, EAF, GBH, ICK, and portions of the three developable surfaces formed by a tangent plane rolling upon each pair of these sheets. These developable surfaces are repre-

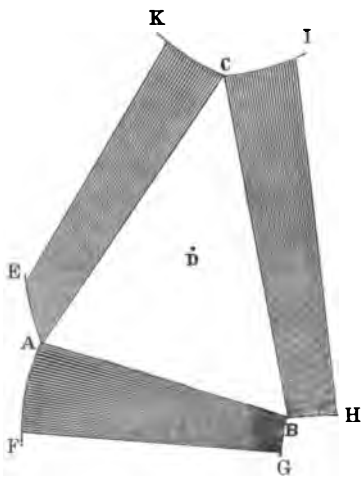

FIG. 2.

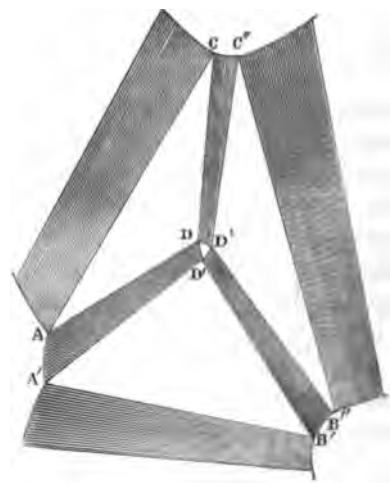

FIG. 3.

sented in the figure by ruled surfaces, the lines indicating the direction of their rectilinear elements. A point within the triangle ABC represents a mass of which the matter is divided, in general, between three or four different phases, in a manner not entirely determined by the position of a point. (The quantities of matter in these phases are such that if placed at the corresponding points, A, B, C, D, their center of gravity would be at the point representing the total mass.)

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Such a mass, if exposed to constant temperature and pressure, would be in neutral equilibrium. A point in the developable surfaces represents a mass of which the matter is divided between two coexisting phases, which are represented by the extremities of the line in the figure passing through that point. A point in the primitive surface represents of course a homogeneous mass.

To determine the effect of a change of temperature without change of pressure upon the general features of the surface of dissipated energy, we must know whether heat is absorbed or yielded by a mass in passing from the phase represented by the point D *in the primitive surface* to the composite state consisting of the phases A, B, and C which is represented by the same point. If the first is the case, an increase of temperature will cause the sheet (D) (i. e., the sheet of the primitive surface to which the point D belongs) to separate from the plane tangent to the three other sheets, so as to be situated entirely above it, and a decrease of temperature, will cause a part of the sheet (D) to protrude through the plane tangent to the other sheets. These effects will be produced by the opposite changes of temperature, when heat is yielded by a mass passing from the homogeneous to the composite state above mentioned.

In like manner, to determine the effect of a variation of pressure without change of temperature, we must know whether the volume for the homogeneous phase represented by D is greater or less than the volume of the same matter divided between the phases A, B, and C. If the homogeneous phase has the greater volume, an increase of pressure will cause the sheet (D) to separate from the plane tangent to the other sheets, and a diminution of pressure will cause a part of the sheet (D) to protrude below that tangent plane. And these effects will be produced by the opposite changes of pressure, if the homogeneous phase has the less volume. All this appears from precisely the same considerations which were used in the analogous case for two component substances.

Now when the sheet (D) rises above the plane tangent to the other sheets, the general features of the surface of dissipated energy are not altered, except by the disappearance of the point D. But when the sheet (D) protrudes below the plane tangent to the other sheets, the surface of dissipated energy will take the form indicated in figure 3. It will include portions of the four sheets of the primitive surface, portions of the six developable surfaces formed by a double tangent plane rolling upon these sheets taken two by two, and portions of three triple tangent planes for these sheets taken by threes, the sheet (D) being always one of the three.

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But when the points of contact with the quadruple tangent plane which represent the four coexistent phases can be joined so as to form a quadrilateral ABCD (fig. 4) without reentrant angles, the surface of dissipated energy will include this plane quadrilateral, portions of the four sheets of the primitive surface which are tangent to it, and portions of the four developable surfaces formed by double

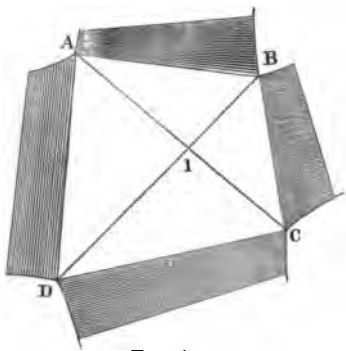

FIG. 4.

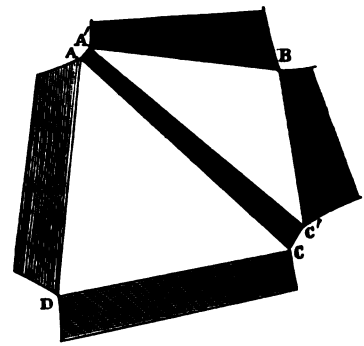

FIG. 5.

tangent planes rolling upon the four pairs of these sheets which correspond to the four sides of the quadrilateral. To determine the general effect of a variation of temperature upon the surface of dissipated energy, let us consider the composite states represented by the point I at the intersection of the diagonals of the quadrilateral. Among these states (which all relate to the same kind and quantity of matter) there is one which is composed of the phases A and C, and another which is composed of the phases B and D. Now if the entropy of the first of these states is greater than that of the second, (i. e., if heat is given out by a body in passing from the first to the second state at constant temperature and pressure,) which we may suppose without loss of generality, an elevation of temperature while the pressure remains constant will cause the triple tangent planes to (B), (D), and (A), and to (B), (D), and (C), to rise above the triple tangent planes to (A), (C), and (B), and to (A), (C), and (D), in the vicinity of the point I. The surface of dissipated energy will therefore take the form indicated in figure 5, in which there are two plane triangles and five developable surfaces besides portions of the four primitive sheets. A diminution of temperature will give a different but entirely analogous form to the surface of dissipated energy. The quadrilateral ABCD will in this case break into two triangles along the diameter BD. The effects produced by

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variation of the pressure while the temperature remains constant will of course be similar to those described. By considering the difference of volume instead of the difference of entropy of the two states represented by the point I in the quadruple tangent plane, we may distinguish between the effects of increase and diminution of pressure.

It should be observed that the points of contact of the quadruple tangent plane with the primitive surface may be at isolated points or curves belonging to the latter. So also, in the case of two component substances, the points of contact of the triple tangent line may be at isolated points belonging to the primitive curve. Such cases need not be separately treated, as the necessary modifications in the preceding statements, when applied to such cases, are quite evident. And in the remaining discussion of this geometrical method, it will generally be left to the reader to make the necessary limitations or modifications in analogous cases.

The necessary condition in regard to simultaneous variations of temperature and pressure, in order that four coexistent phases of three components, or three coexistent phases of two components, shall remain possible, has already been deduced by purely analytical processes. (See equation (129).)

We will next consider the case of two coexistent phases of identical composition, and first, when the number of components is two. The coexistent phases, if each is variable in composition, will be represented by the point of contact of two curves. One of the curves will in general lie above the other except at the point of contact; therefore, when the temperature and pressure remain constant, one phase cannot be varied in composition without becoming unstable, while the other phase will be stable if the proportion of either component is increased. By varying the temperature or pressure, we may cause the upper curve to protrude below the other, or to rise (relatively) entirely above it. (By comparing the volumes or the entropies of the two coexistent phases, we may easily determine which result would be produced by an increase of temperature or of pressure.) Hence, the temperatures and pressures for which two coexistent phases have the same composition form the limit to the temperatures and pressures for which such coexistent phases are possible. It will be observed that as we pass this limit of temperature and pressure, the pair of coexistent phases does not simply become unstable, like pairs and triads of coexistent phases which we have considered before, but there ceases to be any such pair of coexistent phases. The same result has already been obtained analytically on

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page 155. But on that side of the limit on which the coexistent phases are possible, there will be two pairs of coexistent phases for the same values of t and p, as seen in figure 6. If the curve AA′ represents vapor, and the curve BB′ liquid, a liquid (represented by) B may exist in contact with a vapor A, and (at the same temperature and pressure) a liquid B′ in contact with a vapor A′. If we compare these phases in respect to their composition, we see that in one case the vapor is richer than the liquid in a certain component, and in the other case poorer. Therefore, if these liquids are made to boil, the effect on their composition will be opposite. If the boiling is continued under constant pressure, the temperature will rise as the liquids approach each other in composition, and the curve BB′ will rise *relatively* to the curve AA′, until the curves are tangent to each other, when the two liquids become identical in nature, as also the vapors which they yield. In composition, and in the value of ζ per unit of mass, the vapor will then agree with the liquid. But if the curve BB′ (which has the greater curvature) represents vapor, and AA′ represents liquid, the effect of boiling will make the liquids A and A′ differ more in composition. In this case, the relations indicated in the figure will hold for a temperature higher than that for which (with the same pressure) the curves are tangent to one another.

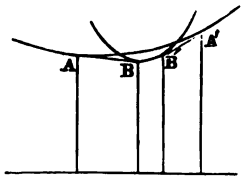

FIG. 6.

When two coexistent phases of three component substances have the same composition, they are represented by the point of contact of two sheets of the primitive surface. If these sheets do not intersect at the point of contact, the case is very similar to that which we have just considered. The upper sheet except at the point of contact represents unstable phases. If the temperature or pressure are so varied that a part of the upper sheet protrudes through the lower, the points of contact of a double tangent plane rolling upon the two sheets will describe a closed curve on each, and the surface of dissipated energy will include a portion of each sheet of the primitive surface united by a ring-shaped developable surface.

If the sheet having the greater curvatures represents liquid, and the other sheet vapor, the boiling temperature for any given pressure will be a maximum, and the pressure of saturated vapor for any given temperature will be a minimun, when the coexistent liquid and vapor have the same composition.

But if the two sheets, constructed for the temperature and pressure of the coexistent phases which have the same composition, intersect

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at the point of contact, the whole primitive surface as seen from below will in general present four re-entrant furrows, radiating from the point of contact, for each of which a developable surface may be formed by a rolling double tangent plane. The different parts of the surface of dissipated energy in the vicinity of the point of contact are represented in figure 7. ATB, ETF are parts of one sheet of the primitive surface, and CTD, GTH are parts of the other. These are united by the developable surfaces BTC, DTE, FTG, HTA. Now we may make either sheet of the primitive surface sink relatively to the other by the proper variation of temperature or pressure. If the sheet to which ATB, ETF belong is that which sinks relatively, these parts of the surface of dissipated energy will be merged in one, as well as the developable surfaces BTC, DTE, and also FTG, HTA. (The lines CTD, BTE, ATF, HTG will separate from one another at T, each forming a continuous curve.) But if the sheet of the primitive surface which sinks relatively is that to which CTD and GTH belong, then these parts will be merged in one in the surface of dissipated energy, as will be the developable surfaces BTC, ATH, and also DTE, FTG.

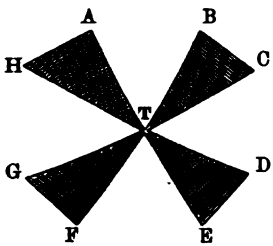

FIG. 7.

It is evident that this is not a case of maximum or minimum temperature for coexistent phases under constant pressure, or of maximum or minimum pressure for coexistent phases at constant temperature.

Another case of interest is when the composition of one of three coexistent phases is such as can be produced by combining the other two. In this case, the primitive surface must touch the same plane in three points in the same straight line. Let us distinguish the parts of the primitive surface to which these points belong as the sheets (A), (B), and (C), (C) denoting that which is intermediate in position. The sheet (C) is evidently tangent to the developable surface formed upon (A) and (B). It may or it may not intersect it at the point of contact. If it does not, it must lie above the developable surface, (unless it represents states which are unstable in regard to continuous changes,) and the surface of dissipated energy will include parts of the primitive sheets (A) and (B), the developable surface joining them, and the single point of the sheet (C) in which it meets this developable surface. Now, if the temperature or pressure is varied so as to make the sheet (C) rise above the

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developable surface formed on the sheets (A) and (B), the surface of dissipated energy will be altered in its general features only by the removal of the single point of the sheet (C). But if the temperature or pressure is altered so as to make a part of the sheet (C) protrude through the developable surface formed on (A) and (B), the surface of dissipated energy will have the form indicated in figure 8. It will include two plane triangles ABC and A′B′C′, a part of each of the sheets (A) and (B), represented in the figure by the spaces on the left of the line aAA′a′ and on the right of the line bBB′b′, a small part CC′ of the sheet (C), and developable surfaces formed upon these sheets taken by pairs ACC′A′, BCC′B′, aABb, a′A′B′b′, the last two being different portions of the same developable surface.

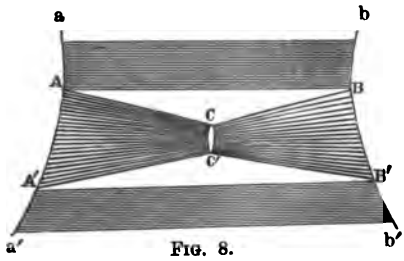

FIG. 8.

But if, when the primitive surface is constructed for such a temperature and pressure that it has three points of contact with the same plane in the same straight line, the sheet (C) (which has the middle position) at its point of contact with the triple tangent plane intersects the developable surface formed upon the other sheets (A) and (B), the surface of dissipated energy will not include this developable surface, but will consist of portions of the three primitive sheets with two developable surfaces formed on (A) and (C) and on (B) and (C). These developable surfaces meet one another at the point of contact of (C) with the triple tangent plane, dividing the portion of this sheet which belongs to the surface of dissipated energy into two parts. If now the temperature or pressure are varied so as to make the sheet (C) sink relatively to the developable surface formed on (A) and (B), the only alteration in the general features of the surface of dissipated energy will be that the developable surfaces formed on (A) and (C) and on (B) and (C) will separate from one another, and the two parts of the sheet (C) will be merged in one. But a contrary variation of temperature or pressure will give a

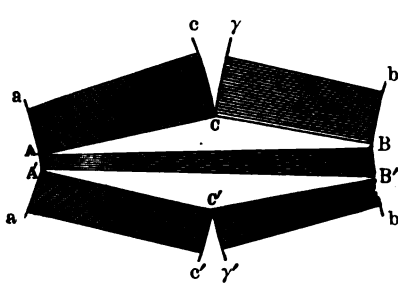

FIG. 9.

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surface of dissipated energy such as is represented in figure (9), containing two plane triangles ABC, A′B′C′ belonging to triple tangent planes, a portion of the sheet (A) on the left of the line aAA′a′, a portion of the sheet (B) on the right of the line bBB′b′, two separate portions cCγ and c′C′γ' of the sheet (C), two separate portions aACc and a′A′C′c′ of the developable surface formed on (A) and (C), two separate portions bBCγ and b′B′C′γ' of the developable surface formed on (B) and (C), and the portion A′ABB′ of the developable surface formed on (A) and (B).

From these geometrical relations it appears that (in general) the temperature of three coexistent phases is a maximum or minimum for constant pressure, and the pressure of three coexistent phases a maximum or minimum for constant temperature, when the composition of the three coexistent phases is such that one can be formed by combining the other two. This result has been obtained analytically on page 156.

The preceding examples are amply sufficient to illustrate the use of the m-ζ surfaces and curves. The physical properties indicated by the nature of the surface of dissipated energy have been only occasionally mentioned, as they are often far more distinctly indicated by the diagrams than they could be in words. It will be observed that a knowledge of the lines which divide the various different portions of the surface of dissipated energy and of the direction of the rectilinear elements of the developable surfaces, *as projected upon the X-Y plane*, without a knowledge of the form of the m-ζ surface in space, is sufficient for the determination (in respect to the quantity and composition of the resulting masses) of the combinations and separations of the substances, and of the changes in their states of aggregation, which take place when the substances are exposed to the temperature and pressure to which the projected lines relate, except so far as such transformations are prevented by passive resistances to change.

CRITICAL PHASES.

It has been ascertained by experiment that the variations of two coexistent states of the same substance are in some cases limited in one direction by a terminal state at which the distinction of the coexistent states vanishes.* This state has been called the *critical state*. Analogous properties may doubtless be exhibited by compounds of variable composition without change of temperature or

* See Dr. Andrews "On the continuity of the gaseous and liquid states of matter." Phil. Trans., vol. 159, p. 575.

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pressure. For if, at any given temperature and pressure, two liquids are capable of forming a stable mixture in any ratio $m_1 : m_2$ less than a, and in any greater than b, a and b being the values of that ratio for two coexistent phases, while either can form a stable mixture with a third liquid in all proportions, and any small quantities of the first and second can unite at once with a great quantity of the third to form a stable mixture, it may easily be seen that two coexistent mixtures of the three liquids may be varied in composition, the temperature and pressure remaining the same, from initial phases in each of which the quantity of the third liquid is nothing, to a terminal phase in which the distinction of the two phases vanishes.

In general, we may define a *critical phase* as one at which the distinction between coexistent phases vanishes. We may suppose the coexistent phases to be stable in respect to continuous changes, for although relations in some respects analogous might be imagined to hold true in regard to phases which are unstable in respect to continuous changes, the discussion of such cases would be devoid of interest. But if the coexistent phases and the critical phase are unstable only in respect to the possible formation of phases entirely different from the critical and adjacent phases, the liability to such changes will in no respect affect the relations between the critical and adjacent phases, and need not be considered in a theoretical discussion of these relations, although it may prevent an experimental realization of the phases considered. For the sake of brevity, in the following discussion, phases in the vicinity of the critical phase will generally be called stable, if they are unstable only in respect to the formation of phases entirely different from any in the vicinity of the critical phase.

Let us first consider the number of independent variations of which a critical phase (while remaining such) is capable. If we denote by n the number of independently variable components, a pair of coexistent phases will be capable of n independent variations, which may be expressed by the variations of n of the quantities $t, p, \mu_1, \mu_2, \ldots \mu_n$. If we limit these variations by giving to $n-1$ of the quantities the constant values which they have for a certain critical phase, we obtain a linear* series of pairs of coexistent phases terminated by the critical phase. If we now vary infinitesimally the values of these $n-1$ quantities, we shall have for the new set of values considered constant a new linear series of pairs of coexistent phases. Now for every pair of phases in the first series, there must be pairs of phases in the

* This term is used to characterize a series having a *single* degree of extension.

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second series differing infinitely little from the pair in the first, and *vice versa*, therefore the second series of coexistent phases must be terminated by a critical phase which differs, but differs infinitely little, from the first. We see, therefore, that if we vary arbitrarily the values of any $n-1$ of the quantities $t, p, \mu_1, \mu_2, \ldots \mu_n$, as determined by a critical phase, we obtain one and only one critical phase for each set of varied values; i. e., a critical phase is capable of $n-1$ independent variations.

The quantities $t, p, \mu_1, \mu_2, \ldots \mu_n$, have the same values in two coexistent phases, but the ratios of the quantities $\eta, v, m_1, m_2, \ldots m_n$, are in general different in the two phases. Or, if for convenience we compare equal volumes of the two phases (which involves no loss of generality), the quantities $\eta, m_1, m_2, \ldots m_n$ will in general have different values in two coexistent phases. Applying this to coexistent phases indefinitely near to a critical phase, we see that in the immediate vicinity of a critical phase, if the values of n of the quantities $t, p, \mu_1, \mu_2, \ldots \mu_n$, are regarded as constant (as well as v), the variations of either of the others will be infinitely small compared with the variations of the quantities $\eta, m_1, m_2, \ldots m_n$. This condition, which we may write in the form

$$\left(\frac{d\mu_n}{dm_n}\right)_{t,\, v,\, \mu_1,\, \ldots\, \mu_{n-1}} = 0, \tag{200}$$

characterizes, as we have seen on page 171, the limits which divide stable from unstable phases in respect to continuous changes.

In fact, if we give to the quantities $t, \mu_1, \mu_2, \ldots \mu_{n-1}$ constant values determined by a pair of coexistent phases, and to $\frac{m_n}{v}$ a series of values increasing from the less to the greater of the values which it has in these coexistent phases, we determine a linear series of phases connecting the coexistent phases, in some part of which μ_n—since it has the same value in the two coexistent phases, but not a uniform value throughout the series (for if it had, which is theoretically improbable, all these phases would be coexistent)—must be a decreasing function of $\frac{m_n}{v}$, or of m_n, if v also is supposed constant. Therefore, the series must contain phases which are unstable in respect to continuous changes. (See page 168.) And as such a pair of coexistent phases may be taken indefinitely near to any critical phase, the unstable phases (with respect to continuous changes) must approach indefinitely near to this phase.

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Critical phases have similar properties with reference to stability as determined with regard to discontinuous changes. For as every stable phase which has a coexistent phase lies upon the limit which separates stable from unstable phases, the same must be true of any stable critical phase. (The same may be said of critical phases which are unstable in regard to discontinuous changes if we leave out of account the liability to the particular kind of discontinuous change in respect to which the critical phase is unstable.)

The linear series of phases determined by giving to n of the quantities $t, p, \mu_1, \mu_2, \ldots \mu_n$ the constant values which they have in any pair of coexistent phases consists of unstable phases in the part between the coexistent phases, but in the part beyond these phases in either direction it consists of stable phases. Hence, if a critical phase is varied in such a manner that n of the quantities $t, p, \mu_1, \mu_2, \ldots \mu_n$ remain constant, it will remain stable in respect both to continuous and to discontinuous changes. Therefore, μ_n is an increasing function of m_n when $t, v, \mu_1, \mu_2, \ldots \mu_{n-1}$ have constant values determined by any critical phase. But as equation (200) holds true at the critical phase, the following conditions must also hold true at that phase:

$$\left(\frac{d^2\mu_n}{dm_n^2}\right)_{t,\, v,\, \mu_1, \ldots \mu_{n-1}} = 0, \tag{201}$$

$$\left(\frac{d^3\mu_n}{dm_n^3}\right)_{t,\, v,\, \mu_1, \ldots \mu_{n-1}} \geqq 0. \tag{202}$$

If the sign of equality holds in the last condition, additional conditions, concerning the differential coefficients of higher orders, must be satisfied.

Equations (200) and (201) may in general be called the equations of critical phases. It is evident that there are only two independent equations of this character, as a critical phase is capable of $n-1$ independent variations.

We are not, however, absolutely certain that equation (200) will always be satisfied by a critical phase. For it is possible that the denominator in the fraction may vanish as well as the numerator for an infinitesimal change of phase in which the quantities indicated are constant. In such a case, we may suppose the subscript n to refer to some different component substance, or use another differential coefficient of the same general form (such as are described on page 171 as characterizing the limits of stability in respect to continuous changes), making the corresponding changes in (201) and (202). We may be certain that some of the formulæ thus formed will not fail. But for a perfectly rigorous method there is an advan-

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tage in the use of η, v, $m_1, m_2, \ldots m_n$ as independent variables. The condition that the phase may be varied without altering any of the quantities t, μ_1, μ_2, . . . μ_n will then be expressed by the equation

$$R_{n+1} = 0, \tag{203}$$

in which R_{n+1} denotes the same determinant as on page 169. To obtain the second equation characteristic of critical phases, we observe that as a phase which is critical cannot become unstable when varied so that n of the quantities t, p, μ_1, μ_2, . . . μ_n remain constant, the differential of R_{n+1} for constant volume, viz.,

$$\frac{dR_{n+1}}{d\eta}\,d\eta + \frac{dR_{n+1}}{dm_1}\,dm_1 \ldots + \frac{dR_{n+1}}{dm_n}\,dm_n \tag{204}$$

cannot become negative when n of the equations (172) are satisfied. Neither can it have a positive value, for then its value might become negative by a change of sign of $d\eta$, dm_1, etc. Therefore the expression (204) has the value zero, if n of the equations (172) are satisfied. This may be expressed by an equation

$$S = 0, \tag{205}$$

in which S denotes a determinant in which the constituents are the same as in R_{n+1}, except in a single horizontal line, in which the differential coefficients in (204) are to be substituted. In whatever line this substitution is made, the equation (205), as well as (203), will hold true of every critical phase without exception.

If we choose t, p, m_1, m_2, . . . m_n as independent variables, and write U for the determinant

$$\begin{vmatrix} \dfrac{d^2\zeta}{dm_1^2} & \dfrac{d^2\zeta}{dm_2 dm_1} & \cdots & \dfrac{d^2\zeta}{dm_{n-1} dm_1} \\ \dfrac{d^2\zeta}{dm_1 dm_2} & \dfrac{d^2\zeta}{dm_2^2} & \cdots & \dfrac{d^2\zeta}{dm_{n-1} dm_2} \\ \cdot & \cdot & \cdots & \cdot \\ \dfrac{d^2\zeta}{dm_1 dm_{n-1}} & \dfrac{d^2\zeta}{dm_2 dm_{n-1}} & \cdots & \dfrac{d^2\zeta}{dm_{n-1}^2} \end{vmatrix} \tag{206}$$

and V for the determinant formed from this by substituting for the constituents in any horizontal line the expressions

$$\frac{dU}{dm_1}, \quad \frac{dU}{dm^2}, \quad \cdots \quad \frac{dU}{dm_{n-1}}, \tag{207}$$

the equations of critical phases will be

$$U = 0, \qquad V = 0. \tag{208}$$

It results immediately from the definition of a critical phase, that an infinitesimal change in the condition of a mass in such a phase

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may cause the mass, if it remains in a state of dissipated energy (i. e., in a state in which the dissipation of energy by internal processes is complete), to cease to be homogeneous. In this respect a critical phase resembles any phase which has a coexistent phase, but differs from such phases in that the two parts into which the mass divides when it ceases to be homogeneous differ infinitely little from each other and from the original phase, and that neither of these parts is in general infinitely small. If we consider a change in the mass to be determined by the values of $d\eta$, dv, dm_1, dm_2, . . . dm_n, it is evident that the change in question will cause the mass to cease to be homogeneous whenever the expression

$$\frac{dR_{n+1}}{d\eta}\,d\eta+\frac{dR_{n+1}}{dv}\,dv+\frac{dR_{n+1}}{dm_1}\,dm_1,\ldots+\frac{dR_{n+1}}{dm_n}\,dm_n \tag{209}$$

has a negative value. For if the mass should remain homogeneous, it would become unstable, as R_{n+1} would become negative. Hence, in general, any change thus determined, or its reverse (determined by giving to $d\eta$, dv, dm_1, dm_2, . . . dm_n the same values taken negatively), will cause the mass to cease to be homogeneous. The condition which must be satisfied with reference to $d\eta$, dv, dm_1, dm_2, . . . dm_n, in order that neither the change indicated, nor the reverse, shall destroy the homogeneity of the mass, is expressed by equating the above expression to zero.

But if we consider the change in the state of the mass (supposed to remain in a state of dissipated energy) to be determined by arbitrary values of $n+1$ of the differentials dt, dp, $d\mu_1$, $d\mu_2$, . . . $d\mu_n$, the case will be entirely different. For, if the mass ceases to be homogeneous, it will consist of two coexistent phases, and as applied to these only n of the quantities t, p, μ_1, μ_2, . . . μ_n will be independent. Therefore, for arbitrary variations of $n+1$ of these quantities, the mass must in general remain homogeneous.

But if, instead of supposing the mass to remain in a state of dissipated energy, we suppose that it remains homogeneous, it may easily be shown that to certain values of $n+1$ of the above differentials there will correspond three different phases, of which one is stable with respect both to continuous and to discontinuous changes, another is stable with respect to the former and unstable with respect to the latter, and the third is unstable with respect to both.

In general, however, if n of the quantities p, t, μ_1, μ_2, . . . μ_n, or n arbitrary functions of these quantities, have the same constant values as at a critical phase, the linear series of phases thus determined will be stable, in the vicinity of the critical phase. But if less

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than n of these quantities or functions of the same together with certain of the quantities η, v, m_1, m_2, . . . m_n, or arbitrary functions of the latter quantities, have the same values as at a critical phase, so as to determine a linear series of phases, the differential of R_{n+1} in such a series of phases will not in general vanish at the critical phase, so that in general a part of the series will be unstable.

We may illustrate these relations by considering separately the cases in which $n=1$ and $n=2$. If a mass of invariable composition is in a critical state, we may keep its volume constant, and destroy its homogeneity by changing its entropy (i. e., by adding or subtracting heat—probably the latter), or we may keep its entropy constant and destroy its homogeneity by changing its volume; but if we keep its pressure constant we cannot destroy its homogeneity by any thermal action, nor if we keep its temperature constant can we destroy its homogeneity by any mechanical action.

When a mass having two independently variable components is in a critical phase, and either its volume or its pressure is maintained constant, its homogeneity may be destroyed by a change of entropy or temperature. Or, if either its entropy or its temperature is maintained constant, its homogeneity may be destroyed by a change of volume or pressure. In both these cases it is supposed that the quantities of the components remain unchanged. But if we suppose both the temperature and the pressure to be maintained constant, the mass will remain homogeneous, however the proportion of the components be changed. Or, if a mass consists of two coexistent phases, one of which is a critical phase having two independently variable components, and either the temperature or the pressure of the mass is maintained constant, it will not be possible by mechanical or thermal means, or by changing the quantities of the components, to cause the critical phase to change into a pair of coexistent phases, so as to give three coexistent phases in the whole mass. The statements of this paragraph and of the preceding have reference only to infinitesimal changes.*

* A brief abstract (which came to the author's notice after the above was in type) of a memoir by M. Duclaux, "Sur la séparation des liquides mélangés, etc." will be found in *Comptes Rendus*, vol. lxxxi. (1875), p. 815.

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ON THE VALUES OF THE POTENTIALS WHEN THE QUANTITY OF ONE OF THE COMPONENTS IS VERY SMALL.

If we apply equation (97) to a homogeneous mass having two independently variable components S_1 and S_2, and make t, p, and m_1 constant, we obtain

$$m_1\left(\frac{d\mu_1}{dm_2}\right)_{t,\,p,\,m_1} + m_2\left(\frac{d\mu_2}{dm_2}\right)_{t,\,p,\,m_1} = 0. \tag{210}$$

Therefore, for $m_2=0$, either

$$\left(\frac{d\mu_1}{dm_2}\right)_{t,\,p,\,m_1} = 0, \tag{211}$$

or

$$\left(\frac{d\mu_2}{dm_2}\right)_{t,\,p,\,m_1} = \infty. \tag{212}$$

Now, whatever may be the composition of the mass considered, we may always so choose the substance S_1 that the mass shall consist solely of that substance, and in respect to any other variable component S_2, we shall have $m_2=0$. But equation (212) cannot hold true *in general* as thus applied. For it may easily be shown (as has been done with regard to the potential on pages 148, 149) that the value of a differential coefficient like that in (212) for any given mass, when the substance S_2 (to which m_2 and μ_2 relate) is determined, is independent of the particular substance which we may regard as the other component of the mass; so that, if equation (212) holds true when the substance denoted by S_1 has been so chosen that $m_2=0$, it must hold true without such a restriction, which cannot generally be the case.

In fact, it is easy to prove directly that equation (211) will hold true of any phase which is stable in regard to continuous changes and in which $m_2=0$, *if m_2 is capable of negative as well as positive values.* For by (171), in any phase having that kind of stability, μ_1 is an increasing function of m_1 when t, p, and m_2 are regarded as constant. Hence, μ_1 will have its greatest value when the mass consists wholly of S_1, i. e., when $m_2=0$. Therefore, if m_2 is capable of negative as well as positive values, equation (211) must hold true for $m_2=0$. (This appears also from the geometrical representation of potentials in the m-ζ curve. See page 177.)

But if m_2 is capable only of positive values, we can only conclude from the preceding considerations that the value of the differential coefficient in (211) cannot be positive. Nor, if we consider the physical significance of this case, viz., that an increase of m_2 denotes an

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addition to the mass in question of a substance not before contained in it, does any reason appear for supposing that this differential coefficient has generally the value zero. To fix our ideas, let us suppose that S_1 denotes water, and S_2 a salt (either anhydrous or any particular hydrate). The addition of the salt to water, previously in a state capable of equilibrium with vapor or with ice, will destroy the possibility of such equilibrium at the same temperature and pressure. The liquid will dissolve the ice, or condense the vapor, which is brought in contact with it under such circumstances, which shows that μ_1 (the potential for water in the liquid mass) is diminished by the addition of the salt, when the temperature and pressure are maintained constant. Now there seems to be no *a priori* reason for supposing that the ratio of this diminution of the potential for water to the quantity of the salt which is added vanishes with this quantity. We should rather expect that, for small quantities of the salt, an effect of this kind would be proportional to its cause, i. e., that the differential coefficient in (211) would have a finite negative value for an infinitesimal value of m_2. That this is the case with respect to numerous watery solutions of salts is distinctly indicated by the experiments of Wüllner* on the tension of the vapor yielded by such solutions, and of Rüdorff† on the temperature at which ice is formed in them; and unless we have experimental evidence that cases are numerous in which the contrary is true, it seems not unreasonable to assume, as a *general* law, that when m_2 has the value zero and is incapable of negative values, the differential coefficient in (211) will have a finite negative value, and that equation (212) will therefore hold true. But this case must be carefully distinguished from that in which m_2 is capable of negative values, which also may be illustrated by a solution of a salt in water. For this purpose let S_1 denote a hydrate of the salt which can be crystallized, and let S_2 denote water, and let us consider a liquid consisting entirely of S_1 and of such temperature and pressure as to be in equilibrium with crystals of S_1. In such a liquid, an increase or a diminution of the quantity of water would alike cause crystals of S_1 to dissolve, which requires that the differential coefficient in (211) shall vanish at the particular phase of the liquid for which $m_2 = 0$.

Let us return to the case in which m_2 is incapable of negative values, and examine, without other restriction in regard to the substances

* Pogg. Ann., vol. ciii. (1858), p. 529; vol. cv. (1858), p. 85; vol. cx. (1860), p. 564.

† Pogg. Ann., vol. cxiv. (1861), p. 63.

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denoted by S_1 and S_2, the relation between μ_2 and $\frac{m_2}{m_1}$ for any constant temperature and pressure and for such small values of $\frac{m_2}{m_1}$ that the differential coefficient in (211) may be regarded as having the same constant value as when $m_2 = 0$, the values of t, p, and m_1 being unchanged. If we denote this value of the differential coefficient by $\frac{-A}{m_1}$, the value of A will be positive, and will be independent of m_1. Then for small values of $\frac{m_2}{m_1}$, we have by (210), approximately,

$$m_2\left(\frac{d\mu_2}{dm_2}\right)_{t,\,p,\,m_1} = A, \tag{213}$$

i. e.,

$$\left(\frac{d\mu_2}{d\log m_2}\right)_{t,\,p,\,m_1} = A. \tag{214}$$

If we write the integral of this equation in the form

$$\mu_2 = A \log \frac{Bm_2}{m_1}, \tag{215}$$

B like A will have a positive value depending only upon the temperature and pressure. As this equation is to be applied only to cases in which the value of m_2 is very small compared with m_1, we may regard $\frac{m_1}{v}$ as constant, when temperature and pressure are constant, and write

$$\mu_2 = A \log \frac{Cm_2}{v}, \tag{216}$$

C denoting a positive quantity, dependent only upon the temperature and pressure.

We have so far considered the composition of the body as varying only in regard to the proportion of two components. But the argument will be in no respect invalidated, if we suppose the composition of the body to be capable of other variations. In this case, the quantities A and C will be functions not only of the temperature and pressure but also of the quantities which express the composition of the substance of which together with S_2 the body is composed. If the quantities of any of the components besides S_2 are very small (relatively to the quantities of others), it seems reasonable to assume that the value of μ_2, and therefore the values of A and C, will be nearly the same as if these components were absent.

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Hence, if the independently variable components of any body are $S_e, \ldots S_g$, and $S_h, \ldots S_k$, the quantities of the latter being very small as compared with the quantities of the former, and are incapable of negative values, we may express approximately the values of the potentials for $S_h, \ldots S_k$ by equations (subject of course to the uncertainties of the assumptions which have been made) of the form

$$\mu_h = A_h \log \frac{C_h m_h}{v}, \tag{217}$$

. . . . .

$$\mu_k = A_k \log \frac{C_k m_k}{v}, \tag{218}$$

in which $A_h, C_h, \ldots A_k, C_k$ denote functions of the temperature, the pressure, and the ratios of the quantities $m_e, \ldots m_g$.

We shall see hereafter, when we come to consider the properties of gases, that these equations may be verified experimentally in a very large class of cases, so that we have considerable reason for believing that they express a general law in regard to the limiting values of potentials.*

ON CERTAIN POINTS RELATING TO THE MOLECULAR CONSTITUTION OF BODIES.

It not unfrequently occurs that the number of proximate components which it is necessary to recognize as independently variable in a body exceeds the number of components which would be sufficient to express its ultimate composition. Such is the case, for example, as has been remarked on page 117, in regard to a mixture at ordinary temperatures of vapor of water and free hydrogen and oxygen. This case is explained by the existence of three sorts of molecules in the gaseous mass, viz., molecules of hydrogen, of oxygen, and of hydrogen and oxygen combined. In other cases, which are essentially the same in principle, we suppose a greater number of different sorts of molecules, which differ in composition, and the relations between

* The reader will not fail to remark that, if we could assume the universality of this law, the statement of the conditions necessary for equilibrium between different masses in contact would be much simplified. For, as the potential for a substance which is only a *possible* component (see page 117) would always have the value $-\infty$, the case could not occur that the potential for any substance should have a greater value in a mass in which that substance is only a possible component, than in another mass in which it is an actual component; and the conditions (22) and (51) might be expressed with the sign of equality without exception for the case of possible components.

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these may be more complicated. Other cases are explained by molecules which differ in the quantity of matter which they contain, but not in the kind of matter, nor in the proportion of the different kinds. In still other cases, there appear to be different sorts of molecules, which differ neither in the kind nor in the quantity of matter which they contain, but only in the manner in which they are constituted. What is essential in the cases referred to is that a certain number of some sort or sorts of molecules shall be equivalent to a certain number of some other sort or sorts in respect to the kinds and quantities of matter which they collectively contain, and yet the former shall never be transformed into the latter within the body considered, nor the latter into the former, however the proportion of the numbers of the different sorts of molecules may be varied, or the composition of the body in other respects, or its thermodynamic state as represented by temperature and pressure or any other two suitable variables, provided, it may be, that these variations do not exceed certain limits. Thus, in the example given above, the temperature must not be raised beyond a certain limit, or molecules of hydrogen and of oxygen may be transformed into molecules of water.

The differences in bodies resulting from such differences in the constitution of their molecules are capable of continuous variation, in bodies containing the same matter and in the same thermodynamic state as determined, for example, by pressure and temperature, as the numbers of the molecules of the different sorts are varied. These differences are thus distinguished from those which depend upon the manner in which the molecules are combined to form sensible masses. The latter do not cause an increase in the number of variables in the fundamental equation; but they may be the cause of different values of which the function is sometimes capable for one set of values of the independent variables, as, for example, when we have several different values of ζ for the same values of t, p, m_1, m_2, . . . m_n, one perhaps being for a gaseous body, one for a liquid, one for an amorphous solid, and others for different kinds of crystals, and all being invariable for constant values of the above mentioned independent variables.

But it must be observed that when the differences in the constitution of the molecules are entirely determined by the quantities of the different kinds of matter in a body with the two variables which express its thermodynamic state, these differences will not involve any increase in the number of variables in the fundamental equation. For example, if we should raise the temperature of the mixture of

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vapor of water and free hydrogen and oxygen, which we have just considered, to a point at which the numbers of the different sorts of molecules are entirely determined by the temperature and pressure and the total quantities of hydrogen and of oxygen which are present, the fundamental equation of such a mass would involve but four independent variables, which might be the four quantities just mentioned. The fact of a certain part of the matter present existing in the form of vapor of water would, of course, be one of the facts which determine the nature of the relation between ζ and the independent variables, which is expressed by the fundamental equation.

But in the case first considered, in which the quantities of the different sorts of molecules are *not* determined by the temperature and pressure and the quantities of the different kinds of matter in the body as determined by its ultimate analysis, the components of which the quantities or the potentials appear in the fundamental equation must be those which are determined by the proximate analysis of the body, so that the variations in their quantities, with two variations relating to the thermodynamic state of the body, shall include all the variations of which the body is capable.* Such cases present no especial difficulty; there is indeed nothing in the physical and chemical properties of such bodies, so far as a certain range of experiments is concerned, which is different from what might be, if the proximate components were incapable of farther reduction or transformation. Yet among the the various phases of the kinds of matter concerned, represented by the different sets of values of the variables which satisfy the fundamental equation, there is a certain class which merit especial attention. These are the phases for which the entropy has a maximum value for the same matter, as determined by the ultimate analysis of the body, with the same energy and volume. To fix our ideas let us call the proximate components $S_1, \ldots S_n$, and the ultimate components $S_a, \ldots S_h$; and let $m_1, \ldots m_n$ denote the quantities of the former, and $m_a, \ldots m_h$, the quantities of the latter. It is evident that $m_a \ldots m_h$ are homogeneous functions of the first degree of $m_1, \ldots m_n$; and that the relations between the substances $S_1, \ldots S_n$ might be expressed by homogeneous equations of the first degree between the units of these substances, equal in number to the difference of the numbers of the proximate and of the ultimate com-

* The terms proximate or ultimate are not necessarily to be understood in an absolute sense. All that is said here and in the following paragraphs will apply to many cases in which components may conveniently be regarded as proximate or ultimate, which are such only in a relative sense.

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ponents. The phases in question are those for which η is a maximum for constant values of ε, v, $m_a, \ldots m_h$; or, as they may also be described, those for which ε is a minimum for constant values of η, v, $m_a, \ldots m_h$; or for which ζ is a minimum for constant values of t, p, $m_a, \ldots m_h$. The phases which satisfy this condition may be readily determined when the fundamental equation (which will contain the quantities $m_1, \ldots m_n$ or $\mu_1, \ldots \mu_n$,) is known. Indeed it is easy to see that we may express the conditions which determine these phases by substituting $\mu_1, \ldots \mu_n$ for the letters denoting the units of the corresponding substances in the equations which express the equivalence in ultimate analysis between these units.

These phases may be called, with reference to the kind of change which we are considering, phases of dissipated energy. That we have used a similar term before, with reference to a different kind of changes, yet in a sense entirely analogous, need not create confusion.

It is characteristic of these phases that we cannot alter the values of $m_1, \ldots m_n$ in any real mass in such a phase, while the volume of the mass as well as its matter remain unchanged, without diminishing the energy or increasing the entropy of some other system. Hence, if the mass is large, its equilibrium can be but slightly disturbed by the action of any small body, or by a single electric spark, or by any cause which is not in some way proportioned to the effect to be produced. But when the proportion of the proximate components of a mass taken in connection with its temperature and pressure is not such as to constitute a phase of dissipated energy, it may be possible to cause great changes in the mass by the contact of a very small body. Indeed it is possible that the changes produced by such contact may only be limited by the attainment of a phase of dissipated energy. Such a result will probably be produced in a fluid mass by contact with another fluid which contains molecules of all the kinds which occur in the first fluid (or at least all those which contain the same kinds of matter which also occur in other sorts of molecules), but which differs from the first fluid in that the quantities of the various kinds of molecules are entirely determined by the ultimate composition of the fluid and its temperature and pressure. Or, to speak without reference to the molecular state of the fluid, the result considered would doubtless be brought about by contact with another fluid, which absorbs all the proximate components of the first, $S_1, \ldots S_n$, (or all those between which there exist relations of equivalence in respect to their ultimate analysis), independently, and without passive resistances, but for which the phase is completely deter-

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mined by its temperature and pressure and its ultimate composition (in respect at least to the particular substances just mentioned). By the absorption of the substances $S_1, \ldots S_n$ independently and without passive resistances, it is meant that when the absorbing body is in equilibrium with another containing these substances, it shall be possible by *infinitesimal* changes in these bodies to produce the exchange of all these substances in either direction and independently. An exception to the preceding statement may of course be made for cases in which the result in question is prevented by the occurrence of some other kinds of change; in other words, it is assumed that the two bodies can remain in contact preserving the properties which have been mentioned.

The term *catalysis* has been applied to such action as we are considering. When a body has the property of reducing another, without limitation with respect to the proportion of the two bodies, to a phase of dissipated energy, in regard to a certain kind of molecular change, it may be called a *perfect catalytic agent* with respect to the second body and the kind of molecular change considered.

It seems not improbable that in some cases in which molecular changes take place slowly in homogeneous bodies, a mass of which the temperature and pressure are maintained constant will be finally brought to a state of equilibrium which is entirely determined by its temperature and pressure and the quantities of its ultimate components, while the various transitory states through which the mass passes, (which are evidently not completely defined by the quantities just mentioned,) may be completely defined by the quantities of certain proximate components with the temperature and pressure, and the matter of the mass may be brought by processes approximately reversible from permanent states to these various transitory states. In such cases, we may form a fundamental equation with reference to all possible phases, whether transitory or permanent; and we may also form a fundamental equation of different import and containing a smaller number of independent variables, which has reference solely to the final phases of equilibrium. The latter are the phases of dissipated energy (with reference to molecular changes), and when the more general form of the fundamental equation is known, it will be easy to derive from it the fundamental equation for these permanent phases alone.

Now, as these relations, theoretically considered, are independent of the rapidity of the molecular changes, the question naturally arises, whether in cases in which we are not able to distinguish such transi-

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tory phases, they may not still have a theoretical significance. If so, the consideration of the subject from this point of view, may assist us, in such cases, in discovering the form of the fundamental equation with reference to the ultimate components, which is the only equation required to express all the properties of the bodies which are capable of experimental demonstration. Thus, when the phase of a body is completely determined by the quantities of n independently variable components, with the temperature and pressure, and we have reason to suppose that the body is composed of a greater number n' of proximate components, which are therefore not independently variable (while the temperature and pressure remain constant), it seems quite possible that the fundamental equation of the body may be of the same form as the equation for the phases of dissipated energy of analogous compounds of n' proximate and n ultimate components, in which the proximate components are capable of independent variation (without variation of temperature or pressure). And if such is found to be the case, the fact will be of interest as affording an indication concerning the proximate constitution of the body.

Such considerations seem to be especially applicable to the very common case in which at certain temperatures and pressures, regarded as constant, the quantities of certain proximate components of a mass are capable of independent variations, and all the phases produced by these variations are permanent in their nature, while at other temperatures and pressures, likewise regarded as constant, the quantities of these proximate components are not capable of independent variation, and the phase may be completely defined by the quantities of the ultimate components with the temperature and pressure. There may be, at certain intermediate temperatures and pressures, a condition with respect to the independence of the proximate components intermediate in character, in which the quantities of the proximate components are independently variable when we consider all phases, the essentially transitory as well as the permanent, but in which these quantities are not independently variable when we consider the permanent phases alone. Now we have no reason to believe that the passing of a body in a state of dissipated energy from one to another of the three conditions mentioned has any necessary connection with any discontinuous change of state. Passing the limit which separates one of these states from another will not therefore involve any discontinuous change in the values of any of the quantities enumerated in (99)–(103) on page 143, if $m_1, m_2, \ldots m_n, \mu_1, \mu_2, \ldots \mu_n$ are

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understood as always relating to the ultimate components of the body. Therefore, if we regard masses in the different conditions mentioned above as having different fundamental equations, (which we may suppose to be of any one of the five kinds described on page 143,) these equations will agree at the limits dividing these conditions not only in the values of all the variables which appear in the equations, but also in all the differential coefficients of the first order involving these variables. We may illustrate these relations by supposing the values of t, p, and ζ for a mass in which the quantities of the ultimate components are constant to be represented by rectilinear coordinates. Where the proximate composition of such a mass is not determined by t and p, the value of ζ will not be determined by these variables, and the points representing connected values of t, p, and ζ will form a solid. This solid will be bounded in the direction opposite to that in which ζ is measured, by a surface which represents the phases of dissipated energy. In a part of the figure, all the phases thus represented may be permanent, in another part only the phases in the bounding surface, and in a third part there may be no such solid figure (for any phases of which the existence is experimentally demonstrable), but only a surface. This surface together with the bounding surfaces representing phases of dissipated energy in the parts of the figure mentioned above forms a continuous sheet, without discontinuity in regard to the direction of its normal at the limits dividing the different parts of the figure which have been mentioned. (There may, indeed, be different sheets representing liquid and gaseous states, etc., but if we limit our consideration to states of one of these sorts, the case will be as has been stated.)

We shall hereafter, in the discussion of the fundamental equations of gases, have an example of the derivation of the fundamental equation for phases of dissipated energy (with respect to the molecular changes on which the proximate composition of the body depends) from the more general form of the fundamental equation.

THE CONDITIONS OF EQUILIBRIUM FOR HETEROGENEOUS MASSES UNDER THE INFLUENCE OF GRAVITY.

Let us now seek the conditions of equilibrium for a mass of various kinds of matter subject to the influence of gravity. It will be convenient to suppose the mass enclosed in an immovable envelop which is impermeable to matter and to heat, and in other respects, except in regard to gravity, to make the same suppositions as on pages 115, 116. The energy of the mass will now consist of two parts, one of

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which depends upon its intrinsic nature and state, and the other u
its position in space. Let Dm denote an element of the mass, D
intrinsic energy of this element, h its height above a fixed horiz
plane, and g the force of gravity; then the total energy of the
(when without sensible motions) will be expressed by the formul

$$\int D\varepsilon + \int g\,h\,Dm, \qquad ($$

in which the integrations include all the elements of the mass;
the general condition of equilibrium will be

$$\delta \int D\varepsilon + \delta \int g\,h\,Dm \geqq 0, \qquad ($$

the variations being subject to certain equations of condition. T
must express that the entropy of the whole mass is constant, tha
surface bounding the whole mass is fixed, and that the total qu
ties of each of the component substances is constant. We shall
pose that there are no other equations of condition, and that
independently variable components are the same throughout
whole mass; and we shall at first limit ourselves to the consider
of the conditions of equilibrium with respect to the changes v
may be expressed by infinitesimal variations of the quantities v
define the initial state of the mass, without regarding the possi
of the formation at any place of infinitesimal masses entirely diff
from any initially existing in the same vicinity.

Let $D\eta$, Dv, $Dm_1, \ldots Dm_n$ denote the entropy of the ele
Dm, its volume, and the quantities which it contains of the va
components. Then

$$Dm = Dm_1 \ldots + Dm_n,$$

and

$$\delta Dm = \delta Dm_1 \ldots + \delta Dm_n.$$

Also, by equation (12),

$$\delta D\varepsilon = t\,\delta D\eta - p\,\delta Dv + \mu_1\,\delta Dm_1 \ldots + \mu_n\,\delta Dm_n. \qquad ($$

By these equations the general condition of equilibrium ma
reduced to the form

$$\int t\,\delta D\eta - \int p\,\delta Dv + \int \mu_1\,\delta Dm_1 \ldots + \int \mu_n\,\delta Dm_n$$
$$+ \int g\,\delta h\,Dm + \int g\,h\,\delta Dm_1 \ldots + \int g\,h\,\delta Dm_n \geqq 0.$$

Now it will be observed that the different equations of cond
affect different parts of this condition, so that we must have,
rately,

$$\int t\,\delta D\eta \geqq 0, \quad \text{if} \quad \int \delta D\eta = 0;$$

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$$-\int p\,\delta Dv + \int g\,\delta h\,Dm \geqq 0, \tag{226}$$

if the bounding surface is unvaried;

$$\left.\begin{array}{l} \int \mu_1\,\delta Dm_1 + \int g h\,\delta Dm_1 \geqq 0, \quad \text{if} \quad \int \delta Dm_1 = 0; \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ \int \mu_n\,\delta Dm_n + \int g h\,\delta Dm_n \geqq 0, \quad \text{if} \quad \int \delta Dm_n = 0. \end{array}\right\} \tag{227}$$

From (225) we may derive the condition of thermal equilibrium,

$$t = \text{Const.} \tag{328}$$

Condition (226) is evidently the ordinary mechanical condition of equilibrium, and may be transformed by any of the usual methods. We may, for example, apply the formula to such motions as might take place longitudinally within an infinitely narrow tube, terminated at both ends by the external surface of the mass, but otherwise of indeterminate form. If we denote by m the mass, and by v the volume, included in the part of the tube between one end and a transverse section of variable position, the condition will take the form

$$-\int p\,\delta dv + \int g\,\delta h\,dm \geqq 0, \tag{229}$$

in which the integrations include the whole contents of the tube. Since no motion is possible at the ends of the tube,

$$\int p\,\delta dv + \int \delta v\,dp = \int d(p\,\delta v) = 0. \tag{230}$$

Again, if we denote by γ the density of the fluid,

$$\int g\,\delta h\,dm = \int g\,\frac{dh}{dv}\,\delta v\,\gamma\,dv = \int g\,\gamma\,\delta v\,dh. \tag{231}$$

By these equations condition (229) may be reduced to the form

$$\int \delta v\,(dp + g\,\gamma\,dh) \geqq 0. \tag{232}$$

Therefore, since δv is arbitrary in value,

$$dp = -g\,\gamma\,dh, \tag{233}$$

which will hold true at any point in the tube, the differentials being taken with respect to the direction of the tube at that point. Therefore, as the form of the tube is indeterminate, this equation must hold true, without restriction, throughout the whole mass. It evidently requires that the pressure shall be a function of the height alone, and that the density shall be equal to the first derivative of this function, divided by $-g$.

Conditions (227) contain all that is characteristic of chemical equilibrium. To satisfy these conditions it is necessary and sufficient that

$$\left.\begin{array}{l} \mu_1 + g h = \text{Const.} \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ \mu_n + g h = \text{Const.} \end{array}\right\} \tag{234}$$

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The expressions $\mu_1, \ldots \mu_n$ denote quantities which we have called the potentials for the several components, and which are entirely determined at any point in a mass by the nature and state of the mass about that point. We may avoid all confusion between these quantities and the potential of the force of gravity, if we distinguish the former, when necessary, as *intrinsic* potentials. The relations indicated by equations (234) may then be expressed as follows:

When a fluid mass is in equilibrium under the influence of gravity, and has the same independently variable components throughout, the intrinsic potentials for the several components are constant in any given level, and diminish uniformly as the height increases, the difference of the values of the intrinsic potential for any component at two different levels, being equal to the work done by the force of gravity when a unit of matter falls from the higher to the lower level.

The conditions expressed by equations (228), (233), (234) are necessary and sufficient for equilibrium, except with respect to the possible formation of masses which are not approximately identical in phase with any previously existing about the points where they may be formed. The possibility of such formations at any point is evidently independent of the action of gravity, and is determined entirely by the phase or phases of the matter about that point. The conditions of equilibrium in this respect have been discussed on pages 128–134.

But equations (228), (233), and (234) are not entirely independent. For with respect to any mass in which there are no surfaces of discontinuity (i. e., surfaces where adjacent elements of mass have finite differences of phase), one of these equations will be a consequence of the others. Thus by (228) and (234), we may obtain from (97), which will hold true of any continuous variations of phase, the equation

$$v\,dp = -\,g\,(m_1 \ldots + m_n)\,dh; \tag{235}$$

or

$$dp = -\,g\,\gamma\,dh; \tag{236}$$

which will therefore hold true in any mass in which equations (228) and (234) are satisfied, and in which there are no surfaces of discontinuity. But the condition of equilibrium expressed by equation (233) has no exception with respect to surfaces of discontinuity; therefore in any mass in which such surfaces occur, it will be necessary for equilibrium, in addition to the relations expressed by equations (228) and (234), that there shall be no discontinuous change of pressure at these surfaces.

This superfluity in the particular conditions of equilibrium which we have found, as applied to a mass which is everywhere continuous

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in phase, is due to the fact that we have made the elements of volume variable in position and size, while the matter initially contained in these elements is not supposed to be confined to them. Now, as the different components may move in different directions when the state of the system varies, it is evidently impossible to define the elements of volume so as always to include the same matter; we must, therefore, suppose the matter contained in the elements of volume to vary; and therefore it would be allowable to make these elements fixed in space. If the given mass has no surfaces of discontinuity, this would be much the simplest plan. But if there are any surfaces of discontinuity, it will be possible for the state of the given mass to vary, not only by infinitesimal changes of phase in the fixed elements of volume, but also by movements of the surfaces of discontinuity. It would therefore be necessary to add to our general condition of equilibrium terms relating to discontinuous changes in the elements of volume about these surfaces,—a necessity which is avoided if we consider these elements movable, as we can then suppose that each element remains always on the same side of the surface of discontinuity.

Method of treating the preceding problem, in which the elements of volume are regarded as fixed.

It may be interesting to see in detail how the particular conditions of equilibrium may be obtained if we regard the elements of volume as fixed in position and size, and consider the possibility of finite as well as infinitesimal changes of phase in each element of volume. If we use the character Δ to denote the differences determined by such finite differences of phase, we may express the variation of the intrinsic energy of the whole mass in the form

$$\int \delta D\varepsilon + \int \Delta D\varepsilon, \tag{237}$$

in which the first integral extends over all the elements which are infinitesimally varied, and the second over all those which experience a finite variation. We *may* regard both integrals as extending throughout the whole mass, but their values will be zero except for the parts mentioned.

If we do not wish to limit ourselves to the consideration of masses so small that the force of gravity can be regarded as constant in direction and in intensity, we may use Υ to denote the potential of the force of gravity, and express the variation of the part of the energy which is due to gravity in the form

$$-\int \Upsilon\, \delta Dm - \int \Upsilon\, \Delta Dm. \tag{238}$$

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We shall then have, for the general condition of equilibrium,

$$\int \delta D\varepsilon + \int \Delta D\varepsilon - \int \Upsilon \delta Dm - \int \Upsilon \Delta Dm \geqq 0; \tag{239}$$

and the equations of condition will be

$$\int \delta D\eta + \int \Delta D\eta = 0, \tag{240}$$

$$\left.\begin{array}{l} \int \delta Dm_1 + \int \Delta Dm_1 = 0, \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \int \delta Dm_n + \int \Delta Dm_n = 0. \end{array}\right\} \tag{241}$$

We may obtain a condition of equilibrium independent of these equations of condition, by subtracting these equations, multiplied each by an indeterminate constant, from condition (239). If we denote these indeterminate constants by T, $M_1, \ldots M_n$, we shall obtain after arranging the terms

$$\int \overline{\delta D\varepsilon - \Upsilon \delta Dm - T \delta D\eta - M_1 \delta Dm_1 \ldots - M_n \delta Dm_n}$$
$$+ \int \overline{\Delta D\varepsilon - \Upsilon \Delta Dm - T \Delta D\eta - M_1 \Delta Dm_1 \ldots - M_n \delta Dm_n} \geqq 0. \tag{242}$$

The variations, both infinitesimal and finite, in this condition are independent of the equations of condition (240) and (241), and are only subject to the condition that the varied values of $D\varepsilon$, $D\eta$, $Dm_1, \ldots Dm_n$ for each element are determined by a certain change of phase. But as we do not suppose the same element to experience both a finite and an infinitesimal change of phase, we must have

$$\delta D\varepsilon - \Upsilon \delta Dm - T \delta D\eta - M_1 \delta Dm_1 \ldots - M_n \delta Dm_n \geqq 0, \tag{243}$$

and

$$\Delta D\varepsilon - \Upsilon \Delta Dm - T \Delta D\eta - M_1 \Delta Dm_1 \ldots - M_n \Delta Dm_n \geqq 0. \tag{244}$$

By equation (12), and in virtue of the necessary relation (222), the first of these conditions reduces to

$$(t - T)\, \delta D\eta + (\mu_1 - \Upsilon - M_1)\, \delta Dm_1 \ldots$$
$$+ (\mu_n - \Upsilon - M_n)\, \delta Dm_n \geqq 0; \tag{245}$$

for which it is necessary and sufficient that

$$t = T, \tag{246}$$

$$\left.\begin{array}{l} \mu_1 - \Upsilon = M_1, \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \mu_n - \Upsilon = M_n. \end{array}\right\}^{*} \tag{247}$$

* The gravitation potential is here supposed to be defined in the usual way. But if it were defined so as to *decrease* when a body falls, we would have the sign + instead of — in these equations; i. e., for each component, the sum of the gravitation and intrinsic potentials would be constant throughout the whole mass.

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Condition (244) may be reduced to the form

$$\Delta D\varepsilon - \Upsilon \Delta D\eta - (\Upsilon + M_1)\Delta Dm_1 \ldots - (\Upsilon + M_n)\Delta Dm_n \geqq 0; \quad (248)$$

and by (246) and (247) to

$$\Delta D\varepsilon - t\Delta D\eta - \mu_1 \Delta Dm_1 \ldots - \mu_n \Delta Dm_n \geqq 0. \quad (249)$$

If values determined subsequently to the change of phase are distinguished by accents, this condition may be written

$$D\varepsilon' - t\,D\eta' - \mu_1\,Dm_1' \ldots - \mu_n\,Dm_n'$$
$$- D\varepsilon + t\,D\eta + \mu_1\,Dm_1 \ldots + \mu_n\,Dm_n \geqq 0, \quad (250)$$

which may be reduced by (93) to

$$D\varepsilon' - t\,D\eta' - \mu_1\,Dm_1, \ldots - \mu_n\,Dm_n' + p\,Dv \geqq 0. \quad (251)$$

Now if the element of volume Dv is adjacent to a surface of discontinuity, let us suppose $D\varepsilon'$, $D\eta'$, Dm_1', ... Dm_n' to be determined (for the same element of volume) by the phase existing on the other side of the surface of discontinuity. As t, μ_1, ... μ_n have the same values on both sides of this surface, the condition may be reduced by (93) to

$$-p'Dv + p\,Dv \geqq 0. \quad (252)$$

That is, the pressure must not be greater on one side of a surface of discontinuity than on the other.

Applied more generally, (251) expresses the condition of equilibrium with respect to the possibility of discontinuous changes of phases at any point. As $Dv' = Dv$, the condition may also be written

$$D\varepsilon' - t\,D\eta' + p\,Dv' - \mu_1\,Dm_1', \ldots - \mu_n\,Dm_n' \geqq 0, \quad (253)$$

which must hold true when t, p, μ_1, ... μ_n have values determined by any point in the mass, and $D\varepsilon'$, $D\eta'$, Dv', Dm_1', ... Dm_n' have values determined by any possible phase of the substances of which the mass is composed. The application of the condition is, however, subject to the limitations considered on pages 128–134. It may easily be shown (see pages 160, 161) that for constant values of t, μ_1, ... μ_n, and of Dv', the first member of (253) will have the least possible value when $D\varepsilon'$, $D\eta'$, Dm_1', ... Dm_n' are determined by a phase for which the temperature has the value t, and the potentials the values μ_1, ... μ_n. It will be sufficient, therefore, to consider the condition as applied to such phases, in which case it may be reduced by (93) to

$$p - p' \geqq 0. \quad (254)$$

That is, the pressure at any point must be as great as that of any phase of the same components, for which the temperature and the

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potentials have the same values as at that point. We may also express this condition by saying that the pressure must be as great as is consistent with equations (246), (247). This condition with the equations mentioned will always be sufficient for equilibrium; when the condition is not satisfied, if equilibrium subsists, it will be at least practically unstable.

Hence, the phase at any point of a fluid mass, which is in stable equilibrium under the influence of gravity (whether this force is due to external bodies or to the mass itself), and which has throughout the same independently variable components, is completely determined by the phase at any other point and the difference of the values of the gravitation potential for the two points.

FUNDAMENTAL EQUATIONS OF IDEAL GASES AND GAS-MIXTURES.

For a constant quantity of a perfect or ideal gas, the product of the volume and pressure is proportional to the temperature, and the variations of energy are proportional to the variations of temperature. For a unit of such a gas we may write

$$p\,v = a\,t,$$
$$d\varepsilon = c\,dt,$$

a and c denoting constants. By integration, we obtain the equation

$$\varepsilon = c\,t + E,$$

in which E also denotes a constant. If by these equations we eliminate t and p from (11), we obtain

$$d\varepsilon = \frac{\varepsilon - E}{c}\,d\eta - \frac{a}{v}\,\frac{\varepsilon - E}{c}\,dv,$$

or

$$c\,\frac{d\varepsilon}{\varepsilon - E} = d\eta - a\,\frac{dv}{v}.$$

The integral of this equation may be written in the form

$$c \log \frac{\varepsilon - E}{c} = \eta - a \log v - H,$$

where H denotes a fourth constant. We may regard E as denoting the energy of a unit of the gas for $t=0$; H its entropy for $t=1$ and $v=1$; a its pressure in the latter state, or its volume for $t=1$ and $p=1$; c its specific heat at constant volume. We may extend the application of the equation to any quantity of the gas, without altering the values of the constants, if we substitute $\frac{\varepsilon}{m}, \frac{\eta}{m}, \frac{v}{m}$ for ε, η, v, respectively. This will give

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$$c \log \frac{\varepsilon - Em}{cm} = \frac{\eta}{m} - H + a \log \frac{m}{v}. \tag{255}$$

This is a fundamental equation (see pages 140–144) for an ideal gas of invariable composition. It will be observed that if we do not have to consider the properties of the matter which forms the gas as appearing in any other form or combination, but solely as constituting the gas in question (in a state of purity), we may without loss of generality give to E and H the value zero, or any other arbitrary values. But when the scope of our investigations is not thus limited, we may have determined the states of the substance of the gas for which $\varepsilon = 0$ and $\eta = 0$ with reference to some other form in which the substance appears, or, if the substance is compound, the states of its components for which $\varepsilon = 0$ and $\eta = 0$ may be already determined; so that the constants E and H cannot in general be treated as arbitrary.

We obtain from (255) by differentiation

$$\frac{c}{\varepsilon - Em} d\varepsilon = \frac{1}{m} d\eta - \frac{a}{v} dv + \left(\frac{cE}{\varepsilon - Em} + \frac{c+a}{m} - \frac{\eta}{m^2} \right) dm, \tag{256}$$

whence, in virtue of the general relation expressed by (86),

$$t = \frac{\varepsilon - Em}{cm}, \tag{257}$$

$$p = a \frac{\varepsilon - Em}{cv}, \tag{258}$$

$$\mu = E + \frac{\varepsilon - Em}{cm^2} (cm + am - \eta). \tag{259}$$

We may obtain the fundamental equation between ψ, t, v, and m from equations (87), (255), and (257). Eliminating ε we have

$$\psi = Em + cmt - t\eta,$$

and

$$c \log t = \frac{\eta}{m} - H + a \log \frac{m}{v};$$

and eliminating η, we have the fundamental equation

$$\psi = Em + mt \left(c - H - c \log t + a \log \frac{m}{v} \right). \tag{260}$$

Differentiating this equation, we obtain

$$d\psi = -m \left(H + c \log t + a \log \frac{v}{m} \right) dt - \frac{amt}{v} dv$$
$$+ \left(E + t \left(c + a - H - c \log t + a \log \frac{m}{v} \right) \right) dm; \tag{261}$$

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whence, by the general equation (88),

$$\eta = m\left(H + c\log t + a\log\frac{v}{m}\right), \tag{262}$$

$$p = \frac{a\,m\,t}{v}, \tag{263}$$

$$\mu = E + t\left(c + a - H - c\log t + a\log\frac{m}{v}\right). \tag{264}$$

From (260), by (87) and (91), we obtain

$$\zeta = Em + m\,t\left(c - H - c\log t + a\log\frac{m}{v}\right) + p\,v,$$

and eliminating v by means of (263), we obtain the fundamental equation

$$\zeta = Em + m\,t\left(c + a - H - (c+a)\log t + a\log\frac{p}{a}\right). \tag{265}$$

From this, by differentiation and comparison with (92), we may obtain the equations

$$\eta = m\left(H + (c+a)\log t - a\log\frac{p}{a}\right), \tag{266}$$

$$v = \frac{a\,m\,t}{p}, \tag{267}$$

$$\mu = E + t\left(c + a - H - (c+a)\log t + a\log\frac{p}{a}\right). \tag{268}$$

The last is also a fundamental equation. It may be written in the form

$$\log\frac{p}{a} = \frac{H-c-a}{a} + \frac{c+a}{a}\log t + \frac{\mu-E}{a\,t}, \tag{269}$$

or, if we denote by e the base of the Naperian system of logarithms,

$$p = a\,e^{\frac{H-c-a}{a}}\,t^{\frac{c+a}{a}}\,e^{\frac{\mu-E}{at}} \tag{270}$$

The fundamental equation between χ, η, p, and m may also be easily obtained; it is

$$(c+a)\log\frac{\chi - Em}{(c+a)m} = \frac{\eta}{m} - H + a\log\frac{p}{a}, \tag{271}$$

which can be solved with respect to χ.

Any one of the fundamental equations (255), (260), (265), (270), and (271), which are entirely equivalent to one another, may be

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regarded as defining an ideal gas. It will be observed that most of these equations might be abbreviated by the use of different constants. In (270), for example, a single constant might be used for $a\,e^{\frac{H-c-a}{a}}$, and another for $\frac{c+a}{a}$. The equations have been given in the above form, in order that the relations between the constants occurring in the different equations might be most clearly exhibited. The sum $c+a$ is the specific heat for constant pressure, as appears if we differentiate (266) regarding p and m as constant.*

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* We may easily obtain the equation between the temperature and pressure of a saturated vapor, if we know the fundamental equations of the substance both in the gaseous, and in the liquid or solid state. If we suppose that the density and the specific heat at constant pressure of the liquid may be regarded as constant quantities (for such moderate pressures as the liquid experiences while in contact with the vapor), and denote this specific heat by k, and the volume of a unit of the liquid by V, we shall have for a unit of the liquid

$$t\,d\eta = k\,dt,$$

whence

$$\eta = k \log t + H',$$

where H' denotes a constant. Also, from this equation and (97),

$$d\mu = -(k \log t + H')\,dt + V\,dp,$$

whence

$$\mu = kt - kt \log t - H'\,t + Vp + E', \tag{A}$$

where E' denotes another constant. This is a fundamental equation for the substance in the liquid state. If (268) represents the fundamental equation for the same substance in the gaseous state, the two equations will both hold true of coexistent liquid and gas. Eliminating μ we obtain

$$\log \frac{p}{a} = \frac{H-H'+k-c-a}{a} - \frac{k-c-a}{a} \log t - \frac{E-E'}{at} + \frac{V}{a}\,\frac{p}{t}.$$

If we neglect the last term, which is evidently equal to the density of the vapor divided by the density of the liquid, we may write

$$\log p = A - B \log t - \frac{C}{t},$$

A, B, and C denoting constants. If we make similar suppositions in regard to the substance in the solid state, the equation between the pressure and temperature of coexistent solid and gaseous phases will of course have the same form.

A similar equation will also apply to the phases of an ideal gas which are coexistent with two different kinds of solids, one of which can be formed by the combination of the gas with the other, each being of invariable composition and of constant specific heat and density. In this case we may write for one solid

$$\mu_1 = k't - k't \log t - H't + V'p + E',$$

and for the other

$$\mu_2 = k''t - k''t \log t - H''t + V''p + E'',$$

and for the gas

$$\mu_3 = E + t\left(c + a - H - (c+a) \log t + a \log \frac{p}{a}\right).$$

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The preceding fundamental equations all apply to gases *of constant composition*, for which the matter is entirely determined by a single

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Now if a unit of the gas unites with the quantity λ of the first solid to form the quantity $1+\lambda$ of the second it will be necessary for equilibrium (see pages 121, 122) that

$$\mu_3 + \lambda\mu_1 = (1+\lambda)\mu_2.$$

Substituting the values of μ_1, μ_2, μ_3, given above, we obtain after arranging the terms and dividing by at

$$\log\frac{p}{a} = A - B\log t - \frac{C}{t} + D\frac{p}{t},$$

when

$$A = \frac{H+\lambda H'-(1+\lambda)H''-c-a-\lambda k'+(1+\lambda)k''}{a},$$

$$B = \frac{(1+\lambda)k''-\lambda k'-c-a}{a},$$

$$C = \frac{E+\lambda E'-(1+\lambda)E''}{a}, \qquad D = \frac{(1+\lambda)V''-\lambda V'}{a}.$$

We may conclude from this that an equation of the same form may be applied to an ideal gas in equilibrium with a liquid of which it forms an independently variable component, when the specific heat and density of the liquid are entirely determined by its composition, except that the letters A, B, C, and D must in this case be understood to denote quantities which vary with the composition of the liquid. But to consider the case more in detail, we have for the liquid by (A)

$$\frac{\zeta}{m} = \mu = kt - kt\log t - H't + Vp + E',$$

where k, H', V, E' denote quantities which depend only upon the composition of the liquid. Hence, we may write

$$\zeta = \mathbf{k}t - \mathbf{k}t\log t - \mathbf{H}t + \mathbf{V}p + \mathbf{E},$$

where $\mathbf{k}$, $\mathbf{H}$, $\mathbf{V}$, and $\mathbf{E}$ denote functions of m_1, m_2, etc. (the quantities of the several components of the liquid). Hence, by (92),

$$\mu_1 = \frac{d\mathbf{k}}{dm_1}t - \frac{d\mathbf{k}}{dm_1}t\log t - \frac{d\mathbf{H}}{dm_1}t + \frac{d\mathbf{V}}{dm_1}p + \frac{d\mathbf{E}}{dm_1}.$$

If the component to which this potential relates is that which also forms the gas, we shall have by (269)

$$\log\frac{p}{a} = \frac{H-c-a}{a} + \frac{c+a}{a}\log t + \frac{\mu_1 - E}{at}.$$

Eliminating μ_1, we obtain the equation

$$\log\frac{p}{a} = A - B\log t - \frac{C}{t} + D\frac{p}{t},$$

in which A, B, C, and D denote quantities which depend only upon the composition of the liquid, viz:

$$A = \frac{1}{a}\left(H - \frac{d\mathbf{H}}{dm_1} - c - a + \frac{d\mathbf{k}}{dm_1}\right),$$

$$B = \frac{1}{a}\left(\frac{d\mathbf{k}}{dm_1} - c - a\right).$$

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variable (m). We may obtain corresponding fundamental equations for a mixture of gases, in which the proportion of the components shall be variable, from the following considerations.

It is a rule which admits of a very general and in many cases very exact experimental verification, that if several liquid or solid substances which yield different gases or vapors are simultaneously in equilibrium with a mixture of these gases (cases of chemical action between the gases being excluded,) the pressure in the gas-mixture is equal to the sum of the pressures of the gases yielded at the same temperature by the various liquid or solid substances taken separately. Now the potential in any of the liquids or solids for the substance which it yields in the form of gas has very nearly the same value when the liquid or solid is in equilibrum with the gas-mixture as when it is in equilibrium with its own gas alone. The difference of the pressure in the two cases will cause a certain difference in the values of the potential, but that this difference will be small, we may infer from the equation

$$\left(\frac{d\mu_1}{dp}\right)_{t,\,m} = \left(\frac{dv}{dm_1}\right)_{t,\,p,\,m}, \qquad (272)$$

which may be derived from equation (92). In most cases, there will be a certain absorption by each liquid of the gases yielded by the others, but as it is well known that the above rule does not apply to cases in which such absorption takes place to any great extent, we may conclude that the effect of this circumstance in the cases with which we have to do is of secondary importance. If we neglect the slight differences in the values of the potentials due to these circumstances, the rule may be expressed as follows:

The pressure in a mixture of different gases is equal to the sum of the pressures of the different gases as existing each by itself at the same temperature and with the same value of its potential.

To form a precise idea of the practical significance of the law as thus stated with reference to the equilibrium of two liquids with a mixture of the gases which they emit, when neither liquid absorbs the gas emitted by the other, we may imagine a long tube closed at each end and bent in the form of a W to contain in each of the descending

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$$C = \frac{1}{a}\left(E - \frac{d\mathbf{E}}{dm_1}\right) \qquad D = \frac{1}{a}\,\frac{d\mathbf{V}}{dm_1}.$$

With respect to some of the equations which have here been deduced, the reader may compare Professor Kirchhoff "Ueber die Spannung des Dampfes von Mischungen aus Wasser und Schwefelsäure," Pogg. Ann., vol. civ. (1858), p. 612; and Dr. Rankine "On Saturated Vapors," Phil. Mag., vol. xxxi. (1866), p. 199.

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loops one of the liquids, and above these liquids the gases which they emit, viz., the separate gases at the ends of the tube, and the mixed gases in the middle. We may suppose the whole to be in equilibrium, the difference of the pressures of the gases being balanced by the proper heights of the liquid columns. Now it is evident from the principles established on pages 203–210 that the potential for either gas will have the same value in the mixed and in the separate gas *at the same level*, and therefore according to the rule in the form which we have given, the pressure in the gas-mixture is equal to the sum of the pressures in the separate gases, *all these pressures being measured at the same level.* Now the experiments by which the rule has been established relate rather to the gases in the vicinity of the surfaces of the liquids. Yet, although the differences of level in these surfaces may be considerable, the corresponding differences of pressure in the columns of gas will certainly be very small in all cases which can be regarded as falling under the laws of ideal gases, for which very great pressures are not admitted.

If we apply the above law to a mixture of ideal gases and distinguish by subscript numerals the quantities relating to the different gases, and denote by Σ_1 the sum of all similar terms obtained by changing the subscript numerals, we shall have by (270)

$$p = \Sigma_1 \left(a_1 \, e^{\frac{H_1 - c_1 - a_1}{a_1}} \, t^{\frac{c_1 + a_1}{a_1}} \, e^{\frac{\mu_1 - E_1}{a_1 t}} \right), \tag{273}$$

It will be legitimate to assume this equation provisionally as the fundamental equation defining an ideal gas-mixture, and afterwards to justify the suitableness of such a definition by the properties which may be deduced from it. In particular, it will be necessary to show that an ideal gas-mixture as thus defined, when the proportion of its components remains constant, has all the properties which have already been assumed for an ideal gas of invariable composition; it will also be desirable to consider more rigorously and more in detail the equilibrium of such a gas-mixture with solids and liquids, with respect to the above rule.

By differentiation and comparison with (98) we obtain

$$\frac{\eta}{v} = \Sigma_1 \left(\left(c_1 + a_1 - \frac{\mu_1 - E_1}{t}\right) e^{\frac{H_1 - c_1 - a_1}{a_1}} \, t^{\frac{c_1}{a_1}} \, e^{\frac{\mu_1 - E_1}{a_1 t}} \right), \tag{274}$$

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$$\left.\begin{aligned} \frac{m_1}{v} &= e^{\frac{H_1-c_1-a_1}{a_1}}\, t^{\frac{c_1}{a_1}}\, e^{\frac{\mu_1-E_1}{a_1 t}}, \\ \frac{m_2}{v} &= e^{\frac{H_2-c_2-a_2}{a_2}}\, t^{\frac{c_2}{a_2}}\, e^{\frac{\mu_2-E_2}{a_2 t}}, \\ &\text{etc.} \end{aligned}\right\} \tag{275}$$

Equations (275) indicate that the relation between the temperature, the density of any component, and the potential for that component, is not affected by the presence of the other components. They may also be written

$$\left.\begin{aligned} \mu_1 &= E_1 + t\left(c_1 + a_1 - H_1 - c_1 \log t + a_1 \log \frac{m_1}{v}\right), \\ &\text{etc.} \end{aligned}\right\} \tag{276}$$

Eliminating μ_1, μ_2, etc. from (273) and (274) by means of (275) and (276), we obtain

$$p = \Sigma_1 \frac{a_1 m_1 t}{v}, \tag{277}$$

$$\eta = \Sigma_1\left(m_1 H_1 + m_1 c_1 \log t + m_1 a_1 \log \frac{v}{m_1}\right). \tag{278}$$

Equation (277) expresses the familiar principle that the pressure in a gas-mixture is equal to the sum of the pressures which the component gases would possess if existing separately with the same volume at the same temperature. Equation (278) expresses a similar principle in regard to the entropy of the gas-mixture.

From (276) and (277) we may easily obtain the fundamental equation between ψ, t, v, m_1, m_2, etc. For by substituting in (94) the values of p, μ_1, μ_2, etc. taken from these equations, we obtain

$$\psi = \Sigma_1\left(E_1 m_1 + m_1 t\left(c_1 - H_1 - c_1 \log t + a_1 \log \frac{m_1}{v}\right)\right). \tag{279}$$

If we regard the proportion of the various components as constant, this equation may be simplified by writing

$$\begin{aligned} m &\text{ for } \Sigma_1 m_1, \\ c\,m &\text{ for } \Sigma_1 (c_1 m_1), \\ a\,m &\text{ for } \Sigma_1 (a_1 m_1), \\ E\,m &\text{ for } \Sigma_1 (E_1 m_1), \\ \text{and} \quad H\,m - a\,m \log m &\text{ for } \Sigma_1 (H_1 m_1 - a_1 m_1 \log m_1). \end{aligned}$$

The values of c, a, E, and H, will then be constant and m will denote the total quantity of gas. As the equation will thus be reduced to the

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form of (260), it is evident that an ideal gas-mixture, as defined by (273) or (279), when the proportion of its components remains unchanged, will have all the properties which we have assumed for an ideal gas of invariable composition. The relations between the specific heats of the gas-mixture at constant volume and at constant pressure and the specific heats of its components are expressed by the equations

$$c = \Sigma_1 \frac{m_1 c_1}{m}, \tag{280}$$

and

$$c + a = \Sigma_1 \frac{m_1 (c_1 + a_1)}{m}. \tag{281}$$

We have already seen that the values of t, v, m_1, μ_1 in a gas-mixture are such as are possible for the component G_1 (to which m_1 and μ_1 relate) existing separately. If we denote by p_1, η_1, ψ_1, ε_1, χ_1, ζ_1 the connected values of the several quantities which the letters indicate determined for the gas G_1 as thus existing separately, and extend this notation to the other components, we shall have by (273), (274), and (279)

$$p = \Sigma_1 p_1, \qquad \eta = \Sigma_1 \eta_1, \qquad \psi = \Sigma_1 \psi_1; \tag{282}$$

whence by (87), (89), and (91)

$$\varepsilon = \Sigma_1 \varepsilon_1, \qquad \chi = \Sigma_1 \chi_1, \qquad \zeta = \Sigma_1 \zeta_1. \tag{283}$$

The quantities p, η, ψ, ε, χ, ζ relating to the gas-mixture may therefore be regarded as consisting of parts which may be attributed to the several components in such a manner that between the parts of these quantities which are assigned to any component, the quantity of that component, the potential for that component, the temperature, and the volume, the same relations shall subsist as if that component existed separately. It is in this sense that we should understand the law of Dalton, that every gas is as a vacuum to every other gas.

It is to be remarked that these relations are consistent and possible for a mixture of gases which are not ideal gases, and indeed without any limitation in regard to the thermodynamic properties of the individual gases. They are all consequences of the law that the pressure in a mixture of different gases is equal to the sum of the pressures of the different gases as existing each by itself at the same temperature and with the same value of its potential. For let p_1, η_1, ε_1, ψ_1, χ_1, ζ_1; p_2, etc.; etc. be defined as relating to the different gases existing each by itself with the same volume, temperature, and potential as in the gas-mixture; if

$$p = \Sigma_1 p_1,$$

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then
$$\left(\frac{dp}{d\mu_1}\right)_{t,\,\mu_2\,\ldots\,\mu_n} = \left(\frac{dp_1}{d\mu_1}\right)_t;$$
and therefore, by (98), the quantity of any component gas G_1 in the gas-mixture, and in the separate gas to which p_1, η_1, etc. relate, is the same and may be denoted by the same symbol m_1. Also
$$\eta = v\left(\frac{dp}{dt}\right)_{\mu_1,\,\ldots\,\mu_n} = v\,\Sigma_1\left(\frac{dp}{dt}\right)_{\mu_1} = \Sigma_1\eta_1;$$
whence also, by (93)–(96),
$$\varepsilon = \Sigma_1\varepsilon_1, \qquad \psi = \Sigma_1\psi_1, \qquad \chi = \Sigma_1\chi_1, \qquad \zeta = \Sigma_1\zeta_1.$$

All the same relations will also hold true whenever the value of ψ for the gas-mixture is equal to the sum of the values of this function for the several component gases existing each by itself in the same quantity as in the gas-mixture and with the temperature and volume of the gas-mixture. For if p_1, η_1, ε_1, ψ_1, χ_1, ζ_1; p_2, etc.; etc. are defined as relating to the components existing thus by themselves, we shall have
$$\psi = \Sigma_1\psi_1,$$
whence
$$\left(\frac{d\psi}{dm_1}\right)_{t,\,v,\,m} = \left(\frac{d\psi_1}{dm_1}\right)_{t,\,v}.^*$$
Therefore, by (88), the potential μ_1 has the same value in the gas-mixture and in the gas G_1 existing separately as supposed. Moreover,
$$\eta = -\left(\frac{d\psi}{dt}\right)_{v,\,m} = -\Sigma_1\left(\frac{d\psi_1}{dt}\right)_{v,\,m} = \Sigma_1\eta_1,$$
and
$$p = -\left(\frac{d\psi}{dv}\right)_{t,\,m} = -\Sigma_1\left(\frac{d\psi_1}{dv}\right)_{t,\,m} = \Sigma_1 p_1,$$
whence
$$\varepsilon = \Sigma_1\varepsilon_1, \qquad \chi = \Sigma_1\chi_1, \qquad \zeta = \Sigma_1\zeta_1.$$

Whenever different bodies are combined without communication of work or heat between them and external bodies, the energy of the body formed by the combination is necessarily equal to the sum of the energies of the bodies combined. In the case of ideal gas-mixtures, when the initial temperatures of the gas-masses which are combined

* A subscript m after a differential coefficient relating to a body having several independently variable components is used here and elsewhere in this paper to indicate that each of the quantities m_1, m_2, etc., unless its differential occurs in the expression to which the suffix is applied, is to be regarded as constant in the differentiation.

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are the same, (whether these gas-masses are entirely different gases, or gas-mixtures differing only in the proportion of their components,) the condition just mentioned can only be satisfied when the temperature of the resultant gas-mixture is also the same. In such combinations, therefore, the final temperature will be the same as the initial.

If we consider a vertical column of an ideal gas-mixture which is in equilibrium, and denote the densities of one of its components at two different points by γ_1 and γ_1', we shall have by (275) and (234)

$$\frac{\gamma_1}{\gamma_1'} = e^{\frac{\mu_1 - \mu_1'}{a_1 t}} = e^{\frac{g(h'-h)}{a_1 t}}. \tag{284}$$

From this equation, in which we may regard the quantities distinguished by accents as constant, it appears that the relation between the density of any one of the components and the height is not affected by the presence of the other components.

The work obtained or expended in any reversible process of combination or separation of ideal gas-mixtures at constant temperature, or when the temperatures of the initial and final gas-masses and of the only external source of heat or cold which is used are all the same, will be found by taking the difference of the sums of the values of ψ for the initial, and for the final gas-masses. (See pages 145, 146). It is evident from the form of equation (279) that this work is equal to the sum of the quantities of work which would be obtained or expended in producing in each different component existing separately the same changes of density which that component experiences in the actual process for which the work is sought.*

We will now return to the consideration of the equilibrium of a liquid with the gas which it emits as affected by the presence of different gases, when the gaseous mass in contact with the liquid may be regarded as an ideal gas-mixture.

It may first be observed, that the density of the gas which is emitted by the liquid will not be affected by the presence of other gases which are not absorbed by the liquid, when the liquid is protected in any way from the pressure due to these additional gases. This may be accomplished by separating the liquid and gaseous

* This result has been given by Lord Rayleigh, (Phil. Mag., vol. xlix, 1875, p. 311). It will be observed that equation (279) might be deduced immediately from this principle in connection with equation (260) which expresses the properties ordinarily assumed for perfect gases.

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masses by a diaphragm which is permeable to the liquid. It will then be easy to maintain the liquid at any constant pressure which is not greater than that in the gas. The potential in the liquid for the substance which it yields as gas will then remain constant, and therefore the potential for the same substance in the gas and the density of this substance in the gas and the part of the gaseous pressure due to it will not be affected by the other components of the gas.

But when the gas and liquid meet under ordinary circumstances, i. e., in a free plane surface, the pressure in both is necessarily the same, as also the value of the potential for any common component S_1. Let us suppose the density of an insoluble component of the gas to vary, while the composition of the liquid and the temperature remain unchanged. If we denote the increments of pressure and of the potential for S_1 by dp and $d\mu_1$, we shall have by (272)

$$d\mu_1 = \left(\frac{d\mu_1}{dp}\right)^{(L)}_{t,\,m} dp = \left(\frac{dv}{dm_1}\right)^{(L)}_{t,\,p,\,m} dp,$$

the index (L) denoting that the expressions to which it is affixed refer to the liquid. (Expressions without such an index will refer to the gas alone or to the gas and liquid in common.) Again, since the gas is an ideal gas-mixture, the relation between p_1 and μ_1 is the same as if the component S_1 existed by itself at the same temperature, and therefore by (268)

$$d\mu_1 = a_1\, t\, d \log p_1.$$

Therefore

$$a_1\, t\, d \log p_1 = \left(\frac{dv}{dm_1}\right)^{(L)}_{t,\,p,\,m} dp. \tag{285}$$

This may be integrated at once if we regard the differential coefficient in the second member as constant, which will be a very close approximation. We may obtain a result more simple, but not quite so accurate, if we write the equation in the form

$$dp_1 = \gamma_1 \left(\frac{dv}{dm_1}\right)^{(L)}_{t,\,p,\,m} dp, \tag{286}$$

where γ_1 denotes the density of the component S_1 in the gas, and integrate regarding this quantity also as constant. This will give

$$p_1 - p_1' = \gamma_1 \left(\frac{dv}{dm_1}\right)^{(L)}_{t,\,p,\,m} (p - p'). \tag{287}$$

where p_1' and p' denote the values of p_1 and p when the insoluble component of the gas is entirely wanting. It will be observed that $p - p'$ is nearly equal to the pressure of the insoluble component, in the phase of the gas-mixture to which p_1 relates. S_1 is not neces-

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sarily the only common component of the gas and liquid. If there are others, we may find the increase of the part of the pressure in the gas-mixture belonging to any one of them by equations differing from the last only in the subscript numerals.

Let us next consider the effect of a gas which is absorbed to some extent, and which must therefore in strictness be regarded as a component of the liquid. We may commence by considering in general the equilibrium of a gas-mixture of two components S_1 and S_2 with a liquid formed of the same components. Using a notation like the previous, we shall have by (98) for constant temperature,

$$dp = \gamma_1\, d\mu_1 + \gamma_2\, d\mu_2,$$

and

$$dp = \gamma_1^{(L)}\, d\mu_1 + \gamma_2^{(L)}\, d\mu_2;$$

whence

$$(\gamma_1^{(L)} - \gamma_1)\, d\mu_1 = (\gamma_2 - \gamma_2^{(L)})\, d\mu_2.$$

Now if the gas is an ideal gas-mixture,

$$d\mu_1 = \frac{a_1 t}{p_1}\, dp_1 = \frac{dp_1}{\gamma_1}, \qquad \text{and} \qquad d\mu_2 = \frac{a_2 t}{p_2}\, dp_2 = \frac{dp_2}{\gamma_2},$$

therefore

$$\left(\frac{\gamma_1^{(L)}}{\gamma_1} - 1\right) dp_1 = \left(1 - \frac{\gamma_2^{(L)}}{\gamma_2}\right) dp_2. \tag{288}$$

We may now suppose that S_1 is the principal component of the liquid, and S_2 is a gas which is absorbed in the liquid to a slight extent. In such cases it is well known that the ratio of the densities of the substance S_2 in the liquid and in the gas is for a given temperature approximately constant. If we denote this constant by A, we shall have

$$\left(\frac{\gamma_1^{(L)}}{\gamma_1} - 1\right) dp_1 = (1 - A)\, dp_2. \tag{289}$$

It would be easy to integrate this equation regarding γ_1 as variable, but as the variation in the value of p_1 is necessarily very small we shall obtain sufficient accuracy if we regard γ_1 as well as $\gamma_1^{(L)}$ as constant. We shall thus obtain

$$\left(\frac{\gamma_1^{(L)}}{\gamma_1} - 1\right)(p_1 - p_1') = (1 - A)\, p_2, \tag{290}$$

where p_1' denotes the pressure of the saturated vapor of the pure liquid consisting of S_1. It will be observed that when $A = 1$, the presence of the gas S_2 will not affect the pressure or density of the gas S_1. When $A < 1$, the pressure and density of the gas S_1 are greater than if S_2 were absent, and when $A > 1$, the reverse is true.

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The properties of an ideal gas-mixture (according to the definition which we have assumed) when in equilibrium with liquids or solids have been developed at length, because it is only in respect to these properties that there is any variation from the properties usually attributed to perfect gases. As the pressure of a gas saturated with vapor is usually given as a little less than the sum of the pressure of the gas calculated from its density and that of saturated vapor in a space otherwise empty, while our formulæ would make it a little more, when the gas is insoluble, it would appear that in this respect our formulæ are less accurate than the rule which would make the pressure of the gas saturated with vapor equal to the sum of the two pressures mentioned. Yet the reader will observe that the magnitude of the quantities concerned is not such that any stress can be laid upon this circumstance.

It will also be observed that the statement of Dalton's law which we have adopted, while it serves to complete the theory of gas-mixtures (with respect to a certain class of properties), asserts nothing with reference to any solid or liquid bodies. But the common rule that the density of a gas necessary for equilibrium with a solid or liquid is not altered by the presence of a different gas which is not absorbed by the solid or liquid, if construed *strictly*, will involve consequences in regard to solids and liquids which are entirely inadmissible. To show this, we will assume the correctness of the rule mentioned. Let S_1 denote the common component of the gaseous and liquid or solid masses, and S_2 the insoluble gas, and let quantities relating to the gaseous mass be distinguished when necessary by the index (G), and those relating to the liquid or solid by the index (L). Now while the gas is in equilibrium with the liquid or solid, let the quantity which it contains of S_2 receive the increment dm_2, its volume and the quantity which it contains of the other component, as well as the temperature, remaining constant. The potential for S_1 in the gaseous mass will receive the increment

$$\left(\frac{d\mu_1}{dm_2}\right)^{(G)}_{t,\,v,\,m} dm_2$$

and the pressure will receive the increment

$$\left(\frac{dp}{dm_2}\right)^{(G)}_{t,\,v,\,m} dm_2.$$

Now the liquid or solid remaining in equilibrium with the gas must experience the same variations in the values of μ_1 and p. But by (272)

$$\left(\frac{d\mu_1}{dp}\right)^{(L)}_{t,\,m} = \left(\frac{dv}{dm_1}\right)^{(L)}_{t,\,p,\,m}.$$

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Therefore,

$$\left(\frac{dv}{dm_1}\right)^{(L)}_{t,\,p,\,m} = \frac{\left(\frac{d\mu_1}{dm_2}\right)^{(G)}_{t,\,v,\,m}}{\left(\frac{dp}{dm_2}\right)^{(G)}_{t,\,v,\,m}}.$$

It will be observed that the first member of this equation relates solely to the liquid or solid, and the second member solely to the gas. Now we may suppose the same gaseous mass to be capable of equilibrium with several different liquids or solids, and the first member of this equation must therefore have the same value for all such liquids or solids; which is quite inadmissible. In the simplest case, in which the liquid or solid is identical in substance with the vapor which it yields, it is evident that the expression in question denotes the reciprocal of the density of the solid or liquid. Hence, when a gas is in equilibrium with one of its components both in the solid and liquid states (as when a moist gas is in equilibrium with ice and water), it would be necessary that the solid and liquid should have the same density.

The foregoing considerations appear sufficient to justify the definition of an ideal gas-mixture which we have chosen. It is of course immaterial whether we regard the definition as expressed by equation (273), or by (279), or by any other fundamental equation which can be derived from these.

The fundamental equations for an ideal gas-mixture corresponding to (255), (265), and (271) may easily be derived from these equations by using inversely the substitutions given on page 217. They are

$$\Sigma_1(c_1 m_1) \log \frac{\varepsilon - \Sigma_1(E_1 m_1)}{\Sigma_1(c_1 m_1)} = \eta + \Sigma_1\left(a_1 m_1 \log \frac{m_1}{v} - H_1 m_1\right), \quad (291)$$

$$\Sigma_1(c_1 m_1 + a_1 m_1) \log \frac{\chi - \Sigma_1(E_1 m_1)}{\Sigma_1(c_1 m_1 + a_1 m_1)}$$
$$= \eta + \Sigma_1\left(a_1 m_1 \log \frac{p\, m_1}{\Sigma_1(a_1 m_1)} - H_1 m_1\right), \quad (292)$$

$$\zeta = \Sigma_1\Big(E_1 m_1 + m_1 t (c_1 + a_1 - H_1)\Big)$$
$$- \Sigma_1(c_1 m_1 + a_1 m_1)\, t \log t + \Sigma_1\left(a_1 m_1 t \log \frac{p\, m_1}{\Sigma_1(a_1 m_1)}\right). \quad (293)$$

The components to which the fundamental equations (273), (279), (291), (292), 293) refer, may themselves be gas-mixtures. We may for example apply the fundamental equations of a binary gas-mixture

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to a mixture of hydrogen and air, or to any ternary gas-mixture in which the proportion of two of the components is fixed. In fact, the form of equation (279) which applies to a gas-mixture of any particular number of components may easily be reduced, when the proportions of some of these components are fixed, to the form which applies to a gas-mixture of a smaller number of components. The necessary substitutions will be analogous to those given on page 217. But the components must be entirely different from one another with respect to the gases of which they are formed by mixture. We cannot, for example, apply equation (279) to a gas-mixture in which the components are oxygen and air. It would indeed be easy to form a fundamental equation for such a gas-mixture with reference to the designated gases as components. Such an equation might be derived from (279) by the proper substitutions. But the result would be an equation of more complexity than (279). A *chemical* compound, however, with respect to Dalton's law, and with respect to all the equations which have been given, is to be regarded as entirely different from its components. Thus, a mixture of hydrogen, oxygen, and vapor of water is to be regarded as a ternary gas-mixture, having the three components mentioned. This is certainly true when the quantities of the compound gas and of its components are all independently variable in the gas-mixture, without change of temperature or pressure. Cases in which these quantities are not thus independently variable will be considered hereafter.

Inferences in regard to Potentials in Liquids and Solids.

Such equations as (264), (268), (276), by which the values of potentials in pure or mixed gases may be derived from quantities capable of direct measurement, have an interest which is not confined to the theory of gases. For as the potentials of the independently variable components which are common to coexistent liquid and gaseous masses have the same values in each, these expressions will generally afford the means of determining for liquids, at least approximately, the potential for any independently variable component which is capable of existing in the gaseous state. For although every state of a liquid is not such as can exist in contact with a gaseous mass, it will always be possible, when any of the components of the liquid are volatile, to bring it by a change of pressure alone, its temperature and composition remaining unchanged, to a state for which there is a coexistent phase of vapor, in which

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the values of the potentials of the volatile components of the liquid may be estimated from the density of these substances in the vapor. The variations of the potentials in the liquid due to the change of pressure will in general be quite trifling as compared with the variations which are connected with changes of temperature or of composition, and may moreover be readily estimated by means of equation (272). The same considerations will apply to volatile solids with respect to the determination of the potential for the substance of the solid.

As an application of this method of determining the potentials in liquids, let us make use of the law of Henry in regard to the absorption of gases by liquids to determine the relation between the quantity of the gas contained in any liquid mass and its potential. Let us consider the liquid as in equilibrium with the gas, and let $m_1^{(G)}$ denote the quantity of the gas existing as such, $m_1^{(L)}$ the quantity of the same substance contained in the liquid mass, μ_1 the potential for this substance common to the gas and liquid, $v^{(G)}$ and $v^{(L)}$ the volumes of the gas and liquid. When the absorbed gas forms but a very small part of the liquid mass, we have by Henry's law

$$\frac{m_1^{(L)}}{v^{(L)}} = A\,\frac{m_1^{(G)}}{v^{(G)}}, \tag{294}$$

where A is a function of the temperature; and by (276)

$$\mu_1 = B + C \log \frac{m_1^{(G)}}{v^{(G)}}, \tag{295}$$

B and C also denoting functions of the temperature. Therefore

$$\mu_1 = B + C \log \frac{m_1^{(L)}}{A\,v^{(L)}}. \tag{296}$$

It will be seen (if we disregard the difference of notation) that this equation is equivalent in form to (216), which was deduced from *a priori* considerations as a probable relation between the quantity and the potential of a small component. When a liquid absorbs several gases at once, there will be several equations of the form of (296), which will hold true simultaneously, and which we may regard as equivalent to equations (217), (218). The quantities A and C in (216), with the corresponding quantities in (217), (218), were regarded as functions of the temperature and pressure, but since the potentials in liquids are but little affected by the pressure, we might anticipate that these quantities in the case of liquids might be regarded as functions of the temperature alone.

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In regard to equations (216), (217), (218), we may now observe that by (264) and (276) they are shown to hold true in ideal gases or gas-mixtures, not only for components which form only a small part of the whole gas-mixture, but without any such limitation, and not only approximately but absolutely. It is noticeable that in this case quantities A and C are functions of the temperature alone, and do not even depend upon the nature of the gaseous mass, except upon the particular component to which they relate. As all gaseous bodies are generally supposed to approximate to the laws of ideal gases when sufficiently rarefied, we may regard these equations as approximately valid for gaseous bodies in general when the density is sufficiently small. When the density of the gaseous mass is very great, but the separate density of the component in question is small, the equations will probably hold true, but the values of A and C may not be entirely independent of the pressure, or of the composition of the mass in respect to its principal components. These equations will also apply, as we have just seen, to the potentials in liquid bodies for components of which the density in the liquid is very small, whenever these components exist also in the gaseous state, and conform to the law of Henry. This seems to indicate that the law expressed by these equations has a very general application.

Considerations relating to the Increase of Entropy due to the Mixture of Gases by Diffusion.

From equation (278) we may easily calculate the increase of entropy which takes place when two different gases are mixed by diffusion, at a constant temperature and pressure. Let us suppose that the quantities of the gases are such that each occupies initially one half of the total volume. If we denote this volume by V, the increase of entropy will be

$$m_1 a_1 \log V + m_2 a_2 \log V - m_1 a_1 \log \frac{V}{2} - m_2 a_2 \log \frac{V}{2},$$

or

$$(m_1 a_1 + m_2 a_2) \log 2.$$

Now

$$m_1 a_1 = \frac{pV}{2t}, \quad \text{and} \quad m_2 a_2 = \frac{pV}{2t}.$$

Therefore the increase of entropy may be represented by the expression

$$\frac{pV}{t} \log 2. \tag{297}$$

It is noticeable that the value of this expression does not depend upon the kinds of gas which are concerned, if the quantities are such as has been supposed, except that the gases which are mixed must be

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of different kinds. If we should bring into contact two masses of the same kind of gas, they would also mix, but there would be no increase of entropy. But in regard to the relation which this case bears to the preceding, we must bear in mind the following considerations. When we say that when two different gases mix by diffusion, as we have supposed, the energy of the whole remains constant, and the entropy receives a certain increase, we mean that the gases could be separated and brought to the same volume and temperature which they had at first by means of certain changes in external bodies, for example, by the passage of a certain amount of heat from a warmer to a colder body. But when we say that when two gas-masses of the same kind are mixed under similar circumstances there is no change of energy or entropy, we do not mean that the gases which have been mixed can be separated without change to external bodies. On the contrary, the separation of the gases is entirely impossible. We call the energy and entropy of the gas-masses when mixed the same as when they were unmixed, because we do not recognize any difference in the substance of the two masses. So when gases of different kinds are mixed, if we ask what changes in external bodies are necessary to bring the system to its original state, we do not mean a state in which each particle shall occupy more or less exactly the same position as at some previous epoch, but only a state which shall be undistinguishable from the previous one in its sensible properties. It is to states of systems thus incompletely defined that the problems of thermodynamics relate.

But if such considerations explain why the mixture of gas-masses of the same kind stands on a different footing from the mixture of gas-masses of different kinds, the fact is not less significant that the increase of entropy due to the mixture of gases of different kinds, in such a case as we have supposed, is independent of the nature of the gases.

Now we may without violence to the general laws of gases which are embodied in our equations suppose other gases to exist than such as actually do exist, and there does not appear to be any limit to the resemblance which there might be between two such kinds of gas. But the increase of entropy due to the mixing of given volumes of the gases at a given temperature and pressure would be independent of the degree of similarity or dissimilarity between them. We might also imagine the case of two gases which should be absolutely identical in all the properties (sensible and molecular) which come into play while they exist as gases either pure or mixed with each other,

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but which should differ in respect to the attractions between their atoms and the atoms of some other substances, and therefore in their tendency to combine with such substances. In the mixture of such gases by diffusion an increase of entropy would take place, although the process of mixture, dynamically considered, might be absolutely identical in its minutest details (even with respect to the precise path of each atom) with processes which might take place without any increase of entropy. In such respects, entropy stands strongly contrasted with energy. Again, when such gases have been mixed, there is no more impossibility of the separation of the two kinds of molecules in virtue of their ordinary motions in the gaseous mass without any especial external influence, than there is of the separation of a homogeneous gas into the same two parts into which it has once been divided, after these have once been mixed. In other words, the impossibility of an uncompensated decrease of entropy seems to be reduced to improbability.

There is perhaps no fact in the molecular theory of gases so well established as that the number of molecules in a given volume at a given temperature and pressure is the same for every kind of gas when in a state to which the laws of ideal gases apply. Hence the quantity $\frac{pV}{t}$ in (297) must be entirely determined by the number of molecules which are mixed. And the increase of entropy is therefore determined by the number of these molecules and is independent of their dynamical condition and of the degree of difference between them.

The result is of the same nature when the volumes of the gases which are mixed are not equal, and when more than two kinds of gas are mixed. If we denote by v_1, v_2, etc., the initial volumes of the different kinds of gas, and by V as before the total volume, the increase of entropy may be written in the form

$$\Sigma_1 (m_1 a_1) \log V - \Sigma_1 (m_1 a_1 \log v_1).$$

And if we denote by r_1, r_2, etc., the numbers of the molecules of the several different kinds of gas, we shall have

$$r_1 = C m_1 a_1, \quad r_2 = C m_2 a_2, \text{ etc.},$$

where C denotes a constant. Hence

$$v_1 : V :: m_1 a_1 : \Sigma_1 (m_1 a_1) :: r_1 : \Sigma_1 r_1;$$

and the increase of entropy may be written

$$\frac{\Sigma_1 r_1 \log \Sigma_1 r_1 - \Sigma_1 (r_1 \log r_1)}{C}. \tag{298}$$

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The Phases of Dissipated Energy of an Ideal Gas-mixture with Components which are Chemically Related.

We will now pass to the consideration of the phases of dissipated energy (see page 200) of an ideal gas-mixture, in which the number of the proximate components exceeds that of the ultimate.

Let us first suppose that an ideal gas-mixture has for proximate components the gases G_1, G_2, and G_3, the units of which are denoted by $\mathfrak{G}_1$, $\mathfrak{G}_2$, $\mathfrak{G}_3$, and that in ultimate analysis

$$\mathfrak{G}_3 = \lambda_1 \mathfrak{G}_1 + \lambda_2 \mathfrak{G}_2, \tag{299}$$

λ_1 and λ_2 denoting positive constants, such that $\lambda_1 + \lambda_2 = 1$. The phases which we are to consider are those for which the energy of the gas-mixture is a minimum for constant entropy and volume and constant quantities of G_1 and G_2, as determined in ultimate analysis. For such phases, by (86),

$$\mu_1 \delta m_1 + \mu_2 \delta m_2 + \mu_3 \delta m_3 \geqq 0. \tag{300}$$

for such values of the variations as do not affect the quantities of G_1 and G_2 as determined in ultimate analysis. Values of δm_1, δm_2, δm_3 proportional to λ_1, λ_2, -1, and only such, are evidently consistent with this restriction: therefore

$$\lambda_1 \mu_1 + \lambda_2 \mu_2 = \mu_3. \tag{301}$$

If we substitute in this equation values of μ_1, μ_2, μ_3 taken from (276), we obtain, after arranging the terms and dividing by t,

$$\lambda_1 a_1 \log \frac{m_1}{v} + \lambda_2 a_2 \log \frac{m_2}{v} - a_3 \log \frac{m_3}{v} = A + B \log t - \frac{C}{t}, \tag{302}$$

where

$$A = \lambda_1 H_1 + \lambda_2 H_2 - H_3 - \lambda_1 c_1 - \lambda_2 c_2 + c_3 - \lambda_1 a_1 - \lambda_2 a_2 + a_3, \tag{303}$$

$$B = \lambda_1 c_1 + \lambda_2 c_2 - c_3, \tag{304}$$

$$C = \lambda_1 E_1 + \lambda_2 E_2 - E_3. \tag{305}$$

If we denote by β_1 and β_2 the volumes (determined under standard conditions of temperature and pressure) of the quantities of the gases G_1 and G_2 which are contained in a unit of volume of the gas G_3, we shall have

$$\beta_1 = \frac{\lambda_1 a_1}{a_3}, \text{ and } \beta_2 = \frac{\lambda_2 a_2}{a_3}, \tag{306}$$

and (302) will reduce to the form

$$\log \frac{m_1^{\beta_1} m_2^{\beta_2}}{m_3 v^{\beta_1 + \beta_2 - 1}} = \frac{A}{a_3} + \frac{B}{a_3} \log t - \frac{C}{a_3 t}. \tag{307}$$

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Moreover, as by (277)

$$p\,v = (a_1\,m_1 + a_2\,m_2 + a_3\,m_3)\,t, \tag{308}$$

we have on eliminating v

$$\log \frac{m_1^{\beta_1}\,m_2^{\beta_2}\,p^{\beta_1+\beta_2-1}}{m_3\,(a_1\,m_1 + a_2\,m_2 + a_3\,m_3)^{\beta_1+\beta_2-1}} = \frac{A}{a_3} + \frac{B'}{a_3}\log t - \frac{C}{a_3\,t}, \tag{309}$$

where

$$B' = \lambda_1\,c_1 + \lambda_2\,c_2 - c_3 + \lambda_1\,a_1 + \lambda_2\,a_2 - a_3. \tag{310}$$

It will be observed that the quantities β_1, β_2 will always be positive and have a simple relation to unity, and that the value of $\beta_1 + \beta_2 - 1$ will be positive or zero, according as gas G_3 is formed of G_1 and G_2 with or without condensation. If we should assume, according to the rule often given for the specific heat of compound gases, that the thermal capacity at constant volume of any quantity of the gas G_3 is equal to the sum of the thermal capacities of the quantities which it contains of the gases G_1 and G_2, the value of B would be zero. The heat evolved in the formation of a unit of the gas G_3 out of the gases G_1 and G_2, without mechanical action, is by (283) and (257)

$$\lambda_1\,(c_1\,t + E_1) + \lambda_2\,(c_2\,t + E_2) - (c_3\,t + E_3),$$

or

$$B\,t + C,$$

which will reduce to C when the above relation in regard to the specific heats is satisfied. In any case the quantity of heat thus evolved divided by $a_3\,t^2$ will be equal to the differential coefficient of the second member of equation (307) with respect to t. Moreover, the heat evolved in the formation of a unit of the gas G_3 out of the gases G_1 and G_2 under constant pressure is

$$B\,t + C + \lambda_1\,a_1\,t + \lambda_2\,a_2\,t - a_3\,t = B'\,t + C,$$

which is equal to the differential coefficient of the second member of (309) with respect to t, multiplied by $a_3\,t^2$.

It appears by (307) that, except in the case when $\beta_1 + \beta_2 = 1$, for any given finite values of m_1, m_2, m_3, and t (infinitesimal values being excluded as well as infinite), it will always be possible to assign such a finite value to v that the mixture shall be in a state of dissipated energy. Thus, if we regard a mixture of hydrogen, oxygen, and vapor of water as an ideal gas-mixture, for a mixture containing any given quantities of these three gases at any given tem-

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perature there will be a certain volume at which the mixture will be in a state of dissipated energy. In such a state no such phenomenon as explosion will be possible, and no formation of water by the action of platinum. (If the mass should be expanded beyond this volume, the only possible action of a catalytic agent would be to resolve the water into its components.) It may indeed be true that at ordinary temperatures, except when the quantity either of hydrogen or of oxygen is very small compared with the quantity of water, the state of dissipated energy is one of such extreme rarefaction as to lie entirely beyond our power of experimental verification. It is also to be noticed that a state of great rarefaction is so unfavorable to any condensation of the gases, that it is quite probable that the catalytic action of platinum may cease entirely at a degree of rarefaction far short of what is necessary for a state of dissipated energy. But with respect to the theoretical demonstration, such states of great rarefaction are precisely those to which we should suppose that the laws of ideal gas-mixtures would apply most perfectly.

But when the compound gas G_3 is formed of G_1 and G_2 without condensation, (i. e., when $\beta_1 + \beta_2 = 1$,) it appears from equation (307) that the relation between m_1, m_2, and m_3 which is necessary for a phase of dissipated energy is determined by the temperature alone.

In any case, if we regard the total quantities of the gases G_1 and G_2 (as determined by the ultimate analysis of the gas-mixture), and also the volume, as constant, the quantities of these gases which appear uncombined in a phase of dissipated energy will increase with the temperature, if the formation of the compound G_3 *without change of volume* is attended with evolution of heat. Also, if we regard the total quantities of the gases G_1 and G_2, and also the pressure, as constant, the quantities of these gases which appear uncombined in a phase of dissipated energy, will increase with the temperature, if the formation of the compound G_3 *under constant pressure* is attended with evolution of heat. If $B = 0$, (a case, as has been seen, of especial importance), the heat obtained by the formation of a unit of G_3 out of G_1 and G_2 without change of volume or of temperature will be equal to C. If this quantity is positive, and the total quantities of the gases G_1 and G_2 and also the volume have given finite values, for an infinitesimal value of t we shall have (for a phase of dissipated energy) an infinitesimal value either of m_1 or of m_2, and for an infinite value of t we shall have finite (neither infinitesimal nor infinite) values of m_1, m_2, and m_3. But if we suppose the pressure instead of the volume to have a given finite value (with

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suppositions otherwise the same), we shall have for infinitesimal values of t an infinitesimal value either of m_1 or m_2, and for infinite values of t finite or infinitesimal values of m_3 according as $\beta_1 + \beta_2$ is equal to or greater than unity.

The case which we have considered is that of a ternary gas-mixture, but our results may easily be generalized in this respect. In fact, whatever the number of component gases in a gas-mixture, if there are relations of equivalence in ultimate analysis between these components, such relations may be expressed by one or more equations of the form

$$\lambda_1 \mathfrak{G}_1 + \lambda_2 \mathfrak{G}_2 + \lambda_3 \mathfrak{G}_3 + \text{etc.} = 0, \tag{311}$$

where $\mathfrak{G}_1$, $\mathfrak{G}_2$, etc. denote the units of the various component gases, and λ_1, λ_2, etc. denote positive or negative constants such that $\Sigma_1 \lambda_1 = 0$. From (311) with (86) we may derive for phases of dissipated energy,

$$\lambda_1 \mu_1 + \lambda_2 \mu_2 + \lambda_3 \mu_3 + \text{etc.} = 0,$$

or

$$\Sigma_1 (\lambda_1 \mu_1) = 0. \tag{312}$$

Hence, by (276),

$$\Sigma_1 \left(\lambda_1 a_1 \log \frac{m_1}{v}\right) = A + B \log t - \frac{C}{t}, \tag{313}$$

where A, B and C are constants determined by the equations

$$A = \Sigma_1 (\lambda_1 H_1 - \lambda_1 c_1 - \lambda_1 a_1), \tag{314}$$

$$B = \Sigma_1 (\lambda_1 c_1), \tag{315}$$

$$C = \Sigma_1 (\lambda_1 E_1). \tag{316}$$

Also, since

$$p v = \Sigma_1 (a_1 m_1) t,$$

$$\Sigma_1 (\lambda_1 a_1 \log m_1) - \Sigma (\lambda_1 a_1) \log \Sigma_1 (a_1 m_1) + \Sigma (\lambda_1 a_1) \log p = A + B' \log t - \frac{C}{t}, \tag{317}$$

where

$$B' = \Sigma_1 (\lambda_1 c_1 + \lambda_1 a_1). \tag{318}$$

If there is more than one equation of the form (311), we shall have more than one of each of the forms (313) and (317), which will hold true simultaneously for phases of dissipated energy.

It will be observed that the relations necessary for a phase of dissipated energy between the volume and temperature of an ideal gas-mixture, and the quantities of the components which take part in the chemical processes, and the pressure due to these components, are not affected by the presence of neutral gases in the gas-mixture.

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From equations (312) and (234) it follows that if there is a phase of dissipated energy at any point in an ideal gas-mixture in equilibrium under the influence of gravity, the whole gas-mixture must consist of such phases.

The equations of the phases of dissipated energy of a binary gas-mixture, the components of which are identical in substance, are comparatively simple in form. In this case the two components have the same potential, and if we write β for $\frac{a_1}{a_2}$ (the ratio of the volumes of equal quantities of the two components under the same conditions of temperature and pressure), we shall have

$$\log \frac{m_1{}^{\beta}}{m_2\, v^{\beta-1}} = \frac{A}{a_2} + \frac{B}{a_2} \log t - \frac{C}{a_2\, t}, \tag{319}$$

$$\log \frac{m_1{}^{\beta}\, p^{\beta-1}}{m_2\, (a_1 m_1 + a_2 m_2)^{\beta-1}} = \frac{A}{a_2} + \frac{B'}{a_2} \log t - \frac{C}{a_2\, t}; \tag{320}$$

where

$$A = H_1 - H_2 - c_1 + c_2 - a_1 + a_2, \tag{321}$$

$$B = c_1 - c_2, \qquad B' = c_1 - c_2 + a_1 - a_2, \tag{322}$$

$$C = E_1 - E_2. \tag{323}$$

Gas-mixtures with Convertible Components.

The equations of the phases of dissipated energy of ideal gas-mixtures which have components of which some are identical in ultimate analysis to others have an especial interest in relation to the theory of gas-mixtures in which the components are not only thus equivalent, but are actually transformed into each other within the gas-mixture on variations of temperature and pressure, so that quantities of these (proximate) components are entirely determined, at least in any permanent phase of the gas-mixture, by the quantities of a smaller number of ultimate components, with the temperature and pressure. Such gas-mixtures may be distinguished as having *convertible components.* The very general considerations adduced on pages 197–203, which are not limited in their application to gaseous bodies, suggest the hypothesis that the equations of the phases of dissipated energy of ideal gas-mixtures may apply to such gas-mixtures as have been described. It will, however, be desirable to consider the matter more in detail.

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In the first place, if we consider the case of a gas-mixture which only differs from an ordinary ideal gas-mixture for which some of the components are equivalent in that there is perfect freedom in regard to the transformation of these components, it follows at once from the general formula of equilibrium (1) or (2) that equilibrium is only possible for such phases as we have called phases of dissipated energy, for which some of the characteristic equations have been deduced in the preceding pages.

If it should be urged, that regarding a gas-mixture which has convertible components as an ideal gas-mixture of which, for some reason, only a part of the phases are actually capable of existing, we might still suppose the particular phases which alone can exist to be determined by some other principle than that of the free convertibility of the components (as if, perhaps, the case were analogous to one of *constraint* in mechanics), it may easily be shown that such a hypothesis is entirely untenable, when the quantities of the proximate components may be varied independently by suitable variations of the temperature and pressure, and of the quantities of the ultimate components, and it is admitted that the relations between the energy, entropy, volume, temperature, pressure, and the quantities of the several proximate components in the gas-mixture are the same as for an ordinary ideal gas-mixture, in which the components are not convertible. Let us denote the quantities of the n' proximate components of a gas-mixture A by m_1, m_2, etc., and the quantities of its n ultimate components by $\mathbf{m}_1$, $\mathbf{m}_2$, etc. (n denoting a number less than n'), and let us suppose that for this gas-mixture the quantities ε, η, v, t, p, m_1, m_2, etc. satisfy the relations characteristic of an ideal gas-mixture, while the phase of the gas-mixture is entirely determined by the values of $\mathbf{m}_1$, $\mathbf{m}_2$, etc., with two of the quantities ε, η, v, t, p. We may evidently imagine such an ideal gas-mixture B having n' components (not convertible), that every phase of A shall correspond with one of B in the values of ε, η, v, t, p, m_1, m_2, etc. Now let us give to the quantities $\mathbf{m}_1$, $\mathbf{m}_2$, etc. in the gas-mixture A any fixed values, and for the body thus defined let us imagine the v-η-ε surface (see page 174) constructed; likewise for the ideal gas-mixture B let us imagine the v-η-ε surface constructed for every set of values of m_1, m_2, etc. which is consistent with the given values of $\mathbf{m}_1$, $\mathbf{m}_2$, etc., i. e., for every body of which the *ultimate* composition would be expressed by the given values of $\mathbf{m}_1$, $\mathbf{m}_2$, etc. It follows immediately from our supposition, that every point in the v-η-ε surface relating to A must coincide with some point of one of the v-η-ε surfaces relating

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to B not only in respect to position but also in respect to its tangent plane (which represents temperature and pressure); therefore the v-η-ε surface relating to A must be tangent to the various v-η-ε surfaces relating to B, and therefore must be an envelop of these surfaces. From this it follows that the points which represent phases common to both gas-mixtures must represent the phases of dissipated energy of the gas-mixture B.

The properties of an ideal gas-mixture which are assumed in regard to the gas-mixture of convertible components in the above demonstration are expressed by equations (277) and (278) with the equation

$$\varepsilon = \Sigma_1 (c_1 m_1 t + m_1 E_1). \tag{324}$$

It is usual to assume in regard to gas-mixtures having convertible components that the convertibility of the components does not affect the relations (277) and (324). The same cannot be said of the equation (278). But in a very important class of cases it will be sufficient if the applicability of (277) and (324) is admitted. The cases referred to are those in which in certain phases of a gas-mixture the components are convertible, and in other phases of the same proximate composition the components are not convertible, and the equations of an ideal gas-mixture hold true.

If there is only a single degree of convertibility between the components, (i. e., if only a single kind of conversion, with its reverse, can take place among the components,) it will be sufficient to assume, in regard to the phases in which conversion takes place, the validity of equation (277) and of the following, which can be derived from (324) by differentiation, and comparison with equation (11), which expresses a necessary relation,

$$[t\,d\eta - p\,dv - \Sigma_1 (c_1 m_1)\,dt]_m = 0.^* \tag{325}$$

We shall confine our demonstration to this case. It will be observed that the physical signification of (325) is that if the gas-mixture is subjected to such changes of volume and temperature as do not alter its proximate composition, the heat absorbed or yielded may be calculated by the same formula as if the components were not convertible.

Let us suppose the thermodynamic state of a gaseous mass M, of such a kind as has just been described, to be varied while within the limits within which the components are not convertible. (The quantities of the proximate components, therefore, as well as of the ulti-

* This notation is intended to indicate that m_1, m_2, etc. are regarded as constant

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mate, are supposed constant). If we use the same method of geometrical representation as before, the point representing the volume, entropy, and energy of the mass will describe a line in the v-η-ε surface of an ideal gas-mixture of inconvertible components, the form and position of this surface being determined by the proximate composition of M. Let us now suppose the same mass to be carried beyond the limit of inconvertibility, the variations of state after passing the limit being such as not to alter its proximate composition. It is evident that this will in general be possible. Exceptions can only occur when the limit is formed by phases in which the proximate composition is uniform. The line traced in the region of convertibility must belong to the same v-η-ε surface of an ideal gas-mixture of inconvertible components as before, continued beyond the limit of inconvertibility for the components of M, since the variations of volume, entropy and energy are the same as would be possible if the components were not convertible. But it must also belong to the v-η-ε surface of the body M, which is here a gas-mixture of convertible components. Moreover, as the inclination of each of these surfaces must indicate the temperature and pressure of the phases through which the body passes, these two surfaces must be tangent to each other along the line which has been traced. As the v-η-ε surface of the body M in the region of convertibility must thus be tangent to all the surfaces representing ideal gas-mixtures of every possible proximate composition consistent with the ultimate composition of M, continued beyond the region of inconvertibility, in which alone their form and position may be capable of experimental demonstration, the former surface must be an envelop of the latter surfaces, and therefore a continuation of the surface of the phases of dissipated energy in the region of inconvertibility.

The foregoing considerations may give a measure of *a priori* probability to the results which are obtained by applying the ordinary laws of ideal gas-mixtures to cases in which the components are convertible. It is only by experiments upon gases in phases in which their components are convertible that the validity of any of these results can be established.

The very accurate determinations of density which have been made for the peroxide of nitrogen enable us to subject some of our equations to a very critical test. That this substance in the gaseous state is properly regarded as a mixture of different gases can hardly be doubted, as the proportion of the components derived from its density on the supposition that one component has the molecular formula

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NO_2 and the other the formula N_2O_4 is the same as that derived from the depth of the color on the supposition that the absorption of light is due to one of the components alone, and is proportioned to the separate density of that component.*

MM. Sainte-Claire Deville and Troost† have given a series of determinations of what we shall call the *relative densities* of peroxide of nitrogen at various temperatures under atmospheric pressure. We use the term *relative density* to denote what it is usual in treatises on chemistry to denote by the term *density*, viz., the actual density of a gas divided by the density of a standard perfect gas at the same pressure and temperature, the standard gas being air, or more strictly, an ideal gas which has the same density as air at the zero of the centigrade scale and the pressure of one atmosphere. In order to test our equations by these determinations, it will be convenient to transform equation (320), so as to give directly the relation between the relative density, the pressure, and the temperature.

As the density of the standard gas at any given temperature and pressure may by (263) be expressed by the formula $\frac{p}{a_s t}$, the relative density of a binary gas-mixture may be expressed by

$$D = (m_1 + m_2)\frac{a_s t}{p v}. \tag{326}$$

Now by (263)

$$a_1 m_1 + a_2 m_2 = \frac{p v}{t}. \tag{327}$$

By giving to m_2 and m_1 successively the value zero in these equations, we obtain

$$D_1 = \frac{a_s}{a_1}, \qquad D_2 = \frac{a_s}{a_2}, \tag{328}$$

where D_1 and D_2 denote the values of D when the gas consists wholly of one or of the other component. If we assume that

$$D_2 = 2 D_1, \tag{329}$$

we shall have

$$a_1 = 2 a_2. \tag{330}$$

From (326) we have

$$m_1 + m_2 = D\frac{p v}{a_s t},$$

* Salet, "Sur la coloration du peroxyde d'azote," Comptes Rendus, vol. lxvii, p. 488.

† Comptes Rendus, vol. lxiv, p. 237.

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and from (327), by (328) and (330),

$$2\,m_1 + m_2 = D_2 \frac{p\,v}{a_{\prime}\,t} = 2\,D_1 \frac{p\,v}{a_{\prime}\,t},$$

whence

$$m_1 = (D_2 - D)\frac{p\,v}{a_{\prime}\,t}, \tag{331}$$

$$m_2 = 2\,(D - D_1)\,\frac{p\,v}{a_{\prime}\,t}. \tag{332}$$

By (327), (331), and (332) we obtain from (320)

$$\log \frac{(D_2 - D)^2\,p}{2\,(D - D_1)\,a_{\prime}} = \frac{A}{a_2} + \frac{B'}{a_2}\log t - \frac{C}{a_{\prime}\,t}. \tag{333}$$

This formula will be more convenient for purposes of calculation if we introduce common logarithms (denoted by $\log_{10}$) instead of hyperbolic, the temperature of the ordinary contigrade scale t_c instead of the absolute temperature t, and the pressure in atmospheres p_{at} instead of p the pressure in a rational system of units. If we also add the logarithm of $a_{\prime}$ to both sides of the equation, we obtain

$$\log_{10} \frac{(D_2 - D)^2\,p_{at}}{2\,(D - D_1)} = \mathbf{A} + \frac{B'}{a_2}\log_{10}(t_c + 273) - \frac{\mathbf{C}}{t_c + 273}, \tag{334}$$

where **A** and **C** denote constants, the values of which are closely connected with those of A and C.

From the molecular formulæ of peroxide of nitrogen NO_2 and N_2O_4, we may calculate the relative densities

$$D_1 = \frac{14 + 32}{2}.0691 = 1.589, \text{ and } D_2 = \frac{28 + 64}{2}.0691 = 3.178. \tag{335}$$

The determinations of MM. Deville and Troost are satisfactorily represented by the equation

$$\log_{10} \frac{(3.178 - D)^2\,p_{at}}{2\,(D - 1.589)} = 9.47056 - \frac{3118.6}{t_c + 273}, \tag{336}$$

which gives

$$D = 3.178 + \Theta - \sqrt{\Theta\,(3.178 + \Theta)}$$

where $$\log_{10}\Theta = 9.47056 - \frac{3118.6}{t_c + 273} - \log_{10} p_{at}.$$

In the first part of the following table are given in successive columns the temperature and pressure of the gas in the several experiments of MM. Deville and Troost, the relative densities calculated from these numbers by equation (336), the relative densities as observed, and the difference of the observed and calculated relative

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densities. It will be observed that these differences are quite small, in no case reaching .03, and on the average scarcely exceeding .01. The significance of such correspondence in favor of the hypothesis by means of which equation (336) has been established is of course diminished by the fact that two constants in the equation have been determined from these experiments. If the same equation can be shown to give correctly the relative densities at other pressures than that for which the constants have been determined, such correspondence will be much more decisive.

t_r	p_{av}	D calculated by eq. (336).	D observed.	diff.	Observers.
26.7	1	2.676	2.65	−.026	D. & T.
35.4	1	2.524	2.53	+.006	D. & T.
39.8	1	2.443	2.46	+.017	D. & T.
49.6	1	2.256	2.27	+.014	D. & T.
60.2	1	2.067	2.08	+.013	D. & T.
70.0	1	1.920	1.92	.000	D. & T.
80.6	1	1.801	1.80	−.001	D. & T.
90.0	1	1.728	1.72	−.008	D. & T.
100.1	1	1.676	1.68	+.004	D. & T.
111.3	1	1.641	1.65	+.009	D. & T.
121.5	1	1.622	1.62	−.002	D. & T.
135.0	1	1.607	1.60	−.007	D. & T.
154.0	1	1.597	1.58	−.017	D. & T.
183.2	1	1.592	1.57	−.022	D. & T.
97.5	1	1.687			
97.5	$\frac{10450}{26397}$	1.631	1.783	+.152	P. & W.
24.5	1	2.711			
24.5	$\frac{18090}{42829}$	2.524	2.52	−.004	P. & W.
11.3	1	2.891			
11.3	$\frac{9265}{44205}$	2.620	2.645	+.025	P. & W.
4.2	1	2.964			
4.2	$\frac{6023}{35438}$	2.708	2.588	−.120	P. & W.

Messrs. Playfair and Wanklyn have published* four determinations of the relative density of peroxide of nitrogen at various temperatures when diluted with nitrogen. Since the relations expressed by equations (319) and (320) are not affected by the presence of a third gas which is different from the gases G_1 and G_2 (to which m_1 and m_2 relate) and neutral to them, (see the remark at the foot of page 233), —provided that we take p to denote the pressure which we attribute to the gases G_1 and G_2, i. e., the total pressure diminished by the pressure which the third gas would exert if occupying alone the same space at the same temperature,—it follows that the relations expressed for

* Transactions of the Royal Society of Edinburg, vol. xxii, p. 441.

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peroxide of nitrogen by (333), (334), and (336) will not be affected by the presence of free nitrogen, if the pressure expressed by p or p_a and contained implicitly in the symbol D (see equation (326) by which D is defined) is understood to denote the total pressure diminished by the pressure due to the free nitrogen. The determinations of Playfair and Wanklyn are given in the latter part of the above table. The pressures given are those obtained by subtracting the pressure due to the free nitrogen from the total pressure. We may suppose such reduced pressures to have been used in the reduction of the observations by which the numbers in the column of observed relative densities were obtained. Besides the relative densities calculated by equation (336) for the temperatures and (reduced) pressures of the observations, the table contains the relative densities calculated for the same temperatures and the pressure of one atmosphere.

The reader will observe that in the second and third experiments of Playfair and Wanklyn there is a very close accordance between the calculated and observed values of D, while in the second and fourth experiments there is a considerable difference. Now the weight to be attributed to the several determinations is very different. The quantities of peroxide of nitrogen which were used in the several experiments were respectively .2410, .5893, .3166, and .2016 grammes. For a rough approximation, we may assume that the probable errors of the relative densities are inversely proportional to these numbers. This would make the probable error of the first and fourth observations two or three times as great as that of the second and considerably greater than that of the third. We must also observe that in the first of these experiments, the observed relative density 1.783 is greater than 1.687, the relative density calculated by equation (336) for the temperature of the experiment and the pressure of one atmosphere. Now the number 1.687 we may regard as established directly by the experiments of Deville and Troost. For in seven successive experiments in this part of the series the calculated relative densities differ from the observed by less than .01. If then we accept the numbers given by experiment, the effect of diluting the gas with nitrogen is to increase its relative density. As this result is entirely at variance with the facts observed in the case of other gases, and in the case of this gas at lower temperatures, as appears from the three other determinations of Playfair and Wanklyn, it cannot possibly be admitted on the strength of a single

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observation. The first experiment of this series cannot therefore properly be used as a test of our equations. Similar considerations apply with somewhat less force to the last experiment. By comparing the temperatures and pressures of the three last experiments with the observed relative densities, the reader may easily convince himself that if we admit the substantial accuracy of the determinations in the two first of these experiments (the second and third of the series, which have the greatest weight), the last determination of relative density 2.588 must be too small. In fact, it should evidently be greater than the number in the preceding experiment 2.645.

If we confine our attention to the second and third experiments of the series, the agreement is as good as could be desired. Nor will the admission of errors of .152 and .120 (certainly not large in determinations of this kind) in the first and fourth experiments involve any serious doubt of the substantial accuracy of the second and third, when the difference of weight of the determinations is considered. Yet it is much to be desired that the relation expressed by (336), or with more generality by (334), should be tested by more numerous experiments.

It should be stated that the numbers in the column of pressures are not quite accurate. In the experiments of Deville and Troost the gas was subject to the actual atmospheric pressure at the time of the experiment. This varied from 747 to 764 millimeters of mercury. The precise pressure for each experiment is not given. In the experiments of Playfair and Wanklyn the mixture of nitrogen and peroxide of nitrogen was subject to the actual atmospheric pressure at the time of the experiment. The numbers in the column of pressures express the fraction of the whole pressure which remains after substracting the part due to the free nitrogen. But no indication is given in the published account of the experiments in regard to the height of the barometer. Now it may easily be shown that a variation of $\frac{13}{760}$ in the value of p can in no case cause a variation of more than .005 in the value of D as calculated by equation (336). In any of the experiments of Playfair and Wanklyn a variation of more than 30^{mm} in the height of the barometer would be necessary to produce a variation of .01 in the value of D. The errors due to this source cannot therefore be very serious. They might have been avoided altogether in the discussion of the experiments of Deville and Troost by using instead of (336) a formula expressing the relation between the relative density, the temperature, and the actual density, as the reciprocal of the latter quantity is given for each ex-

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periment of this series. It seemed best, however, to make a trifling sacrifice of accuracy for the sake of simplicity.

It might be thought that the experiments under discussion would be better represented by a formula in which the term containing log t (see equation (333)) was retained. But an examination of the figures in the table will show that nothing important can be gained in this respect, and there is hardly sufficient motive for adding another term to the formula of calculation. Any attempt to determine the *real* values of A, B', and C in equation (333), (assuming the absolute validity of such an equation for peroxide of nitrogen,) from the experiments under discussion would be entirely misleading, as the reader may easily convince himself.

From equation (336), however, the following conclusions may deduced. By comparison with (334) we obtain

$$\mathbf{A} + \frac{B'}{a_2} \log_{10} t - \frac{\mathbf{C}}{t} = 9.47056 - \frac{3118.6}{t},$$

which must hold true approximately between the temperatures 11^c and 90^c. (At higher temperatures the relative densities vary too slowly with the temperatures to afford a critical test of the accuracy of this relation.) By differentiation we obtain

$$\frac{MB'}{a_2 t} + \frac{\mathbf{C}}{t^2} = \frac{3118.6}{t^2},$$

where M denotes the modulus of the common system of logarithms. Now by comparing equations (333) and (334) we see that

$$\mathbf{C} = \frac{MC}{a_2} = .43429 \frac{C}{a_2}.$$

Hence

$$B' t + C = 7181\, a_2 = 3590\, a_1,$$

which may be regarded as a close approximation at 40^c or 50^c, and a tolerable approximation between the limits of temperature above mentioned. Now $B' t + C$ represents the heat evolved by the conversion of a unit of NO_2 into N_2O_4 under constant pressure. Such conversion cannot take place at constant pressure without change of temperature, which renders the experimental verification of the last equation less simple. But since by equations (322)

$$B' = B + a_1 - a_2 = B + \tfrac{1}{2} a_1,$$

we shall have for the temperature of 40^c

$$B t + C = 3434\, a_1.$$

Now $B t + C$ represents the decrease of energy when a unit of NO_2 is transformed into N_2O_4 without change of temperature. It therefore

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represents the excess of the heat evolved over the work done by external forces when a mass of the gas is compressed at constant temperature until a unit of NO_2 has been converted into N_2O_4. This quantity will be constant if $B = 0$, i. e., if the specific heats at constant volume of NO_2 and N_2O_4 are the same. This assumption would be more simple from a theoretical stand-point and perhaps safer than the assumption that $B' = 0$. If $B = 0$, $B' = a_2$. If we wish to embody this assumption in the equation between D, p, and t, we may substitute

$$6.5228 + \log_{10}(t_c + 273) - \frac{2977.4}{t_c + 273}$$

for the second member of equation (336). The relative densities calculated by the equation thus modified from the temperatures and pressures of the experiments under discussion will not differ from those calculated from the unmodified equation by more than .002 in any case, or by more than .001 in the first series of experiments.

It is to be noticed that if we admit the validity of the volumetrical relation expressed by equation (333), which is evidently equivalent to an equation between p, t, v, and m (this letter denoting the quantity of the gas without reference to its molecular condition), or if we admit the validity of the equation only between certain limits of temperature and for densities less than a certain limit of density, and also admit that between the given limits of temperature the specific heat of the gas at constant volume may be regarded as a constant quantity when the gas is sufficiently rarefied to be regarded as consisting wholly of NO_2,—or, to speak without reference to the molecular state of the gas, when it is rarefied until its relative density D approximates to its limiting value D_1,—we must also admit the validity (within the same limits of temperature and density) of all the calorimetrical relations which belong to ideal gas-mixtures with convertible components. The premises are evidently equivalent to this,—that we may imagine an ideal gas with convertible components such that between certain limits of temperature and above a certain limit of density the relation between p, t, and v shall be the same for a unit of this ideal gas as for a unit of peroxide of nitrogen, and for a very great value of v (within the given limits of temperature) the thermal capacity at constant volume of the ideal and actual gases shall be the same. Let us regard t and v as independent variables; we may let these letters and p refer alike to the ideal and real gases, but we must distinguish the entropy η' of the ideal gas from the entropy η of the real gas. Now by (88)

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$$\frac{d\eta}{dv} = \frac{dp}{dt}, \tag{337}$$

therefore

$$\frac{d}{dv}\frac{d\eta}{dt} = \frac{d}{dt}\frac{d\eta}{dv} = \frac{d}{dt}\frac{dp}{dt} = \frac{d^2p}{dt^2}. \tag{338}$$

Since a similar relation will hold true for η', we obtain

$$\frac{d}{dv}\frac{d\eta}{dt} = \frac{d}{dv}\frac{d\eta'}{dt}, \tag{339}$$

which must hold true within the given limits of temperature and density. Now it is granted that

$$\frac{d\eta}{dt} = \frac{d\eta'}{dt} \tag{340}$$

for very great values of v at any temperature within the given limits, (for the two members of the equation represent the thermal capacities at constant volume of the real and ideal gases divided by t,) hence, in virtue of (339), this equation must hold true in general within the given limits of temperature and density. Again, as an equation like (337) will hold true of η', we shall have

$$\frac{d\eta}{dv} = \frac{d\eta'}{dv}. \tag{341}$$

From the two last equations it is evident that in all calorimetrical relations the ideal and real gases are identical. Moreover the energy and entropy of the ideal gas are evidently so far arbitrary that we may suppose them to have the same values as in the real gas for any given values of t and v. Hence the entropies of the two gases are the same within the given limits; and on account of the necessary relation

$$d\varepsilon = t\,d\eta - p\,dv,$$

the energies of the two gases are in like manner identical. Hence the fundamental equation between the energy, entropy, volume, and quantity of matter must be the same for the ideal gas as for the actual.

We may easily form a fundamental equation for an ideal gas-mixture with convertible components, which shall relate only to the phases of equilibrium. For this purpose, we may use the equations of the form (312) to eliminate from the equation of the form (273), which expresses the relation between the pressure, the temperature, and the potentials for the proximate components, as many of the potentials as there are equations of the former kind, leaving the

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potentials for those components which it is convenient to regard as the ultimate components of the gas-mixture.

In the case of a binary gas-mixture with convertible components, the components will have the same potential, which may be denoted by μ, and the fundamental equation will be

$$p = a_1 L_1 t^{\frac{c_1+a_1}{a_1}} e^{\frac{\mu-E_1}{a_1 t}} + a_2 L_2 t^{\frac{c_2+a_2}{a_2}} e^{\frac{\mu-E_2}{a_2}}, \tag{342}$$

where

$$L_1 = e^{\frac{H_1-c_1-a_1}{a_1}}, \quad L_2 = e^{\frac{H_2-c_2-a_2}{a_2}}. \tag{343}$$

From this equation, by differentiation and comparison with (98), we obtain

$$\frac{\eta}{v} = L_1\left(c_1 + a_1 - \frac{\mu-E_1}{t}\right) t^{\frac{c_1}{a_1}} e^{\frac{\mu-E_1}{a_1 t}}$$

$$+ L_2\left(c_2 + a_2 - \frac{\mu-E_2}{t}\right) t^{\frac{c_2}{a_2}} e^{\frac{\mu-E_2}{a_2 t}}, \tag{244}$$

$$\frac{m}{v} = L_1 t^{\frac{c_1}{a_1}} e^{\frac{\mu-E_1}{a_1 t}} + L_2 t^{\frac{c_2}{a_2}} e^{\frac{\mu-E_2}{a_2 t}}. \tag{345}$$

From the general equation (93) with the preceding equations the following is easily obtained,—

$$\frac{\varepsilon}{v} = L_1 (c_1 t + E_1) t^{\frac{c_1}{a_1}} e^{\frac{\mu-E_1}{a_1 t}} + L_2 (c_2 t + E_2) t^{\frac{c_2}{a_2}} e^{\frac{\mu-E_2}{a_2 t}}. \tag{346}$$

We may obtain the relation between p, t, v, and m by eliminating μ from (342) and (345). For this purpose we may proceed as follows. From (342) and (345) we obtain

$$p - a_2 t \frac{m}{v} = (a_1 - a_2) L_1 t^{\frac{c_1+a_1}{a_1}} e^{\frac{\mu-E_1}{a_1 t}} \tag{347}$$

$$a_1 t \frac{m}{v} - p = (a_1 - a_2) L_2 t^{\frac{c_2+a_2}{a_2}} e^{\frac{\mu-E_2}{a_2 t}}; \tag{348}$$

and from these equations we obtain

$$a_1 \log\left(p - a_2 t \frac{m}{v}\right) - a_2 \log\left(a_1 t \frac{m}{v} - p\right) = (a_1 - a_2) \log (a_1 - a_2)$$

$$+ a_1 \log L_1 - a_2 \log L_2 + (c_2 - c_2 + a_1 - a_2) \log t - \frac{E_1 - E_2}{t}. \tag{349}$$

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(In the particular case when $a_1 = 2\,a_2$ this equation will be equivalent to (333)). By (347) and (348) we may easily eliminate μ from (346).

The reader will observe that the relations thus deduced from the fundamental equation (342) without any reference to the different components of the gaseous mass are equivalent to those which relate to the phases of dissipated energy of a binary gas-mixture with components which are equivalent in substance but not convertible, except that the equations derived from (342) do not give the quantities of the proximate components, but relate solely to those properties which are capable of direct experimental verification without the aid of any theory of the constitution of the gaseous mass.

The practical application of these equations is rendered more simple by the fact that the ratio $a_1 : a_2$ will always bear a simple relation to unity. When a_1 and a_2 are equal, if we write a for their common value, we shall have by (342) and (345)

$$p\,v = a\,m\,t, \tag{350}$$

and by (345) and (346)

$$\frac{\varepsilon}{m} = \frac{L_1(c_1 t + E_1) + L_2(c_2 t + E_2)t^{\frac{c_2 - c_1}{a}} e^{\frac{E_1 - E_2}{at}}}{L_1 + L_2 t^{\frac{c_2 - c_1}{a}} e^{\frac{E_1 - E_2}{at}}}. \tag{351}$$

By this equation we may calculate directly the amount of heat required to raise a given quantity of the gas from one given temperature to another at constant volume. The equation shows that the amount of heat will be independent of the volume of the gas. The heat necessary to produce a given change of temperature in the gas at constant pressure, may be found by taking the difference of the values of χ, as defined by equation (89), for the initial and final states of the gas. From (89), (350), and (351) we obtain

$$\frac{\chi}{m} = \frac{L_1(c_1 t + at + E_1) + L_2(c_2 t + at + E_2)t^{\frac{c_2 - c_1}{a}} e^{\frac{E_1 - E_2}{at}}}{L_1 + L_2 t^{\frac{c_2 - c_1}{a}} e^{\frac{E_1 - E_2}{at}}}. \tag{352}$$

By differentiation of the two last equations we may obtain directly the specific heats of the gas at constant volume and at constant pressure.

The fundamental equation of an ideal ternary gas-mixture with a single relation of convertibility between its components is

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$$
\begin{aligned}
p = a_1\, & e^{\frac{H_1 - c_1 - a_1}{a_1}}\, t^{\frac{c_1 + a_1}{a_1}}\, e^{\frac{\mu_1 - E_1}{a_1 t}} \\
+ a_2\, & e^{\frac{H_2 - c_2 - a_2}{a_2}}\, t^{\frac{c_2 + a_2}{a_2}}\, e^{\frac{\mu_2 - E_2}{a_2 t}} \\
+ a_3\, & e^{\frac{H_3 - c_3 - a_3}{a_3}}\, t^{\frac{c_3 + a_3}{a_3}}\, e^{\frac{\lambda_1 \mu_1 + \lambda_2 \mu_2 - E_3}{a_3 t}},
\end{aligned}
$$

where λ_1 and λ_2 have the same meaning as on page 230.

(*To be continued.*)

ERRATA.

Page 167, formula (168), for m_1 read μ_1.

" formula (169), for $m_1, \ldots m_{n-1}$ read $\mu_1, \ldots \mu_{n-1}$.

Page 239, formula (333), for $\frac{C}{t}$ read $\frac{C}{a_2 t}$.

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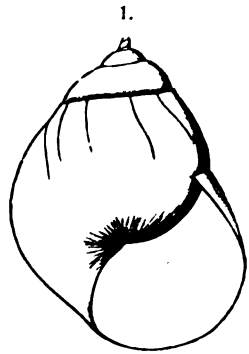

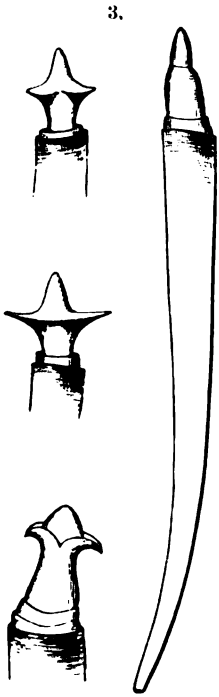

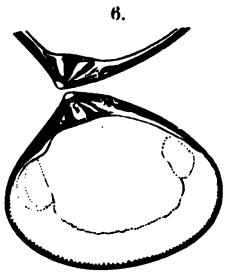

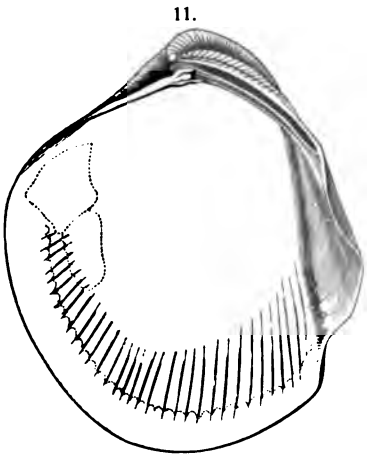

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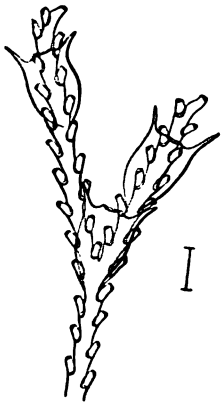

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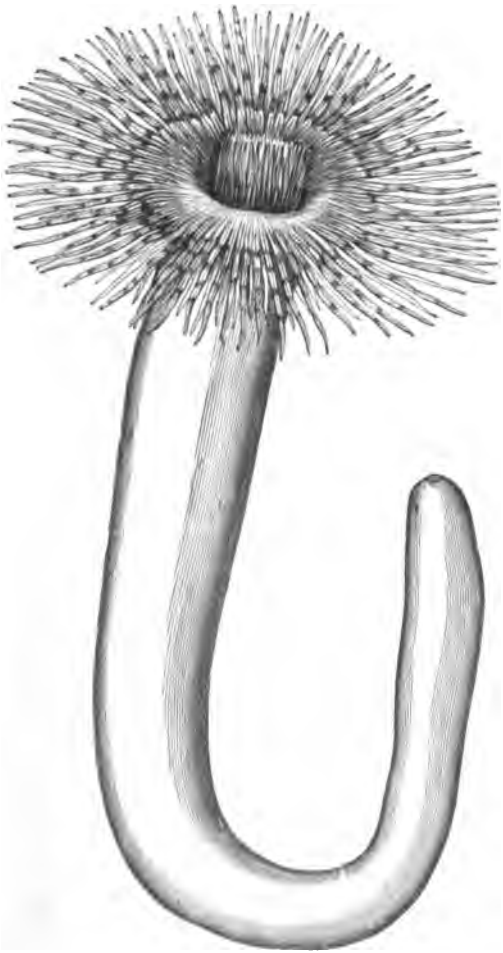

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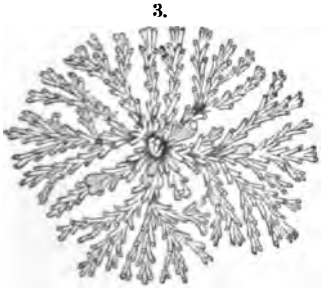

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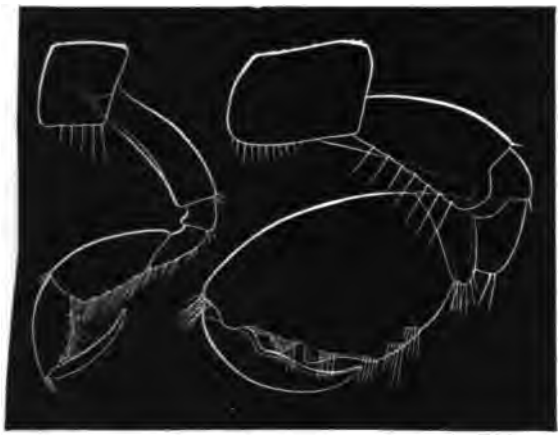

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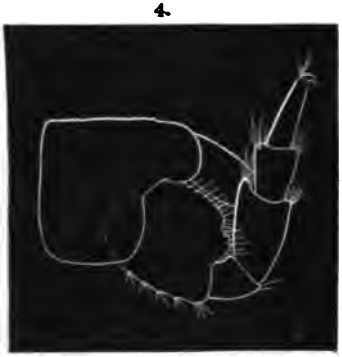

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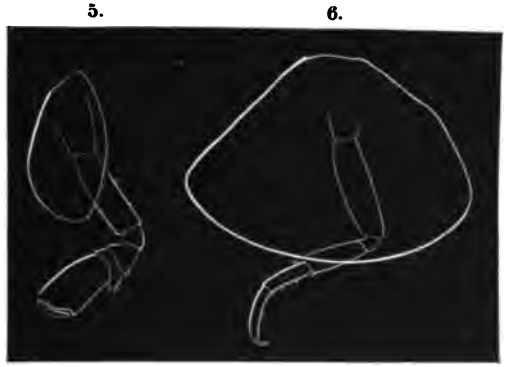

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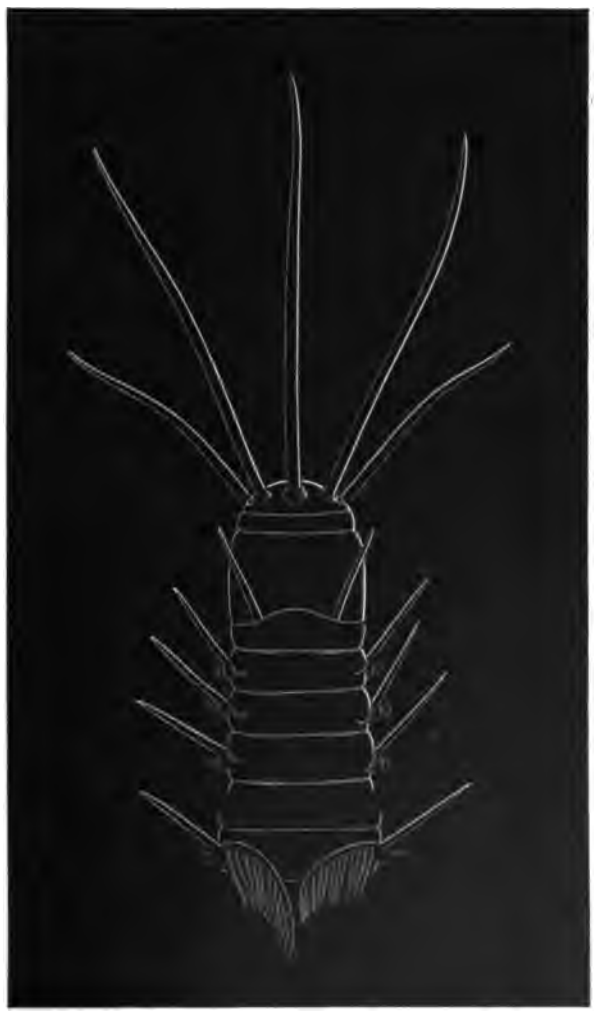

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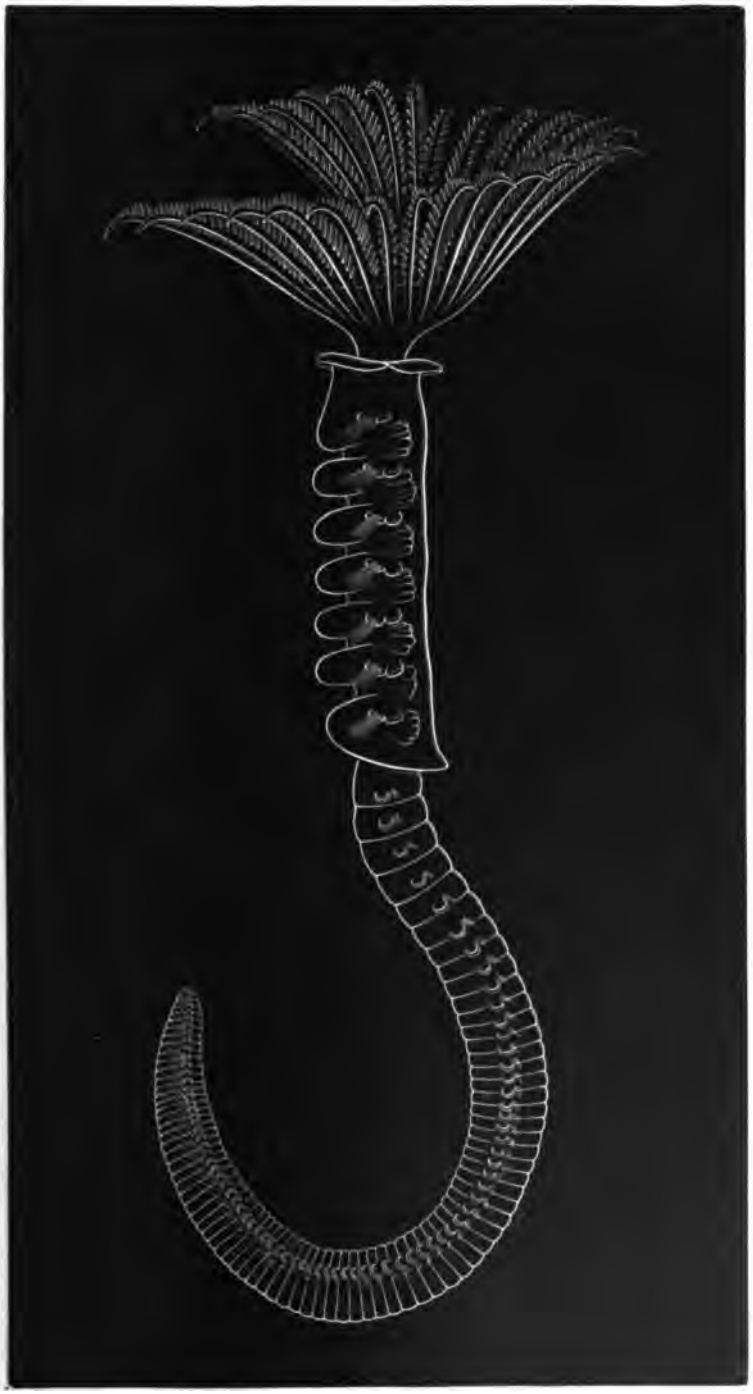

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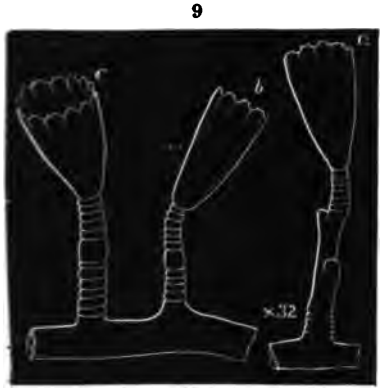

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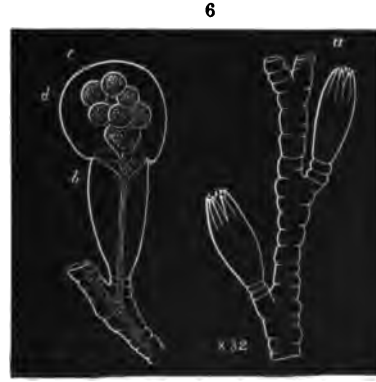

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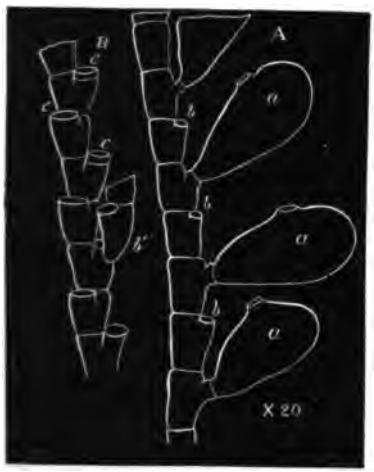

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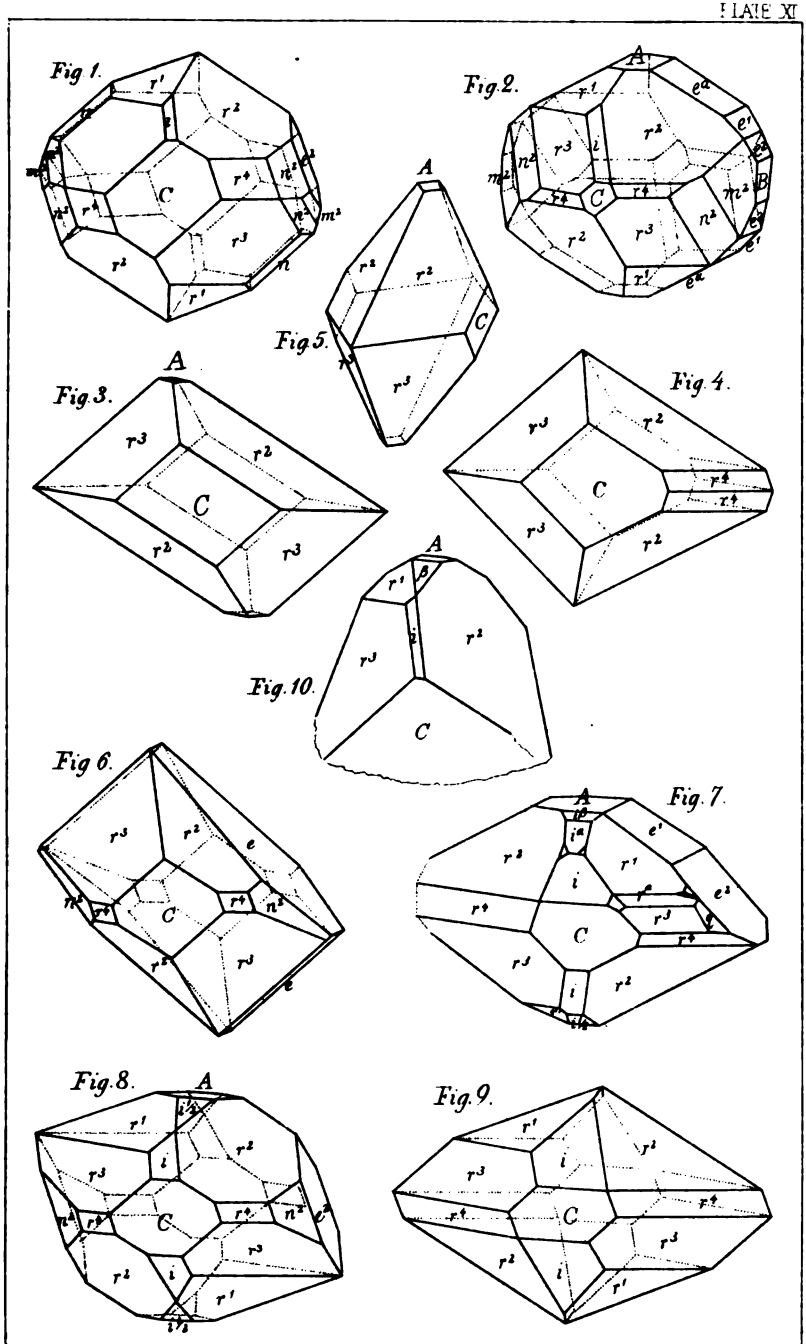

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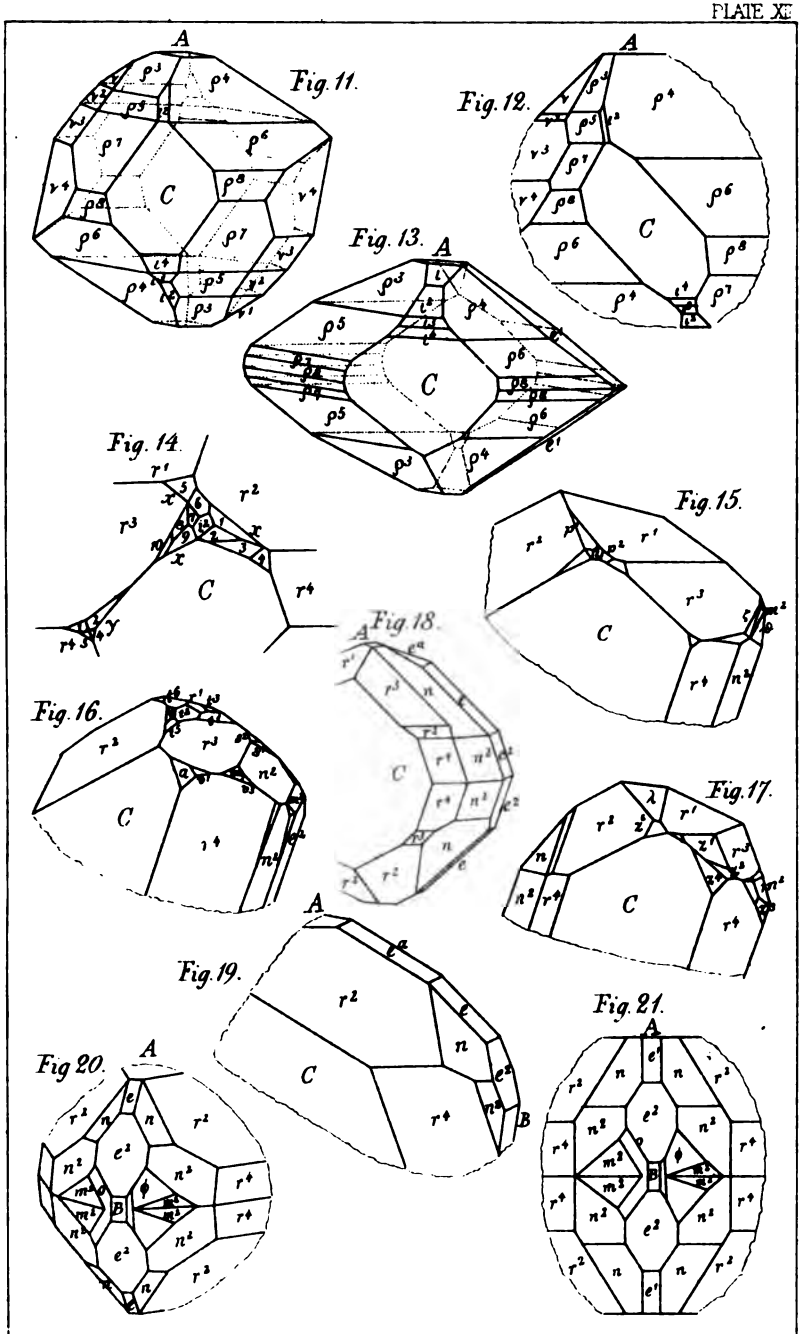

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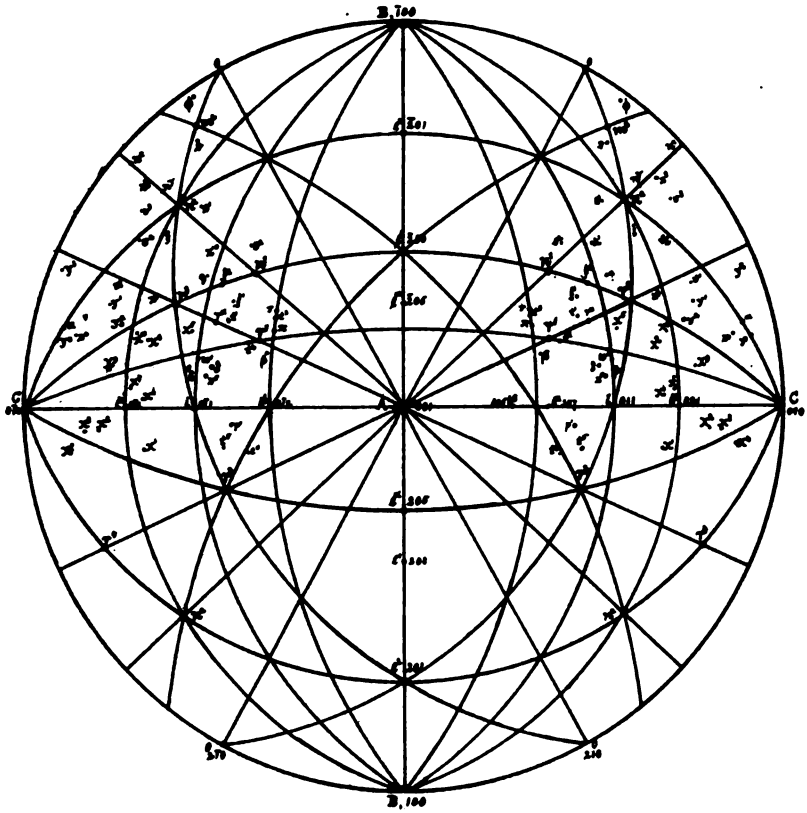

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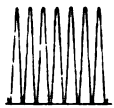
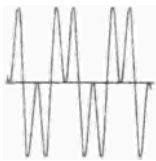
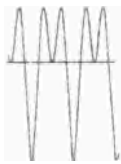
Curves whose equations are $y = a \sin x \sin mx$.



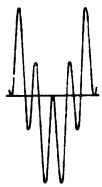
1. $m = 1$.



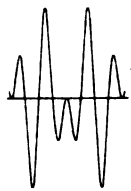
2. $m = 1/2$.



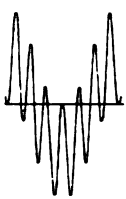
3. $m = 1/3$.



4. $m = 2/3$.

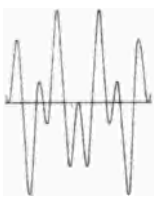


5. $m = 1/4$.

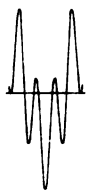


6. $m = 3/4$.

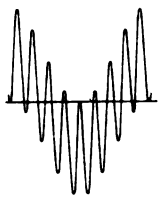
7. $m = 1/5$.



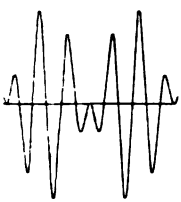
8. $m = 2/5$.



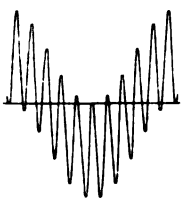
9. $m = 3/5$.



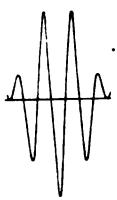
10. $m = 4/5$.



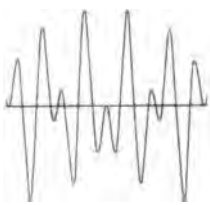
11. $m = 1/6$.



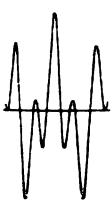
12. $m = 5/6$.



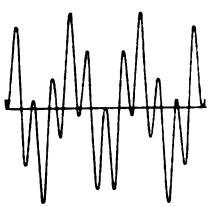
13. $m = 1/7$.



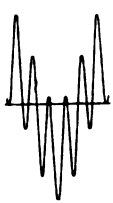
14. $m = 2/7$.



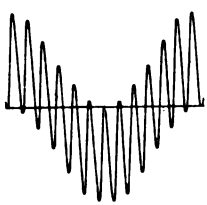
15. $m = 3/7$.



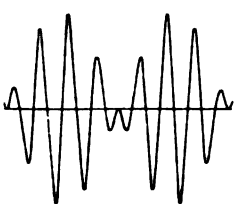
16. $m = 4/7$.



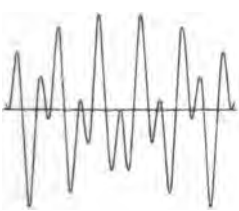
17. $m = 5/7$.



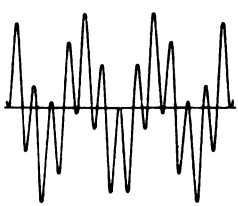
18. $m = 6/7$.



19. $m = 1/8$.



20. $m = 3/8$.



21. $m = 5/8$.

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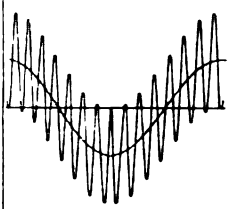
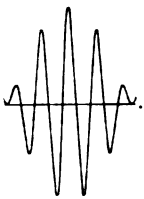
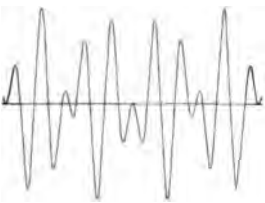
Curves whose equations are $y = a \sin x \sin mx$.



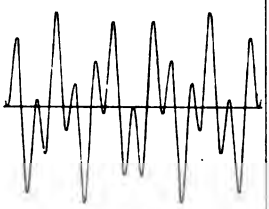
22. $m = 7/8$.



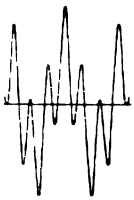
23. $m = 1/9$.



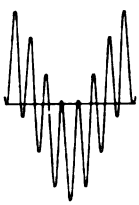
24. $m = 2/9$.



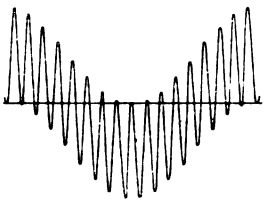
25. $m = 4/9$.



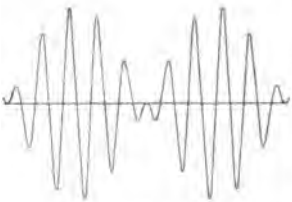
26. $m = 5/9$.



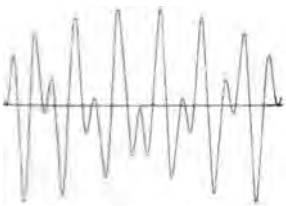
27. $m = 7/9$.



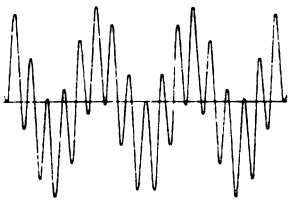
28. $m = 8/9$.



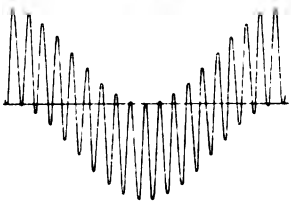
29. $m = 1/10$.



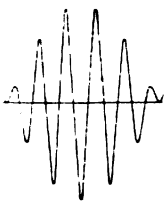
30. $m = 3/10$.



31. $m = 7/10$.

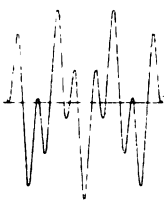


32. $m = 9/10$.

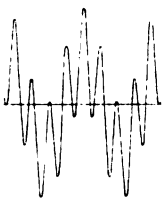


33. $m = 1/11$.

34. $m = 3/11$.



35. $m = 5/11$.



36. $m = 7/11$.

37. $m = \sqrt{1/5}$.

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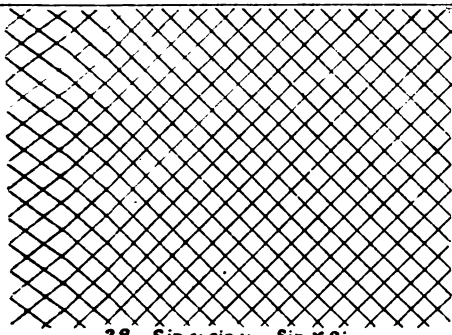
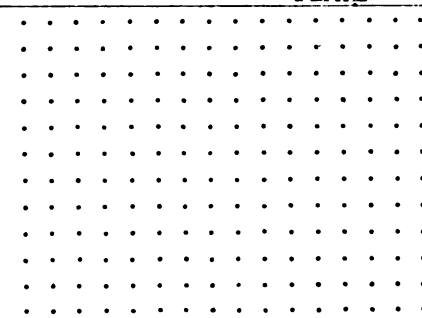
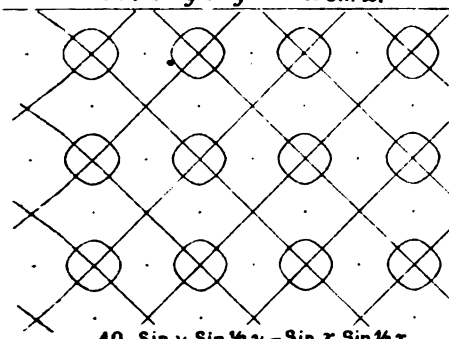


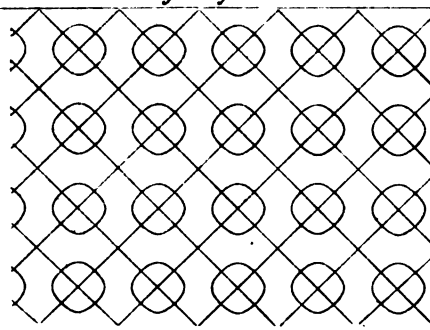
38. Sin y sin y = Sin x Sin x.



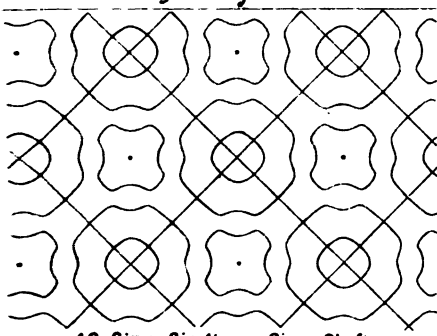
39. Sin y Sin y = −Sin x Sin x.



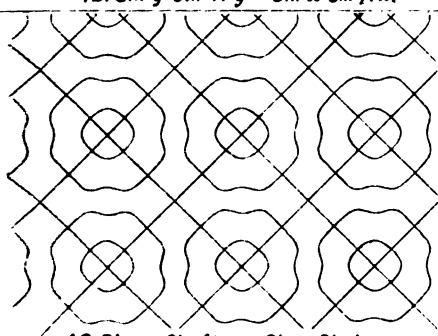
40. Sin y Sin ½ y = Sin x Sin ½ x.



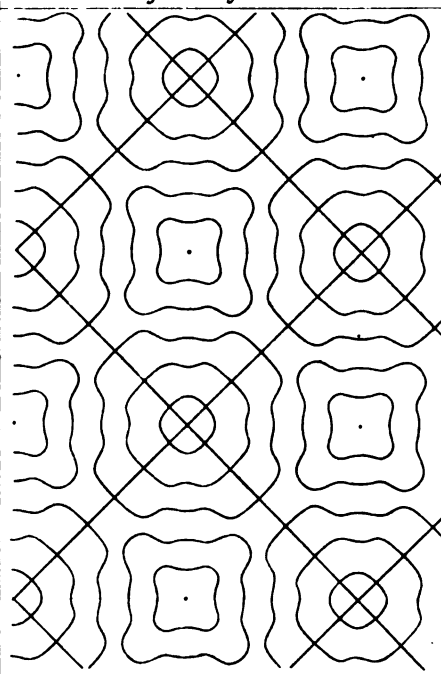
41. Sin y Sin ⅓ y = Sin x Sin ⅓ x.



42. Sin y Sin ¼ y = Sin x Sin ¼ x.



43. Sin y Sin ⅕ y = Sin x Sin ⅕ x.



44. Sin y Sin ⅙ y = Sin x Sin ⅙ x.

Fig. 62 B/A

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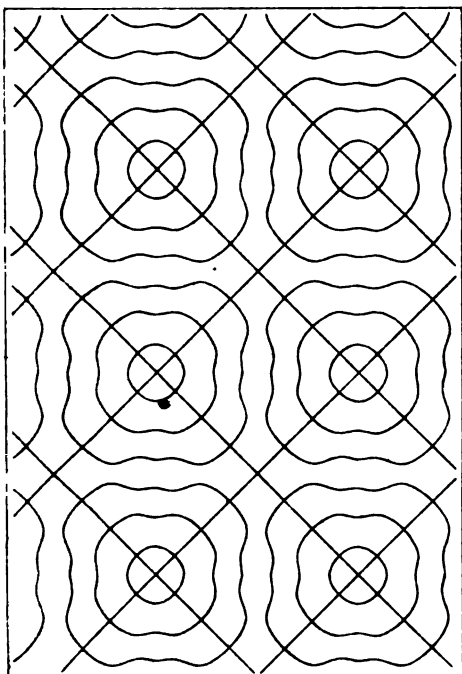
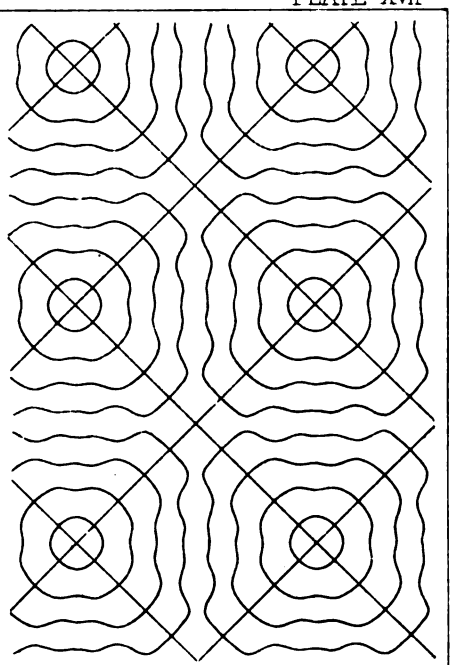
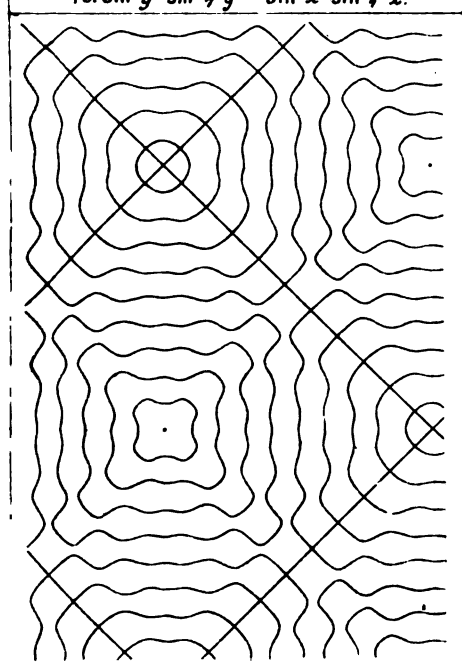


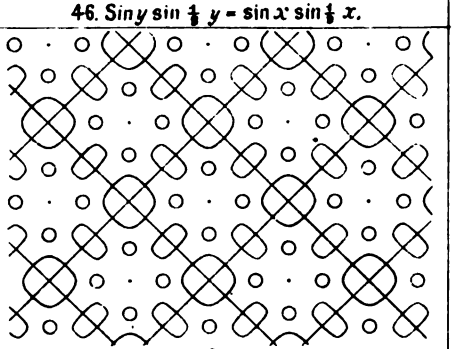
45. Sin y sin $\frac{1}{7}$ y = sin x sin $\frac{1}{7}$ x.



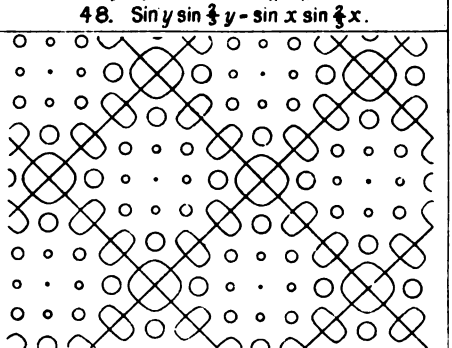
46. Sin y sin $\frac{1}{5}$ y = sin x sin $\frac{1}{5}$ x.



47. Sin y sin $\frac{1}{10}$ y = sin x sin $\frac{1}{10}$ x.



48. Sin y sin $\frac{2}{5}$ y = sin x sin $\frac{2}{5}$ x.



49. Sin y sin $\frac{3}{7}$ y = sin x sin $\frac{3}{7}$ x.

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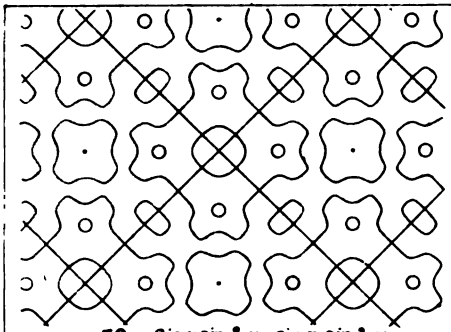
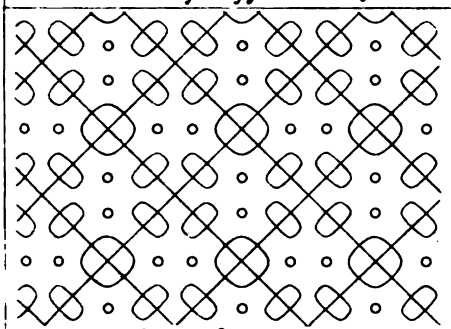
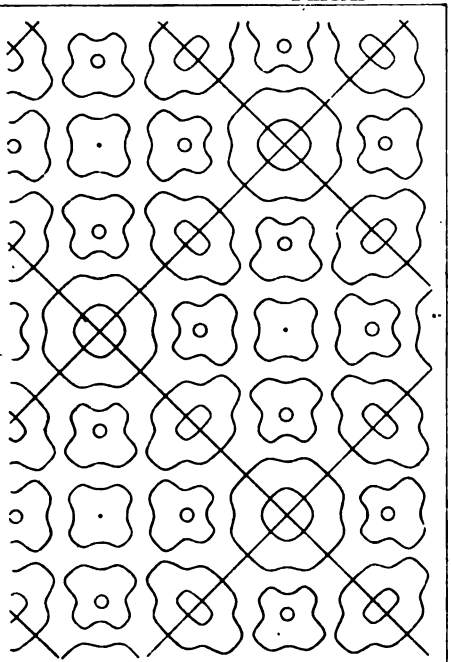


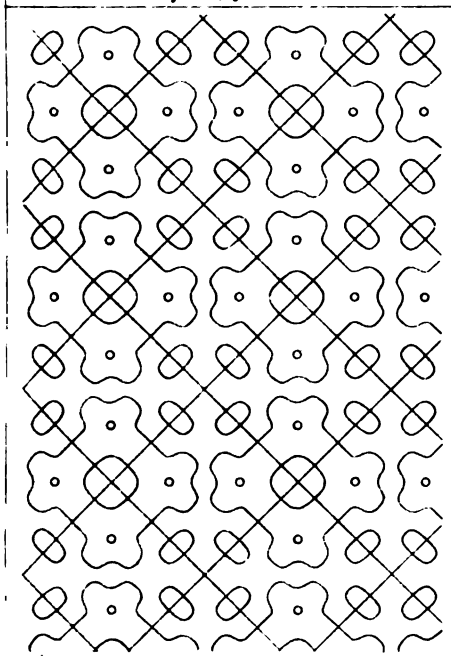
50. $\sin y \sin \frac{2}{3} y = \sin x \sin \frac{2}{3} x$.



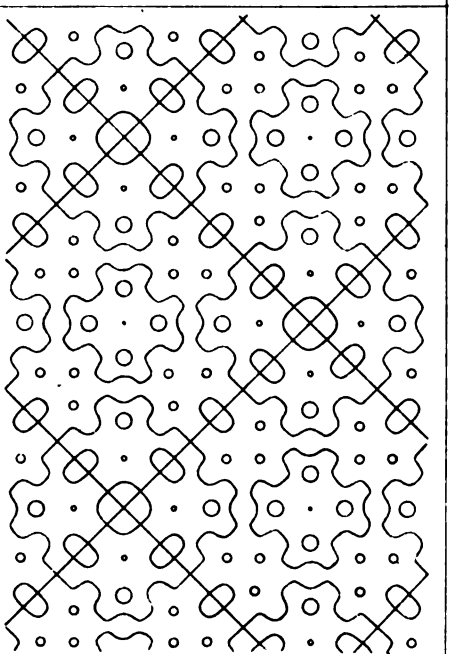
51. $\sin y \sin \frac{3}{5} y = \sin x \sin \frac{3}{5} x$.



52. $\sin y \sin \frac{2}{7} y = \sin x \sin \frac{2}{7} x$.



53. $\sin y \sin \frac{4}{7} y = \sin x \sin \frac{4}{7} x$.



54. $\sin y \sin \frac{4}{7} y = \sin x \sin \frac{4}{7} x$.

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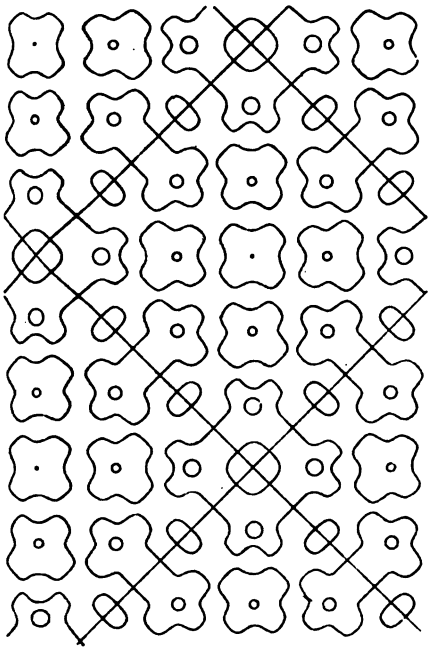
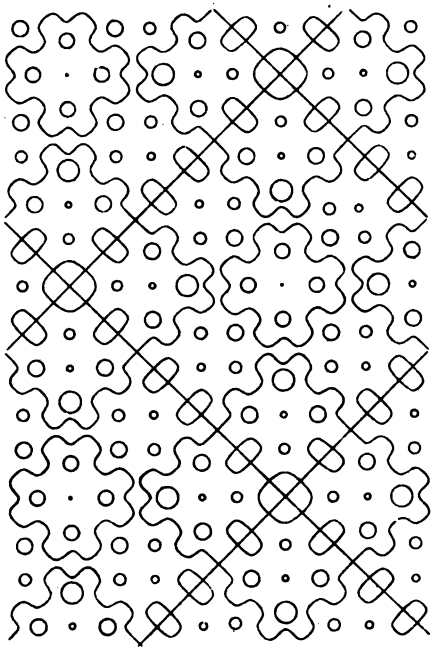
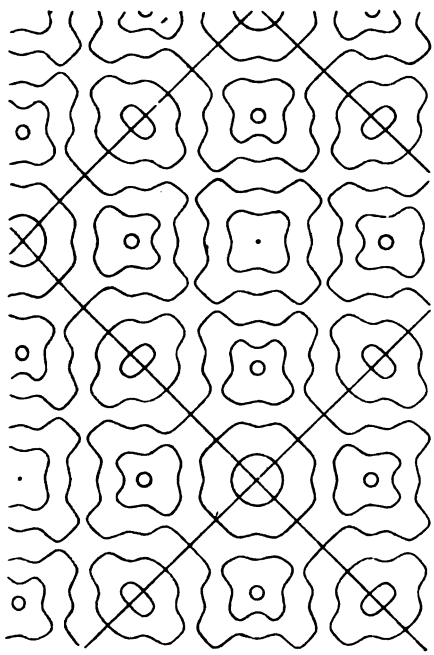


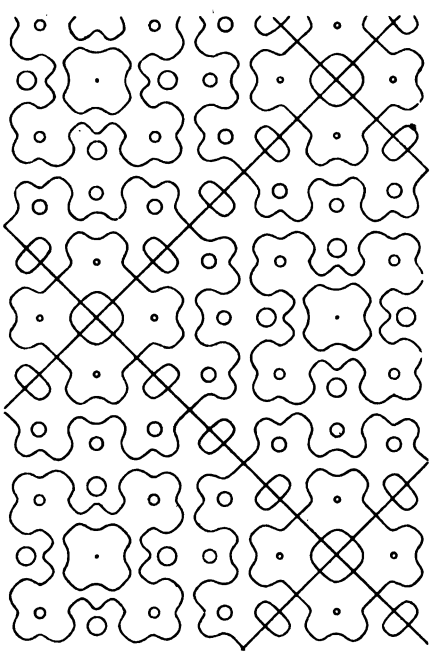
55. $\sin y \sin \frac{3}{8} y = \sin x \sin \frac{3}{8} x$.



56. $\sin y \sin \frac{5}{8} y = \sin x \sin \frac{5}{8} x$.



57. $\sin y \sin \frac{7}{8} y = \sin x \sin \frac{7}{8} x$.



58. $\sin y \sin \frac{4}{8} y = \sin x \sin \frac{4}{8} x$.

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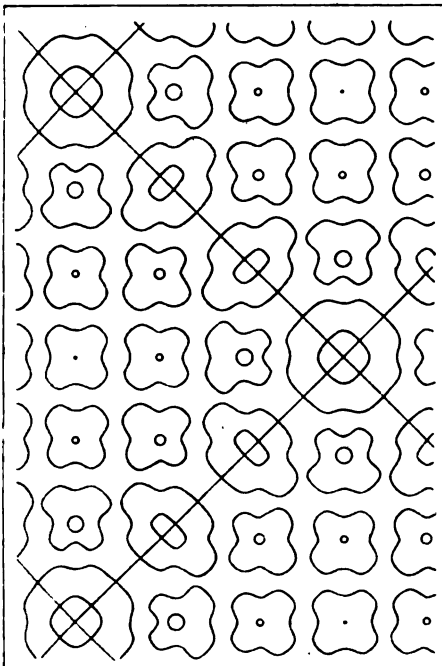
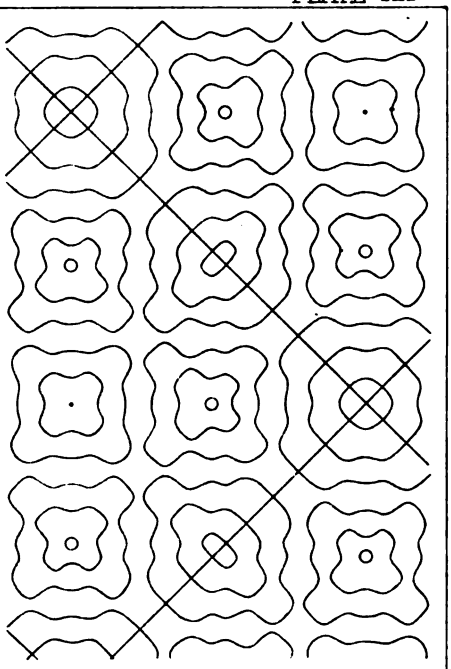
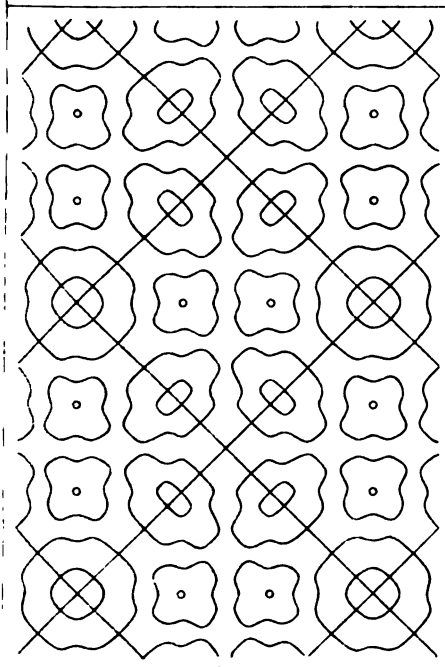


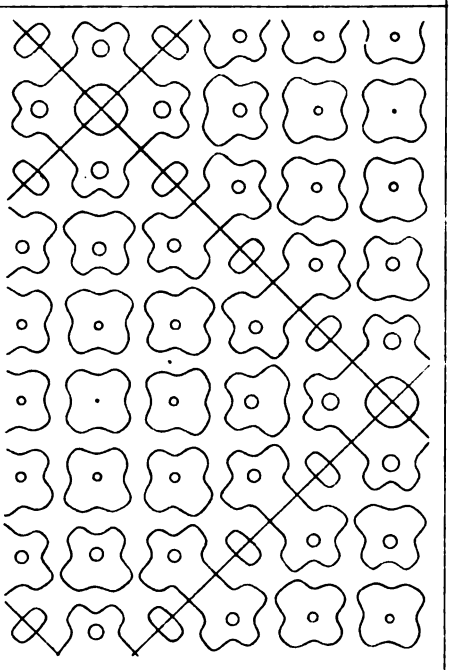
59. $\sin y \sin \frac{3}{10} y - \sin x \sin \frac{3}{10} x$.



60. $\sin y \sin \frac{2}{11} y - \sin x \sin \frac{2}{11} x$.



61. $\sin y \sin \frac{3}{11} y - \sin x \sin \frac{3}{11} x$.



62. $\sin y \sin \frac{4}{11} y - \sin x \sin \frac{4}{11} x$.

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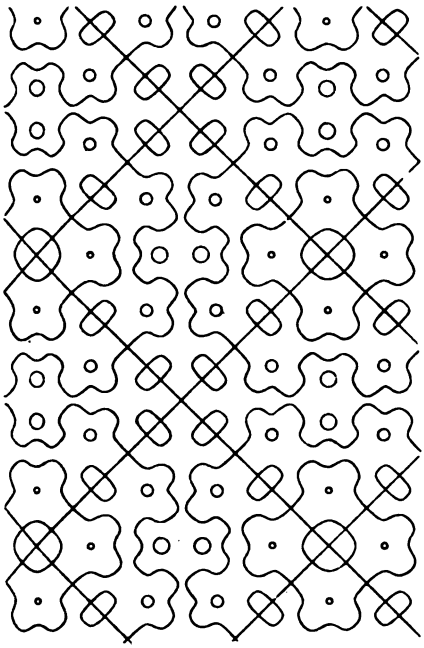
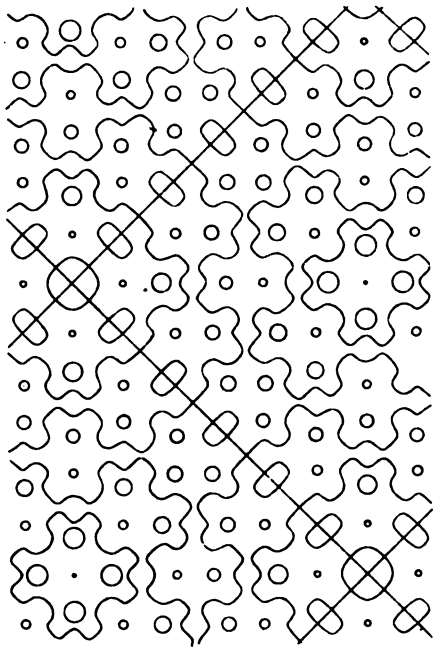
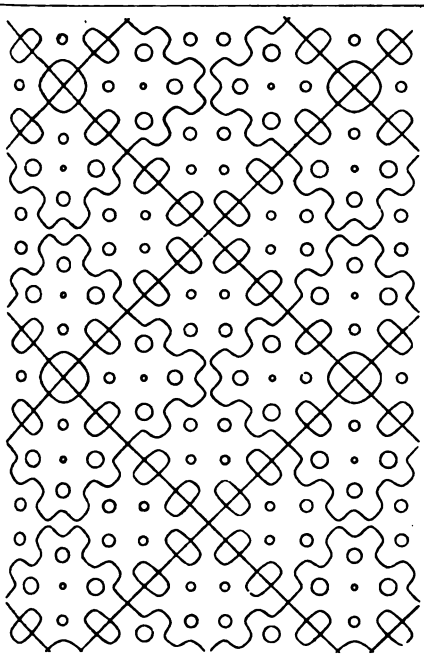


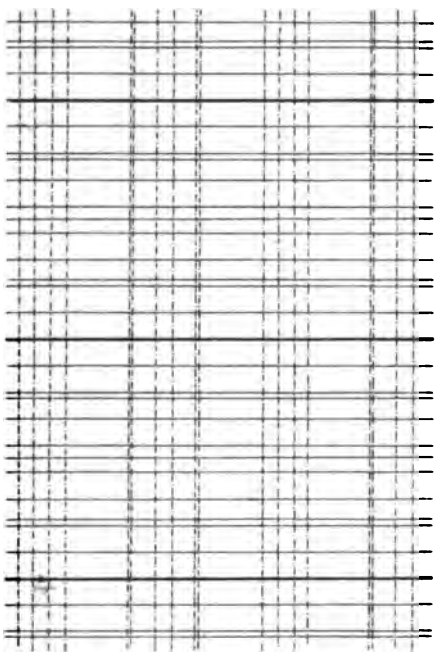
63. $\sin y \sin \frac{5}{11} y = \sin x \sin \frac{5}{11} x.$



64. $\sin y \sin \frac{8}{11} y = \sin x \sin \frac{8}{11} x.$



65. $\sin y \sin \frac{7}{11} y = \sin x \sin \frac{7}{11} x.$



66. $\sin y \sin \frac{4}{5} y (\sin x \sin \frac{5}{8} x - .40) = 0.$

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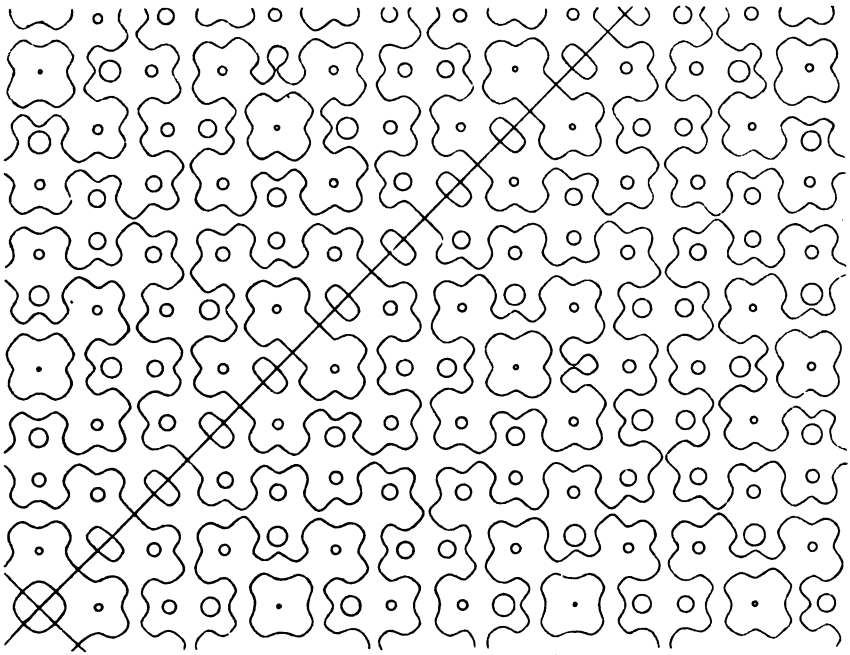
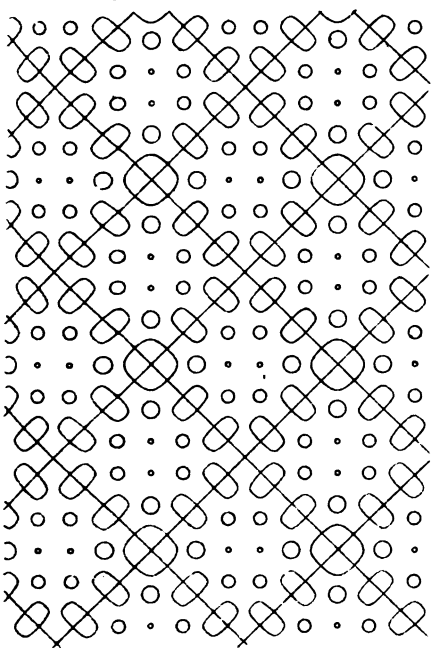
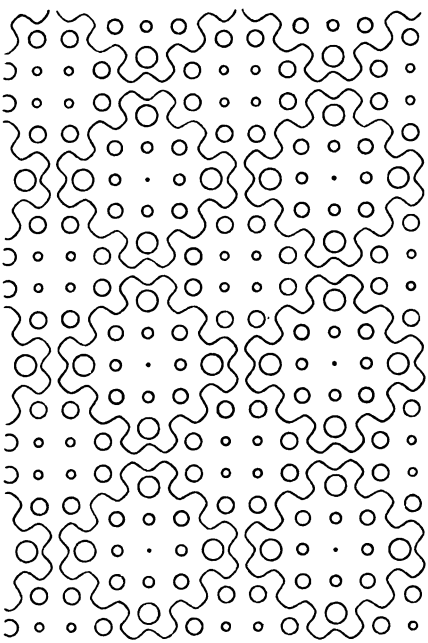


67. Sin $y \sin \sqrt{\tfrac{1}{2}}\, y = \sin x \sin \sqrt{\tfrac{1}{2}}\, x$.



68. Sin $y \sin \tfrac{3}{7} y = \sin x \sin \tfrac{3}{7} x$.



69. Sin $y \sin \tfrac{5}{7} y = -\sin x \sin \tfrac{5}{7} x$.

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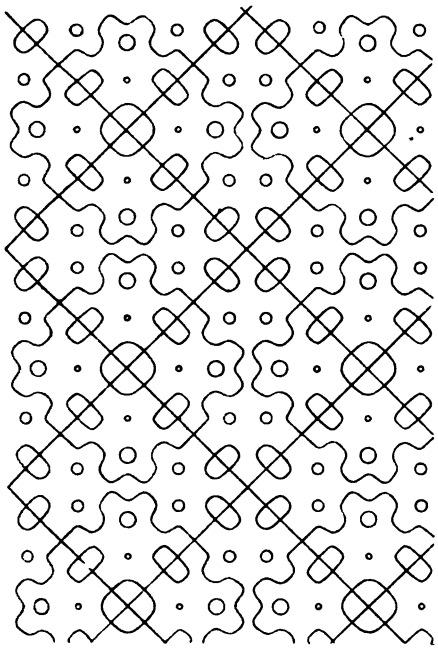
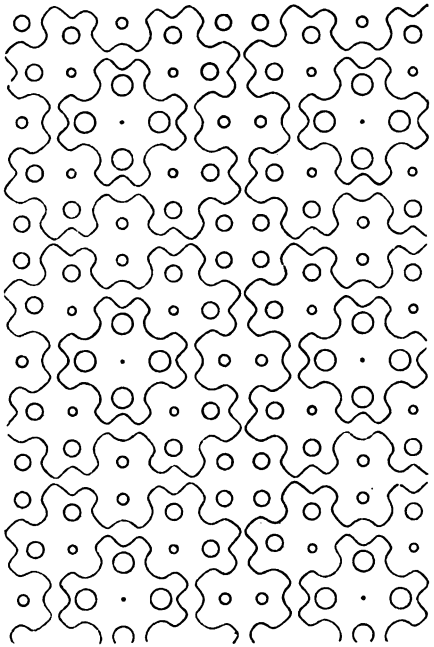
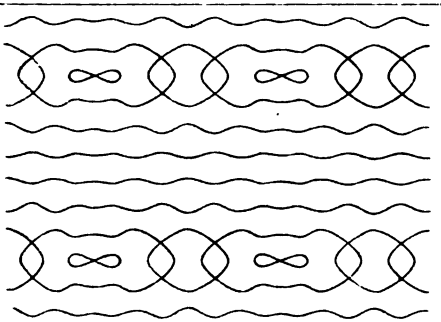


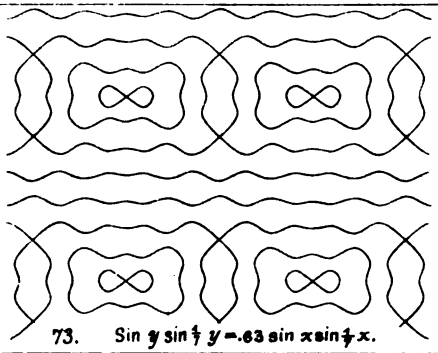
70. $\sin y \sin \frac{5}{8} y = \sin x . \sin \frac{3}{8} x.$



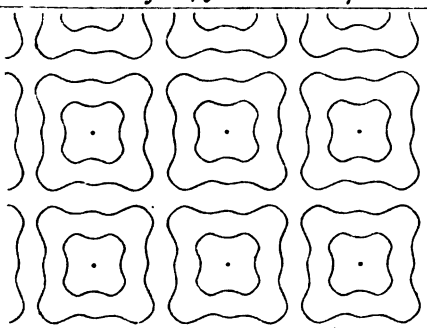
71. $\sin y \sin \frac{5}{8} y = -\sin x \sin \frac{3}{8} x.$



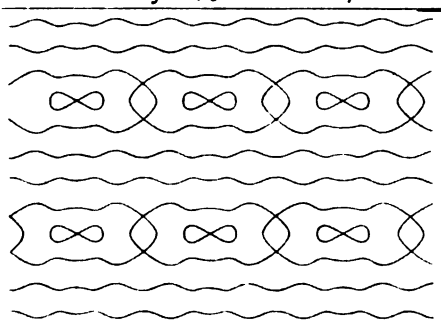
72. $\sin y \sin \frac{1}{7} y = .28 \sin x \sin \frac{1}{7} x.$



73. $\sin y \sin \frac{1}{7} y = .63 \sin x \sin \frac{1}{7} x.$



74. $\sin y \sin \frac{1}{5} y = -\sin x \sin \frac{1}{5} x.$



75. $\sin y \sin \frac{1}{5} y = .35 \sin x \sin \frac{1}{5} x.$

Plate 17th

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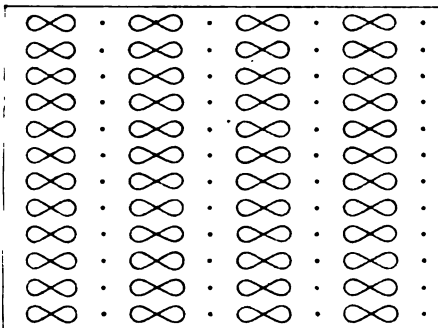
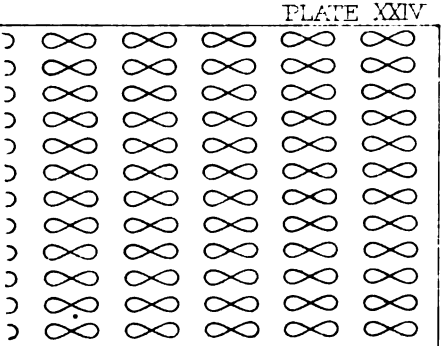
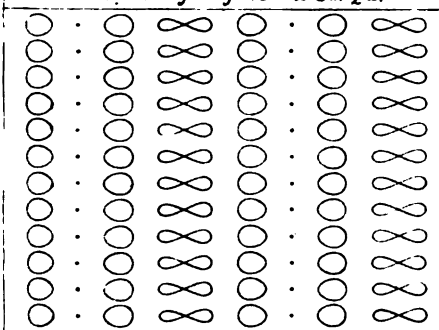


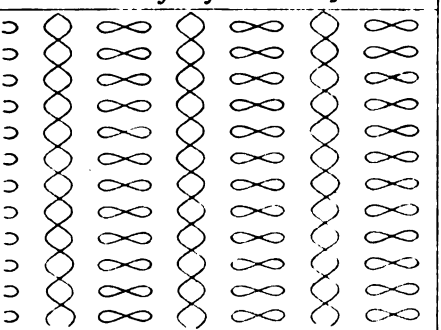
76. Sin y sin y = sin x sin $\frac{1}{2}x$.



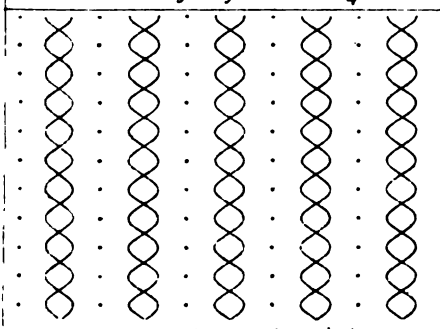
77. Sin y sin y = sin x sin $\frac{1}{3}x$.



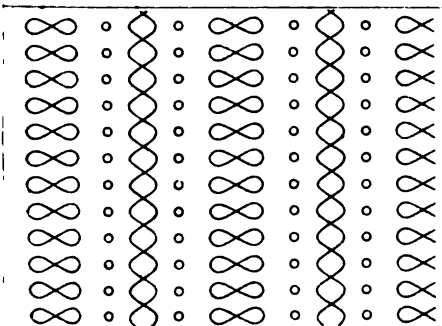
78. Sin y sin y = sin x sin $\frac{1}{4}x$.



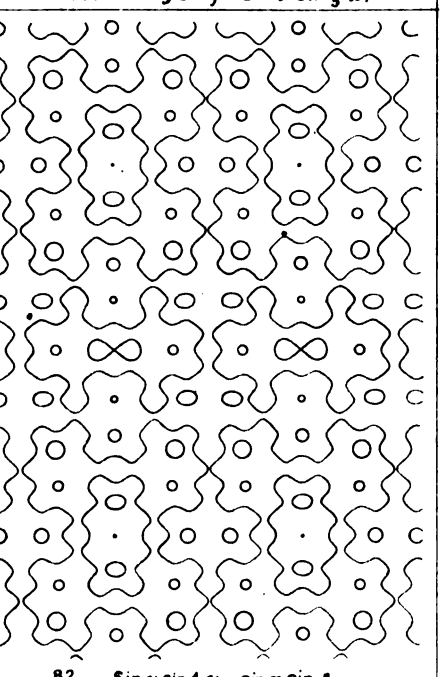
79. Sin y sin y = sin x sin $\frac{1}{5}x$.



80. Sin y sin y = −sin x sin $\frac{1}{8}x$.



81. Sin y sin y = sin x sin $\frac{3}{7}x$.



82. Sin y sin $\frac{4}{7}y$ = sin x sin $\frac{3}{7}x$.

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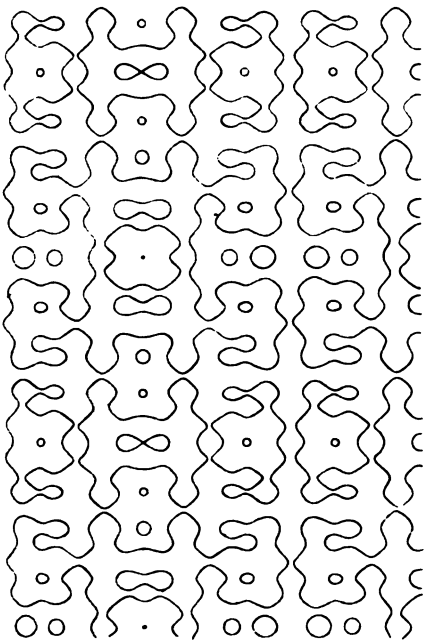
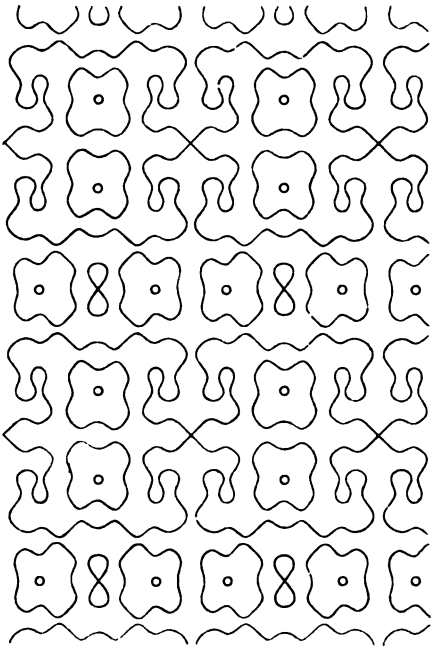
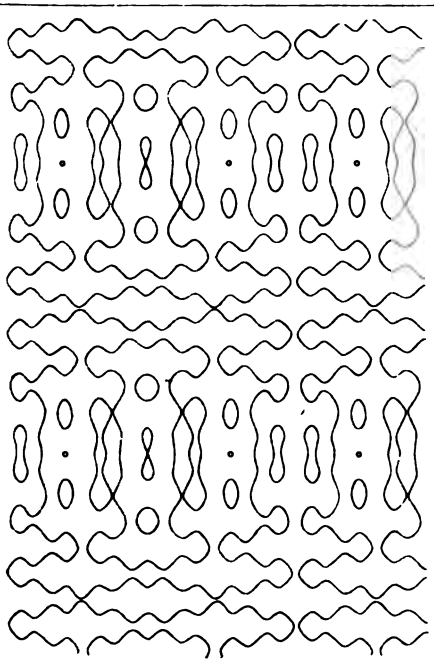


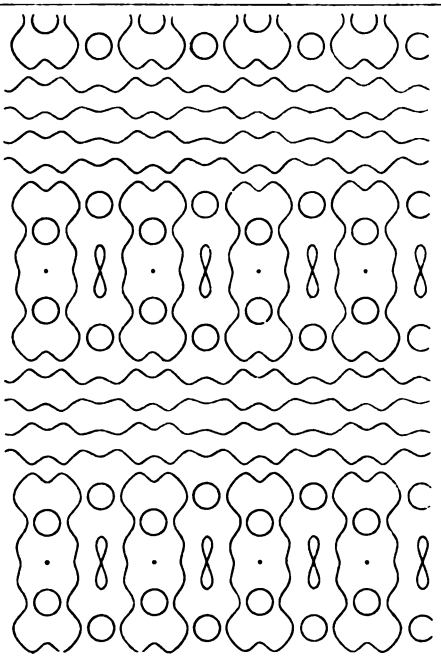
83. $\sin y \sin \frac{4}{7} y = \sin x \sin \frac{3}{11} x.$



84. $\sin y \sin \frac{3}{11} y = \sin x \sin \frac{3}{7} x.$



85. $\sin y \sin \frac{4}{11} y = \sin x \sin \frac{7}{11} x.$



86. $\sin y \sin \frac{1}{11} y = \sin x \sin \frac{1}{2} x.$

Photo 19th

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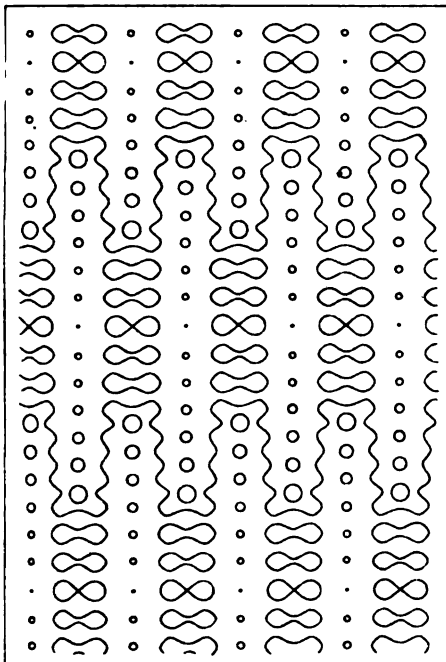
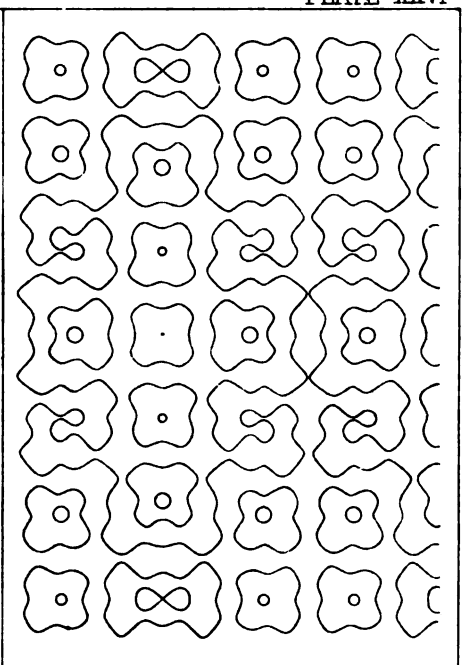
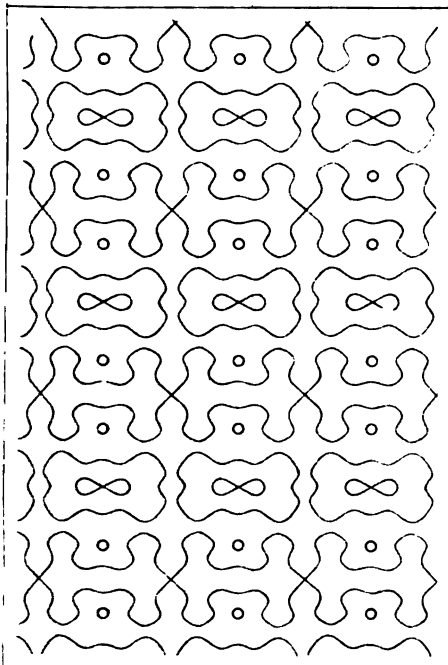


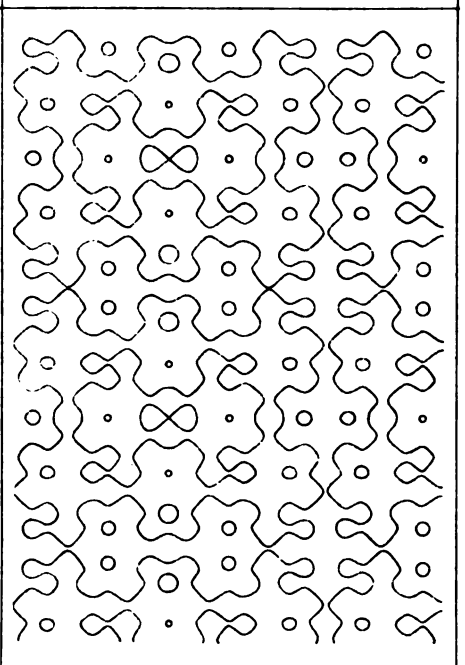
87. $\sin y \sin \frac{3}{10} y = \sin x \sin \frac{1}{2} x.$



88. $\sin y \sin \frac{3}{10} y = \sin x \sin \frac{2}{11} x.$



89. $\sin y \sin \frac{3}{7} y = \sin x \sin \frac{1}{3} x.$



90. $\sin y \sin \frac{3}{8} y = \sin x \sin \frac{3}{11} x.$

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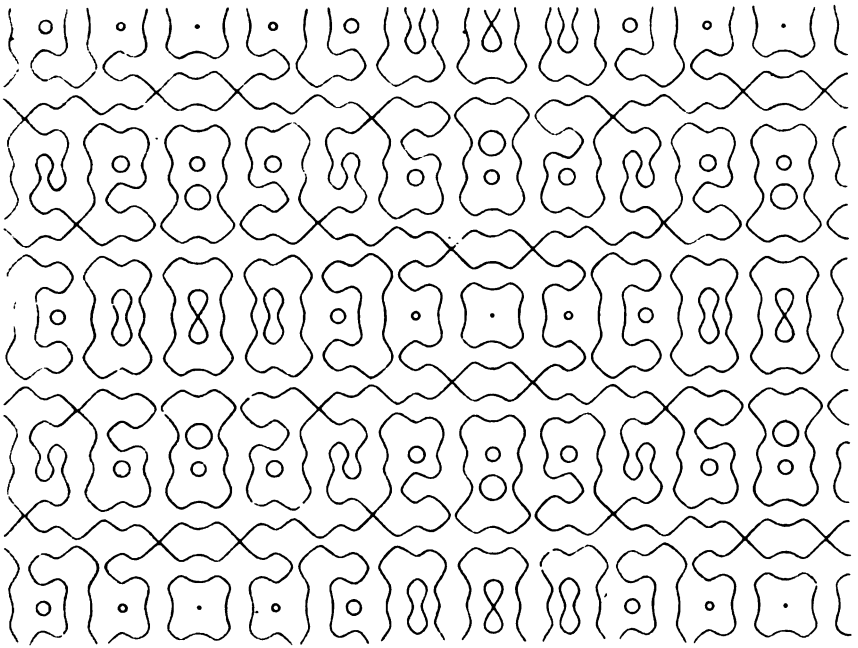
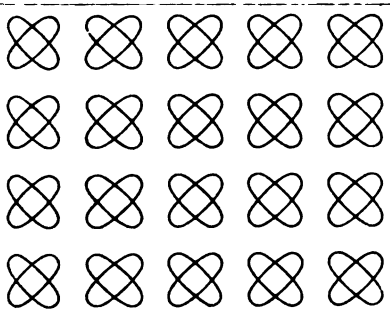
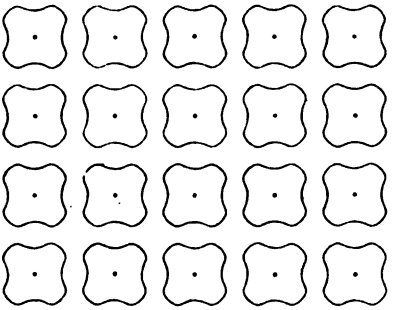


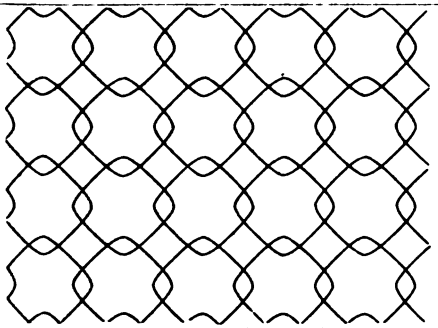
91. $\sin y \sin \frac{3}{11} y = \sin x \sin \frac{4}{11} x$.



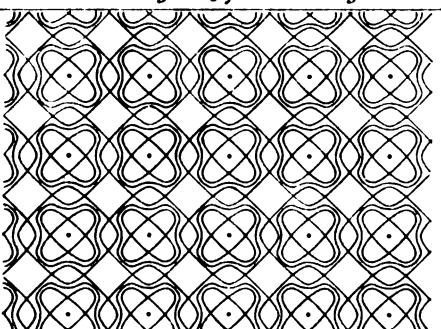
92. $\sin y \sin \frac{1}{3} y = -\sin x \sin \frac{1}{3} x + .56$.



93. $\sin y \sin \frac{1}{3} y = -\sin x \sin \frac{1}{3} x$.



94. $\sin y \sin \frac{1}{3} y = -\sin x \sin \frac{1}{3} x - .44$.



95. $\sin y \sin \frac{1}{3} y = -\sin x \sin \frac{1}{3} x + k$.

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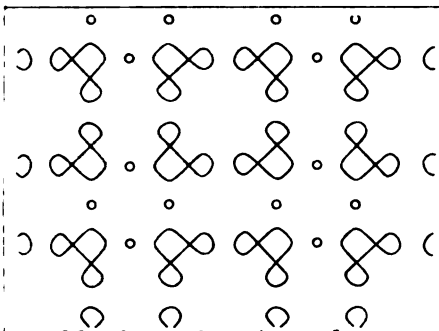
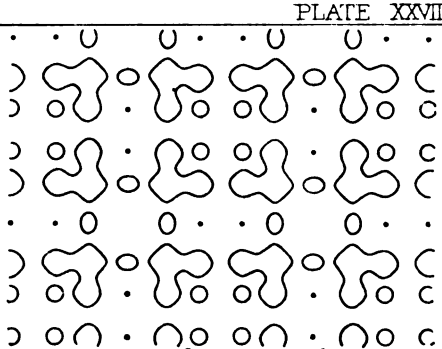
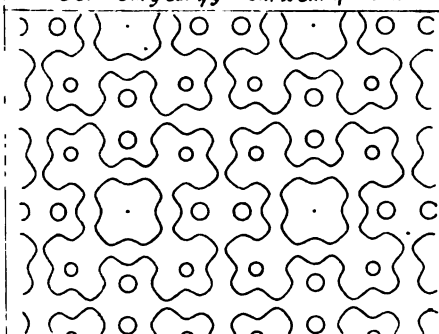


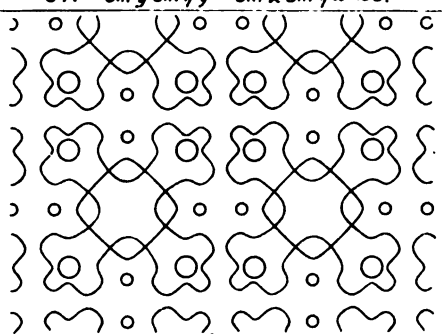
96. $\sin y \sin \frac{3}{7} y = -\sin x \sin \frac{3}{7} x - .80$



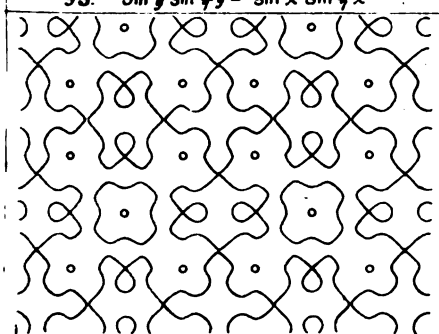
97. $\sin y \sin \frac{3}{7} y = -\sin x \sin \frac{3}{7} x - .38$



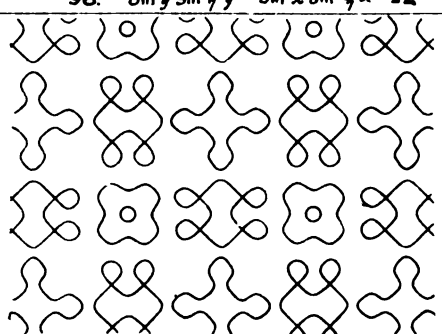
99. $\sin y \sin \frac{3}{7} y = -\sin x \sin \frac{3}{7} x$



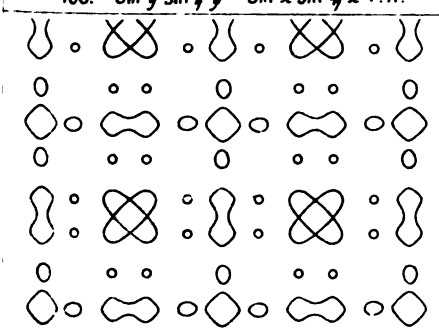
98. $\sin y \sin \frac{3}{7} y = -\sin x \sin \frac{3}{7} x - .22$



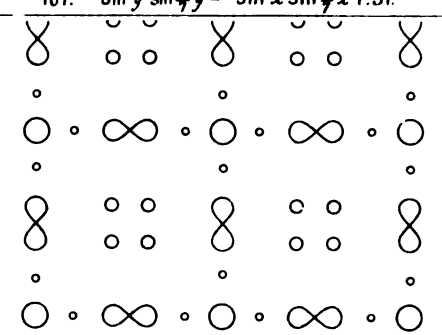
100. $\sin y \sin \frac{3}{7} y = -\sin x \sin \frac{3}{7} x + .11$



101. $\sin y \sin \frac{3}{7} y = -\sin x \sin \frac{3}{7} x + .31$



102. $\sin y \sin \frac{3}{7} y = -\sin x \sin \frac{3}{7} x + .69$



103. $\sin y \sin \frac{3}{7} y = -\sin x \sin \frac{3}{7} x + 1.00$

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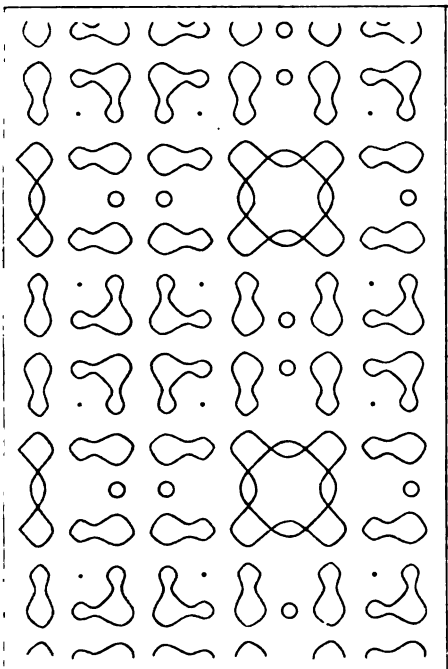
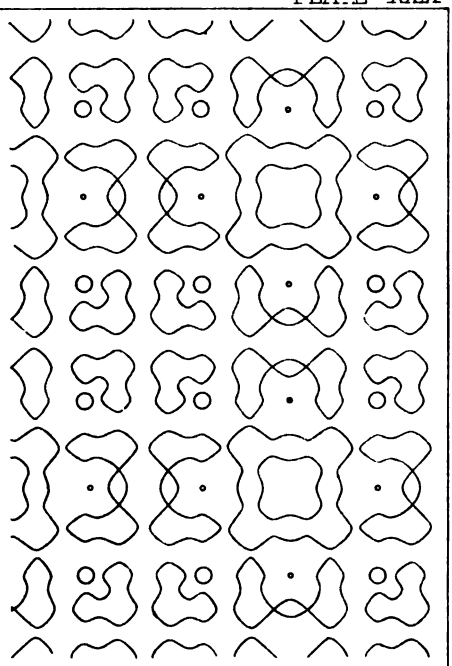
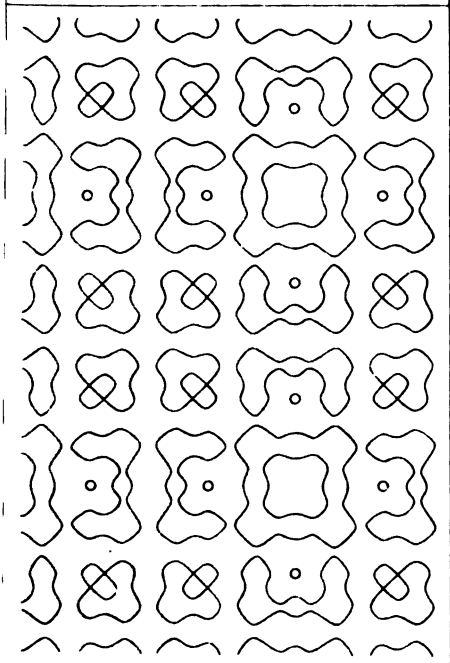


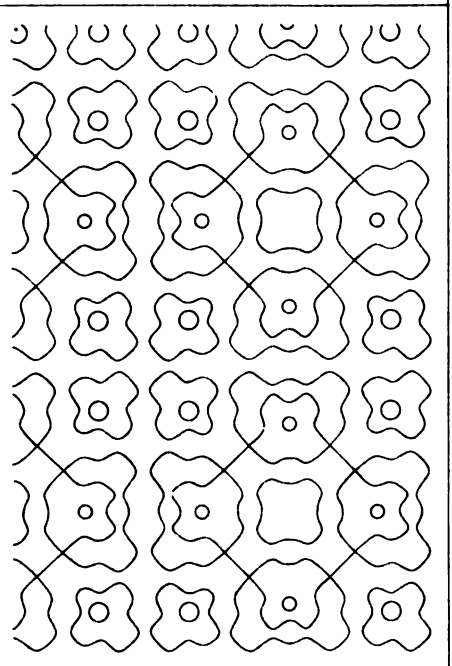
104. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x - .19.$



105. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x - .21.$



106. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x - .18.$



107. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x - .11.$

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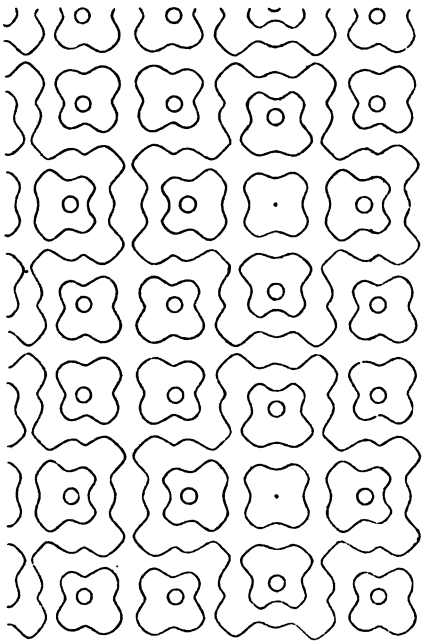
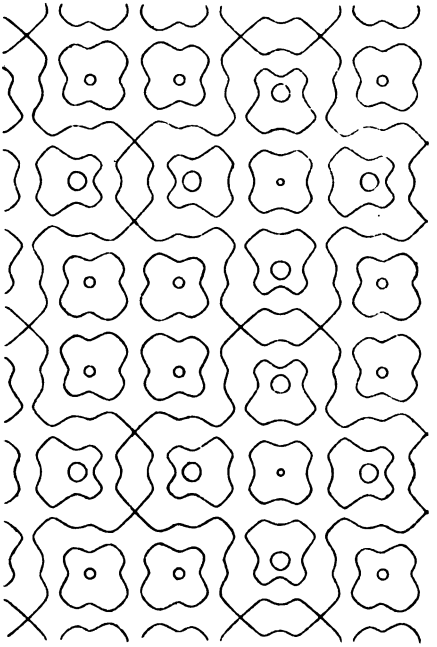
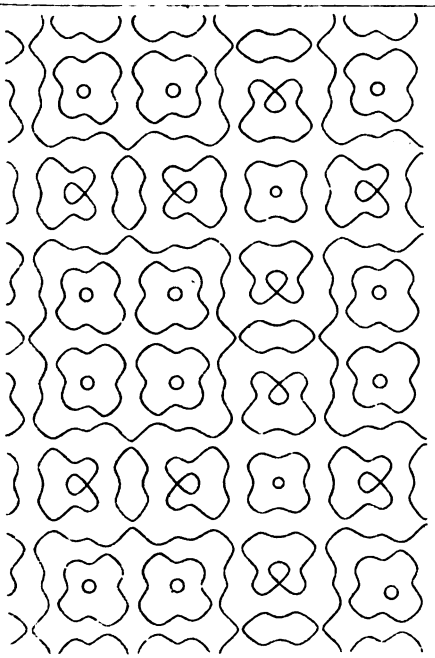


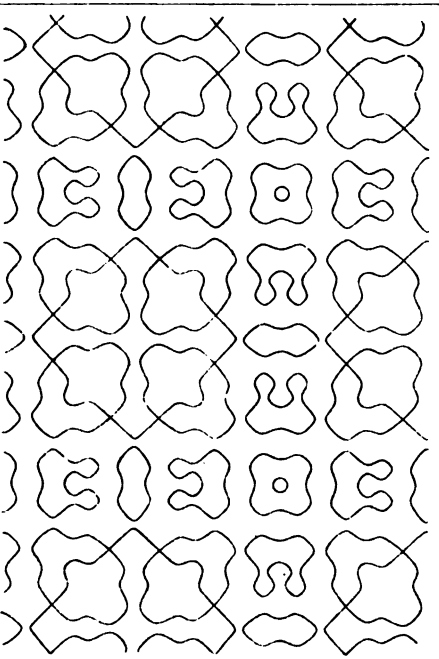
108. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x.$



109. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x + .04.$



110. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x + .07.$



111. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x + .17.$

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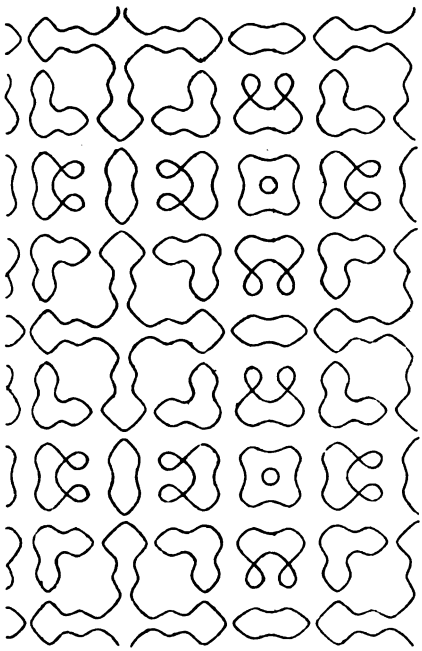
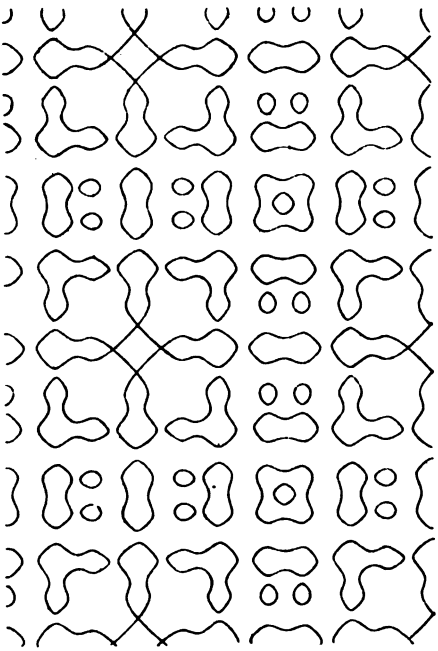
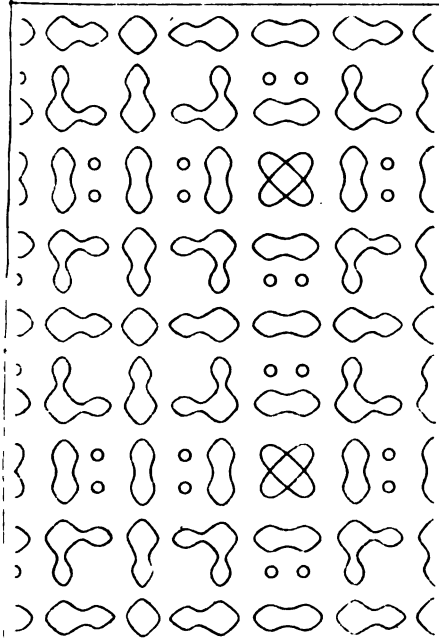


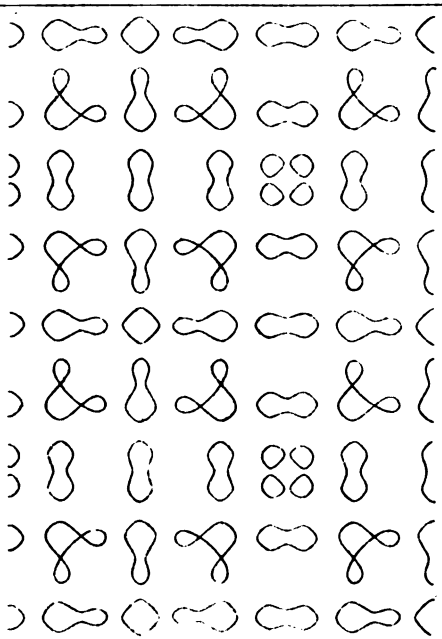
112. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x + .22.$



113. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x + .32.$



114. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x + .47.$



115. $\sin y \sin \frac{3}{11} y = -\sin x \sin \frac{3}{11} x + .60.$

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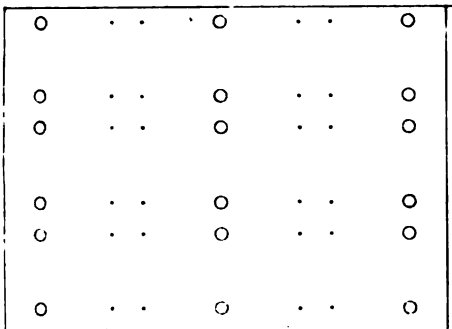
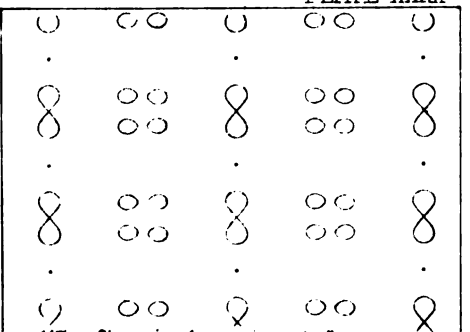
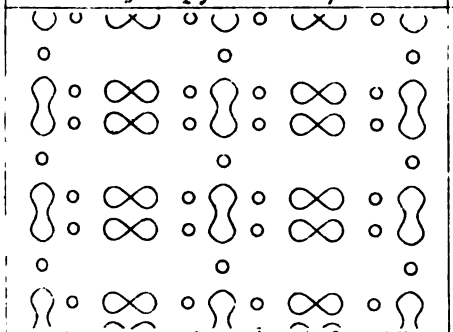


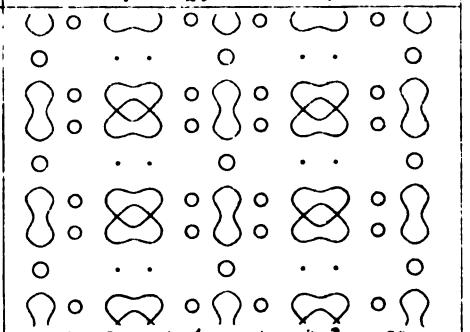
116. $\sin y \sin \frac{1}{2} y = \sin x \sin \frac{3}{7} x - 1.46.$



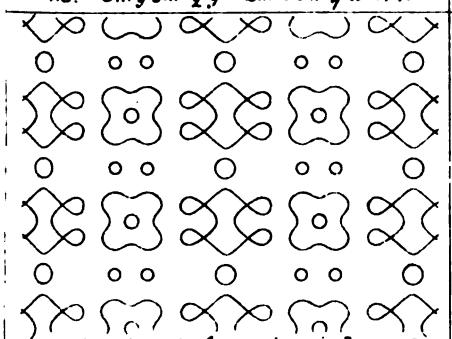
117. $\sin y \sin \frac{1}{2} y = \sin x \sin \frac{3}{7} x - 1.00.$



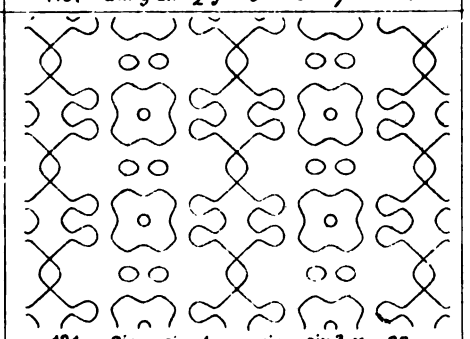
118. $\sin y \sin \frac{1}{2} y = \sin x \sin \frac{3}{7} x - .77.$



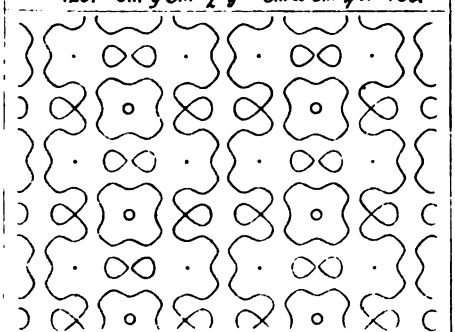
119. $\sin y \sin \frac{1}{2} y = \sin x \sin \frac{3}{7} x - .69.$



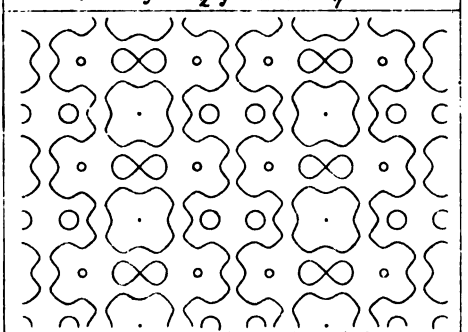
120. $\sin y \sin \frac{1}{2} y = \sin x \sin \frac{3}{7} x - .39.$



121. $\sin y \sin \frac{1}{2} y = \sin x \sin \frac{3}{7} x - .23.$



122. $\sin y \sin \frac{1}{2} y = \sin x \sin \frac{3}{7} x - .11.$



123. $\sin y \sin \frac{1}{2} y = \sin x \sin \frac{3}{7} x.$

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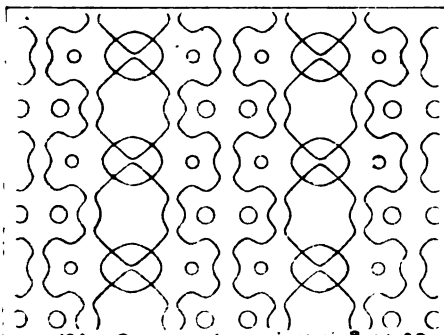
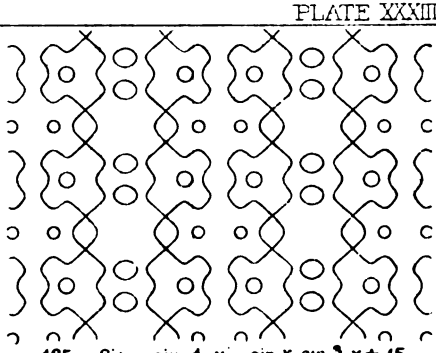
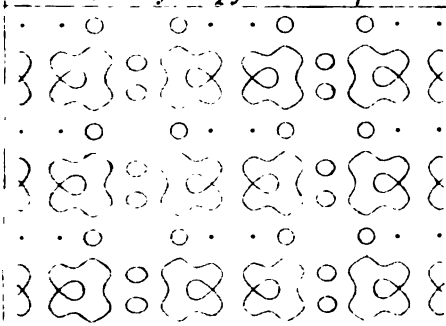


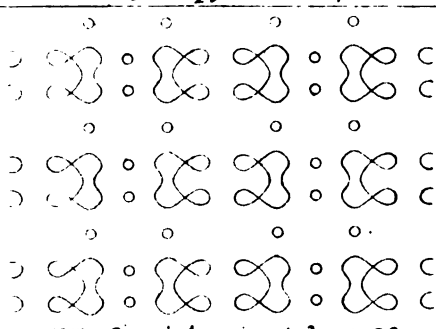
124. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{3}{7}x$ + .08.



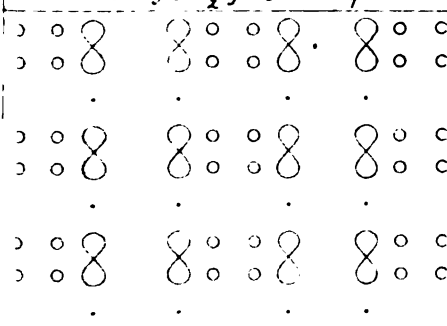
125. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{3}{7}x$ + .15.



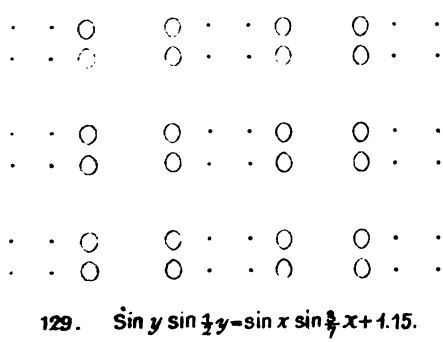
126. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{3}{7}x$ + .38.



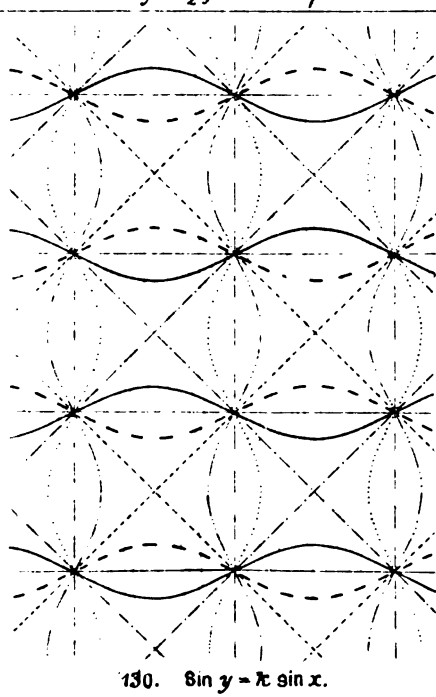
127. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{3}{7}x$ + .66.



128. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{3}{7}x$ + .92.



129. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{3}{7}x$ + 1.15.



130. Sin y = π sin x.

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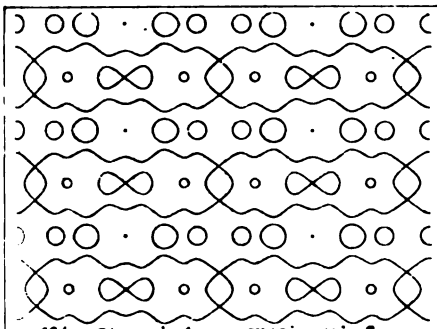
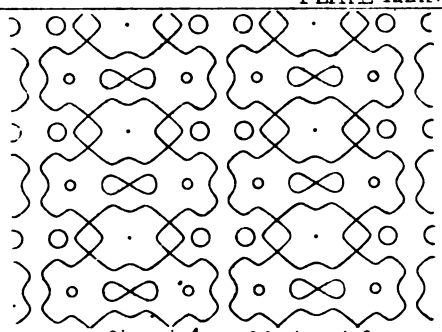
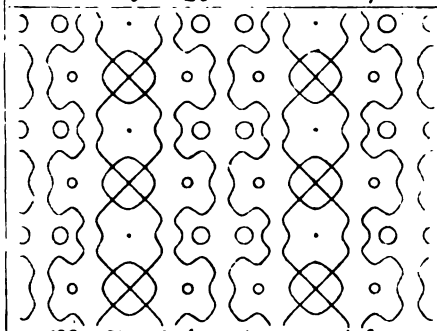


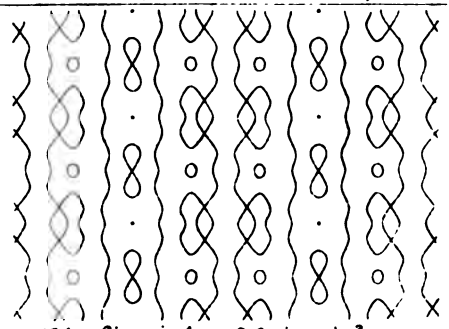
131. $\sin y \sin \frac{1}{2} y = .77 \sin x \sin \frac{3}{7} x.$



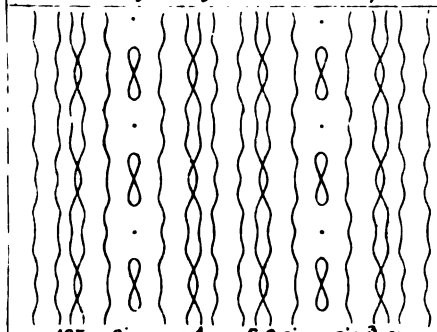
132. $\sin y \sin \frac{1}{2} y = .84 \sin x \sin \frac{3}{7} x.$



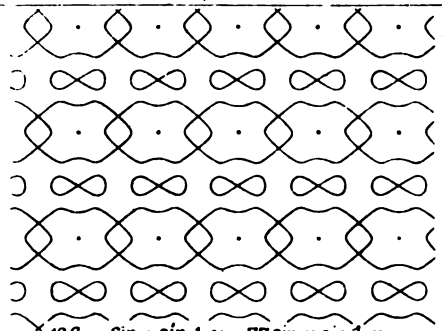
133. $\sin y \sin \frac{1}{2} y = 1.11 \sin x \sin \frac{3}{7} x.$



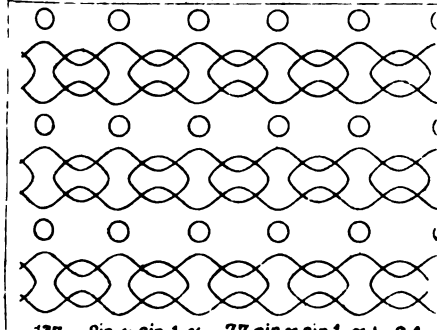
134. $\sin y \sin \frac{1}{2} y = 2.0 \sin x \sin \frac{3}{7} x.$



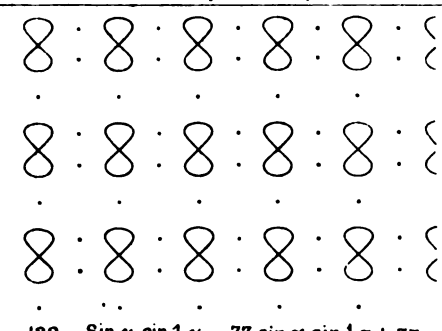
135. $\sin y \sin \frac{1}{2} y = 6.9 \sin x \sin \frac{3}{7} x.$



136. $\sin y \sin \frac{1}{2} y = .77 \sin x \sin \frac{1}{3} x.$



137. $\sin y \sin \frac{1}{2} y = .77 \sin x \sin \frac{1}{3} x + .34.$



138. $\sin y \sin \frac{1}{2} y = .77 \sin x \sin \frac{1}{3} x + .77.$

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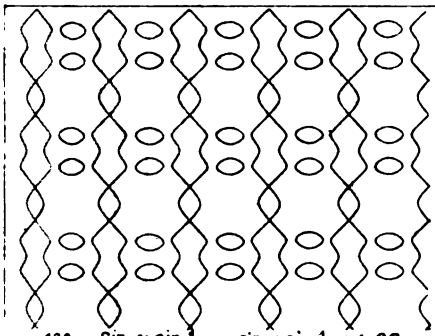
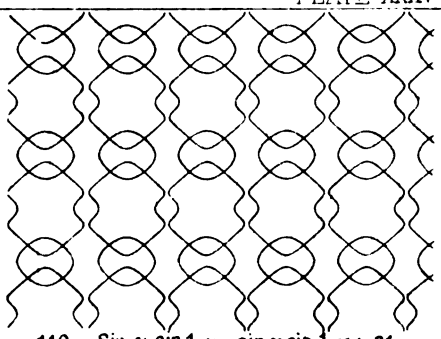
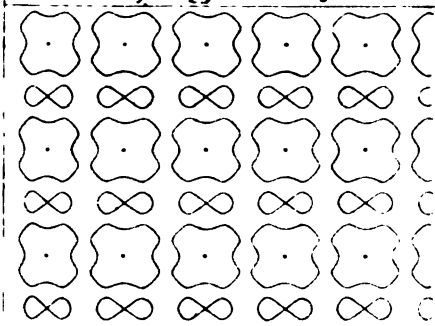


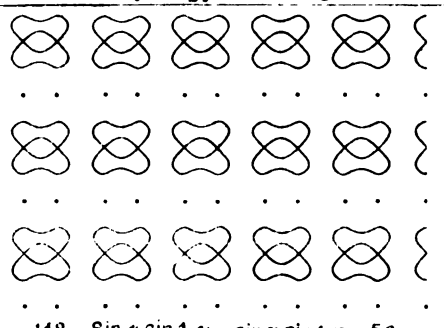
139. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{1}{3}x$ + .23.



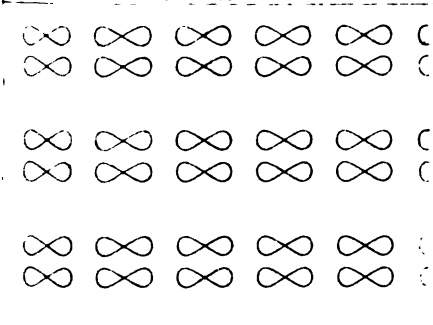
140. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{1}{3}x$ + .21.



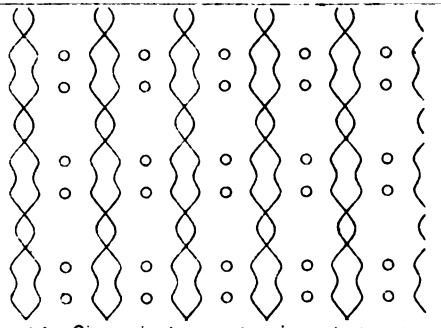
141. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{1}{3}x$.



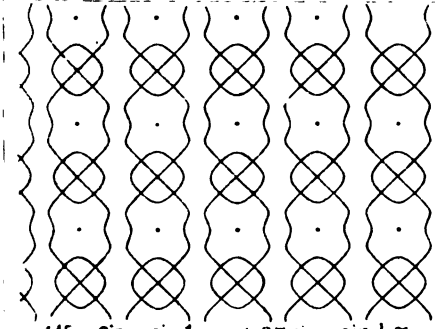
142. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{1}{3}x$ − .56.



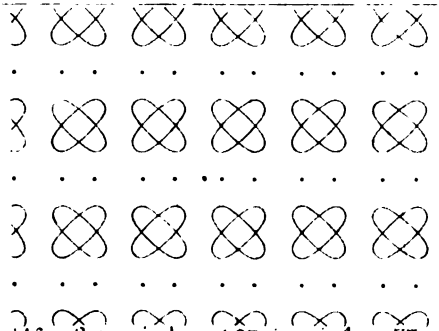
143. Sin y sin $\frac{1}{2}y$ = sin x sin $\frac{1}{3}x$ − .77.



144. Sin y sin $\frac{1}{2}y$ = 1.37 sin x sin $\frac{1}{3}x$ + .60.



145. Sin y sin $\frac{1}{2}y$ = 1.37 sin x sin $\frac{1}{3}x$.



146. Sin y sin $\frac{1}{2}y$ = 1.37 sin x sin $\frac{1}{3}x$ − .77.

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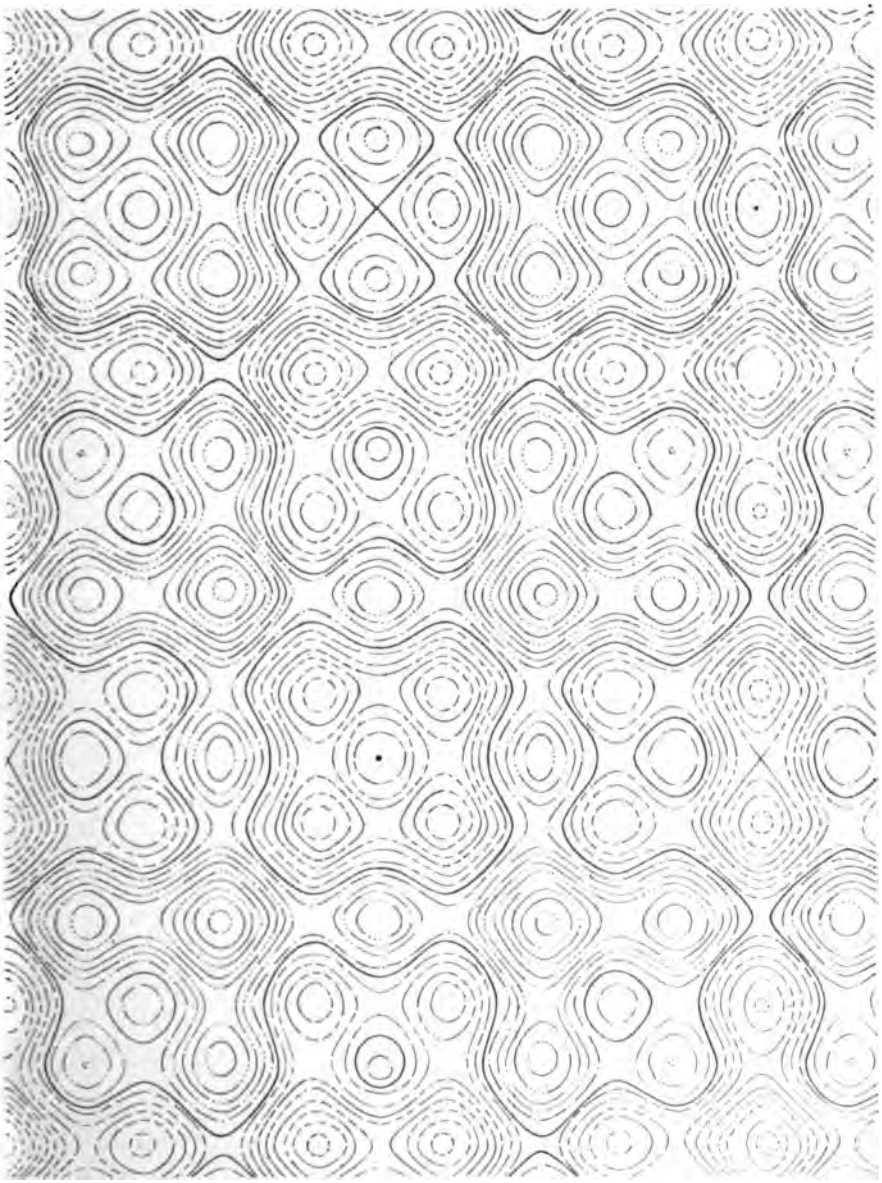

147. $\sin y \sin \frac{8}{5} y = \sin x \sin \frac{3}{7} x + k.$

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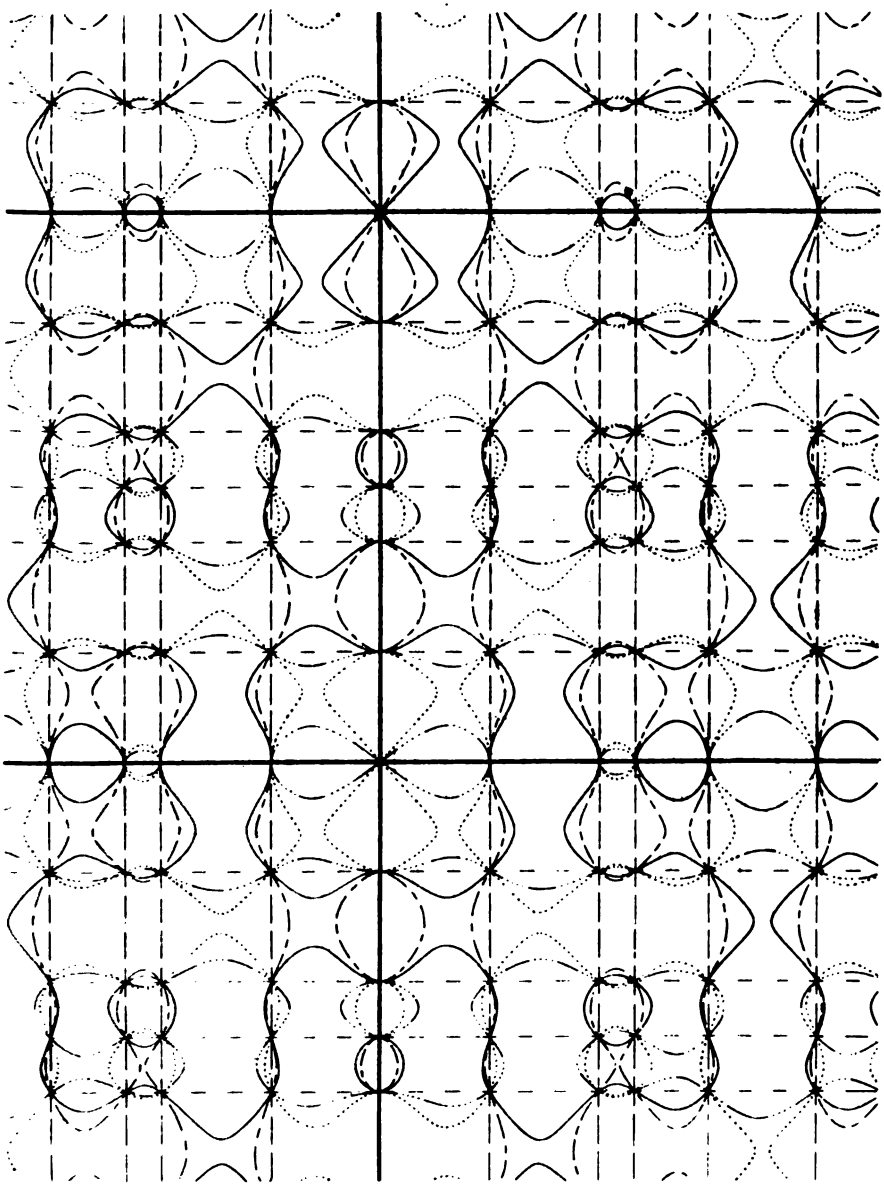

148. $\sin y \sin \frac{2}{3} y = k \sin x \sin \frac{3}{7} x$.

Fig. 148.

Bender and Orrand New Haven Ct.

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TRANSACTIONS

OF THE

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OF

ARTS AND SCIENCES.

VOLUME III, PART 2.

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TRANSACTIONS

OF THE

CONNECTICUT ACADEMY OF ARTS AND SCIENCES,

NEW HAVEN, CONN.

Vol. I, Part 1, 1866. 246 pp. and 3 plates. Price $2.50.

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Extracts from an auroral register kept at New Haven, Conn., by Mr. FRANCIS BRADLEY. pp. 139–154.

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III.—HADLEY, J. On Bekker's digammated text of Homer. pp. 173–193.

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Vol. I, Part 2, 1867–71. 367 pp. and 7 plates. Price $3.50.

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No. 8.—Additional observations on Echinoderms, chiefly from the Pacific coast of America. pp. 568–593.

No. 9.—On the Echinoderm-fauna of the Gulf of California and Cape St. Lucas. pp. 593–596.

Vol. II, Part 1, 1870. 208 pp. and 7 plates. Price $3.00.

Contents.—I.—SMITH, S. I. Notice of the Crustacea collected by Prof. C. F. Hartt on the coast of Brazil in 1867. pp. 1–31, plate 1.

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1878. Jan. 8.

VI. The Hydroids of the Pacific Coast of the United States, south of Vancouver Island. With a Report upon those in the Museum of Yale College. By S. F. Clark.

Read Jan. 19. 1876.

The Museum is indebted for its collection of Californian Hydroids chiefly to Prof. D. C. Eaton, who has presented during the last two or three years, a large number of specimens, that were received by him with dried algæ from that coast. They were collected and sent to him by Dr. C. W. Anderson, Santa Cruz, Cal.; Dr. L. N. Dimmick, Santa Barbara, Cal.; Mrs. Ellwood Cooper, Santa Barbara, Cal.; and Miss Mitchell of Vancouver Island. All the specimens received from these sources were collected in tide-pools along the shore or attached to algæ, washed in from deeper water. A few alcoholic specimens have also been received from San Diego, Cal., collected on the piles of the wharves and along the shore, by Dr. E. Palmer, and a fine specimen of *Plumularia setacea* was dredged in six to eight fathoms, off San Diego, by Mr. Henry Hemphill.

Some of the species, including most of the *Sertularidæ*, do not seem to be injured by being dried, but others, as the *Campanularidæ*, are usually rendered useless for description. The specimens of the two species of *Campanularia* described below are unusually well preserved, both hydrothecæ and gonothecæ being in good condition.

There has been very little published on the hydroids of the western coast of North America, up to the present time. In 1857 Dr. Trask* described and figured nine new species of Zoöphytes from the Bay of San Francisco and adjacent localities. Five of these are Bryozoa; the remaining four represent three genera of the family Sertularidæ, as follows: *Sertularia anguina*, *S. furcata*, *Sertularella turgida* and *Hydrallmania Franciscana*, all of which, so far as I am aware, are peculiar to that coast; unless indeed the last named species prove to be identical with *H. falcata* of Europe,

* Proceedings of the California Academy of Natural Sciences, vol. i, March, 1857. Dr. J. B. Trask.

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Africa and New England. In 1860 Andrew Murray* described and figured five species from the Californian coast, of which three are new, and the other two, *Sertularia labrata* and *Plumularia gracilis*, are synonymous with two of Trask's species, viz: *Sertularia anguina* and *Plumularia Franciscana.* Mr. Alexander Agassiz† in 1865 described seven species and recorded seven others from the Bay of San Francisco; and he had three of the same from the Gulf of Georgia, W. T. Five species were also mentioned by him from the North Pacific. Two of these five northern species, *Bougainvillia Mertensii* Ag. and *Cotulina Greenei* A. Ag., are also found at San Francisco. The latter species having also been collected at Santa Barbara, Cal., has the wide range of nearly three thousand miles upon our western coast. Professor Allman mentions having found sixteen species in a collection from the Californian coast, submitted to him for examination; two of them, *Lafoëa dumosa* and *Sertularia pumila*, are common on the European and New England coasts, and the former species is also recorded from South Africa. The collection in the Museum of Yale College contains twelve species and one variety. Of these four are new; nine are recorded only from the Pacific coast of North America, as yet; and three, *Halecium tenellum*, *Sertularia argentea* and *Plumularia setacea*, are also common on the European shores; the first two of these have also been found on the New England coast, from Maine to Long Island Sound. The most common form on the Californian coast is the showy *Aglaophenia struthionides*, which is apparently as abundant there as *Sertularia argentea* and *S. pumila* are upon our eastern shores, for it forms the bulk of every package sent to us from the western coast. The following table gives a list of all the Hydroids known on the western coast of the United States, from Vancouver's Island to San Diego, with the range of the different species and the names of some of the collectors.

List of Hydroida known to occur between San Diego and Vancouver Island.

Coryne rosaria *A. Ag.*	Bay of San Francisco, Cal. (A. Agassiz).
Tubularia elegans *Clark.*	San Diego, Cal. (Dr. E. Palmer).
Thamnocnidia tubularoides *A. Ag.*	Bay of San Francisco, Cal. (A. Agassiz).

* The Annals and Magazine of Natural History, Series 3, No. XXVIII, April, 1860. Descriptions of new species of Hydroids from the Californian Coast. By Andrew Murray.

† Illustrated Catalogue of the Museum of Comparative Zoölogy. No. II. North American Acalephæ. By Alexander Agassiz. 1865.

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Parypha microcephala *A. Ag.*	Bay of San Francisco, Cal. (A. Agassiz).
Bimeria gracilis *Clark.*	San Diego, Cal. (E. Palmer).
Bougainvillia Mertensii *Agassiz.*	Bay of San Francisco, Cal. (A. Agassiz).
Eudendrium, sp.	Santa Cruz, Cal. (C. W. Anderson).
Campanularia everta *Clark.*	San Diego, Cal. (H. Hemphill), to Vancouver Island (J. M. Dawson).
Campanularia fusiformis *Clark.*	Vancouver Island (J. M. Dawson).
Campanularia cylindrica *Clark.*	Santa Cruz, Cal. (C. W. Anderson).
Laomedea rigida *A. Ag.*	Bay of San Francisco, Cal. (A. Agassiz).
Laomodea Pacifica *A. Ag.*	Gulf of Georgia (A. Ag.) to Bay of San Francisco (A. Agassiz).
Lafoëa dumosa *Sars.*	
Halecium tenellum *Hincks.*	San Diego, Cal. (Dr. E. Palmer).
Sertularia anguina *Trask.*	Santa Cruz, Cal. (C. W. Anderson), to Vancouver Island (J. M. Dawson).
Sertularia anguina, var robusta *Clark.*	San Diego, Cal. (H. Hemphill), to Vancouver Island (J. M. Dawson).
Sertularia argentea *E.* and *S.*	Santa Barbara, Cal. (Mrs. Ellwood Cooper).
Sertularia pumila *Linn.*	
Sertularia Greenei *Murray.*	Santa Barbara, Cal. (Mrs. Ellwood Cooper), to Vancouver Island (J. M. Dawson).
Sertularia furcata *Trask.*	San Diego (Dr. E. Palmer), to Bay of San Francisco (J. B. Trask).
Sertularia corniculata *Murray.*	Bay of San Francisco (A. Murray).
Sertularella turgida *Clark* (Trask).	San Diego, Cal., to Vancouver Island (J. M. Dawson).
Plumularia setacea *Lamarck.*	San Diego, Cal. (Dr. E. Palmer), to Vancouver Island (J. M. Dawson).
Aglaophenia struthionides *Clark* (Murray).	San Diego, Cal. (D. C. Cleveland), to Vancouver Island (Miss Mitchell).

This list of twenty-four species is very small compared with that of the eastern coast, from Maine to New York, the fauna of the latter region containing five times as many species as that of the former, notwithstanding that the region included on the western coast is over thirteen hundred miles in length, while that of the New England coast is only about eight hundred. It should be borne in mind however that most of the collecting on the Pacific coast has been done along the shore, the dredge having been little used, and there is little doubt that when the fauna has been more thoroughly investigated the number of Hydroids may be at least doubled. Such a variety as exists on the New England coast can hardly be expected from our Pacific shores south of Vancouver Island, for the waters there do not afford the same diversity in temperature.

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Bimeria (?) gracilis, sp. nov.

Plate XXXVIII, figure 3.

Stems clustered, rooted by a creeping stolon, erect, simple, delicate, not divided by distinct joints, thickly branched; branches suberect, the larger ones reaching to the end of the stem and resembling the main stalk, the smaller ones bear but one or two hydranths and are also unjointed; perisarc extending over the hydranths and partially covering the tentacles, annulated at the base of each branch and branchlet. Sporosacs developed from the hydrophyton, a single one at the base of each hydranth-bearing branchlet, oval or ovate, supported by a short peduncle consisting of one or two annulations. Hydranths large, tapering uniformly from the distal end to the base, provided with about ten or twelve tentacles and with a large, rounded or slightly conical proboscis. Height of best specimen, 55^{mm}.

Collected on the piles of wharves at San Diego, Cal., by Dr. E. Palmer, 1875.

Our specimens were not in a good condition when they arrived, having been crowded in a tin can with many other things, which pressed them all out of shape, and the quantity of alcohol not being sufficient to preserve so much animal matter, the hydroids suffered considerably; the hydranths and sporosacs especially were in a very worn and mutilated state. It is not easy to determine just how far the perisarc extends upon the hydranth, but it certainly covers the body of the latter, and it must, I think, be developed over a portion of the tentacles, for after soaking them in a dilute solution of caustic potash for forty-eight hours the tentacles still retained their normal position, nor did they show any decrease in size. The potash seemed to act very slowly, for after being in the warm solution forty-eight hours the hydranths were not entirely dissolved out. The fact of the tentacles being unaffected would seem to indicate that they are entirely protected by chitin, but tentacles so protected would be of little or no use to the animal, and I think it more probable that the distal portions are free and may be contracted into the basal covering. It is impossible to determine from our specimens how the tentacles are held, whether in a single erect verticil as in *Garveia* or with each alternate tentacle depressed, as in *Bimeria vestita* of Wright. With such imperfect data I feel some doubt about placing this species in the genus *Bimeria*, and only do so provisionally.

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Tubularia elegans, sp. nov.

Plate XXXVIII, figure 2.

Stems clustered, rooted by a creeping stolon, erect, unbranched, more or less annulated at intervals toward the base. Hydranths large, with about thirty tentacles in the proximal set and twenty to twenty-four in the distal. Gonophores borne in clusters just inside the proximal tentacles, twelve to twenty in a cluster, each of the larger ones crowned with four conical tubercles. Height of finest specimen, 75^{mm}.

Collected on the piles of the wharf at San Diego, by Dr. E. Palmer, 1875. Intermingled with it and often attached to it were numerous shoots of *Bimeria.* Many of the young had attached themselves to the parent stalk, giving at first sight the appearance of branching stems.

The specimens from which this species is described were crowded in the same can with the Bimeria described above, and are in the same dilapidated condition. There is a Tubularian, *Thamnocnidia tubularoides*, from the Bay of San Francisco, described by A. Agassiz (Cat. of N. A. Acalephæ, p. 196), which he says "is readily distinguished from its eastern congeners by the stoutness of the stem and large size of the head." The description is a very meagre one, but from these two characters I conclude that it must be distinct from *T. elegans*, for the latter species has neither a stouter stem nor larger head than *Thamnocnidia spectabilis* of the New England coast.

Eudendrium, sp.

Plate XXXVIII, figure 1.

We have also received from the California coast the perisarc or chitinous portion of what I take to be a species of Eudendrium.

Stems stout, erect, dark horn color, strongly annulated throughout, rather sparingly branched; branches sub-erect, springing from all sides of the stem and much divided. Hydranths borne at the extremity of the short ramuli. The entire perisarc is strongly ringed, giving it a close resemblance to the tracheæ of an insect. Height of largest specimen, 80^{mm}.

Santa Cruz, Bay of Monterey, Cal.,—Dr. C. W. Anderson.

Campanularia everta, sp. nov.

Plate XXXIX, figure 4.

Stems rather stout, arising at intervals from the creeping stolon, with two annulations at the base of the hydrothecæ, the lower one

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smaller than the upper; the remainder of the stem has a wavy outline or is slightly annulated. Hydrothecæ broadly campanulate, not deep, tapering more or less gradually from the distal end to the base, the rim strongly everted and bearing about fifteen rather shallow teeth. Gonothecæ, large, turgid, nearly cylindrical, tapering a little at the base, borne on short, stout peduncles and with the aperture terminal, small and cylindrical.

Found creeping on an Alga from San Diego, Cal.,—H. Hemphill.

This is a very pretty form and may readily be distinguished by the broad hydrothecæ with their strongly everted, toothed rims. The peculiar shape of the gonothecæ is also very characteristic.

Campanularia cylindrica, sp. nov.

Plate XXXIX, figures 1–1[d].

Stems are simple, unbranched pedicels, of very variable length, more or less annulated over the entire length and with a single well-marked ring at the base of the hydrothecæ, rooted by a creeping, twisted stolon. Hydrothecæ campanulate, nearly cylindrical, tapering but very slightly toward the base, varying greatly in depth, rim armed with about fifteen very shallow, sharply pointed teeth. The gonothecæ also show considerable variation in size, there being occasionally one or two which are at least twice the size of the ordinary form; they are subfusiform, tapering slightly more toward the proximal than the distal end, supported on short pedicels with one or two annulations.

Campanularia fusiformis, sp. nov.

Plate XXXIX, figures 2–2[g].

Hydrocaulus simple, creeping, bearing the pedicels at irregular intervals; pedicels of variable length, usually two or three times the length of the hydrothecæ, never more than six times their length, with a more or less wavy outline. Hydrothecæ small, deeply campanulate, tapering at the base, rim ornamented with about twelve stout, shallow, acute teeth, a single distinct annulation at the base. Gonothecæ small, fusiform, constricted at both ends, sessile, aperture small, terminal.

Vancouver Island,—J. M. Dawson. Found growing on *Sertularia anguina* var. *robusta.*

This species is closely allied to *C. cylindrica* of the Californian coast from which it may be distinguished by the size of the hydro-

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thecæ and by their shape, not being rounded at the base; by the form of the gonothecæ, which are sessile and have a circular terminal aperture.

Found creeping on the old stems of a *Eudendrium*-like form, taken at Santa Cruz, Cal., Bay of Monterey, by Dr. C. W. Anderson.

The variation in the length of the stems is very great; sometimes they are about equal to the length of the hydrothecæ, and again they will be five or six times that length. The stolon is quite uniformly twisted and is at least twice the diameter of the stems.

Halecium tenellum Hincks.

Halecium tenellum Hincks, Annals and Mag. of Nat. Hist., 3, VIII, 252, pl. VI.

Plate XXXIX, figure 5.

Some very good specimens of this delicate species have been received from San Diego. There were no gonothecæ but the hydrosome is so exactly similar to our New England specimens and to the figure and description of Hincks that I do not hesitate to call it the same. A glance at our figure will show how exactly it corresponds.

Found parasitic on a species of *Bimeria*, collected on the piles of wharves, San Diego, Cal.,—Dr. E. Palmer, 1875.

Sertularia anguina Trask.

Sertularia anguina Trask, Proc. Cal. Acad. Nat. Sci., 112, Plate V, fig. 1. March 30, 1857.

Sertularia labrata Murray, Ann. and Mag. for April, 1860, 250, Plate XI, fig. 2.

Plate XL, figures 1, 1[a], 2.

Stems clustered, simple, erect, straight from the proximal end to the first branch, above the first branch flexuous, becoming more and more so toward the distal end, sparingly branched, divided by transverse joints into short internodes, those below the first branch bearing a single pair of nearly opposite hydrothecæ, while those above the first branch have three hydrothecæ and give origin to a single branch; branches regularly alternate, ascending, slightly curved, mostly short, a few have a much larger growth and exactly imitate the main stems in every particular; color corneous. Hydrothecæ nearly opposite, somewhat flask-shaped or tapering evenly to the distal end without any constriction or flask-shaped neck; aperture usually entire, slightly oblique, facing toward the stem, or with the outer margin much more produced than the inner and in some cases showing a dis-

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tinctly sinuous outline. Gonothecæ unknown. Height of largest specimen, 75^{mm}. Plentiful on a large species of *Mytilus* from Monterey, Tomales Pt., Punta Reyes, and on old shells, Bay of San Francisco (Dr. Trask); Santa Cruz,—Dr. Anderson; Vancouver Island, —J. M. Dawson.

Our specimens of *S. anguina* agree so closely with Hinck's description and figures of *S. filicula* that I cannot separate the trophosomes, but the gonosomes being unknown, I prefer to let the species remain distinct rather than to unite them on such incomplete data. Murray also noticed the similarity to *S. filicula* although he only possessed "a minute portion without vesicles."

Dr. Trask says of his specimens, "Their affinity is with that of *S. fallax* of Johnston more nearly than with any other species with which I am acquainted." He could not have known of *S. filicula* at that time or he would at once have noticed the much closer resemblance to that form. He also says that his specimens have four hydrothecæ between each pair of branchlets, while ours have but three: one pair opposite each other and one odd cell in the axil of the branch. His description and figure agree so well, however, in every other respect that I am inclined to regard this as an error of observation.

This description has been made from specimens which were dried before they were sent to us and have since been soaked out in warm water. The perisarc being very stout and durable I do not think that it can have changed to any great extent.

Sertularia anguina, variety robusta Clark.

Plate XL, figures 3, 4, 5.

The variety *robusta* differs from the ordinary form in having a stouter stem, larger hydrothecæ, longer pinnæ, and in being in every way a much larger and stouter form. The mode of growth, the branching, the shape and arrangement of the hydrothecæ are the same as in the normal form. Gonothecæ borne on the pinnæ, more or less fusiform, arising from just below the hydrothecæ, distal extremity slightly curved to one side, the terminal aperture, large, circular. Length of largest specimens, 100^{mm}.

San Diego, Cal.,—Henry Hemphill; taken from kelp roots washed ashore during a storm.

San Diego, Cal.,—D. C. Cleveland, 1875; Santa Cruz, Cal., Bay of Monterey,—Dr. C. W. Anderson; Vancouver Island,—J. M. Dawson.

This variety is very similar to *S. abietina* of Linnæus in many

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respects. It has the same mode of growth, the same robust habit, the same style and arrangement of hydrothecæ. There is quite a difference in the natural size figures of *S. abietina* given by Hincks and by Johnston. Our specimens from the New England coast agree very well with Johnston's figure, which is just about twice the size of Hincks' figure.

Sertularia argentea Ellis and Solander (Linn.)

Three fine specimens of this widely distributed species have been found at Santa Barbara, California; they are in good condition and loaded with gonothecæ. The only characteristic which shows any variation is the gonothecæ, which are proportionally a trifle longer and also have the orifice a little larger than in our east coast specimens. This slight difference being the only one and this character varying considerably in the same specimen, I should not think of separating them.

Santa Barbara, Cal.,—Mrs. Ellwood Cooper. Height of largest specimen, 160^{mm}.

Sertularia Greenei Murray.

Sertularia tricuspidata Murray, Ann. and Mag. for April, 1860, p. 250, Pl. XI, fig. 1.
Sertularia Greenei Murray, Ann. and Mag., v, p. 504, 1860.
Cotulina Greenei A. Agassiz, Cat. of N. Amer. Acalephæ, 1865, p. 147.

Plate XXXVIII, figure 6.

Stems erect, slender, densely clustered, simple, thickly branched, basal portion straight, above the first branch becoming slightly flexuous, forming a graceful arc between each two branches, color corneous, usually darkest at the base; joints placed at right angles to the stem and very irregularly distributed, forming internodes which bear from one to eight pairs of hydrothecæ; branches alternate, erect, many of them short; some of the lower ones equal in length to the main stem and closely resembling it in every respect; others from the middle portion of the stem are of a medium length and, like the longest branches from the lower part of the stem, reach to the extremity of the main stem forming a corymb-like structure. The branchlets, like the branches, are mostly short, but a few are of considerable length, extending to the ends of the branches. Hydrothecæ sub-alternate, tapering uniformly to the distal end, with oblique, toothed apertures, which face toward the stem; on the outer edge of the aperture are two large, prominent teeth separated by a deep notch.

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Gonothecæ borne in rows on the upper sides of the branchlets; the upper portion cylindrical, the proximal half tapering toward the base, aperture terminal, in a small cylindrical process elevated from the center of the distal end. Height of largest specimens, 90^{mm}.

Bay of San Francisco, Cal.,—Murray; Santa Cruz, Cal.,—Dr. C. W. Anderson; Santa Barbara, Cal.,—Mrs. Ellwood Cooper; Vancouver Island,—J. M. Dawson.

This is an interesting form as it is the only member of the *Sertularidæ* on the American coast having the peculiar aperture to the gonothecæ, by Agassiz called bottle-shaped, though it is by no means an uncommon form among our *Campanularidæ*,—*Obelia gelatinosa*, *O. geniculata* and *O. dichotoma* having the same general form of gonothecæ.

A peculiar discrepancy occurs in the descriptions of Murray and A. Agassiz in regard to the number of teeth on the rims of the hydrothecæ; the former describes them with three teeth, the central one being larger than the two lateral, while Agassiz describes them with four, two prominent exterior points and two smaller ones near the stem. We have quite a large supply of specimens in a good state of preservation and after having carefully examined them all, I cannot find a single hydrotheca that would afford any reason for changing the above description of *two* teeth upon the rim of each cell.

I should judge from Murray's figure that his specimens were not well preserved and by contracting had thrown out the inner margin of the rim, giving it, in some views, the appearance of a tooth. But how he made out one tooth to be much larger than the other two, I am at a loss to understand. And the fact of Agassiz having seen four teeth I am at present unable to account for.

Sertularia furcata Trask.

Sertularia furcata Trask, Proc. Cal. Acad. Nat. Sci., Mar. 30, 1857, 112, Plate V, figs. 2, a, b, c, d, e.

Plate XXXIX, figure 3.

Stems short, unbranched, rooted by a creeping stolon, simple, spreading in every direction forming dense verticillated clusters around the pieces of fucus on which it is usually found, attached to the stolon by a short, slender, twisted process about the length of an internode, divided by transverse joints into short regular internodes each bearing a single pair of hydrothecæ, color corneous. Hydrothecæ opposite, deeply immersed in the stem, with two large, sharp teeth on

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the outer margin and a large aperture generally reaching to the stem. Gonothecæ large, sessile, generally borne near the base of the stems though occasionally found scattered over the entire length, of an elongated oval form, sometimes slightly compressed, with a large circular, terminal aperture. Height of largest specimen, 45^{mm}.

Bay of San Francisco and Farallone Islands,—Trask; Santa Cruz, Cal., Bay of Monterey,—C. W. Anderson; San Diego, Cal.,—Dr. E. Palmer; Santa Barbara, Cal.,—Mrs. Ellwood Cooper; Santa Barbara, Cal.,—Dr. L. N. Dimmick.

S. furcata seems to be more nearly allied to *S. Greenei* than to any other Sertularian of the west coast known to us, both having the same style of hydrothecæ, arranged in about the same manner and with similarly toothed apertures. It is the same style seen in *S. operculata* of Linnæus.

S. furcata may be readily distinguished from *S. Greenei* by its entirely different gonothecæ, by the different size of the aperture in the hydrothecæ, by the extent to which the latter are immersed in the stem and by the habit or mode of growth. In general appearance it strongly reminds one of the *Sertularia pumila* of the New England coast.

Sertularella turgida Clark (Trask).

Sertularia turgida Trask, Proc. Cal. Acad. Nat. Sci., Mar. 30, 1857, 113, Plate IV, fig. 1.

Sertularia turgida A. Agassiz, Cat. N. Am. Acalephæ, p. 145, 1865.

Plate XXXVIII, figures 4, 5.

Stems attached by a creeping stolon, sparingly branched, attached to the stolon by a pedicel consisting of three or four rings, short, stout, simple, spreading in every direction from the branches of fucus and pieces of laminaria which seem to be the favorite stations of this species, divided by oblique joints into short, stout internodes each bearing a single hydrotheca, color light corneous; branches stout, erect, usually about half the length of the main stem and very irregularly arranged, in some cases alternately, in others all the branches spring from one side of the stem and sometimes there seems to be no regular arrangement. Hydrothecæ large, full, alternate, deeply immersed in the stem, the inner angle of the proximal end extending more deeply into the stem than the outer, aperture large, armed with three stout teeth, two of which are larger than the other and are situated on the outer side of the rim, facing the stem, the third tooth

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is on the inner margin of the rim midway between the other two. Gonothecæ, arising in the axils of the hydrothecæ, are large and of two forms; the larger form, similar to the gonotheca of *Sertularella polyzonias* Gray (Linn.), is obovate, sessile, armed with a few stout, blunt spines around the distal end, aperture, terminal and at the outer end of a small cylindrical process formed by a constriction or a very sudden tapering near the extremity and surrounded by a number of the largest spines; the smaller form is supplied with about twice as many spines as the larger form and is shorter and proportionally broader, the broadest portion being nearer to the distal end; this variation in form undoubtedly indicates a sexual difference, the smaller form probably being the male and the larger the female gonothecæ; both forms have the surface more or less roughened by transverse wrinkles. Length of largest specimen, 38^{mm}.

Bay of San Francisco, Monterey, Tomales Point, Cal,, on mollusca and algæ,—Trask; Santa Cruz, Bay of Monterey, Cal.,—Dr. C. W. Anderson; San Diego, Cal.,—D. C. Cleveland; Vancouver Island, —J. M. Dawson.

The nearest ally of *S. turgida* is the *S. polyzonias* of Gray, to which in many respects it bears a striking resemblance.

Hydrallmania Franciscana Clark (Trask).

Plumularia Franciscana Trask, Proc. Cal. Acad. of Nat. Sci., vol. i, p. 113, Pl. IV, fig. 3.

Plumularia gracilis Murray, Ann. and Mag. of Nat. Hist. for April, 1860, p. 251, Pl. XII, fig. 1.

Trask and Murray both had representatives of a species which they referred to the genus *Plumularia* and which, from their descriptions and figures, undoubtedly belongs to Hinck's genus *Hydrallmania*, though at the time their descriptions were published, this genus had not been recognized. This species is certainly very close to *H. falcata* of Hincks, but Murray, who had an opportunity of comparing the two, says they are distinct.

Unfortunately we have had no specimens.

Bay of San Francisco, Cal., among rejectamenta on the beach, —Trask.

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Plumularia setacea Lamarck.

Sertularia pinnata, β, Linn., Syst. Nat., 1312.
Sertularia setacea Pallas, Elench., p. 148.
Plumularia setacea Lamk., An. s. Vert. (2d ed.), ii, 165.

Plate XLI, figures 1, 2.

Stems simple, slender, erect, rooted by a creeping stolon and divided by transverse joints into short internodes of uniform size, regularly branched; pinnæ alternate, regularly arranged, one from each internode, arising from the stem by a prominent process produced from the outer and upper side of each internode, divided by transverse or slightly oblique joints into internodes of two sizes arranged alternately. In large specimens 150^{mm}. to 300^{mm}. long, the main stems are considerably branched, the branches alternately arranged and clustered, extending quite or nearly to the distal end of the stem; the branches give off branchlets, which like themselves resemble the main stems in every particular. Hydrothecæ with an even rim, small, borne on the larger sized internodes of the pinnæ. Nematophores compound, those on the pinnæ not quite equal in length to the hydrothecæ, those on the main stems a little longer than the hydrothecæ; three on each internode of the stem, two in the axil of each pinna, one on the opposite side of the internode near the base, one only on the upper side of the smaller joints of the pinnæ and three on the larger joints, one just below the hydrotheca, and one on each side of the aperture Gonothecæ sessile, borne in the axils of the pinnæ; female elongate oval, produced at the distal end into a tubular neck with a discoidal, terminal orifice; male, smaller than the female, fusiform, and with a much smaller aperture.

Santa Cruz, Cal.,—C. W. Anderson; San Diego, Cal.,—Dr. E. Palmer; San Diego, Cal.,—H. Hemphill; Vancouver Island,—Dawson.

Most of our specimens consist of dense clusters of the delicate shoots, about 50^{mm}. to 80^{mm}. long, and usually attached to some large, coarse alga. Ellis' old name of "*Sea Bristles*" was well chosen, for it conveys quite an accurate idea of the appearance of these smaller forms. The larger forms are more branched, usually of a darker color and have a closer resemblance to hair than to bristles; one of our largest specimens from San Diego consists of a tuft 200^{mm}. in length composed of about three hundred branched shoots; this had been washed ashore and was found by Dr. Palmer; a still larger specimen was dredged off San Diego in six to ten fathoms by Mr. Hemphill, which measures 300^{mm}. in length and forms a thick cluster of about a thousand shoots.

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Aglaophenia struthionides Clark (Murray).

Plumularia struthionides Murray, Ann. and Mag. of Nat. Hist. for April, 1870, 251, Plate XII, fig. 2.

Aglaophenia franciscana A. Agassiz, Cat. N. A. Acalephæ, p. 140, 1865.

Plate XLI, figures 3-3[d].

Stems rooted by a creeping stolon, simple, erect or spreading in every direction, divided by slightly oblique joints into very short internodes of equal length, each bearing a single pinna, varying from the lightest to the darkest horn-color; shoots tall, stout, plumose, tapering slightly toward the base, the distal end abruptly pointed; pinnæ slightly curved, sub-erect, unbranched, not in the same plane, the sides bearing the hydrothecæ curving toward each other, divided into short internodes by slightly oblique joints, each internode bearing a single hydrotheca. Hydrothecæ large, cup-shaped, expanding toward the distal end, aperture large, patulous, rim denticulated, armed usually with eleven, sometimes nine, sharp, uneven teeth. Nematophores tubular, the lateral ones of medium size, projecting ear-like from the sides of the hydrothecæ, the anterior one long, adnate for the greater part of its length, free near the distal end, extending nearly or quite to the edge of the toothed rim, aperture small, discoidal, terminal; those upon the corbulæ are a trifle larger than the lateral ones and are arranged in transverse rows, the ends of which do not meet. Corbulæ large, cylindrical, with numerous ridges (ten to sixteen) composed of oblique rows of nematophores; usually from two to six hydrothecæ at the base of the corbula. In luxurious specimens the corbulæ are very abundant, there being between seventy and eighty on a single shoot. Length of largest specimen, 150[mm].

Bay of San Francisco,--Trask; San Francisco, Cal.,—A. Agassiz; Santa Barbara, Cal.,—L. F. Dimmick; Santa Barbara, Cal.,—Mrs. Ellwood Cooper; Santa Cruz, Cal., Bay of Monterey,—C. W. Anderson; San Diego, Cal.,—D. C. Cleveland; Vancouver Island,—Miss Mitchell; Vancouver Island,—J. M. Dawson.

In the various lots of Hydroids which we have received from the western coast, this species has always been the most abundant. It seems to be as common and as widely distributed on the western coast of the United States as *Sertularia pumila* is upon the eastern coast. Both are also very often found parasitic on algæ, but *A. struthionides* is often found in deeper water than *S. pumila*.

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As Murray remarks, this species is nearly allied to *Plumularia cristata*, the *P. pluma* of Linnæus, but is much closer in habit, the hydrothecæ are wider-mouthed and shallower and the teeth upon the rim are unequal. An important error occurs in the synonymy of this species given by Mr. A. Agassiz. He has, under the name *Aglaophenia Franciscana* the following synonymy:

Plumularia Franciscana Trask.

Plumularia struthionides Murray.

The *Plumularia Franciscana* of Trask belongs to the genus *Hydrallmania* of Hincks, as a glance at the figures and description of Trask will show; and it is synonymous with the *Plumularia gracilis* of Murray. Hence the name of the above described species should not be *A. Franciscana*, but *A. struthionides*.

EXPLANATION OF PLATES.

Plate XXXVIII.

Figure 1.—*Eudendrium, sp.*; from Santa Cruz, Cal.

Figure 2.—*Tubularia elegans*; *a*, cluster of medusæ buds; *a*, *a'*, and *a''*, buds in different stages of development; *a'''*, an actinula escaping.

Figure 3.—*Bimeria gracilis*; *a*, *a'* and *a''*. sporosacs.

Figure 4.—*Sertularella turgida*; *a*, the gonangium or gonotheca.

Figure 5.—*Sertularella turgida*; another form of gonotheca.

Figure 6.—*Sertularia Greenei*; *a*, and *a'*, the gonangia (gonothecæ); *h*, hydrotheca.

Plate XXXIX.

Figure 1.—*Campanularia cylindrica*; 1*a*, the same showing the full length of the pedicel; 1*b*, the female gonotheca; 1*c*, an abnormally developed hydrotheca; 1*d*, the male gonotheca.

Figure 2.—*Campanularia fusiformis*; 2*a*, 2*b*, and 2*d*, the same showing the amount of variation in the width and depth of the hydrothecæ; 2*e*, 2*f*, and 2*g*. the gonothecæ; *r*, the rootstock or creeping stem.

Figure 3.—*Sertularia furcata*; *a*, and *a'*, the gonothecæ.

Figure 4.—*Campanularia everta*; *a*, and *a'*, the gonothecæ; *r*, the creeping stem.

Figure 5.—*Halecium tenellum*; from San Diego, Cal.; *r*, the creeping stem.

Plate XL.

Figure 1.—*Sertularia anguina*; 1*a*, a single hydrotheca showing the outline of the outer margin of the rim.

Figure 2.—*Sertularia anguina*; a portion of the main stem.

Figure 3.—*Sertularia anguina*, var. *robusta*; a portion of the main stem.

Figure 4.—The same; portion of a branch; *a*, gonotheca.

Figure 5.—The same; with a monstrosity, *a*, at the extremity of the branch.

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PLATE XLI.

Figure 1.—*Plumularia setacea;* portion of a branch with nematophores and female gonothecæ, *a*.

Figure 2.—The same; a portion of the main stem; *n*, nematophores; *h*, hydrothecæ; 2*a*, male gonotheca.

Figure 3.—*Aglaophenia struthionides;* a portion of a pinna; 3*a*, 3*b*, and 3*c*, different views of the same; *n* and *n'*, nematophores; *h*, hydrotheca; 3*d*, corbula; *a*, the body of the corbula; *b*, the wing-like processes at the base; *n*, and *n'*, the nematophores; *h*, the hydrothecæ.

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VII. On the Anatomy and Habits of Nereis virens. By Frederick M. Turnbull.

Read January 19, 1876.*

The *Nereis* (*Alitta*) *virens*, which is one of the largest and most common of our marine annellids, is found under stones or burrowing in the sand and mud of sheltered shores, both at low-water mark, and at a considerable distance farther up. It grows to the length of eighteen inches or more, and is quite stout in its proportions.

It is very active and voracious, feeding on other worms and various kinds of marine animals which it finds when burrowing in the sand.† It will even devour its own immediate relatives, if hungry when it meets them. It suddenly thrusts out its proboscis and seizes its prey with the two powerful jaws, then withdraws the proboscis, the jaws closing at the same time. In this way it will tear large pieces from the body of its victim, being able, at one bite, to cut in two a worm of its own size. One which I had confined in a small dish of water, bit its own body in two pieces at the middle. As the proboscis is turned inside out, when it is protruded, whatever has been siezed by the jaws will be drawn by them inside the proboscis as soon as the latter resumes its natural position, the proboscis then acting as a sort of gizzard.

These worms, by secreting a viscid fluid, will surround themselves in a few minutes with a translucent sheath which binds the grains of sand together, forming a loose and flexible tube. They remain most of the time in these tubes, which are nearly always situated in sand and mud or under flat stones, and they move in them with consider-

* Abstract of a graduation thesis presented at the Sheffield Scientific School, June, 1875.

† Later observations show that this species does not restrict itself to an animal diet. Several large specimens, taken by me in October, 1875, had the intestine completely filled with algæ of several species, among which *Ulva latissima* was most abundant. The algæ were torn into fragments and large shreds and rolled together into long pellets, but even after passing through the intestine their nature could be easily recognized.—A. E. VERRILL.

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able freedom and rapidity, pushing themselves along by means of their aciculæ, setæ and ligulæ, the aciculæ being controlled by special muscles. They hold their tentacular cirri straight out in front of them, as they move, in order to have warning of anything that they may approach.

The tautog, scup and other fishes dig them out of the sand and devour them eagerly. But at certain times, especially at night, they leave their burrows, and swim about like eels or snakes, in large numbers, and at such times fall an easy prey to many kinds of fishes. This habit seems to be connected with the season of reproduction. They were thus observed swimming at the surface in the day time, near Newport, in April, 1872, by Mr. T. M. Prudden and Mr. T. H. Russell, and several times by Professor Verrill, later in the season. At Watch Hill, R. I., April 12th, 1873, Professor Verrill found great numbers of the males swimming in the pools among the rocks at low-water, and discharging their milt. The males were also seen swimming in the tide-pools and shallow waters at Savin Rock, April, 1875, by Professor D. C. Eaton and Mr. Kleeberger. The *Nereis virens* is abundant at all seasons of the year, in most places along the sandy and muddy shores, both of the sounds and estuaries, burrowing near low-water mark. It occurs all along the coast, from New York to the Arctic Ocean, and is also found on the northern coasts of Europe.*

The body consists of a large number of rings or segments. This number varies with the size and age of the worm. It may be less than one hundred, or as many as two hundred. The increase in length takes place by the addition of new posterior segments, in advance of the caudal segment. New segments may also be formed when a part of the body is broken off, and in this way a considerable part of the posterior portion of the body can be completely reproduced. The head (figs. 1 and 17) is very fully developed, being provided with two pairs of eyes and two pairs of antennæ. It is attached to the dorsal side of the first segment, which is called the buccal or mouth-ring (figs. 1 and 15, *d*), because it contains the mouth (fig. 15, *m*). There are also four feelers, called tentacular cirri (fig. 1, *ee*, *ee'*, *e* and *e'*), arising from the buccal ring on each side of the head.

The abdominal rings (fig. 1, *g*) follow the buccal ring. Each one of these has a pair of lateral lamelliform appendages (figs. 1 and 17, *h*), used as paddles in swimming and also serving the purpose of gills. The last segment or ring of the body bears a pair of cirri, similar to the tentacular cirri, and also contains the anal orifice.

* See, for habits of this and allied species, Professor Verrill's report in First Report of U. S. Commission of Fish and Fisheries, p. 318. 1873.

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The males are easily distinguished from the females by differences in color and in the form of the side appendages. The color of the body in the male is an intense steel-blue, which blends into green at the base of the lateral appendages. These have a rich and brilliant sea-green color which is heightened by the complimentary effect of the numerous red blood-vessels they contain. The latter are especially noticeable toward the posterior end of the body where the skin is thinner and less opaque, so that the appendages, with their network of capillaries, appear bright red.

In the female the body is of a dull greenish color, with a slight tinge of orange and red. The appendages are orange-green at the base, and become bright orange toward their extremities; but sometimes they are greenish throughout. The whole surface of the skin, in both male and female, is iridescent, reflecting bright hues when placed in the light.

The head (figs. 1 and 15, *a*) is small, and flattened on the dorsal and ventral sides. From the position of the anterior eyes the sides taper toward the anterior extremity, where it is rounded off and terminated by a pair of small antennæ (figs. 1 and 15, *b*, *b*). There are two pairs of eyes on the upper surface of the head, one pair near its base and another pair more anterior and farther apart. The anterior eyes are situated near the middle and on the broadest portion of the head. On each side of the head, attached to its anterior half and also to the buccal ring, is a large antenna or palpus, as it is sometimes called (figs. 1 and 15, *c*). These are stout, fleshy and somewhat contractile organs. Each has a small rounded lobe at the tip (figs. 1 and 15, *c'*).

The buccal segment and the head constitute the cephalic or head region. The tegument about the mouth is wrinkled and folded longitudinally, presenting an appearance like the mouth of a purse, when drawn together by strings. The tentacular cirri are long, slender and quite flexible. They receive their nerves from the first abdominal nerve-ganglion, whereas the antennæ receive theirs from the head-ganglion.

There are four pairs of tentacular cirri, two dorsal (fig. 1, *ee*, *ee'*) and two ventral (fig. 1, *e*, *e'*). The ventral ones are situated nearer to the palpi than the dorsal, and hence the two are called respectively the internal and external tentacular cirri. The relative lengths of the tentacular cirri are shown in fig. 1. The two large dorsal tentacular cirri (*ee*, *ee*) are longer in the male than in the female. In the male they will reach to about the middle of the ninth segment, when laid along the back. In the female they will reach to about the middle of the

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fifth segment. The other tentacular cirri have more nearly the same length in both sexes.

The abdominal segments increase in size to about the eighth, and then remain nearly the same for some distance along the middle region, but the posterior rings decrease in size, causing the body to taper until it becomes quite slender. The appendages, also, are longer and broader along the middle region than toward either extremity. On the ventral side of the segments the part continuous with the feet is smooth, but the other parts show the strong transverse muscles (fig. 26), which, by their contraction, lessen the size of the body cavity.

The lateral appendages or feet of *Nereis virens* are quite complicated and wonderful organs. They are biramous (fig. 12), having two rami, one dorsal (A) and the other ventral (B). The aciculæ of each foot arise from a crypt which is attached by shroud-like muscles to the base of the foot. The setæ arise from the interior of the two rami. The feet are complicated by the addition of other organs, serving for locomotion, sensation and respiration.

The respiratory organs, often called ligulæ, are more or less flattened lobes with their teguments very thin and filled with a rich vascular network (figs. 22, *s*, 25, 26). The upper ramus has two ligulæ, a superior one (fig. 12, *b*) on its upper, and an inferior one (fig. 12, *d*) on its lower side. At the base of the superior ligula, on a sort of shoulder of its upper edge, is the dorsal cirrus (fig. 12, *a*). There are also two setigerous lobes smaller than the ligulæ, one on each side of the opening through which the setæ protrude (fig. 12, *c* and *k*). The anterior (*c*) is longer than the other (*k*) and is connected with the inferior ligula (*d*) of the upper ramus. The acicula is attached to the inner walls of the ramus forming a partition, which terminates with the end of the acicula between the lobes *c* and *d* (fig. 12) and generally forms, in the middle and posterior parts of the body, a third and smaller lobe (fig. 12, *c'*).

The lower ramus lacks a superior ligula, but it has an inferior one (fig. 12, *g*) more rounded and not so broad as the others. In the lower ramus, as in the upper, there is normally only one fascicle of setæ, but in the lower ramus it is divided into two clusters by the acicula, which, by its attachment to the inner walls of the ramus, forms a partition. Here, as in the upper ramus, there are two flattened setigerous lobes, about equal in length, one on each side of the opening for the setæ (fig. 12, *f* and *e*), and the partition formed by the acicula extends to the extremity of the anterior one, dividing it into two small lobes (fig. 12, *e* and *e'*).

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The inferior ligula of the upper ramus, with its two lobes (fig. 12, *c*, *d*) corresponds, apparently to the lobe (*e*) and its divisions, while the lobe (*k*) of the upper ramus corresponds to the lobe (*f*) of the lower ramus. In the upper ramus, the acicula and its partition do not divide the bunch of setæ, as it comes out above the acicula. The ventral cirrus arises from a slight protuberance of the inferior side of the lower ramus (fig. 12, *h*).

The feet are not all alike from one end of the body to the other; they change in form most along the anterior region, and in the first five the variation is considerable; along the middle and posterior regions, it is slight in comparison.

I have figured the first five, the forty-fifth, and the one hundred-and-ninth feet of a female worm, having one hundred and twenty-one segments; also the first, fifth, forty-fifth, and one hundred-and-ninth in a male worm of nearly equal size, but having one hundred and sixty-one segments. These figures show the posterior view of the feet. Those of the female will be first described, and then compared with those of the male.

In the first foot of the female (fig. 3) the upper ramus has only the dorsal cirrus and the superior ligula developed. The former is about one-fifth longer and a little less than one-third as broad as the latter, being very well developed, while the ligula is somewhat rudimentary. The ligula is rounded and simple in shape, having a constriction at its base. Of the lower ramus, all the parts are present. The posterior setigerous lobe (*f*) is longer than the anterior setigerous lobe (*e*), and is oval and flattened. The anterior setigerous lobe (*e*) is seen to be divided by the acicula into two smaller terminal lobes (*e* and *e'*), indicated by dotted lines where they are covered by the posterior setigerous lobe (*f*). The inferior ligula (*g*), of the lower ramus, is of about the same size and shape as the superior ligula (*b*), and the inferior cirrus (*h*), is like the superior cirrus (*a*). The setæ of the lower ramus, as shown in the figure, extend just beyond the posterior setigerous lobe (*f*). They are somewhat rudimentary and are for the most part blunt. The acicula is also rather small.

The second foot (fig. 4) has the anterior setigerous lobe (*e*), and the acicula a little larger and better developed, and the dorsal cirrus is seen to be a little shorter than that of the first foot; otherwise the second foot is scarcely different from the first.

In the third foot (fig. 5) the upper ramus has developed an inferior ligula, setæ and acicula. It also has an anterior setigerous lobe (*c*). This lobe is a part of the inferior ligula (*d*). It shows more

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distinctly in some of my figures of the feet of the male. The superior ligula is of about the same size and shape as the inferior ligula. The lower ramus is a little more developed than it is in either the first or second feet, but the inferior cirrus (*h*) is shorter.

The fourth foot (fig. 6) is more highly developed than the third; but the inferior cirrus is shorter in comparison with the lower ramus.

The fifth foot (fig. 7) is still more highly developed than the fourth, and the superior ligula is larger than it is in the preceding feet. The dorsal and ventral cirri are both smaller. If we now compare the figure of the fifth with the figures of the first and second feet, the difference is seen to be considerable, particularly in the length of the cirri.

The superior ligula (*b*) begins to increase in size at the fifth foot, and continues to do so, until in the forty-fifth (fig. 8, *b*) it is larger than any other part of the foot. It has also become flat and pointed. The shoulder (*i*) is much larger. The inferior ligula (*d*) is also flat and pointed. A posterior setigerous lobe (*k*) is now large enough to be easily seen. The posterior setigerous lobe (*f*) of the lower ramus is somewhat pointed, and its lower edge is oblique. The anterior setigerous lobe (*e*) is now as long as the posterior setigerous lobe, and its two divisions are nearly equally developed. The inferior ligula (*g*) is rounded and somewhat tapering toward its extremity. The dorsal and ventral cirri are now quite small, particularly the latter, while otherwise the foot is much larger. The setæ have become longer and more perfect, from the anterior toward the posterior, attaining their maximum in the middle region of the body. The aciculæ are also larger here than in either the anterior or posterior regions; although, in the posterior region they are longer in comparison with the size of the foot than anywhere else.

The one hundred and ninth foot (fig. 9), as shown by the figures, is nearly the same in form as the forty-fifth, but smaller.

On comparing the first, fifth, forty-fifth and hundred and ninth feet of the female with the same in the male, we find that there is a marked difference in the cirri. The dorsal cirrus in the first foot of the male (fig. 10) is one-half longer than it is in the female, and it is larger in proportion. In the fifth foot (fig. 11) the difference is the same. In the forty-fifth (fig. 12) the dorsal cirrus of the male is longer and also slenderer than it is in the female. In the one hundred and ninth foot (fig. 13) the dorsal cirri are about the same in length, but those of the male are more slender.

The difference between the dorsal cirri decreases toward the posterior end of the body, being greatest in the anterior segments.

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The *ventral* cirrus of the first foot of the male (fig. 10) is not quite so long as the *dorsal* cirrus, but it is a little longer than the ventral cirrus of the female. In the fifth foot (fig. 11) it is a little longer and much more slender than it is in the female. In the forty-fifth (fig. 12) it is twice as long and of the same diameter as it is in the female. In the one hundred and ninth (fig. 13) there is the same difference which we see in the forty-fifth.

The difference between the ventral cirri of the male and female, increases from anterior to posterior, being greatest in the posterior segments.

Beginning with the forty-fifth foot (fig. 12) a shoulder (x) appears on the dorsal side of the lower ramus in the male. It increases in size and definiteness farther back, but finally disappears. This shoulder is never seen in the female, and is a good character for distinguishing the sexes, but it requires microscopic examination. The difference between the cirri of the male and female is sufficient to distinguish them without the use of a lens.

The setæ under the microscope are very delicate and beautiful (figs. 2, 2*a*). They consist of two parts, the shaft (*a*) and the blade (*b*). The shaft has a transversely striated appearance, which is exceedingly regular. The blade is held in a sort of socket in the end of the shaft, and one edge is toothed like a saw. There are two forms: one in which the blade is short, having its extremity blunt and slightly hooked (fig. 2), and one with the blade tapering to a fine point, the blade being long and slender (fig. 2*a*). The length varies and with it the delicacy of the point. In most of the setæ the latter is so sharp that it seems to vanish, and can be seen only with a high power. Those setæ which are hooked have the hooks and toothed edge turned upward; and these are always confined to the lower ramus, and to the lower side of the latter in both bunches. In the middle and posterior regions these hooked setæ disappear, their place being taken by the other kind; but the latter are shorter than those in the upper part of the bunch. The same forms of setæ are found in both male and female. Those of the anterior feet are shorter than those in the middle region. When viewed by transmitted light the bunches present all the colors of the spectrum.

The aciculæ (figs. 12 and 22) are simple thorns, in the form of an elongated cone, generally a little bent. They are black, except at the base, showing through the translucent integuments of the foot. At the base they are hollow and therefore lighter colored.

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Muscular System.

The walls of the body contain two muscular layers, which are well defined. The first is thick on the ventral side of the body (fig. 22, *k*), but is thinner elsewhere. It forms in each ring a muscular plane of transverse fibres. The second is placed within the preceding and is formed of great fasciæ which are attached to the interannular partitions (fig. 16, *b*). This layer does not exist on the ventral side, between the bases of the feet. The interannular partitions (fig. 16, *a*) are attached to the intestine, which they hold in place, dividing the general cavity of the body into a series of chambers (fig. 16); but as the inner subcutaneous layer does not extend over the ventral floor of the cavity, the chambers open into one another beneath the intestine. I have already mentioned the muscles attached to the fleshy knobs, which hold the aciculæ. These muscles (fig. 22), when they contract all together, force the aciculæ outward. When different ones contract they move the aciculæ to one side.

The *Nereis virens* in swimming moves its body laterally, like a snake. It sometimes has also an undulatory movement, up and down. These motions are all produced by the subcutaneous muscles. The lateral appendages are used as paddles, but their principal use is to push the worm along in its tube, and for crawling. They are greatly aided by the stiff aciculæ, controlled by their special muscles (fig. 22), and by the setæ. When burrowing, the proboscis is used to push away the sand in front and is then withdrawn, while the body is moved forward partly by a vermicular motion and partly by the side appendages.

Alimentary System.

The proboscis is a very remarkable organ and constitutes a formidable weapon. It is divided by M. DeQuatrefages into three regions: the pharyngeal, the dental, and the œsophageal (figs. 16–18). The mouth has already been partially described. The walls of its cavity are connected by several small, delicate muscles, with the walls of the body cavity, as I have shown in figure 16, *m* and *n*. The pharyngeal region commences immediately back of the buccal cavity (figs. 16 and 20, *x*), and has two muscular partitions (fig. 16, *c*).

The dental region (20 and 16, *p*) is very muscular, and is provided with a considerable number of small teeth, or denticles, which are arranged in groups on the anterior, inner surface of its walls. It also has two large and powerful jaws attached by their hollow bases to the muscles of the posterior inner surface. The worm has the power of turning this dental region inside out.

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When the proboscis is inside the body, it takes the position shown in figures 16 and 20, the œsophagus being curved and pushed back against the intestine; but when it is forced out, the œsophagus straightens, and that part of the proboscis which is protruded takes the position shown in figures 17 and 18, these figures showing the arrangement of the jaws and denticles, figure 17 giving the dorsal view and figure 18 the ventral view; the buccal ring is very much stretched; and the head, antennæ and tentacular cirri are forced over on the back (fig. 17). The jaws are imbedded in and attached to the special muscles, in such a manner that the more the proboscis is protruded the farther apart their points move, and when the proboscis is withdrawn they close like a pair of scissors, their points crossing. The jaws (fig. 19) are curved like hooks, and have their inner concave edge denticulated with about ten teeth. They and the denticles are composed of a black chitinous material. The denticles, which are conical and pointed, are not attached to muscles, but are simply imbedded in the surface. The number and size of the denticles, and even their positions, vary considerably in the different specimens examined. I think it would be hard to find two specimens exactly alike in this respect. Among seven worms, I found five with one denticle, one with two denticles, and one with seven denticles on the median anterior area of the dorsal side (fig. 17, *o*).

Among six worms, I found two with two denticles and four with none at all on the posterior median area of the dorsal side (fig. 17, *t*). On the left submedian anterior area of the dorsal side (fig. 17, *l*), the number of denticles varied from three to eight; and on the right submedian anterior area of the dorsal side (fig. 17, *l'*) from two to eight. In only one case did the two last areas have the same number of denticles. On the left submedian posterior area of the dorsal side (fig. 17, *s*) the number of denticles varied from none to five; and on the right submedian posterior area of the dorsal side (fig. 17, *s'*) from one to four.

The denticles on the lateral and ventral posterior areas (fig. 17 and 18, *r*) vary considerably in number, position and size. Among four worms, the number of denticles on the right lateral anterior area (figs. 17 and 18, *n*) varied from twelve to twenty-five; and on the left lateral anterior area (figs. 17 and 18, *n'*) from eleven to thirty.

Among five worms the number of denticles on the anterior median area of the ventral side (fig. 18, *y*) varied from four to twelve. On each of the two submedian areas of the ventral side (fig. 18, *x'*) there was one denticle.

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In some of the large worms the denticles are as small as those of smaller worms, while in other cases they are much larger. The large denticles probably are knocked off in some way and new ones grow in their place. This would account in part for the large number of very small denticles, and also for the variations in number.

Attached to the anterior end of the œsophagus, one on each side, are two salivary glands (figs. 16 and 20, *j*). These are free except at one end, and are ciliated on their outer surface. The intestine proper (figs. 16 and 20, *r*) is straight and is constricted somewhat by the muscular partitions of each segment through which it passes. It is brilliant greenish yellow in color and is surrounded by a regular capillary network of blood vessels (fig. 20).

The internal surface of the œsophagus is tessellated with low, rounded papillæ or tubercles. These are regular in shape and equal in size. Their sides are diagonal to the length of the œsophagus. They show through the walls of the œsophagus, so that its outside appears tessellated with dark squares, and as the œsophagus is stretched or contracted they become diamond shaped. The tubercles are of a dull color, between brown and yellowish green.

The interior surface of the intestine is also covered with regular longitudinal rows of low, rounded tubercles, which are much smaller than those of the œsophagus. They are greenish-yellow like the outside of the intestine. The end of the œsophagus projects into the cavity of the intestine, and its opening, which has sphincter and also longitudinal muscles, can be enlarged or contracted to a considerable extent. The outer surface of this end of the œsophagus is continuous with and like the internal surface of the intestine. The latter secretes a brown fluid in its interior and probably acts as a hepatic organ. The dental portion of the proboscis acts like a gizzard, and the œsophagus is probably a sort of stomach.

Circulation and Respiration.

The circulatory system is highly developed and complicated. The blood is red, and the vascular system is complete and closed. The principal vessels have a longitudinal course, occupying the whole length of the median line of the body, one as a dorsal (figs. 20–24, *a*), and the other as a ventral vessel (*b*). They are contractile, and by a sort of peristaltic motion the blood is pushed forward in the dorsal vessel, and in the opposite direction by the ventral vessel. The dorsal vessel is visible for nearly its whole length through the more or less transparent walls of the body, and its blood can be seen

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moving in a series of waves toward the head. The ventral vessel sends off, in each segment of the body, except a few in the region of the proboscis, two smaller vessels, one on each side. These two vessels fork, each sending a branch to the inferior ramus of the foot of the next segment to the rear (figs. 20–24, *f*), and another larger branch (*c*) around the intestine, by the side of the transverse partition, to the dorsal vessel, receiving, also, on its way, a vessel from the upper ramus of the foot of its own segment (*d*). Besides these principal lateral vessels, there are five other vessels on each side in each segment, coming from the ventral vessel (fig. 20). These form a loose but regular network that surrounds the intestine and is connected with five other convoluted vessels, which join the dorsal vessel. This network on the intestine probably supplies the hepatic organ with material for its secretion, and very likely may receive nutritive material from the digested food. The blood moves in waves, at regular intervals, through the peripheral vessels (figs. 20–24, *c*) to the dorsal vessel, but I could not see in which direction the blood moved in the network. The blood is forced into it at each pulsation of the dorsal vessel, but the normal flow may be in the opposite direction. The peripheral vessels are also connected with this network (fig. 20). The dorsal and ventral vessels are connected at the posterior extremity of the body by a simple peripheral vascular ring (fig. 23, *c*), in which the blood flows from the ventral to the dorsal vessel. In the region of the proboscis, the ventral vessel sends lateral branches directly to all the feet but the first three (figs. 20 and 21). It then sends a pair of vessels to the œsophagus (figs. 20 and 21, *e*), which pass back along the œsophagus, one on each side, as far as the intestine, being connected with smaller vessels on the surface of the œsophagus. The ventral vessel next sends off a pair of vessels which expand into capillary networks, one on each side (figs. 20, 21, *s* and *g*). Each of these networks sends small branches to the first three feet on its own side, and then merges into a vessel (figs. 20 and 21, *k*), which goes to the base of the tentacular cirri. The ventral vessel now goes upward to the under surface of the proboscis, and there divides into three branches (figs. 20, 21, *t*, *t* and *l*). The middle branch (fig. 21, *l*) passes under a muscle and along the median line of the ventral surface, as far as the pharynx, where it divides into two, forming a small vascular ring (fig. 21, *n*) about the latter. The two lateral branches pass upward and backward on the proboscis, each expanding into a remarkably rich and delicate network on its own side of the proboscis (figs. 20 and 21, *u*). From each of these networks a vessel

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(*v*) passes to the base of the head, where it joins the dorsal vessel (*a*), thus completing the circulation. From this junction small vessels probably go into the head and antennæ. The lateral vessel passing to the lower ramus divides into branches ramifying on that portion which is continuous with the foot on the ventral side of the segment, and also over the lower ramus (figs. 22 and 26). There a connection is made with vessels of the upper ramus, and I think this is done by the vessel marked *x* in figures 22 and 24, because it is quite large at the base of the inferior ligula of the upper ramus, and grows smaller at first and then swells out again before joining the vessels of the upper ramus, in the superior ligula. The branch *d* (figs. 20, 22 and 24), coming from the dorsal ramus, receives blood from that organ and also from a peculiar and beautiful arrangement of capillaries on the dorsal side of the body (fig. 25).

In the first four segments, in the region of the proboscis, the dorsal vessel has no branches, but in the remaining segments, commencing with the fifth, it has five pairs of long peripheral branches (fig. 20, *c*, *c'*), corresponding to the peripheral vessels of the posterior part of the body. They are not attached to the proboscis, but are simply connected with the dorsal and ventral vessels by their ends. The one coming from the dorsal vessel in the fifth segment is connected with the ventral vessel in the fourth segment (fig. 20.) The first three feet probably do not act as gills, as very little blood is sent to them. The two networks (figs. 20 and 21, *u*) on the proboscis are probably for carrying on the exchange between the blood and the liquid of the body cavity.

The respiration is carried on by the red fluid in the beautiful arrangement of capillaries on the body and feet, especially the latter. The flat ligulæ of the feet are exceedingly delicate in structure and take the place of gills, absorbing the oxygen from the water to purify the blood received from the ventral vessel, which then returns to the dorsal vessel.

The disposition of this respiratory arrangement is shown in figures 22, 24, 25 and 26.

The Nervous System.

The nervous system of *Nereis virens* (figs. 27 and 28) is complicated and well developed, being composed of a series of ganglia, sending out branches and connected by nervous cords. It lies mainly on the ventral floor of the body beneath the large ventral vessel. The first and largest ganglion (figs. 27 and 28, *a*), analogous to the brain of higher animals, is situated in the head. It is composed of several

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smaller ganglia joined together. It bears the eyes, on four short nervous peduncles, on its dorsal side. In front it sends four nerves to the antennæ (*b*, *b* and *c*, *c*, figs. 27 and 28). Laterally it sends out two branches called the connectives (*d*, *d*), which pass around the mouth and proboscis to join the first of the abdominal ganglia (*h*, fig. 27). Near the junction of the connective with the head ganglion, is a small ganglion sending nerves to the internal tentacular cirri (*e*, *e*, figs. 27 and 28). The connectives, near their lower extremity, send two nerves (*g*, *g*, fig. 27) to a series of ganglia and nerves on the ventral side of the proboscis (*w*, *w*, fig. 27). There is also an accessory connective (figs. 27 and 28, *d'*) on each side, passing from the first abdominal ganglion to the ganglion supplying the external tentacular cirri (*e'*, *e'*, figs. 27 and 28). This accessory connective also has a ganglion (*n'*, fig. 27) at the middle, sending nerves to the muscular partitions of the proboscis.

Each of the first three abdominal ganglia sends, from its anterior portion, on each side, a nerve that forks, one branch (fig. 27, *n*) going to the muscular partition and the other (*o*) passing through the partition to the preceding segment. In the remaining abdominal ganglia, beginning with the fourth, the branches *n* and *o* become separate nerves (fig. 27). From the posterior portion of these ganglia a nervous trunk on each side (*m*), goes to each foot, where there is a small ganglion (*k*) sending off a cutaneous branch and a branch (*i*), supplying nerves to the foot.

The ganglia (fig. 28, *e*, *e*) and the head-ganglion (*a*) send some very slender nerves (fig. 28, *z*, *z*) to a series of ganglia on the dorsal side of the proboscis (*y*, *y*). In figure 27 the series of ganglia (*w*, *w*) are drawn as if the proboscis had been revolved about a line drawn through its anterior end, so that the ventral surface would be uppermost. In figure 28 the ganglia (*y*, *y*) are in their natural position. The dorsal ganglia (fig. 28, *x*, *x*) are connected with the ventral ganglia (fig. 27, *v*, *v*) by means of nervous cords; the dorsal ganglia (*s*, *s*, fig. 28) with the ventral ganglia (fig. 27, *l*, *l*), by means of nerves passing around the proboscis outside the points of the retracted jaws (*f*, *f*); and the dorsal ganglia (*t*, *t*, fig. 28) with the ventral ganglia (*p*, *p*, fig. 27) by means of two short, thick nervous commissures which send off the nerves (*u*, *u*, figs. 27 and 28). These two nerves (*u*, *u*) terminate in the ganglia (*r*, *r*, figs. 27 and 28).

These ganglia and nerves of the proboscis lie on its walls, underneath the muscles.

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The sense-organs are the four eyes, the four antennæ, the tentacular cirri, and the dorsal and ventral cirri of the feet; also the long slender cirri of the posterior extremity. The antennæ and cirri are organs of touch.

Organs of Reproduction.

The sexes are separate, and the genital organs appear as simple glandular bodies, ovaries or spermaries, which project from the ventral surface into the cavity of the body, between the transverse muscular partitions. At the sexual period they are filled with eggs or spermatic particles, although at other times they can scarcely be seen. Neither the spermaries nor the ovaries have special excretory ducts, which open upon the surface of the body. The sperm and ova are discharged into the cavity of the body, which at this period is often filled with them. At the base of the lower ramus of each foot (fig. 22, *g*) is a glandular body, called the segmental organs. Some of these are normally kidneys, as urea has been found in them, but some are usually modified to act as oviducts, having a trumpet-shaped mouth opening into the body cavity and communicating with the exterior. I found the segmental organs all along the body beyond the region of the proboscis, but was unable to find the trumpet-shaped tubes. These are probably situated in the posterior segments, as Professor Verrill has seen the male worms discharging their milt from that portion of the body. The fecundation takes place in the water.

EXPLANATION OF PLATES.

Plate XLII.

Figure 1.—*Nereis virens*, female; dorsal view of the anterior portion of the body; *a*, head, with four eyes; *b*, *b*, antennæ; *c*, *c*, palpi; *c′*, *c′*, lobes of palpi; *d*, buccal ring; *ee*, *ee*, longer dorsal pair of tentacular cirri; *ee′*, *ee′*, shorter dorsal pair of tentacular cirri; *e*, *e*, longer ventral pair of tentacular cirri; *e′*, *e′*, shorter ventral pair of tentacular cirri; *h*, lateral appendages; *g*, abdominal rings.

Fig. 2 and 2*a*.—Two forms of setæ; *a*, shaft; *b*, blade.

Fig. 3.—First lateral appendage of female, posterior view; lettering the same as in fig. 8.

Fig. 4.—Second lateral appendage of female, posterior view; lettering the same as in fig. 8.

Fig. 5.—Third lateral appendage of female, posterior view; lettering the same as in fig. 8.

Fig. 6.—Fourth lateral appendage of female, posterior view; lettering the same as in fig. 8.

Fig. 7.—Fifth lateral appendage of female, posterior view; lettering the same as in fig. 8.

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Fig. 8.—Forty-fifth lateral appendage of female, posterior view; A, upper ramus; B, lower ramus; *a*, dorsal cirrus; *b*, superior ligula of upper ramus; *c*, anterior setigerous lobe: *k*, posterior setigerous lobe; *d*, inferior ligula of upper ramus; *e*, *e*,′ divisions of anterior setigerous lobe of lower ramus; *f*, posterior setigerous lobe; *g*, inferior ligula; *h*, ventral cirrus; *i*, shoulder of upper ramus; *s*, *s*′, setæ; *y*, *y*′, aciculæ.

Fig. 9.—One hundred and ninth lateral appendage of female, posterior view; lettering the same as in fig. 8.

Fig. 10.—First lateral appendage of male, posterior view; lettering the same as in fig. 8.

Fig. 11.—Fifth lateral appendage of male, posterior view; lettering the same as in fig. 8.

Fig. 12.—Forty-fifth lateral appendage of male, posterior view; *c*, extra division of anterior setigerous lobe; *x*, shoulder, peculiar to the male, on the dorsal edge of the lower ramus; otherwise the lettering is the same as in fig. 8.

Fig. 13.—One hundred and ninth lateral appendage of male, posterior view; lettering the same as in fig 8.

PLATE XLIII.

Fig. 15.—*Nereis virens;* ventral view of the head and mouth, the proboscis withdrawn; *a*, head; *b*, *b*, antennæ: *c*, *c*, palpi; *c*′, *c*′, lobes of palpi; *d*, buccal ring; *ee*′, *ee*′, shorter dorsal pair of tentacular cirri; *e*, *e*, longer ventral pair of tentacular cirri; *e*,′ *e*,′ shorter ventral pair of tentacular cirri; *m*, mouth; *g*, abdominal rings; *h*, *h*, lateral appendages.

Fig. 16.—*Nereis virens;* the walls of the body are cut through longitudinally on the dorsal side, so as to show the perivisceral cavity with the alimentary canal; *m*, mouth; *n*, muscles of mouth; *x*, pharyngeal region of the proboscis; *c*, muscular partitions of proboscis; *b*, perivisceral cavity: *p*, dental region of proboscis; *o*, œsophageal region of proboscis; *j*, salivary glands; *r*, intestine proper; *a*, muscular partitions.

Fig. 17.—Head of *Nereis virens*, with the proboscis protruded, dorsal view; *a*, head; *b*, *b*, antennæ; *c*, *c*, palpi; *c*′, *c*′, lobes of palpi; *d*, buccal ring; *ee*, longer dorsal pair of tentacular cirri; *ee*′, *ee*′, shorter dorsal pair of tentacular cirri; *e*, *e*, longer ventral pair of tentacular cirri; *e*′, *e*′, shorter ventral pair of tentacular cirri; *f*, *f*, jaws; *o*, anterior median area of dorsal side; *l*, *l*′, left and right anterior sub-median areas of dorsal side; *n*, *n*′, anterior lateral areas; *t*, posterior median area of dorsal side; *s*, *s*′, left and right posterior sub-median areas of dorsal side; *r*, posterior lateral and ventral areas.

Fig. 18.—Protruded proboscis, ventral side; *f*, *f*, jaws; *n*, *n*′, anterior lateral areas; *y*, anterior median area of ventral side; *x*, *x*′, left and right anterior sub-median areas of ventral side; *r*, posterior lateral and ventral areas.

Fig. 19.—Jaw of *Nereis virens*, much enlarged.

Figs. 27 and 28.—Nervous system of *Nereis virens;* *h*, abdominal ganglia; *n*, *n*, nerves to muscular partitions; *o*, *o*, nerves passing through partition to preceding segment; *m*, *m*, nervous trunks to feet; *k*, *k*, ganglia sending off a cutaneous branch and a branch (*i*) supplying nerves to the feet; *d*, *d*, connectives; *d*′, *d*′, accessory connectives; *g*, *g*, nerves communicating with the ganglia of the proboscis; *e*, *e*, ganglia sending branches to the internal or ventral tentacular cirri; *e*′, *e*′, ganglia sending

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branches to the external or dorsal tentacular cirri; *a*, head-ganglion with four eyes; *b*, *b*, nerves to antennæ; *c*, *c*, nerves to palpi; *w*, *w*, series of ganglia and nerves on the ventral side of proboscis; *f*, *f*, jaws, ventral side; *y*, *y*, the series of ganglia and nerves on the dorsal side of the proboscis; *f*, *f*, jaws, dorsal side.

In figure 27 the proboscis has been revolved about a line passing through *g*, *g*, so that the ventral side is uppermost. The head and abdominal ganglia are in their natural position. The position of figure 28 is reversed so as to show the relations of the dorsal ganglia *y*, *y*, to the ventral ganglia *w*, *w* (fig. 27).

Plate XLIV.

Fig. 20.—Circulation of blood in *Nereis virens*, and also the alimentary canal in its natural position; *m*, mouth; *x*, pharyngeal region of proboscis; *p*, dental region of proboscis; *o*, œsophageal region of proboscis; *r*, intestine, covered with a vascular network, which is connected in each segment with the large dorsal and ventral vessels by short branches; *j*, salivary glands; *a*, large dorsal vessel; *b*, large ventral vessel; *c*, *c'*, c^6, peripheral vessels; *d*, branches from the dorsal side of the feet; *e*, branch to œsophagus; *f*, f^6, branches to the ventral side of the feet; *s*, lateral branch, supplying the vascular network (*g*) and first three lateral appendages; *k*, vessel from network (*g*) to base of tentacular cirri; *t*, branch from the vascular network (*u*) on the proboscis, to the large ventral vessel (*b*); *v*, branch from the large dorsal vessel (*a*), at base of head, to the network (*u*) on the proboscis.

Fig. 21.—Diagram, showing the disposition of the large ventral vessel, and its branches on both sides, in the anterior portion of the body; *l*, continuation of the large ventral vessel along the median ventral line beneath the muscles of the proboscis; *n*, vascular ring surrounding the pharyngeal region of proboscis; otherwise the lettering is the same as in fig. 20. The arrows indicate the direction in which the blood flows.

Fig. 22.—Diagram to show the circulation of the blood, and also the relative position of the parts, in one segment of the body; *i*, intestine; *p*, perivisceral cavity; *h*, crypt from which aciculæ grow; *l*, muscles of crypt, which are attached to the base of the foot; *g*, *g*, segmental organs; *k*, walls of body; *n*, a ganglion of the abdominal chain; *a*, large dorsal vessel; *b*, large ventral vessel; *c*, peripheral vessel; *f*, branch to ventral side of foot; *d*, branch from dorsal side of foot.

Fig. 23.—Circulation of the blood in the last posterior segment; *a*, large dorsal vessel; *b*, large ventral vessel; *c*, vascular ring, with no branches.

Fig. 24.—Lateral view of the circulation in one segment; *d*, branch from the dorsal side of foot: *f*, branch to the ventral side of foot in the adjacent posterior segment.

In the last two figures arrows indicate the direction in which the blood flows.

Fig. 25.—Dorsal view of two segments showing the vascular network in the lateral appendages and beneath the skin of the back; lettering the same as in fig. 22; the large dorsal vessel, *a*, and the peripheral vessels *c*, *c*, show through the translucent walls of the body.

Fig. 26.—Segment showing the vascular network beneath the skin of the ventral side and in the lateral appendages; lettering as in fig. 22.

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VII. Median and Paired Fins, a Contribution to the History of Vertebrate Limbs. By James K. Thacher.

Median Fins in Amphioxus.

The quadrate markings seen at the base of the median fin in Amphioxus extend on the dorsal side from one extremity of the animal to the other, or nearly so, and on the ventral side from the porus abdominalis aborad toward the extremity of the tail. They are largest and most distinct in the middle of the body, and become smaller and less clearly marked (as seen from the outside), toward the head and tail until they seem to fade out entirely as they closely approach these extremities.

As Stieda* has shown these are but the external marks of a series of cavities, containing what is described as "a transparent, wholly structureless mass, resembling a coagulation."

Thus the relation of these bodies to the somewhat similarly placed primordial fin rays, or "interneural spines," of the Craniote fishes is not so simple and direct as indicated in the mistaken representations of Rathke† and of Müller.‡ Still they occupy a position similar to that of the primordial fin rays of other fishes, and the fact that they do not agree with the segmentation of the lateral muscles, seems to have some pertinency here, and to this alone I wish to call attention.

In the middle of the back there are about five of these bodies to a single segment, and on the ventral side just aborad of the abdominal pore there are about four to each. As we shall see hereafter, the structures of the median line (genuine fin-rays except in Amphioxus) exhibit quite generally throughout the fishes, a total disregard of the segmentation of the lateral muscles, and are more numerous than those segments.

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* Studien über den *Amphioxus lanceolatus* von Dr. Ludwig Stieda, Mem. de l'Acad. Imp. des Sciences de St. Petersbourg, VIIe Serie, Tome xix, No. 7.

† Rathke, Bemerkungen über den Bau des *Amphioxus lanceolatus*. Königsberg, 1841.

‡ Johannes Müller, Ueber den Bau und die Lebenserscheinungen des *Branchiostoma lubricum*. Abhandl. der Berliner Academie, 1842.

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Median Fins in Myxine.

In *Myxine glutinosa* the median fin extends but a short distance forward. In a specimen 24 centimeters long, from the Bay of Fundy, the fin reaches 4 cm. from the extremity of the tail on the dorsal side, and 2·5 on the ventral.

The fin-rays, now unquestionable homologues of the primordial fin-rays of Gnathostomes, though not yet having assumed the histological structure of true cartilage, support the thin fold of skin which forms the fin. They are simple tapering rods, extending distally to the edge of the fin, and proximally scarcely dipping below the general body contours.

The only deviation from simple rods which I have been able to find is the dichotomous splitting of some of the rods where the fin rounds the extremity of the tail.

The numerical relation between these rays and the corresponding muscular segments is as three to one on the dorsal side, and as two and a half to one on the ventral.

I have been unable to detect any muscular fibers in the composition of the fin.

Median Fins in Petromyzon.

Here the median fins are much better developed. In a specimen (*Petromyzon marinus*, from the Connecticut River), 77 cm. long, the caudal fin extends forward along the dorsal side 7·5 cm., sloping downward nearly to the body, then the second dorsal rises abruptly and runs orad 16 cm., where it reaches by a gentle slope the general outline of the body. There follows a finless space 3·5 cm. in length which is succeeded by the first dorsal, whose extent is 9 cm., being therefore shorter as it is lower than the second dorsal. The anus is opposite the orad part of the second dorsal.

The fins, therefore, take up almost the whole of the hinder half of the mid-dorsal line. In Myxine only one-sixth was thus occupied. On the ventral side we have only the caudal, extending about as far here as it does above.

These fins are supported by a series of chondroid rays, lying quite close to one another in the median plane. They are straight and slope aborad from the fatty-fibrous ridge-pole of the myelonal canal (Pl. XLIX, fig. 1, a,) to the very edge of the fin. They are found in all the fins. Their form is represented in Pl. XLIX, fig. 1, where only one ray is drawn complete. As shown, it bifurcates twice and thus ends distally in four fine branches. This figure is from the central part of

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the large second dorsal, and shows one of the longest rays. Where they are shorter we may have only one bifurcation, or one of the two primary branches, that toward the longer rays, may again divide, while the other toward the shorter remains uncleft. Farther forward at the beginning of the second dorsal, where the rays are still shorter, they do not divide at all, but end somewhat bluntly though compressed from side to side.

Each ray is largest in the middle and here lies quite close to its adjacent rays; below they grow more slender, and therefore are somewhat spaced, but expand somewhat to a foot resting on the myelonal canal.

I have seen no cases of concrescence between adjacent rays. With the exception of the variation in the branching and length in different parts of the fin, before alluded to, the rays are all similar and parallel one to another.

This branching is plainly a true dichotomy and not the product of concrescence, as is evidenced by the total absence of anything else resembling concrescence, by the similarity in size between two adjacent differently branched rays, and by the regularity of the branching.

On each side of the row of skeletal elements are muscular bundles of a somewhat blacker color than the two great masses of lateral muscles. The muscles of the median fin wedge themselves into the angle between the lateral muscles of the two sides along the median line. They are sharply distinguished from these. The fibers of the lateral muscles run longitudinally, while those of the fin muscles are parallel to the primordial median fin-rays. There is absolutely no continuity between the two in any part. Moreover the bundles of the fin-muscles show no relations to the segments of the lateral muscles. A cross section, Pl. XLIX, fig. 3, shows the relation between the fin-muscle bundles and fin-rays.

The numerical relation between the fin-rays and the segments of the lateral muscle is shown in Pl. XLIX, fig. 2, where we have a little less than four of the former to one of the latter.

The relation between the fin-rays and the neural arches is shown in figure 1, where we have 35 rods and 23 arches. These neural arches rise from the sheath of the notochord, to stiffen the fibrous sides of the myelonal canal and to apply themselves to its fatty-fibrous ridge-pole. The fin-rays abut on the same ridge-pole in the mid-dorsal line, but they are in no way connected with the neural arches. I have met with no cases even of concrescence between the two.

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Figure 4, Pl. XLIX, shows the relation between the neural arches and the segments of the lateral muscles. There are two arches to one segment. In figure 1, then, there must have been 11½ segments, which gives us a very little more than *three* as the ratio between the fin-rays and the segments. From figure 2 we obtained a little under *four*. Both results are necessarily correct. There is considerable variation in the relation between the fin-rays and the muscular segments and the neural arches. This is exhibited in the following table of observations on a single specimen.

	Fin-rays.	Arches.	Ratio of fin-rays to arches.	Ratio of fin-rays to segments.
In the orad half of 1D.	19	8	2·4	4·8
In the aborad half of 1D.	20	8	2·5	5·
In the orad part of 2D.	20	11	1·8	3·6
In the next 35 rays of 2D.	35	23	1·5	3·
In the next 27 rays of 2D.	27	17	1·6	3·2
In the next 12 rays of 2D.	12	7	1·7	3·4

The Lampreys then have advanced beyond the Myxines toward the Gnathostomes as regards the structure of the median fins, in the greater development and efficiency of those organs, and in the addition of special fin-muscles which seem to be wanting in the lower group. And while in Myxine the original independence of the median fin skeleton and the axial skeleton is shown by the existence of the median fin-rays before any neural arches have appeared, and by their want of agreement with the muscular segments to which the axial skeleton will conform when it does arise, in Petromyzon we have the same independence more strikingly reaffirmed by the simultaneous existence of neural arches and median fin-rays, and their perfect indifference to one another.

*Median Fins of Elasmobranchii.**

We shall here consider the skeleton of the dorsal and anal fins alone. That of the caudal fin has on the ventral side undergone peculiar modifications by the union of fin-rays with hæmal spines.

* The term Elasmobranchii includes the Chimæroids with the Sharks and Rays. I have had no opportunity of examining the fin skeletons of the former. But Chimæra seems to me to be but a divergent form of Sharks and to have its nearest living relative in Cestracion. They are quite specialized forms, Cestracion the less so.

I will state here that the Elasmobranchs described came from Wood's Hole, Mass. The names used are those given by Dr. Gill in the U. S. Fish Comm. Rep. for 1871–72.

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In general the structure of the median fin resembles what we have seen in *Petromyzon*, but there are important differences. The rays are of hyaline cartilage and they do not usually reach down to the ridge-pole of the myelonal canal. In *Eulamia*, for example, in the smaller of the specimens figured, the rays approach within a centimeter and a half at the orad extremity of the first dorsal fin, but are three and a half centimeters distant at the other extremity. In *Squalus* (*Acanthias*) they come closer.

This ridge-pole consists of a cord of rather peculiar white longitudinal fibers, constituting now a "*ligamentum longitudinale.*" It appears to me undoubtedly homologous with the fatty-fibrous body in *Petromyzon.* The cartilaginous arches unite under, and do not extend around over it, though they clasp it somewhat.

The rays are segmented, usually twice. Dichotomy is rare if not altogether absent. Concrescence of adjacent rays is by no means uncommon. The reduction of rays in size is exhibited in all degrees.

Calcification presents itself in a thin superficial layer on each side of the somewhat flattened ray, but it fails on the edges, i. e. as we come close to the median plane.

The muscles of the fin, as in *Petromyzon*, are in total independence of the large masses of segmented lateral muscles, but they are in more definite relation with the skeleton of the fin. This is accurately represented in Pl. LIX, fig. 66, though that is a section of a pectoral and not of a median fin. We see that each ray has on each side a special muscle, separated from its fellows by the fibrous sheet which runs from between the rays to the integument. Each little muscle developes in its median line a flat tendon, which, parallel to the surface of the fin, inserts itself in the fascia covering the extremities of the fin-rays and the proximal ends of the well known horny fibers, which here supplement the primordial skeleton, as the secondary fin-rays of Ganoids and Teleosts do.

The relation between the number of fin-rays and that of the vertebræ opposite to them is similar to what we have seen in the lower forms. In the Nictitantes, for example, there are on the average about 2·5 rays to one vertebra. But there is considerable variation, even in individuals of the same species. The extreme numbers, so far as I have observed, are 3·5 in an anal of *Sphyrna*, and 2 in a first dorsal of *Eulamia.*

We turn now to the more minute examination of several species.

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First Dorsal of Mustelus canis, Pl. XLIX and L, figs. 5–10.

In Pl. XLIX, fig. 5, we have 24 separate rods, unless 3 be the terminal piece of 2; but its conformation seems to testify to its independent but reduced character. The number here then is 24 or 23.

In Pl. XLIX, fig. 6. we have again the same alternative, without quite so strong a case for 24, but still quite strong.

In Pl. L, fig. 7, we have the choice between 24 and 25, but in favor of the latter.

In Pl. L, fig. 8, we have 23 or 24, but the former has the greater probability.

Pl. L, fig. 9 gives us 22, 23 or 24, 23 being more probable.

Pl. L, fig. 10 exhibits 23 or 24, dependent on the view taken of rays 5, 6, and 7. The probability seems in favor of 24.

I think we may sum up then with regard to the number of rays constituting the first dorsal of *Mustelus canis* thus: it has usually 24 rays but this may vary to 23 or 25.

Nearly all the rods are segmented twice. The distal line of segmentations fails in the one or two orad; and the proximal, in the two or three aborad ones. Additional segmentations are very rare. What might be reckoned as such are seen in fig. 7, ray 4; fig. 6, rays 5 and 6. This makes an average of ·0+.*

The union of adjacent rays is rare. I estimate it at ·04 of the total possible concrescence.†

Shortening or reduction in size is likewise rare. We have first those questionable cases of which fig. 5, rod 3 seems the least questionable; and then plainer but less extreme instances in fig. 7, ray 5; fig. 9, ray 18; fig. 10, ray 7, then we have the usual shortening of the rays at the extremities of the fin. When these aborad rays shorten up, those next in front of them have a remarkable tendency to grow up under them. This is well shown in fig. 9.

Moreover when in the aborad rays the proximal joint becomes very short, it is sometimes divided into two lateral halves. This is the case for example in rod 22 of fig. 10.

Occasionally we have a minute piece or pieces of cartilage forming a tip to a ray. It cannot act as an extra joint, by giving increased flexibility to the ray. And it seems doubtful whether the origin of the two is to be referred to the same causes. Yet intermediate forms occur so as to raise the question whether they are to be referred to one or the other category. These tips seem to be exhibited in fig. 7, ray 4; fig. 8, rays 2 and 3; fig. 9, ray 3. This gives $\frac{4}{6 \times 24} = \cdot 0+$.

I now find the ratio of the proximal piece of the middle ray of each fin to the middle piece of the same. The average of these ratios is ·6. The method gives ·3 as the ratio between the terminal and middle joints.

We have then for the first dorsal of *Mustelus canis*:

Number of rays 24. Extra segmentations ·0+.

Concrescence ·04. Betipping ·0+. Ratio of proximal to middle piece of middle ray ·6. Ratio of distal to middle piece ·3.

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* The decimal is obtained by dividing the number of additional segments by the number of rays.

† The amount of concrescence between two adjacent rays is the ratio between the length of the union and the whole distance through which they are adjacent and might have united. The sum of these fractions divided by the number of rays less the number of fins, gives the estimate of the concrescence.

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Second Dorsal of Mustelus canis, Pl. L and LI, figs. 11–15.

Fig. 11 has 24 rays, fig. 12 has 22, fig. 13 has 23, fig. 14 has 24, fig. 15 has 24. Thus of the five examined three have 24, one 23 and one 22 rays. It seems probable that a wider examination would give us forms having 25 or more rarely 26. Thus we have for the second dorsal 24 rays with some slight variation.

Extra segmentations appear only in fig. 11, ray 5; and fig. 14, ray 2. This gives ·0+.

I estimate the concrescence at ·06. The concrescence is mostly confined to the proximal row, and is more frequent at the two ends than in the middle of the series.

Betipping is seen only in fig. 14, ray 23. This gives ·0+.

Ratio of proximal piece of middle ray to middle piece ·6.

Ratio of distal piece of middle ray to middle piece ·4.

The downward prolongation of the proximal parts of one or two of the orad rays is noticeable, being quite pronounced in all the cases except that represented in fig. 11.

The reduction of the rays is rare, but shown to an extreme extent in fig. 11, ray 1; and fig. 15, ray 1.

Anal of Mustelus canis, Pl. LI and LII, figs. 16–19.

Figs. 16 and 17 have each 18 rays. Fig. 18 has 17 or 18 according as the last ray is double or not. Fig. 19 has 18 or 19 under the same conditions. The great width of the last ray in the last two cases makes the larger number probable. Thus we have 18 as the normal number, with probably slight variations.

Extra segmentations are seen in fig. 17, ray 5; and fig. 19, ray 3. This gives ·0+.

The concrescence I estimate at ·09. Betipping occurs in fig. 16, ray 3. This gives ·0+.

Ratio of proximal piece of middle ray to middle piece is ·7.

Ratio of distal piece of middle ray to middle piece is ·6.

First Dorsal of Galeocerdo tigrinus, Pl. LII, fig. 20.

In this sole specimen there are 25 rays.

Extra segmentation occurs in 10, 20, 21, 22, 23, which gives ·2. It should be noticed that this extra segmentation is in each case here a doubling of the *proximal* line of segmentations.

Concrescence is estimated at ·06. Betipping is seen in 18, 19 and 25. This gives ·1.

Ratio of proximal piece of middle ray to middle piece 1·1.

Ratio of distal piece of middle ray to middle piece ·6.

Shortening is seen in 6 and 22. In the latter the proximal piece is excluded from the edge of the fin by *a*, the proximal piece of 23, and by the proximal piece of 21. The piece *a* consists of two lateral halves.

Second Dorsal of Galeocerdo tigrinus, Pl. LII, fig. 21.

Number of rays 13. Extra segmentation in 3, giving ·1.

Concrescence is estimated at ·01. Betipping, in 5, 8 and 13, gives ·2.

Ratio of proximal piece of middle ray to middle piece 1·3.

Ratio of distal piece of middle ray to middle piece ·6.

Anal of Galeocerdo tigrinus, Pl. LII, fig. 22.

Number of rays 12. Extra segmentation in 6, 7, 8 and 10, gives ·3.

Concrescence is estimated at ·05. Betipping in 4 gives ·1.

Ratio of proximal piece of middle ray to middle piece 1·1.

Ratio of distal piece of middle ray to middle piece ·7.

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First Dorsal of Eulamia Milberti, Pl. LII and LIII, figs. 23 and 24.

Number of rays 28 or 29.

As indicated by the numbering of the rays, I take number 21 in each figure to be a single ray, which has widened at the top, and been segmented in the way figured.

Extra segmentations in fig. 23, rays 20 and 21 (2 extra segmentations in the latter) in fig. 24, rays 4 and 21. This gives ·1.

Concrescence is estimated at ·09. Betipping is absent.

Ratio of proximal to middle piece ·9.

Ratio of distal to middle piece ·9.

Second Dorsal of Eulamia Milberti, Pl. LIII, figs. 25 and 26.

The number of rays differs remarkably in the two specimens, being 12 in the one and 16 in the other. It must, however, be remembered that the second dorsal has become very small and of very little physiological importance. Organs which have thus become functionless are peculiarly prone to vary. They thus secure more easily some other and new function. We will take the average number 14 as the normal one for the rays of this fin.

Extra segmentation occurs in fig. 25, ray 6 (twice), and in fig. 26, ray 3 This gives ·1.

Concrescence is estimated at ·09.

Betipping is absent.

Ratio of proximal to middle piece of middle ray ·8.

Ratio of distal to middle piece of middle ray ·5.

Anal of Eulamia Milberti, Pl. LIII, figs. 27 and 28.

Number of rays 17 or 18.

Extra segmentations in fig. 27, rays 7, 10 and 12 (twice in the latter); in fig. 28, twice in 12, once in 16, give ·2.

This implies a certain interpretation of the ambiguous rays 11 and 12 in figure 27. In fig. 28 we seem to have a plain case. Here the ray 12 is broadened at the top, and its distal piece divided by two intersecting cuts into four pieces. Ray 12 in fig. 27 is explained in the same way. Ray 11 is a little shortened, and excluded from the edge by the tips of 10 and of 12. This appears to me the most probable view of the case.

Concrescence is estimated at ·12. Betipping absent.

Ratio of proximal to middle piece of middle ray ·7.

Ratio of distal to middle piece of middle ray ·4.

First Dorsal of Sphyrna zygæna, Pl. LIII and LIV, figs. 29 and 30.

Number of rays 33 and 34.

I regard the three pieces at the extremity of 28 as belonging to that ray. It has been widened and divided like the instances in *Eulamia*.

Extra segmentations, fig. 29, rays 2 and 3, twice; rays 4 and 5; ray 28, twice; fig. 30, ray 28, twice, give ·2.

Concrescence is estimated at ·07. Betipping absent.

Ratio of proximal piece of middle ray to middle piece ·7.

Ratio of distal to middle piece of middle ray 2·4.

In fig. 29 the proximal line of segmentation fails in rays 9–17, except in the joined rays 11 and 12 where it is present. In fig. 30 it fails in rays 8–16.

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Second Dorsal of Sphyrna zygæna, Pl. LIV, fig. 31.

Number of rays 14. Extra segmentations amount to 1·0.

Concrescence is estimated at ·07.

Betipping is absent.

Ratio* of proximal to middle piece of middle ray ·3.

Ratio of distal to middle piece of middle ray ·3.

The last ray, both in the second dorsal and the anal, is large and round.

Anal of Sphyrna zygæna, Pl. LIV, fig. 32.

Number of rays, 27.

Extra segmentations 8 (1), 9 (1), 10 (1), 11 (1), 12 (1), 13 (1), 14 (1), 15 (1), 16 (1), 17 (1), 18 (2), 19 (1), 20 (1). This gives ·5.

Concrescence is estimated at ·03. Betipping none.

Ratio of proximal to middle piece of middle ray ·4.

Ratio of distal to middle piece of middle ray ·3.

First Dorsal of Eugomphodus litoralis, Pl. LIV, and LV, figs. 33–39.

Specimens figured in figs. 33, 34 and 36 have plainly 16 rays. Those in figs. 37 and 39 have plainly 17. Those in figs. 35 and 38 have 16 separate rays, but the last is quite broad. Where we have plainly 17 rays, figs. 37 and 39, the last two rays have united with the exception of their distal joints. We may fairly conclude that figs. 35 and 36 present a more complete concrescence of those rays. We have then as the number of rays 16 or 17, the former in three cases, the latter in four.

Extra segmentation is estimated at ·7. Concrescence is estimated to be ·05.

Betipping reaches ·4, each separate piece being counted. These small nodules of cartilage sometimes seem very evidently to be a continuation of a ray upon the following ray. But not infrequently they seem to be scattered rather irregularly along the edge of the fin. It will be noticed that they are most frequent in the orad part of the fin, though not on the first two or three rays. It is very probable that the estimate of their frequency should be higher than given, for they are easily lost in the preparation of the specimen.

Ratio of proximal to middle piece of middle ray ·5. Ratio of distal to middle piece of middle ray ·4.

Second Dorsal of Eugomphodus litoralis, Pl. LV and LVI, figs. 40–46.

In fig 42 we have 16; in 45, 17; in 40 and 46 we have 18 rays. These are all plain cases. Fig. 43 exhibits 17 rays, but raises a suspicion of 18 by the breadth of the last ray. Fig. 41 gives 16 or 17, probably the latter. Fig. 44 leaves us in doubt between 14, 15 and 16, with, as it seems to me 15, the most probable. We may take 17 as the normal number. As far as the evidence here goes the second dorsal is more liable to vary than the first. We see that in each the greater the number of rays, the greater is the amount of concrescence.

Extra segmentation amounts to ·6.

Concrescence amounts to ·10. Betipping amounts to ·4.

Ratio of proximal to middle piece of middle ray is ·3.

Ratio of distal to middle piece of middle ray is ·3.

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* Where a segmentation is double the point half way between the joints is taken as the limit between the middle and extreme piece. Where it is triple the middle segmentation is taken.

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Anal of Eugomphodus litoralis, Pl. LVI and LVII, figs. 47–50.

The number of rays is 21 in figs. 47 and 49. In 48 we have 22, and in 50, 20 rays. We may take then 21 as the normal number.

Extra segmentations amount to ·4.

Concrescence is estimated at ·10. Betipping amounts to ·3.

Ratio of proximal to middle piece of middle ray 4.

Ratio of distal to middle piece of middle ray ·5.

We may sum up the results of this investigation of the resemblances and differences of the forms so far examined in the following table.

		No. of rays.	Extra segmentations.	Con-crescence.	Betipping.	Ratio of prox. to mid. piece of mid. ray.	Ratio of distal to mid. piece of mid. ray.
First Dorsal.	Eugomphodus	17	·7	·05	·4	·5	·4
	Mustelus	24	·0	·04	·0	·6	·3
	Galeocerdo	25	·2	·06	·1	1·1	·6
	Eulamia	29	·1	·09	·0	·9	·9
	Sphyrna	34	·2	·07	·0	·7	2·4
Second Dorsal.	Eugomphodus	17	·6	·10	·4	·3	·3
	Mustelus	24	·0	·06	·0	·6	·4
	Galeocerdo	13	·1	·01	·2	1·3	·6
	Eulamia	14	·1	·09	·0	·8	·5
	Sphyrna	14	1·0	·07	·0	·3	·3
Anal.	Eugomphodus	21	·4	·10	·3	·4	·5
	Mustelus	18	·0	·09	·0	·7	·6
	Galeocerdo	12	·3	·05	·1	1·1	·7
	Eulamia	18	·2	·12	·0	·7	·4
	Sphyrna	27	·5	·03	·0	·4	·3

The changes which are presented consist chiefly in concrescence. This takes place in various ways. The proximal portions more frequently unite than the distal, but we may have concrescence of the distal points while the proximal portions are separate. The reduction of rays is exhibited in all degrees from the slightest shortening to the extreme degree shown in Pl. LVII, fig. 50, ray 9.

It is perhaps noteworthy that the changes which would have sufficed to differentiate the fin of one species from another, if they had continuously advanced for a few generations, are changes which in no slight degree are now taking place between parents and children. I mean changes of number, segmentation, concrescence, and relative lengths of parts of rays.

I exhibit some figures of the dorsal fins of *Squalus*, Pl. LVII, figs. 51–56, and Pl. LVIII, fig. 57, *Raia*, Pl. LVIII, figs. 58, 59, and *Myliobatis*, Pl. LVIII, fig. 60. It is perfectly certain that they have been derived from a series of parallel rays, and that the principal process of change has been in the way of concrescence. They tell their own story as far as it can be told without the investigation of other closely related forms.

The last ray in the first dorsal of *Raia*, Pl. LVIII, fig. 58, is remarkable, and it is represented in the second dorsal, fig. 59, by a series of separate nodules.

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I also add figures of the dorsal, Pl. LVIII, fig. 61, and anal, Pl. LIX, fig. 62, of *Acipenser*. These are very similar to the simpler shark dorsals. It will be noticed that we have the predominant division into three pieces, but the terminal piece is very short.

Conclusions regarding Median Fins.

The primordial median fin-rays in whatever form they occur are derivatives from a series of simple parallel chondroid rods, which grew up in the median fold in total independence of the cartilaginous arches above and below the notochord. These earliest representatives of these parts were from two to four times as numerous as the vertebræ opposite them. In the Gnathostomi true hyaline cartilage replaced the lower form of tissue seen in *Myxine* and *Petromyzon*.

Segmentation and concrescence, as well as reduction in size, were common changes in the Gnathostomes, and here a division into three parts is the usual though not invariable rule.

Hence it is seen that Gegenbaur's* statement that, in their simplest form, the primordial fin-rays are mere prolongations of the neural spines is incorrect. It has been demonstrated that this was not the *earliest* form. The Dipnoans, however, seem to offer an example where the primordial median fin-rays are mere prolongations of the neural spines. They demand a moment's consideration.

True neural spines are first found in the Ganoids. They are absent in the Elasmobranchs and Agnathostomi. Pl. LIX, fig. 63 represents the projection of a section of a vertebral segment of Acipenser cut through the middle line of the arch and neural spine. As the latter slopes backward, it is considerably foreshortened in the figure.

Now the cartilaginous arches springing from the sheath of the notochord pass upwards to lay themselves on each side of the fibrous cord *b*, and here they spread inward to meet one another on the ventral side of *b*, and also prolong themselves above to almost or quite meet, and then they are followed by the dorsal spine *a*, from which they are separated by a segmentation.

Now the cord *b*, which is the same as the *ligamentum longitudi-*

* Grundriss der Vergleichenden Anatomie, 1873, p. 488. Gegenbaur's assertion in the same place that they usually correspond in number to the vertebræ opposite, is very strange. We have already seen that they do not do this in the earlier and more significant forms; and the statement of Gegenbaur would decidedly misrepresent what we find, for example, in the figures of fish skeletons in Agassiz's Poissons Fossiles.

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nale in *Ceratodus*,* is also, without the slightest doubt, homologous with the chord which lies entirely above the neural arch in the Elasmobranchs. The peculiar fibrous character is almost exactly the same in each. We have seen that this cord in the sharks is in all probability homologous with the fibrous fatty ridge-pole of the neural canal in *Petromyzon*. Thus while in the latter the arches of one side and the other are entirely separate, in the sharks they have spread beneath the ligament so as to meet, and in the higher Gnathostomes they have also joined above it, or nearly joined, for the origin of *a* is still to be discussed.

There are two possibilities with regard to the neural spine *a*. Either it is formed by the union of a median fin-ray with the neural arches, the ray thus constituting the keystone of the arch, or else by the union of the neural rods from each side and their prolongation dorsad.

But the junction between *a* and *b* is quite close; the neural spines correspond in number and position with the lateral parts of the arch; while fig. 61 shows conclusively the absolute independence of neural spines and primordial median fin-rays.

The second of the two possibilities is then the true one. *Thus neither are median fin-rays derived from neural spines, nor neural spines, where they occur, from primordial fin-rays.*

But the cartilaginous supports of the median fold in the Dipnoans are very long and segmented. They are simply elongated neural spines and are not primordial fin-rays in any homological sense. If they were formed by the reduction in number of the primordial fin-rays and their coalescence with the neural spines it is impossible that we should not have here and there an extra one, and some evidence in the case of others of such a junction. But there is nothing of the kind, either in the descriptions of Günther in the case of *Ceratodus*,† or in those of Owen‡ and Peters§ in that of *Protopterus annectens*, or in those of Bischoff‖ in that of *Lepidosiren paradoxa*. Gün-

* Günther's Description of *Ceratodus*, Phil. Trans., vol. clxi, pt. ii, Pl. XXXVIII, Figs. 3–9, é.

† Phil. Trans., vol. clxi, pt. ii, 1871. Günther, Description of *Ceratodus*. In Günther's fig. 2, Pl. XXX, the proximal joint of the 14th neural spine seems to bear two "interneurals," one orad of the other. But as no notice is taken in the text of this, which would be a very noteworthy fact, if it were fact, and as the description of these parts there given is such as would demand a notice of this exception, it is evident that it must be an inaccuracy in the figure.

‡ Trans. Linnean Soc., vol. xviii, pt. iii. Owen, Description of *Lepidosiren annectens*.

§ Müller's Archiv., 1845. Peters, Ueber einen dem *Lepidosiren annectens* verwandten, Fisch von Quellimane.

‖ Ann. Sc. Nat., xiv, 1840. Bischoff, Sur le *Lepidosiren paradoxa*.

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ther's denomination, then, of the ultimate and penultimate joints of the neural spines of *Ceratodus* as "interneural first" and "interneural second," is ill chosen, and rests on a mistake in homology. That great genetic group, then, consisting of Dipnoi, Amphibia and Amniota, seems to have entirely lost those primordial median fin-rays which appeared so early and are found even in *Myxine.*

Limb-skeleton of Air-breathing Vertebrata.

In 1864, Gegenbaur* set forth the splendid results of a widely extended investigation of the limb-skeleton of the air-breathing vertebrates. Herein was established the typical form of these parts for this large group, consisting of Amphibia and Amniota. Inasmuch as there is no doubt of the natural, that is the genetic, character of this group, and inasmuch as it is marked out from all other vertebrates by the development of a *fenestra ovalis* and the modification of the proximal part of the second post-oral, or hyoid, arch into a stapes in connection therewith, I venture to use the name Stapedifera in place of the circumlocutory air-breathing Vertebrates. For the Stapedifera, then, the typical limb-skeleton was established; typical in the sense of the older anatomists, as that ideal form from which we could in our minds easily derive the various actual forms now living; but typical also in the newer sense, as that actual form, the limb-skeleton of the latest common ancestors of all Stapedifera, from which have been developed the corresponding parts in all living Stapedifera.

The same form belongs to both fore and hind limbs. Using the names applicable to the former, we have, as is well known, humerus, radius and ulna, radiale, intermedium and ulnare, a centrale, and then set around these, five carpalia followed by their metacarpals and phalanges. Moreover, the strong suspicion of the double nature of the centrale, as evidenced in the descriptions of *Cryptobranchus Japonicus*,† by Schmidt, Goddard, and J. Van der Hoeven, is later, 1865, confirmed by the careful observations and clear presentation of the anatomy of that animal by Hyrtl.‡ The Ichthyosaurs§ and

* Gegenbaur, Untersuchungen zur vergleichenden Anatomie der Wirbelthiere, Hft. 1, Carpus und Tarsus.

† Gegenbaur, Unters., Hft. 1, p. 57.

‡ Hyrtl, Schodiasma anatomicum. 1865. Gegenbaur, Unters., Hft. 2, p. 165.

§ Gegenb., Unters., Hft. 2, p. 165, and Jena Zeitschr., Bd. v, Hft. 2, 1870. Gegenbaur, Ueber das Gliedmaassenskelet der Enaliosaurier. In this last a further modification is made in the recognition of the pisiform as the remains of a sixth row, and as being an essential part of the carpus and not merely a sesamoid bone.

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Plesiosaurs give too their unambiguous evidence in the same direction.

This limb of the latest common ancestors of the Stapedifera, this typical limb of that group, has been named by Huxley, *chiropterygium*,* and the term will be found convenient.

The Chiropterygium and the Fins of Fishes.

The homology between the paired fins of fishes and the limbs of Stapedifera has long been recognized; but the special homologies of the skeletal parts of each has been the subject of much controversy. For a historical sketch of the various divergent opinions endorsed by the highest authorities I must refer to the second volume of Gegenbaur's Untersuchungen.

Two pairs of limbs are found throughout the great genetic group of the Gnathostomi. The chiropterygium having been established, the determination of that earlier form typical for all the Gnathostomi became a more pressing question.

To an answer to this question the investigations of Gegenbaur now begin to lead.

The first part of the second volume of the Untersuchungen discusses the shoulder girdle, and the result is that we are now able to trace clearly and surely the primordial shoulder girdle, the scapulo-coracoid, throughout the Vertebrata. We are no longer in doubt as to what part of the fish fin and girdle corresponds to limb and what to girdle of the Stapedifera. The results of Gegenbaur's work with which we are here concerned were confirmed by the later but independent researches of Parker.†

Development of the Archipterygium Theory.

This preliminary question having been satisfactorily answered, the derivation of the chiropterygium is next attempted.

The second half of the second volume of the Untersuchungen (1865) takes up the pectoral fin of fishes. Quite a number of very excellent figures of numerous Elasmobranchs, Ganoids and Teleosts are given, and the limb skeleton of *Protopterus* is discussed. The conclusions drawn are as follows:

We may take as the most generalized form of limb that of the Elasmobranchs, where its various parts are most plainly presented in

* Proc. Zool. Soc. London, 1876, pt. i, p. 56. T. H. Huxley, On *Ceratodus Forsteri*.

† Parker, Shoulder-girdle and Sternum. Ray Soc., 1868.

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the rays. It is divided into three parts, Propterygium, Mesopterygium and Metapterygium. Each of these consists of a *basale*, which articulates with the shoulder girdle, and a number of rays set on its edge.

The fin of *Protopterus* is derived from this by the destruction of the pro- and mesopterygium. The metapterygium is here represented by the long articulated rod, which alone remains in *Lepidosiren paradoxa.* The row of cartilages along its sides are the metapterygial rays.

In the Ganoids, *Polypterus* alone has the three divisions represented. In this, neither metapterygium nor propterygium bears rays. These are confined to the mesopterygium, which is excluded from the articulation with the shoulder girdle.

In the other Ganoids the propterygium fails. Between the mesopterygium and metapterygium a number of rays are brought into articulation with the shoulder girdle, resembling what is seen in some of the Rays.

The Teleosts in the main resemble this second group of Ganoids.

The chiropterygium is derived from the metapterygium alone, and thus resembles the limb of *Protopterus.* The fore limb will serve as an example. The *Stammreihe* or *basale metapterygii* is presented by the humerus, radius, radiale, carpale radii, the metacarpal and phalanges of the thumb. The other bones are the rays belonging to this, and their arrangement will be best understood by looking at the *Ichthyosaurus* limb, fig. 70. In 1870,* Gegenbaur published his explanation of the limb of the Enaliosaurs. The unbroken lines in fig. 70 of *Ichthyosaurus* exhibit his view of the relations of the fin with that of fishes. This may be regarded as closing the first stage of the development of the theory in Gegenbaur's publications.

The second immediately opens. For in the next number† of the Jena Zeitschrift there appears an extended article on the ventral fins of Elasmobranchs. The previous view is modified as follows. The fin-skeleton of the latest common ancestors of all Gnathostomes, is represented pure and simple in the fore limb of *Protopterus annectens*, and with only slight modification in the ventral fins of Elasmobranchs. It now has a name given to it. It is called *archipterygium.* There is a limb girdle, complete ventrad. On each side is articulated to this

* Jenaische Zeitschr., Bd. v, Hft. 3. Gegenbaur, Ueber das Gliedmaassenskelet der Enaliosaurier, Feb., 1870.

† Jen. Zeitschr., Bd. v, Hft. 4. Gegenbaur, Ueber der Gliedmaassen der Wirbelthiere im Allgemeinen und der Hintergliedmaassen der Selachier insbesondere, May, 1870.

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the stem-row (*Stamm-reihe*), a long tapering many-jointed cartilaginous rod which bears on the outer side a series of rays. This evidently calls for no change of view regarding the Enaliosaurs or Stapedifera. But the fin-skeleton of fishes exhibits everywhere, except in *Protopterus* and *Scymnus*, a slipping off of the rays from the stem-row and their articulation with the girdle, and very commonly their articulation with one another and considerable fusion (concrescence).

Still another change awaits the primordial limb, even the named archipterygium. In 1871, Günther* published his description of *Ceratodus*. Here the stem-row has a series of rays down each side. The archipterygium is modified to accord with this in the Jena Zeitschrift published April 22, 1872,† where Gegenbaur adopts the "*Biseriale Archipterygium*" as the parent form, and attempts to show that there are some traces of the median row of rays in the pectoral fins of some Elasmobranchs. With the exception of *Ceratodus* and the questionable exception of these Elasmobranch pectorals, the biserial has been everywhere reduced to the uniserial form, and still further reduced as heretofore explained.

In the third volume of the Untersuchungen, dated May, 1872, a suggestion‡ is made of the possible origin of the Archipterygium and the limb-girdles. They are assimilated to the branchial arches and their diverging rays, where rays move up upon, and articulate with, the longest middle ray. It is but justice to say that the suggestion is a little vaguely and hesitatingly made.

In confirmation of Gegenbaur's views, Bunge,§ in 1874, published a further investigation of the pectoral fin of Elasmobranchs, showing a number of rays which might be regarded as median, in several species not examined by Gegenbaur. Finally, in 1876, Huxley‖ took up the question, and, while he accepted the archipterygium, he modified the interpretation of a large number of the forms.

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* Proc. Roy. Soc., 1871, p. 378, and more fully, with a figure of the fin-skeleton, in Ann. and Mag. of Nat. Hist., March, 1871. To these Gegenbaur refers, Jen. Zeitschr., Bd. vii, Hft. 2, p. 132, note. But a much fuller description is given by Günther, Phil. Trans., vol. clxi, pt. ii, pp. 511–572. This was published early in 1872.

† Jen. Zeitschr., Bd. vii, Hft. 2, pp. 131–141. Gegenbaur, Ueber das Archipterygium.

‡ Gegenbaur, Unters., Hft. III. Kopfskelet der Selachier, p. 181, note. 1872.

§ A. Bunge, Jena. Zeitschr., Bd. 8, Hft. 2, 1874. Ueber die Nachweisbarkeit eines biserialem Archipterygium bei Selachiern und Dipnoern. Bunge also calls attention to the fact that the fringing rays in *Protopterus annectens* are on the median (i. e., ventral,) side of the axis, and not, as in Elasmobranchs, on the lateral (i. e., dorsal,) side.

‖ T. H. Huxley, Proc. Zool. Soc. Lon. for 1876, Pt. 1. On *Ceratodus Forsteri*.

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Most of the modifications introduced by Huxley, though perhaps not all, spring from a question which is independent of any theory with regard to the skeleton, archipterygium or other, but which, superior to them, must determine the application of them to the passage from the fish limb to that of Stapedifera.

If an Elasmobranch pectoral fin, for example, of *Mustelus*, be removed and laid on the corresponding hand, with the propterygial edge toward the thumb, and the metapterygial edge toward the little finger, then the ventral surface of the fin will look in the same direction as the palmar surface of the hand. But if it be turned over so that the metapterygial edge corresponds to the thumb and the propterygial to the little finger, then the dorsal surface of the fin will correspond to the palmar surface of the hand.

One or the other of these views must be taken. There is no third possible. Huxley takes the first, Gegenbaur the second. This, however is no new question and no new difference of opinion. Cuvier, following Bakker, named the two ossifications of the scapulo-coracoid which are so generally found in osseous fishes, radius and ulna. Owen simply reversed this nomenclature and Mettenheimer followed him. The question was the same as now respecting the homologies of faces and edges of fin and limb. On the one side, then, we have Bakker, Cuvier and Huxley; on the other, Owen, Mettenheimer and Gegenbaur. The weight of evidence seems to me to be in favor of the view entertained by the latter group, namely, that the metapterygial edge of the fish fin corresponds with the radial or thumb side of the hand, and consequently that the dorsal surface of the fish fin is the palmar (or plantar) surface. But I have no new facts.

By reviewing Gegenbaur's work it will be seen that this theory of his rests upon the form of the limbs in the Elasmobranchii and Dipnoi. In the former group it is the hind limbs which furnish nearly all the evidence. The fore limbs (pectorals) are brought in merely to testify to the *biserial* character of the archipterygium, of which no Elasmobranch *ventral* gives a sign. That is to say, the ventrals having testified to the archipterygium, and that having been accepted, the pectorals find use for themselves in showing that it was fringed down the median as well as the lateral side. If then the same form of limb is found in Elasmobranch and Dipnoan, the same form was undoubtedly possessed by their common ancestors. But as their common ancestors were also undoubtedly common ancestors of all Gnathostomes, therefore all Gnathostome limbs must have been derived from this form.

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The testimony of the Ganoids and Teleosts seems to me to be somewhat adverse to the theory. Again it is impossible to think that that of the Stapedifera can be very clearly in its favor, when Huxley, while accepting the archipterygium as the parent form, gives an explanation of the chiropterygium entirely distinct from and utterly inconsistent with that of Gegenbaur.

Any opinion adverse to the archipterygium theory will have difficulty in maintaining itself, so long as it does not show that the resemblance between the fins of sharks, and those of Dipnoi is a merely superficial one, and is not able to suggest how a certain show of resemblance might have arisen in two entirely distinct and different series of developments.

Another View of the Origin of Vertebrate Limbs.

Into competition with this theory, which sees in the fin of *Ceratodus* that from which all other limbs have been derived, I bring a second which sees in the same only a special development peculiar to the Dipnoi. It is this.

As the dorsal and anal fins were specializations of the median folds of Amphioxus, so the paired fins were specializations of the two lateral folds which are supplementary to the median in completing the circuit of the body. These lateral folds, then, are the homologues of the Wolffian ridges, in embryos of higher forms. Here, as in the median fins, there were formed chondroid and finally cartilaginous rods. These became at least twice segmented. The orad ones, with more or less concrescence proximally, were prolonged inwards. The cartilages spreading met in the middle line, and a later extension of the cartilages dorsad completed the limb girdle.

If now we seek to determine the form of limb for the Protognathostomi, that is to say, for that time for which the archipterygium in its entirety is proposed, we should propose this.

The limbs of the Protognathostomi consisted of a series of parallel articulated cartilaginous rays. They may have coalesced somewhat proximally and orad. In the ventral pair they had extended themselves mesiad until they had nearly or quite met and formed the hip girdle. They had not here extended themselves dorsad. In the pectoral limb the same state of things prevailed, but was carried a step further, namely, by the dorsal extension of the cartilage constituting the scapular portion, thus more nearly forming a ring or girdle.

This theory naturally divides itself into two parts, namely, the derivation of the Gnathostome limb from a series of simple parallel

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cartilages, and the derivation of the latter from the lateral folds of *Amphioxus.*

Though the last mentioned portion of the theory would derive considerable strength from the establishment of the first, it is not a necessary consequence of it, and the first might be true even if the last were false. If the last be true, of course the first must be true.

The establishment of the derivation from the lateral folds of *Amphioxus* is made difficult from the absence of limbs or anything representing them in the two groups which (in a sense) stand between *Amphioxus* and the Elasmobranchs, namely, the Myxinoids and Lampreys.

As will be seen, it assumes the essential correctness of Huxley's suggestion with regard to the relation between the folds which grow down to inclose the atrial cavity of *Amphioxus* and the body walls of higher vertebrates. But it is equally consistent with Huxley's* entire suggestion, as put forth by him, or with Ray Lankester's† modification of it.

On the other hand, it is inconsistent with Kowalewsky's‡ view of the homology between these and opercular folds. This must perhaps be considered still an open question, though Rolph's§ arguments on the other side seem to me of much less weight than they do to Semper.‖

Yet even if this homology with the lateral folds should have to be given up (the embryology of the Marsipobranchs will throw considerable light on it), the very frequent occurrence of the formation of external lateral folds parallel to the axis of the body in the bilateral animals in general and in the Vertebrates in particular, renders it quite possible that the paired fins may have had a *similar* origin. At present, however, I am strongly of the opinion that they are

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* Journ. of Linn. Soc., vol. xii, No. 59, May, 1875. Huxley, Classification of the Animal Kingdom.

† Quarterly Journ. of Micr. Sc., New series, No. 59, July, 1875. Ray Lankester, New Points in the Structure of *Amphioxus.*

‡ Mem. St. Petersb. Acad., VII Series, tome xi, No. 4, 1867. A. Kowalewsky, Entwickelungsgeschichte des *Amphioxus lanceolatus.*

§ Sitzungsberichte der Naturforschenden Gesellschaft zu Leipzig. Jahrg. II, No. 1, Jan. 29, 1875. Rolph, Unters. über den Bau des *Amphioxus lanceolatus.* See also for a complete account of his investigations, under the same title, Morph. Jahb., Bd., ii, Hft. 1, 1876.

‖ C. Semper, Die Verwandtschaftsbeziehungen der gegliederten Thiere, 1875, p. 317. (Sep. Abdr. aus Semper: Arbeiten a. d. Zoolog-zootom Institut zu Würzburg, Bd. II).

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derived directly from the lateral folds of *Amphioxus*. These in their turn may be referred to a reduplication of the process which has already formed the atrial space, but which is not carried so far here, in the lateral folds.

Ventral Fin of Acipenser.

The ventral fin of a young specimen of *Acipenser brevirostris* is exhibited in Pl. LIX, figs. 64 and 65. The fin of one side is separate from that of the other, no synchondrosis uniting the two halves of the girdle. But the part *b* approaches closely its fellow. The same separateness of the two sides obtains in the shoulder girdle. The composition of the fin is perfectly evident. Beginning at the aborad end of the row, we have first three separate and parallel rays. The proximal joints increase in length from the first to the last of the three. In the remaining rays these basal joints, increasing still more in length, have united with each other to form the large pelvic piece *ab*. The composite nature of this is confirmed by the groovings of the surface, which extend about a centimeter before they finally fade out.

The iliac process, *a*, is half a centimeter high.

In fig. 64 the proximal joint of the penultimate ray grows up a little under the last ray, in the manner familiar in the median fins. It does not happen to occur in the fin of the other side, fig. 65.

The predominant three-fold division obtains. But the penultimate ray in fig. 64 has a tip or an extra segmentation, and *c* is without the distal segmentation.

The breadth and the outline of *c* raises the suspicion of its double character.

I have had no opportunity of examining other Ganoid fins, and this one of *Acipenser* seems, on the whole, that which most nearly approaches the parent form of the Gnathostomes. But while in the independence of the two sides, in the separateness of the rays, and the simple segmentation, it gives us the early form more complete than is elsewhere found, in the number of rays and in the absence of the iliac process the shark ventrals are less advanced.

Elasmobranch Ventrals.

We now turn to the derivation of the ventral fins of sharks, one of the two abutments of the broad span of the archipterygium theory.

The series of steps by which I conceive them to have been derived

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from the row of parallel rays is presented in woodcuts A, B, C and D.

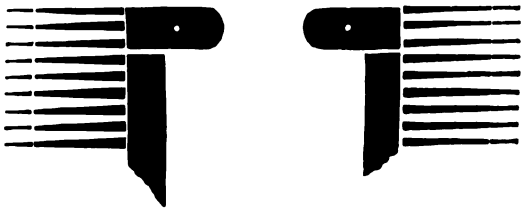
A

B

The kind of change invoked is simply concresence, with some spreading of the cartilage. The former of these processes is abundantly shown in the case of the median fins, while something of the latter process is seen in Pl. LI, figs. 12, 13, 14, 15; Pl. LIII, fig. 27; Pl. LVI, fig. 46; Pl. LVII, fig. 49. And it is noteworthy that here the rays which prolong themselves proximally are the orad ones, just as they are in the ventral fins. As for the concrescence, this has been carried much farther in the dorsal fins of *Squalus Americanus*, *Myliobatis* and *Raia levis* than it has in the shark ventrals. It is barely possible that the definiteness and constancy of the concrescence in the latter may be in whole or in part determined by the copulatory function of the last part of the fin in male Elasmobranchs.

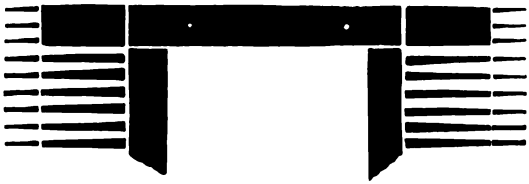
While the derivation of the ventral fins is thus easy from a series of parallel cartilages, we find much greater difficulty in the case of some of the median fins, in *Raia levis*, for example, which is, unless my own preconceptions deceive me, a far better case of a biserial archipterygium than any furnished in the paired fins, aside from *Ceratodus*.

C



Indeed I may state that the origin of this paper lay in an observation of a fin of another species of *Raia*, not however well enough preserved

D



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for drawing. The very striking similarity to the uniserial archipterygium raised the question whether the median fins, at any rate the dorsal and anal fins, might not have arisen from the same archipterygium. The result of my investigations was a decided negative. It has been absolutely proved that they did not so originate, and the way in which they did originate has been clearly shown.

While then, on the one hand it has been shown that the development of a pair of fins, whose skeleton consists of a series of parallel rays clothed on each side with a layer of muscle, as a specialization of the lateral folds (metapleura of Ray Lankester) of *Amphioxus*, contains no steps which have not been taken in the same animals in the case of the median fins, so also it has been shown that the development, of the ventral fins and the pelvic girdle of sharks from such a series exhibits no processes or kinds of change which are not also exhibited in the median fins of those same fishes. When we contrast the changes from a series of parallel rays to the completed ventral fin of the shark, as it has been given above, with the changes which Gegenbaur supposes to have made it out of the archipterygium, namely, the stripping off of every one of the median rays, for no sign of them is ever found in the ventral fins aside from the Dipnoi, and the slipping off of the orad portion of the rays to immediately articulate with the shoulder girdle, I hardly think that those changes of his will appear so well evidenced as these changes which I believe to have taken place. And when the utter darkness that covers the development of the archipterygium itself (for it does not seem fair to the archipterygium to make much account of the suggestion of Gegenbaur respecting the branchial arches, Unters., Hft. iii, p. 181, note) is contrasted with the familiar changes which would have brought these Selachian fins out of the lateral folds *Amphioxus*, I hardly think the advantage can lie with the archipterygium.

*Homodynamism of Median and Paired Fins.**

Let us compare the ventral with the dorsal fins, say in *Mustelus canis.*

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* Since this paper was written, I have found a paper of Humphrey's on the Homological Relations of Mesial and Lateral Fins of Osseous Fishes, Journ. of Anat. and Phys., Nov., 1870. Here a comparison between the fins in question is made in the case of the Pike, and the "Iliac" or "Pubic" bones in osseous fishes are assimilated to the interneural spines or to the prominal part of them. Goodsir had made some earlier comparisons without valuable result. See Anatomical Memoirs, vol. ii, p. 106.

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In each there is a layer of muscle on each side of the cartilaginous skeleton; this flat mass is in each divided into separate muscles by septa running from between the skeletal rods straight to the integument, in the way exhibited in Pl. LIX, fig. 66; in each these muscles develop a tendon in their middle plane parallel to the sides of the fin; this is inserted in the fascia over the terminal cartilages where the horny fibers begin. These last are the same in nature and arrangement in each.

The skeletal elements remain (see Pl. LIX, fig. 67). We have a short terminal piece in each, then comes a longer middle piece. There is left in the median fin a proximal row of cartilages, for the most part separate, which are again much shorter than the middle pieces. In the ventral fin the solid basale metapterygii and a half of the girdle correspond to these in every particular, except in not being of separate rays. The similarity between the two fins is complete except in a single particular. And that exception would be removed by a process which is familiar in both, namely concrescence. Even as they stand, I think that a ventral fin with one half the girdle resembles the dorsal more than it does the pectoral.

A certain amount of similarity warrants us in inferring an earlier state when the similarity was greater. It has been proved that at that earlier time the median fins were composed of separate rays. The greater similarity, then, can only be attained by the resolution of the basale metapterygii into its component parts.

In the Elasmobranchs, as is well known, the primordial fin-skeleton is supplemented in both median and paired fins by the well known horny fibers. In the higher fishes these are replaced by the dermal rays. The presence of horny fibers in the adipose fin of the salmon, shows that the horny fibers were the earlier form. Therefore the same changes have been taking place in the median and paired fins at the same time. The same general result, i. e., of concomitant variation in median and paired fins is confirmed in the sub-groups of Elasmobranchs, with regard to minor changes in the primordial skeleton.

The homodynamism of median and paired fins comes out strikingly in *Centrina Salviani*. Here, according to descriptions, a fold of skin is raised along the median line of the back, recalling the early continuous fold of skin along the back in *Amphioxus*. But similarly there appear two folds of skin along the sides, recalling the continuous lateral folds of *Amphioxus*. It is at once a proof of the homodynamism of the two, and a confirmation of the views here presented of their origin.

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The Dipnoan Fin.

In the Teleosts and Ganoids, which are in a sense intermediate between the Dipnoi and Elasmobranchii, the limb skeleton has been comparatively little modified from the form in this paper set forth as the typical limb skeleton of Gnathostomi. Scarcely any other processes than reduction of the number of rays and concrescence are concerned.

Now in the fin of *Ceratodus* the archipterygium form has certainly been developed, and if the previous views be correct, it has been developed from this series of parallel rays.

Günther* has suggested one way, and Gegenbaur† another, in which a row of parallel rays might transform themselves into an archipterygium form. But it is possible that these fringing rays are new developments. They look very much like it in *Protopterus.* Peters‡ has called attention, in this connection, to the similar structure of the dorsal finlets of *Polypterus*, and these might throw some light on the subject. For myself, I am strongly inclined to suspect that the three portions of the second piece of the limb of *Ceratodus*, which Günther describes, indicate three fin-rays, and that the feathering of one of these is a later development. The fact that Huxley could find no sign of division in his specimens seems of little weight in view of the complete fusion which we know takes place here and there in *median* fins.

The Limb of Stapedifera.

Pl. LX, fig. 70 (*Ichthyosaurus*,) exhibits my view of the composition of the limb of air-breathing vertebrates. The dotted lines indicate the separate rays of which it is composed. But there are other ways in which it may have been derived from a series of parallel rays, and I offer this merely as the most probable interpretation so far as I can now see. Fig. 71 does the same for the hind limb of *Cryptobranchus Japonicus.* The curvature of the rays has been exhibited in a marked degree quite frequently in Elasmobranch median fins.

The Innervation of the Paired Fins.

I have made complete and definite observations of the innervation only in a single case, namely in the pectoral fin of *Mustelus canis.* This fin is supplied by the first 15 myelonal nerves together with a very small branch from the vagus. The simplest condition is seen in the aborad nerves.

* Phil. Trans., vol. clxi, Pt. ii, p. 534. † Unters., Hft. iii, p. 181, note.
‡ Muller's Archiv, 1845, p. 3.

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The aborad four (12–15) branches, coming directly from the myelon, advance, each by itself, close to the metapterygium, where they each bifurcate, sending one branch to the dorsal and the other to the ventral side of the fin. The next four (8–11) unite to form a rather loose plexus, which separates again into four nerves, which then sub-divide in the same way as the last mentioned four, except that the orad of these behaves a trifle differently in a manner hereafter to be described.

Now the first seven nerves unite with one another and with the minute branch of the vagus in the following way. The vagal branch emerges from the skull with that nerve, but already rolled up as a separate branch and easily to be separated from it; this joins the first myelonal nerve and this the second, and their sum the third, and so on, until we have a cord formed of the vagal and first seven myelonal branches. This sends off a branch to the muscles and integument in front of the shoulder girdle, but the main part of it proceeds on its way to enter the foramen called by Gegenbaur, *Eintrittsöffnung*, and then divides within the cartilage of the girdle in the way which he has described, and similarly to the aborad nerves, which he has left unnoticed, sending one branch to the dorsal and the other to the ventral muscles of the fin. Now the eighth nerve sends off its ventral branch like those aborad of it, but the dorsal branch enters the entrance-opening with the cord of the vagus and 1–7 spinal nerves; but it does not unite with this cord till after the latter has divided, and then unites with its dorsal branch and emerges with that from the cartilage on the dorsal side of the fin. In another specimen, this dorsal branch of the eighth nerve enters the cartilage by a minute separate foramen, but unites with the dorsal branch of the anterior cord, as in this case.

As stated, my observations in the other cases have not been as thorough, and I cannot give the number of nerves, but in the ventral fin the arrangement is as follows. A number of nerves are gathered together to form the orad cord. This, on coming opposite the foramen in the pelvic girdle, divides and sends its branch to the ventral side of the fin through that. Then the other aborad nerves coming out, each by itself, to the metapterygium divide into two branches for the two sides of the fin, just as in the case of the pectoral fin. This is in *Mustelus canis*.

In *Eugomphodus litoralis*, see Pl. LX, fig. 60, from the articulation of two or three rays with the girdle, aborad of those which by their concrescence mark themselves out as the propterygium, it appears

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that the articulation (or failure of concrescence) of the metapterygium and girdle has taken place farther aborad, and consequently a greater number of rays devote their basal parts to the formation of the pelvic girdle. Expectedly then, we find that the cartilage does in fact spread around the branches of the next two nerves. Through the foramina the ventral branches of these nerves pass, while the branches to the dorsal side pass along to that side above the cartilage.

The difference in respect to the relation between the cartilage and the nerves in the pectoral and ventral limb is, that in the former the cartilage thickens so as to include the branching place of the first nerve or bundle of nerves, while in the pelvic limb, it is thinner and merely transmits the ventral branch. In his Memoir on the Shoulder Girdle, Gegenbaur has called attention to the two branching canals or two pairs of openings in the shoulder girdle of the Batoidei. He states that he has not observed whether the aborad one is traversed by a nerve. I have examined this in the case of *Raia erinaceus*, and found that both fore and after openings transmit nerves in the same way. This is evidently what would be anticipated from what has been herein said. We have here what we had in the ventral fin (and girdle) of *Eugomphodus;* a greater number of rays are devoted to girdle building, and another bundle of nerves is included in the spreading cartilage.

The observations of Rolph on the innervation of *Amphioxus** are in complete agreement with what would be required by the view here advocated. He says "Der ventrale Ast verläuft herab bis in die Seitenfalten. Beim Eintritt in dieselben theilt er sich in zwei Aeste, deren einer (n_2) an der Aussenwand der Seitenfalte hinzieht; der andere durchläuft die Seitenfalte in querer Richtung, um in die Bauchmuskulatur über zu treten, in der er sich nicht weiter verfolgen lassen. Zuvor jedoch gibt er noch einen Zweig ab (n_3), welcher, n_2 parallel, an der inneren Wand der Seitenfalte verläuft."

The manner of innervation, then, seems to me as totally inconsistent with the Archipterygium theory as it is in thorough and telling harmony with the view which I have here presented.

Addendum.

Since the views expressed in the foregoing pages were complete in my own mind six or eight months ago, I had looked for confirmation of them in the brilliant investigations of Balfour on the development

* Morph. Jahrb., Bd. ii, Hft. 1, p. 107, 1876.

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of Elasmobranchs. The preliminary account, however, in the Journal of Microscopical Science, contained nothing bearing on the point, and the papers in the Journal of Anatomy and Physiology I have been able to obtain only irregularly. Immediately after the last proof of the preceding pages had been received, the number of that Journal for October, 1876, came into my hands. Here Balfour devotes three or four pages to the limbs. He says: "If the account just given of the development of the limb is an accurate record of what really takes place, it is not possible to deny that some light is thrown by it upon the first origin of the vertebrate limbs. The fact can only bear one interpretation, viz: *that the limbs are the remnants of continuous lateral fins.*"

"The development of the limbs is almost identically similar to that of the dorsal fins." He goes on to state that while none of his researches throw any light on the nature of the skeletal parts of the limb, they certainly lend no support to Gegenbaur's view of their derivation from the branchial skeleton. Thus these results have not only been reached independently, but from two different classes of facts. To the belief in the original continuity of the lateral fins and the homodynamism of median and paired fins I was led by observations on adult forms, and particularly on the skeleton. Balfour comes to the same results from embryological investigations, in that group from which on general grounds an answer was most to be expected; nor do these investigations regard the skeleton.

I have also just received the last number of the Morph. Jahrb. It contains a paper by Wiedersheim* confirming Gegenbaur's view respecting the double nature of the centrale. This had previously been shown only in the tarsus of *Cryptobranchus Japonicus*, (and in the Enaliosaurs). Wiedersheim shows its double character in three Siberian species of Urodela, in both carpus and tarsus. This is a very important confirmation of the chiropterygium, and relieves us of suspicions with regard to its correctness when we push our inquiries into earlier history and more simple forms.

In the same number of the Jahrbuch is a paper by Gegenbaur† on the archipterygium theory. He modifies his explanation of the Stapediferal limb to accord with Huxley's view of the homology of edges and faces of limb and fin. He says that while he does not

* Morph. Jahrb., Bd. ii, Hft. 3. R. Wiedersheim, Die ältesten Formen des Carpus und Tarsus der heutigen Amphibien.

† C. Gegenbaur, Zur Morphologie der Gliedmaassen der Wirbelthiere.

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think the correctness of this view fully demonstrated, still he thinks there is a decided balance of probability in its favor. Therefore the ulnar side of the arm now appears as the *Stammreihe.* In other particulars Gegenbaur reaffirms his previous views. He proceeds to devote considerable space to the discussion of the origin of the archipterygium, and again proposes to assimilate the limb and limb-girdles to the gill-arches with their rays. He supports this suggestion with considerable argumentation. To this position the archipterygium theory leads him.

I take this opportunity for expressing my sense of the great advantages furnished by the U. S. Fish Commission for the study of marine life on our coasts, and in particular by the biological laboratory at Wood's Hole, established in connection with that Commission, and also for acknowledging my personal indebtedness to Professor Baird, through whose courtesy I have enjoyed these facilities for a number of summers.

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EXPLANATION OF PLATES.

The figures are all drawn with a camera, and photo-lithographed. They are all three-fourths of the size of the originals, except figs. 1, 2, 3, 4, 20, 21, 22, 60; figs. 20, 21 and 22 are three-eighths, and 60 is a little less than twice natural size.

PLATE XLIX.

Figure 1.—*Petromyzon marinus.* *a*, Ligamentum longitudinale, or ridge-pole of myelonal canal; *b*, notochord; *c*, neural arches.

Figure 2.—*Petromyzon marinus.* *a*, fin-rays; *b*, intermuscular septa.

Figure 3.—*Petromyzon marinus.* *a*, fin-ray; *b*, fin-muscles.

Figure 4.—Section of *Petromyzon marinus*, to show the relation of the neural arches to the muscular segments; *a*, intermuscular septum; *b*, neural arch; *c*, blood-vessel; *d*, muscular segment; *e*, fatty-fibrous ridge-pole of neural canal.

Figures 5, 6.—*Mustelus canis.* First dorsal.

PLATE L.

Figures 7–10.—*Mustelus canis.* First dorsal.

Figure 11.—*Mustelus canis.* Second dorsal.

PLATE LI.

Figures 12–15.—*Mustelus canis.* Second dorsal.

Figures 16, 17.—*Mustelus canis.* Anal.

PLATE LII.

Figures 18, 19.—*Mustelus canis.* Anal.

Figure 20.—*Galeocerdo tigrinus.* First dorsal.

Figure 21.—*Galeocerdo tigrinus.* Second dorsal.

Figure 22.—*Galeocerdo tigrinus.* Anal.

Figure 23.—*Eulamia Milberti.* First dorsal.

PLATE LIII.

Figure 24.—*Eulamia Milberti.* First dorsal.

Figures 25, 26.— " " Second dorsal.

Figures 27, 28.— " " Anal.

Figure 29.—*Sphyrna zygæna.* First dorsal.

PLATE LIV.

Figure 30.—*Sphyrna zygæna.* First dorsal.

Figure 31.— " " Second dorsal.

Figure 32.— " " Anal.

Figures 33–35.—*Eugomphodus* (*Odontaspis*) *litoralis.* First dorsal.

PLATE LV.

Figures 36–39.—*Eugomphodus litoralis.* First dorsal.

Figures 40, 41.— " " Second dorsal.

PLATE LVI.

Figures 42–46.—*Eugomphodus litoralis.* Second dorsal.

Figures 47, 48.— " " Anal.

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PLATE LVII.

Figures 49, 50.—*Eugomphodus litoralis.* Anal.

Figures 51–54.—*Squalus (Acanthias) Americanus.* First dorsal.

Figures 55, 56.— " " " Second dorsal. The spine is removed in figs. 53, 55, 57; but not in figs. 51, 52, 54, 56.

PLATE LVIII.

Figure 57.—*Squalus Americanus.* Second dorsal.

Figure 58.—*Raia levis.* First dorsal.

Figure 59.— " " Second dorsal.

Figure 60.—*Myliobatis Fremenvillei.* Sole dorsal.

Figure 61.—*Acipenser brevirostris.* Sole dorsal; *a*, neural arch; *b*, intercalary cartilages; *c*, neural spine; *d*, foramen for ventral branch of spinal nerve; *e*, foramen for dorsal branch of spinal nerve.

PLATE LIX.

Figure 62.—*Acipenser brevirostris.* Anal. Opposite 2½ vertebræ.

Figure 63.—Section of vertebrarium of *Acipenser brevirostris*; *a*, neural spine; *b*, ligamentum longitudinale; *c*, neural arch.

Figure 64.—*Acipenser brevirostris*, left ventral, from above; *a*, iliac process.

Figure 65.—*Acipenser brevirostris*, portion of right ventral, from below.

Figure 66.—Section of pectoral fin of *Mustelus canis.*

Figure 67.—*Mustelus canis*, ventral.

Figure 68.—*Mustelus canis*, pectoral detached from girdle.

PLATE LX.

Figure 69.—*Eugomphodus litoralis.* Ventrals; *a*, *b* and *c* nerve-foramens.

Figure 70.—Pectoral limb of *Ichthyosaurus*, after Cuvier. *Car.*, carpalia; *Cen.*, centralia; *u*, ulnare; *i*, intermedium; *r*, radiale; *U*, ulna; *R*, radius; *H*, humerus.

I take, though with some hesitation, Gegenbaur's indentification of the radial and ulnar sides in this limb of *Ichthyosaurus.*

Figure 71.—*Cryptobranchus Japonicus.* Hind foot, after Hyrtl, Schediasma anatomicum, 1865.

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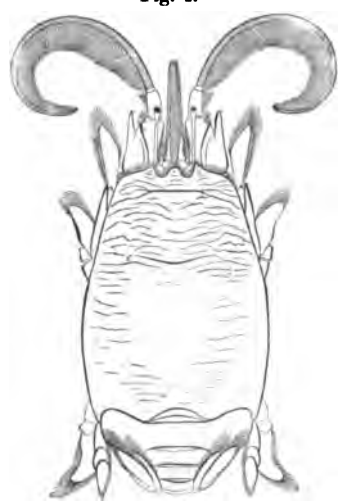
VIII.—The Early Stages of Hippa talpoida, with a note on the Structure of the Mandibles and Maxillæ in Hippa and Remipes. By Sidney I. Smith.

The biological station, established under the auspices of the United States Commissioner of Fish and Fisheries, at Wood's Hole, Massachusetts, during the summer of 1875, afforded several naturalists, and among them the writer, excellent facilities for studying the marine animals of Vineyard Sound and the adjacent waters. The locality is very favorable for obtaining in abundance the free-swimming larvæ of a great variety of marine animals. Among the young of numerous species of crustacea, the zoeæ of *Hippa* were particularly interesting, and I succeeded in obtaining a nearly complete series of the post-embryonal stages of that peculiar genus.

Since almost nothing has been published in regard to the habits of any of the species of Hippidæ or Albunidæ, a few words in regard to the habits of the only species, of either of these groups, living upon the coast of New England may not be out of place here.

Fig. 1.*



Hippa talpoida inhabits the entire eastern coast of the United States from Cape Cod southward to the west coast of Florida; Egmont Key being its most southern and western habitat known to me. At what point it is met or replaced by the Brazilian *H. emerita*, I am unable to determine, never having seen specimens of either species from, or the record of their occurrence in, the West Indies or Central America, although some species of the genus probably inhabits both these regions. On the sandy coasts of the southern United States the *H. talpoida* is apparently very abundant, while on the coast of New England it is much less common, being found only in special localities, although,

* *Hippa talpoida*, adult female with the antennæ extruded, dorsal view, enlarged about two diameters.

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from its gregarious habits, it is usually found in abundance in such localities. The northern range of this, as well as of numerous other, southern species is undoubtedly restricted by the extreme cold of the winters; and exceptionally cold seasons probably destroy a large part of the individuals over considerable portions of the coast. During the summer of 1870 not a specimen of the adult or half grown *Hippa* could be found at Fire Island Beach, Long Island, although the extensive sandy beaches of that region offer specially favorable localities, which were thoroughly searched; but during the last of August and early September, the young just changed from the zoea, and also in a little later stage, appeared abundantly upon the beaches. During the following summer no fully grown specimens were found on the shores of Vineyard Sound, though half grown specimens (perhaps from the young of the previous season) were common. During the summer of 1875, fully grown specimens of both sexes were found in great abundance at a single, very restricted locality near Nobska Point, on the shore of Vineyard Sound, although at this time none could be found at the particular locality where they were common in 1871.

Upon our shores, as far as I have observed, the *Hippa* inhabits sandy beaches which are somewhat exposed to the action of the waves. It seems to prefer only a narrow zone of the shore, at or very near low water mark, where it lives gregariously, burrowing in the loose and changing sands. At the locality near Nobska Point above referred to, it was obtained in great abundance by digging over the sand just at the edge of the receding waves. Several individuals were often thrown out at a single stroke of the spade, but the wonderful rapidity with which these animals burrow made it extremely difficult to secure more than one or two of them at a time. The smooth, oval form of the animal, with the peculiar structure of the short and stout second, third, and fourth pairs of thoracic legs, enables them to burrow with far greater rapidity than any other crustacean I have observed. Like many other sand-dwelling crustaceans, they burrow only backwards; and the wedge-shaped posterior extremity of the animal, formed by the abrupt bend in the abdomen, adapts them admirably for movement in this direction. When thrown upon the wet beach, they push themselves backward with the burrowing thoracic legs and, by digging with the appendages of the sixth segment of the abdomen slightly into the surface, direct the posterior extremity of the body downward into the sand. Upon the beaches, at least where there are any waves, they seem usually to be buried completely

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beneath the surface. Occasionally, however, they are found swimming about in pools left by the tide, and they undoubtedly, when undisturbed, sometimes come out and swim in the same way along the shore, though they probably never venture far from the bottom.

When first placed in an aquarium with a few inches of sand at the bottom, they invariably plunged at once entirely beneath the sand, but, after a few moments of quiet, usually worked themselves gradually towards the surface, resting in a nearly perpendicular position with just the tips of the antennulæ and eyes at the surface, while the excurrent water from the branchiæ formed a small opening and a slightly boiling motion in the sand. Occasionally, when entirely undisturbed, they would suddenly leave the sand and swim rapidly round the top of the aquarium for a moment and then dive suddenly to the bottom and bury themselves in the sand. In swimming, as well as in burrowing, the telson was carried appressed to the sternum and they invariably moved backward, the motion being apparently produced by the appendages of the sixth abdominal segment and the anterior thoracic legs, while the latter served also as steering organs.

During all the ordinary motions of swimming and burrowing, I have never seen the antennæ extruded, although the peculiar arrangement of the peduncular segments and their complex system of muscles are apparently specially adapted for extending and withdrawing these beautiful organs. When the animals are thrown into alcohol however, the antennæ are sometimes thrown out convulsively and then immediately retracted. In life the antennæ are most of the time held in the position in which they are usually found in alcoholic specimens, that is, between the second and external maxillipeds, with the peduncles crossed in front, and the flagella curved down and entirely round the mouth so that their dense armament of setæ all project inward. When extruded, the distal segments of the peduncle are revolved half way round on the proximal ones, so as to carry the whole appendage to its own side of the animal and throw the curve of the flagellum into a reversed position. Judging from the peculiarly armed setæ of the flagella, one of the principal offices of the antennæ is the removal of parasitic growths and all other foreign substances from the appendages of the anterior portion of the animal.

The mouth parts of the adult are not adapted for ordinary prehension or mastication, but I am unable to make any positive statement in regard to the food of these animals. In all specimens examined the alimentary canal was filled with fine sand which seemed to be nearly free from animal or vegetable matter. The material from the

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stomach, however, shew, under the microscope, a small quantity of vegetable matter, and it seems probable that the sand is swallowed for the nutritive matter it may contain.

Upon the beaches of Vineyard Sound the two sexes appeared to occur in about equal numbers, although in museum collections the males are often rare. This is probably due to the great inequality in size between the male and female, the length of the carapax in the larger females from Vineyard Sound being 20 to 22^{mm}, while in the largest males it does not exceed 14^{mm}. The sexes differ also in the form of the telson (Plate XLVIII, figs. 7, 8) which is narrower and more triangular in the male than in the female.

Females carrying eggs were found during the entire month of August, and during that period the embryos within the eggs were nearly fully developed in many of them. Undoubtedly, however, the term of carrying eggs extends over a much longer period than this. The eggs are nearly spherical, $\cdot 40$ to $\cdot 45^{mm}$ in diameter, and the yolk mass is orange yellow while the formed tissues of the embryo are nearly colorless. Numerous attempts to obtain newly hatched young, by keeping egg-carrying females in aquaria, failed from the parent invariably casting off the eggs before they were fully matured. Consequently I failed to secure the earliest stage of the zoea, for the youngest individuals taken in the towing net were apparently in the second stage.

Very nearly fully developed embryos, when removed from the egg, were found to possess all the normal articulated appendages of the fully formed zoeæ, but there was no appearance of lateral spines upon the carapax and the rostrum was broad and obtuse. In this stage the embryo agrees almost perfectly with the figure of the zoea of *Hippa emerita* from the coast of Brazil, given by Fritz Müller in his work entitled "Für Darwin."* The difference between the embryo in this stage and the second zoea-stage (Plate XLV, fig. 1), in which

* English translation, London, 1869, p. 54, fig. 25. The figure is accompanied by the following paragraph: "The Zoëa of the Tatuira [*Hippa*] also appears to differ but little from those of the true Crabs, which it likewise resembles in its mode of locomotion. The carapax possesses only a short, broad frontal process; the posterior margin of the tail is edged with numerous short setæ." This, as far as I am aware, is the only published account of the development of any species of Hippidæ, except a note by myself (in an article on "The Metamorphoses of the Lobster and other Crustacea," in the Report of the U. S. Commissioner of Fish and Fisheries, Part I, 1873, p. 530) recording the occurrence, at the surface in Vineyard Sound, of the young in what is described further on in these pages as the megalops-stage.

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the rostrum and lateral spines are enormously developed, suggests the possibility that Müller had observed only imperfectly developed young zoeæ in which the rostrum and lateral spines were not expanded. It seems scarcely probable that such a difference could exist between the first stage of the zoea, when the veiling membrane, in which, on first escaping from the egg, the young are usually enveloped, has been entirely cast off and the lateral spines and the rostrum are fully expanded, and the second zoea-stage about to be described. The three later, true zoea-stages obtained are evidently contiguous steps in the development and are here designated the second, third, and last stages of the zoea. From this last stage the zoea passes at once into a stage closely resembling the adult in general form, but with the eyes still very large and the abdomen furnished with powerful swimming legs. This condition of the animal corresponds perfectly to the Brachyuran megalops and may properly be designated as the megalops-stage.

Second zoea-stage.

In this stage the young (Plate XLV, fig. 1, ventral view) are a little over 3^{mm} in length, from tip of rostrum to the posterior margin of the carapax, and a little over 2^{mm} between the tips of the lateral spines. In general form the carapax is oval, with the smaller end forward, and its surface is very smooth and regularly rounded. The dorsal surface of the carapax is strongly convex but very regularly rounded and wholly devoid of any rudiment of a dorsal spine, which is so generally characteristic of the zoeæ of Brachyura. At the bases of the ocular peduncles the carapax is sharply contracted laterally into an exceedingly long, very slender, and slightly tapering rostrum curved regularly downward until, toward the tip, it becomes nearly parallel with the posterior margin of the carapax. The lateral spines are nearly as long as the diameter of the carapax, are situated far back and low down on the sides of the carapax, and are directed downward and obliquely outward, but are not strongly curved. Beneath, the carapax curves inward on all sides, leaving a comparatively small opening which is wholly inferior, with its anterior portion about as broad as the telson, but posteriorly contracted into a narrow abdominal sinus, of which the rounded posterior margin is nearly on a line between the lateral spines. This shortening of the inferior opening, carrying the abdomen forward and wholly beneath the carapax, together with the absence of the dorsal spine, gives the animal an appearance unlike ordinary Brachyuran zoeæ.

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The ocular peduncles are stout, regularly tapering to near the bases, and are usually carried perpendicular to the mesial plain, though they admit of considerable motion in all directions. The cornea is considerably larger than the diameter of the peduncle, its diameter being nearly a third of the horizontal diameter of the carapax, and, when the peduncle is held straight out, reaches slightly beyond the lateral margin of the carapax.

The antennulæ (Plate XLVI, fig. 1) are still rudimentary, simple, sack-like, unarticulated appendages, tapering toward the tip, which is furnished, as usual in this stage of development, with three stout, filiform, obtuse setæ, differing slightly in length, diameter, and amount of curvature, and of which the longest is about half the length of the antennula itself.

The antennæ (Plate XLVI, fig. 2) are of about the same length as the antennulæ, but of nearly the same diameter throughout, and are armed distally, at the outer edge, with an acute, dentiform process (*a*, fig. 2) directed straight forward and itself armed with a minute, setiform spine on the inner edge near the tip. Between the base of this process and a slight, rounded prominence (*c*, fig. 2), situated at the extremity of the inner margin, and which represents the rudimentary flagellum, there is a similar, but slightly more slender, process (*b*) attached at its base by an oblique articulation and armed, near the tip, with a minute, setiform spine like that upon the outer process.

The oral appendages differ very little from their condition in the last zoea-stage, under which they are fully described. The labrum and labium differ scarcely at all, except in size, in the three zoea-stages here described. The labrum, as seen from beneath, is a broad, somewhat triangular prominence between the bases of the antennulæ and the tips of the mandibles. The labium is deeply bilobed, though far less deeply than in the adult, with the lobes broadly rounded and the entire margin clothed with microscopic hairs.

The mandibles are nearly as in the last zoea-stage. They are stout at the bases, but taper to very slender tips, which are only slightly different on the right and left side. There is no molar area, but the crown of the mandible is longest in a vertical direction and is armed inferiorly with four long, but blunt, teeth which decrease rapidly in size as they approach the middle of the crown, where they are met by a series of six or seven long, slender, seta-like processes which occupy the superior half of the coronal margin.

The first pair of maxillæ (Plate XLVI, fig. 11) are symmetrical and composed of the same parts as in the adult. The inner lobe

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(*a*, fig. 11) is small and tipped with three long setæ. The outer lobe (*b*, fig. 11) is broader than the inner and armed at the extremity with three nearly equal, long and slender teeth, of which the distal one appears like a process from the margin, showing no line of articulation at its base. The palpus (*c*, fig. 11) is very small, composed of a single segment and tipped with a long, plumose seta.

The second pair of maxillæ (Plate XLVII, fig. 1) are very imperfect. The protognath (*a*, fig. 1) is a small, obtuse lobe tipped with three short setæ. The scaphognath (*c*, fig. 1) projects beyond it anteriorly as a slightly larger lobe, while posteriorly it is broad but short and truncated, and the anterior lobe and the outer edge are, as yet, alone furnished with setæ.

The first and second pairs of maxillipeds, or natatory legs, (Plate XLV, fig. 1, second pair) are similar in structure to those of most zoeæ and differ only slightly from each other. In both pairs the basal portion, or protognath, is alike stout, about as long as the exognath, and unarmed, except by three or four minute setæ on the distal portion of the inner margin. The exognaths, or natatory branches, alike in both pairs, are nearly cylindrical, but flattened at the tips, where they each bear a series of eight slender, plumose setæ, which, in the middle, are as long as the exognath itself but decrease in length to the outer ones, which are scarcely more than two-thirds as long. The inner branch, or endognath, in both pairs, is composed of four cylindrical segments subequal in length. In the first pair, however, the inner branch is shorter than the exognath, while in the second pair it is considerably longer. In both pairs the three proximal segments of the endognath are each armed with two or three small setæ on the inner side, and the distal segment, which is much more slender than the others, is tipped with four setæ, of which two are nearly as long as the segment itself and pectenated with minute, setiform spinules along one side, and the two others shorter and apparently unarmed.

The third pair of maxillipeds and the four anterior pairs of thoracic legs are, even at this early state, represented by a series of clearly defined, though entirely unsegmented, processes situated just above and back of the bases of the second maxillipeds and entirely within the carapax, but visible through it, in a lateral view of the animal, in a line nearly parallel with the posterior margin of the carapax. In the single specimen examined, no lobes representing the slender, posterior thoracic legs of the adult could be discovered. Above each of the processes representing the first four pairs of thoracic legs there is a minute, papilliform process apparently representing one of the branchial appendages belonging to these legs in the later stages.

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The abdomen is smaller and has much less freedom of motion in the mesial plane than in most Brachyuran zoeæ. The first segment is not clearly differentiated from the thorax. The second, third, fourth, and fifth segments are entirely without appendages; the second and third are nearly equal in length and sub-cylindrical; the fourth is slightly shorter and is expanded considerably at the posterior extremity; and the fifth is about as long as the fourth, compressed vertically, and broadly expanded at the postero-lateral angles so that it is about twice as broad as long. The sixth segment is consolidated with the telson, forming a broad, lamelliform, caudal appendage about as long as the middle breadth of the carapax. The appendages of the sixth segment (Plate XLVIII, fig. 10) are small, rudimentary, appressed to the under side of the telson so as to be hidden from above, and are each composed of a stout basal segment and a single narrow lamella (the outer) tipped with two slender setæ, of which the outer is about as long as the lamella itself and the inner much longer.

The lateral margins of the telson are slightly curved outward and unarmed, but project posteriorly into a stout tooth each side of the strongly arcuate posterior margin. This posterior margin has, in all the zoea-stages here described, a remarkably complex armament of ciliated spines and minute teeth (Plate XLVIII, figs. 13, 14, 15). In a considerable number of specimens in the third and the last stages, the number of these ciliated spines is usually twenty-six, of which the eighth, counting from either side, is the largest, and the sixth and tenth usually the next in size. One of the specimens in the second stage (Plate XLVIII, fig. 13) conforms strictly with this: there are two sub-median spines (*a*, *a*, fig. 13) separated by a single denticle, then each side a slightly larger spine (*b*, *b*) separated from the sub-median ones by a single denticle, then four alternately smaller and larger spines (*c*, *d*, *e*, *f*) separated from each other by two denticles at each interspace, the outer (*f*) of these four spines being the eighth, counting from either side, and the largest. Outside this large spine there are, each side, seven smaller spines separated by interspaces which increase toward the outer margin and are armed with from two to nine denticles. The space between the outer spine each side and the tooth of the lateral margin is greater than any of the interspaces between the spines and is armed with twelve or thirteen denticles. In the other specimen in this stage there are only twenty-five spines, a single median spine (Plate XLVIII, fig. 14, *a*) taking the place of the two sub-median spines and the denticle between them; otherwise the spines and denticles are essentially as in

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the first specimen and as in the succeeding stages. This is apparently an abnormal variation in the armament of the telson. The arrangement of these spines, and especially whether they be odd or even in number, I have usually found a constant character for distinguishing the larval forms in different groups of Podophthalmia.

Of this stage only two specimens were secured, one taken on the evening of September 4, the other in the day-time the next day. In coloration and habits they agreed essentially with the young in the succeeding zoea-stages.

Third zoea-stage.

In general form and appearance the zoeæ in this stage very closely resemble those in the second, although they have increased considerably in size, and especially in the length of the rostrum, which is relatively longer and more slender. They are about $4{\cdot}5^{mm}$ in length, from the tip of the rostrum to the posterior margin of the carapax, and nearly 3^{mm} between the tips of the lateral spines.

The ocular peduncles and eyes have increased only slightly in absolute size and are relatively smaller than in the second stage.

The antennulæ have changed very little. There is, as yet, apparently no distinction of peduncle and flagellum, although the two or three distal segments of the latter are faintly indicated, and, on the inner side, there are two or three filiform setæ on the penultimate segment in addition to the three on the terminal segment.

The antennæ (Plate XLVI, fig. 3) have increased in size but show no indication of segmentation. The two dentiform processes (*a*, *b*, fig. 3) have each two or three minute spinules at the tip, but are otherwise unchanged; the flagellum (*c*, fig. 3), however, has increased so as to project beyond the tips of the dentiform processes and show plainly its true character.

The labrum, labium, mandibles, and first maxillæ, except in size, do not differ appreciably from their condition in the second stage. The second maxillæ differ but little, the scaphognath being a little more elongated posteriorly, so as to approach slightly its form in the next stage.

The first and second pairs of maxillipeds differ from those of the first stage only in the exognaths, which are each furnished with ten instead of eight terminal setæ.

The lobes representing the third pair of maxillipeds and the four anterior pairs of thoracic legs have increased much in size, are

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curled up closely beneath the sternum, and, in all the specimens examined, show the segments of the succeeding stage faintly indicated within. The posterior thoracic legs appear to be represented by a small process each side, just back of, and nearly hidden by, the rudimentary fourth pair. Four pairs of gills on each side are represented by two slender processes, one above the other, at the bases of each of the four anterior pairs of legs.

The proximal segments of the abdomen are almost exactly as in the previous stage, except there are very slight elevations beneath the second, third, fourth, and fifth segments, where the rudimentary legs are to appear in the succeeding stage. The sixth segment is still consolidated with the telson. Its appendages (Plate XLVIII, fig. 11) have increased much in size and the inner lamella (*c*, fig. 11) has appeared as a small, sack-like appendage at the base of the outer lamella (*b*, fig. 11), which is twice as long as in the previous stage, very narrow, only slightly expanded in the middle, and sub-truncate at the extremity, where it is furnished with four slender and curved setæ, of which the median ones are longer than the lamella itself, while the outer are little more than half as long. The telson is of the same form as in the previous stage and has the same number of ciliated spines in the armament of the posterior border, while the number of denticles in the interspaces has considerably increased, though they are not as numerous as in the succeeding stage. In one specimen there is the same abnormal arrangement of the spines described under the second stage, that is, there are only twenty-five spines in all, one median spine taking the place of the two sub-median spines and the denticles separating them.

The young in this stage were taken on several occasions, both in the day-time and evening, from August 28 to September 8. Their habits and coloration in life were the same as in the last zoea-stage.

Last zoea-stage.

The length from the tip of the rostrum to the posterior margin of the carapax and the breadth between the tips of the lateral spines are nearly twice as great as in the second stage, while the rostrum is relatively considerably longer than in either the second or the third stage, its entire length being nearly twice that of the carapax proper. The general form and appearance of the young in this stage are shown upon Plate XLV, figs. 2, 3, 4.

The eyes and ocular peduncles are very little larger absolutely than in the last stage. The diameter of the cornea is scarcely a

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fourth the horizontal diameter of the carapax and the ocular peduncles are slightly shorter, proportionally, than in the earlier stages.

In the antennulæ (Plate XLVI, fig. 5) the segmentation of the flagellum is carried nearly or quite to the peduncle, which, however, shows no division into segments and no clear separation from the flagellum. The flagellum itself is composed of six or seven segments which are a little broader than long and of which the terminal one is furnished with three filiform setæ, the penultimate and antepenultimate with two or three each which are situated upon the inner side at the distal articulations, while there are two similarly situated, but small and rudimentary setæ, upon the fourth segment from the tip.

The antennæ (Plate XLVI, fig. 4) have increased very much in size, and the flagellum is much longer than the peduncle. The peduncle shows but one distinct articulation, which is near the bases of the dentiform processes and apparently represents the articulation between the second and third segments of the fully developed appendage. The dentiform processes (*a*, *b*, fig. 4) are much more slender and proportionally smaller than in the third stage, but are armed with the same number of spines at the tips. The flagellum (*c*, fig. 4) externally shows no indication of segmentation, but, in all the specimens examined, the articulations of the flagellum of the succeeding megalops-stage is distinctly visible beneath the integument, as shown in the figure.

The labrum (Plate XLVI, fig. 5, *b*), as seen from beneath, is a conspicuous, somewhat triangular prominence between the bases of the antennulæ and the mandibles, with the margins and the inferior surface regularly rounded and without emarginations at any point. The labium (*d*, fig. 5) is deeply bilobed, the regularly rounded lobes projecting each side of the oral opening nearly to the tips of the mandibles and having the edges clothed with microscopic hairs throughout.

The mandibles (Plate XLVI, fig. 5, *c*; and fig. 6) are almost exactly as in the earlier stages. They are wholly without molar areas and terminate in narrow crowns, which are only slightly different on the right and left sides. In each mandible the inferior half of the coronal margin is armed with four stout teeth, the two most inferior of which are stout and obtusely pointed, the inferior being straight and much longer than the next, which slightly overlaps it at base, as seen from before or behind, and curves upward at the tip; while the two succeeding ones are short, triangular, more acute, and separated from each other by a considerable sinus. On the right side the inferior

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tooth is a little more anterior than the others, while on the left side it is more posterior, so that these inferior marginal teeth overlap each other when the mandibles are brought together. The posterior side of the crown of the right mandible is slightly convex while the same side of the left is a little concave. On the superior half of the crown of each mandible there is a series of six slender processes, or teeth, of which the five inferior are very slender, setæ-like, and twice as long as the two triangular teeth just mentioned, while the superior, or marginal, one is stouter, acutely triangular, and divided at tip. In the specimen figured, there is also a small supplementary tooth on the right mandible, between the bases of the third and fourth processes of the superior half of the margin.

The first pair of maxillæ (Plate XLVI, fig. 5, *e*; and fig. 12) have changed scarcely at all, except in size, from their simple form in the second stage. The inner lobe has a single rudimentary seta, on the inner edge near the tip, in addition to the three terminal ones, the three teeth of the outer lobe are relatively a little shorter and very slightly stouter, and the palpus is considerably larger.

The second pair of maxillæ (Plate XLVII, fig. 2) have advanced considerably in their development and have apparently partially assumed their adult function. The protognath shows a slight indication of division into two lobes (*a*, *b*, fig. 2), of which the outer projects as far forward as the scaphognath, from which it is much more deeply separated than in the earlier stages. The scaphognath is twice as long as the protognath, and its posterior portion is elongated and narrowed at the extremity as in the later stages, but its inner edge is not yet margined with the characteristic plumose setæ.

The first and second pairs of maxillipeds (Plate XLV, figs. 2 and 4, and Plate XLVII, fig. 5) have the same form and structure as in the preceding stages, the only noticeable difference being the addition of still another pair of setæ at the tip of each exognath, making twelve in all, of which the outer are scarcely more than half as long as the middle ones, which are not quite equal in length to the exognath itself.

The third pair of maxillipeds and all the thoracic limbs (Plate XLV, fig. 4, and Plate XLVIII, fig. 9) are still curved inward beneath the sternum and entirely inclosed within the posterior portion of the carapax, although the four anterior pairs of legs have begun to show clearly the form which they assume in the megalops and later stages. The third pair of maxillipeds (*b*, Plate XLVIII, fig. 9) are still sub-cylindrical and in each the meral segment is only

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a little stouter than the palpus, of which only the elongated dactylus and the stout propodus are clearly distinguishable. The first four pairs of thoracic legs (*c*, *d*, *e*, *f*, fig. 9) show clearly only the distal segments, but, of these, the propodus and dactylus have already assumed the peculiar form which characterizes them in the megalops-stage and in the adult, the propodus in the first, second, and third pairs being elongated into a prominent lobe on the inside at the distal extremity, while in the fourth pair (*f*, fig. 9) this segment is truncated at the distal end. The posterior thoracic legs (*g*, fig. 9) are slender, cylindrical, and the three distal segments are nearly equal in length, but there is no indication of the cheliform character which they assume in the megalops-stage. The branchial processes (*h*, fig. 9) above the bases of the legs have become more conspicuous than in the earlier stages and represent the full number of branchiæ in the adult, there being one above the base of each external maxilliped and two each upon the four anterior thoracic legs, making nine pairs in all.

The first segment of the abdomen is still in the same condition as in the second stage, not distinctly differentiated from the thorax, and the remaining segments themselves retain very nearly the same form as before. The abdominal legs of the second, third, fourth, and fifth segments (Plate XLV, fig. 4) are as long as the segments to which they belong, but are still sack-like, the base separated from the terminal portion by an obscure articulation, but with no indication of the separation of the outer from the inner lamella, and the appendages are evidently of no functional importance. The sixth segment is still closely united with the telson, although some of the specimens show a slight indication of the approaching articulation. The appendages of the sixth segment (Plate XLVIII, fig. 12) have increased very much in size, and the outer lamella (*b*) has become narrow-oval in outline, about a third as broad as long, with the tip rounded and furnished with six strongly curved setæ, of which the third from the outside is much longer than the lamella itself, while all the others are much shorter and decrease in length each way from the longest. The inner lamella (*c*, fig. 12) is two-thirds as long as the outer, nearly as broad, and regularly oval in outline, without hairs or setæ.

The telson (Plate XLV, figs. 3 and 4) has the same form as in the previous stages. Its posterior margin (Plate XLVIII, fig. 15) is armed with twenty-six ciliated spines having the same arrangement as in the earlier stages. The number of denticles in the interspaces between the spines is very much increased, there being three between

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sub-median spines, two each side between these and the next outside of them, from three to twelve in each interspace between the other spines, and from thirty to forty between the outer spine each side and the tooth of the lateral margin.

The zoeæ in this stage were frequently taken at the surface, both in the day-time and evening, from August 7th to September 10th. In life, the entire animal is translucent with a slight greenish tint, except a brilliant spot of orange pigment with metallic lustre at the base of the rostrum and at the base of each of the lateral spines. These bright spots would often catch the eye in looking into the water when the rest of the animal was nearly or quite invisible. Their motions in the water are similar to those of zoeæ in general, except that the movements are much less rapid, and they seem comparatively sluggish in habit. In aquaria they were usually seen swimming slowly about the surface, seldom whirling off with the mad, gyratory motion so characteristic of many Brachyuran zoeæ.

Individuals in this last zoea-stage, when kept in confinement, often changed, at a single molt, to the megalops-stage described beyond, although many died during the process of molting.

The following table shows the relative measurements of specimens in each of the zoea-stages described:

	Second Stage.	Third Stage.	Last Stage.
Length from tip of rostrum to posterior margin of carapax,	3·2mm.	4·4mm.	6·2mm.
Breadth between tips of lateral spines,	2·2	2·8	4·3
Length of rostrum from tip to front of ocular peduncles,	1·8	2·8	4·7
" " lateral spines, about,	·8	1·1	1·3
" " carapax from front of ocular peduncles to posterior margin,	1·3	1·8	2·5
Breadth of carapax in the middle,	·9	1·2	1·7
" " telson,	·7	1·0	1·4

Megalops-stage.

In specimens recently changed from the last zoea-stage, the length of the carapax is slightly over 3^{mm} and that of the abdomen, when fully extended, a little less, while the breadth of the carapax is about 2^{mm}. In general form, the young in this stage resemble the adult, but differ essentially in much the same way that the Brachyuran megalops differs from its adult; the eyes still being relatively large, with short and thick peduncles, as in the last stage of the zoea, while the second, third, fourth, and fifth abdominal segments, as well as the sixth, are provided with strong swimming appendages.

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The form of the carapax resembles that of the adult very closely, but is broader in proportion, being a half longer than broad; the lobes of the frontal margin (Plate XLV, fig. 5) are more obtuse and much broader; and the lateral lobes form the antero-lateral angles of the carapax, instead of being separated from the angles by a transverse portion, each side, nearly as long as the thickness of the peduncles of the antennæ, while the lateral margins of the carapax are more regular in outline and do not project downward so much between the first and fourth pairs of legs.

The ocular peduncles are still very short and stout, though the eyes have increased only a very little in absolute size since the last zoea-stage. The cornea is a little elongated and occupies the lateral portion of the tip of the peduncle; its greater diameter is still about a fourth of the horizontal diameter of the carapax and more than half the length of the peduncle.

The antennulæ (Plate XLVIII, fig. 1) reach to the extremities of the peduncles of the antennæ and show a marked advance over the previous stages. The segments of the peduncle are fully differentiated and the second segment (*b*, fig. 1) has already a marked prominence, tipped with a few plumose hairs, in place of the elongation so conspicuous upon the inferior side of the distal extremity of the same segment in the adult (Plate XLVIII, fig. 3). The flagellum (*d*, fig. 1) is composed of eight or nine short and stout segments, all, except one or two of the most proximal, armed inferiorly with plumose, or pectinate, setæ. The secondary flagellum (*e*, fig. 1) is represented by a single, minute segment tipped with a plumose seta.

The antennæ (Plate XLV, fig. 5, and Plate XLVI, figs. 7, 7*a*) are slightly longer than the carapax and have assumed all the important features of the adult. The segments of the peduncle (Plate XLVI, figs. 7, 7*a*) are fully differentiated and essentially the same as in the adult; and they have the same peculiar structure adapted to folding the antennæ across in front of the mouth and within the external maxillipeds. To aid in accomplishing this, there is a supplemental segment or rod (*f*, figs. 7, 7*a*) on the outer side of the third segment—and apparently a separately calcified part of it—which articulates proximally just within the lateral spine of the second segment and distally with the outer edge of the fourth segment, and is so separated, by non-calcified, flexible integument, from the third segment itself, as to move independently of it. When the antenna is extruded, this supplemental rod lies parallel with the outer margin of the third segment, from which it is then separated by a considerable space of

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the non-calcified integument; but when the antenna is folded away within the external maxillipeds, the rod is transverse to its former position and is almost or quite hidden between the second and fourth segments, the outer, calcified portion of the latter segment folding over the previously exposed area of thin integument. The flagellum is stout, tapers very gradually to an obtuse and rather thick tip, and is composed of about twenty-five segments, which are very short proximally, but increase regularly in length distally until, near the tip, they are longer than broad. Each segment is armed upon one side with a fascicle of long, pectinated setæ. As seen in a transverse section of the flagellum (Plate XLV, fig. 6), the outer one of these setæ, on each side of the fascicle, is strongly curved inward at the extremity and much longer than the inner ones, which are nearly straight and armed, for only a part of their length, with much shorter, stronger, flattened, and blade-shaped teeth. In this stage there are usually from five to seven setæ to each segment, the median one or two often being simply acicular, or even very small and rudimentary.

This structure of the flagellum of the antenna is essentially the same as in the adult, where, however, the number and size of the satæ, as well as the extent of their armament, is enormously increased upon each segment, while the number of segments is several times greater than in the megalops-stage. In the ordinary adult specimens, there are one hundred to one hundred and fifty segments in the flagellum and eight to twelve setæ to each segment. There are two forms of these setæ in each fascicle, as in the megalops-stage. The outer one, each side of every fascicle, is very long and convolutely curved inward at the extremity; while all the others are shorter, though varying much in length among themselves, and nearly straight. The long, outer setæ are armed, for nearly their whole length, with very long, almost filiform, secondary setæ, which are arranged in a double series along the inner side of the curve. These secondary setæ are exceedingly slender, very slightly tapering, from ·2 to ·25^{mm} long,—the longer ones being on the distal half though not at the extremity,—and are placed so thickly that, in the middle portion of the seta, there are two hundred in the space of a millimeter. The extremity of the shaft of the seta itself is unarmed for a very short distance, curved sharply so as to be nearly parallel with the last of the secondary setæ, and tapers to a very slender and acute tip. The remaining setæ of each fascicle are nearly straight, or very slightly bent, and armed upon the outside of the curve. The teeth, as in the outer setæ, are arranged in two series, which here, however,

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approach so closely that the bases are nearly in the same line, those of one series alternating with those of the other. The teeth themselves are much shorter than the secondary setæ of the outer setæ, being ·05 to ·07^{mm} long, much stouter and less crowded, so that there are about an hundred in the space of a millimeter in the middle portion of the setæ. Near the base of the seta, a few of the teeth are very slender and hair-like, but the rest are stout, truncated at the tips, lamelliform, and placed with their bases transverse to the shaft of the seta, but with the blades twisted nearly half way round, so that the surface at the tip is nearly at a right angle to the base. The shaft of the seta itself extends a little way beyond the teeth in an acutely cultriform tip.

The oral appendages (Plate XLVI, figs. 8, 9, 13; Plate XLVII, figs. 3, 4, 6, 7) have undergone a transformation even more wonderful than the usual change from the zoea to the megalops. The mandibles have lost, almost entirely, the structure and function usual to them in all the Thoracostraca, and, together with the other oral appendages, have assumed very nearly the adult form.

The labrum, as seen from beneath (Plate XLVI, fig. 8, *a*), is much more elongated than in the zoea-stages, being about as broad as long, but with the sides still arcuate outward and not at all incurved as in the adult. The labium (*e*, fig. 8) is divided very deeply into two long and obtuse lobes, fringed with microscopic hairs along the oral margins, and projecting forward, each side of the mouth, to the coronal portions of the mandibles.

The mandibles (Plate XLVI, fig. 8, *b*, *c*, *d*) have become thin and foliaceous and completely consolidated with the walls of the oral opening. They have become differentiated, however, into two portions, apparently corresponding to protognath and palpus, or endognath, though these parts are not separated by distinct articulations. The protognathal portion (*b*, fig. 8), corresponding to the entire non-palpigerous mandible of the zoea-stages, is coalesced with the lateral walls of the mouth, except at the broad foliaceous tip, which scarcely projects into the oral opening. The distal margin of this is, however, obscurely dentate, the denticulation varying somewhat in different individuals in the same stage (fig. 8, *b*, and fig. 9), although the mandibles have apparently ceased to perform any of the usual mandibular functions. The endognathal portion (*c*, *d*, fig. 8) evidently represents a palpus with its segments completely coalesced, although the fold between the terminal (*d*) and the outer, spinous portion (*c*) apparently marks the union of the two distal segments.

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The distal extremity of the palpus is foliaceous, the margins obtusely rounded and armed with long, plumose setæ, which extend over the oral edge of the labrum and the anterior edge of the mouth itself. The outer margin (*c*), which is thickly spinous in the adult, projects considerably laterally, but is, as yet, armed with only three or four spines.

The first pair of maxillæ (Plate XLVI, fig. 13) have assumed the general form and arrangement of parts which they present in the adult. The inner lobe (*a*, fig. 13) is relatively larger than in the zoea-stages, being considerably broader than the outer lobe; and is armed at the tip, and a little way down the inner margin, with long and stout setæ, most of which are slightly spinulose or plumose distally. The outer lobe (*b*) is long, rather narrow, and the terminal margin is obtusely rounded and armed with setiform spines, which are short and stout toward the inner margin but increase in length outward, and, at the outer margin, grade suddenly into very long and slender, plumose setæ, of which the outermost one is nearly three times as long as the width of the lobe itself. On the inner, lateral margin there are several plumose hairs and on the outer margin, a single short spinule. The palpus (*c*) projects laterally from near the base of the outer lobe as an irregular, sack-like appendage. The articulation at the base of the inner lobe is much less distinct than in the adult, although the articulation at the base of the outer lobe is conspicuous.

The peculiar structure of the mandibles and maxillæ of the adult are more fully discussed in a special note at the end of this article.

The second pair of maxillæ (Plate XLVII, fig. 3) have assumed nearly the adult form. The lobes of the protognath (*a*, *b*, fig. 3) are deeply separated, and a small, supplementary, papiliform lobe (*a'*), tipped with a long, plumose seta, has appeared between the inner and outer lobe, arising near the base of the inner. The terminal margin of the inner lobe is armed with plumose setæ, arranged in three series of varying lengths, the longest, situated upon the very edge, being nearly as long as the lobe itself. The outer lobe is similarly armed, but the setæ are all much shorter and less plumose, and some of them are stout and serrate. Between the outer lobe and the anterior projection of the scaphognath, there is a small, triangular lobe (*d*) apparently representing the endognath. The characteristic fringing of plumose hairs has extended round upon the inner margin of the broad, posterior portion of the scaphognath (*c*), which has changed comparatively little since the last zoea-stage, but still is of

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nearly the same form as in the adult, and has apparently fully acquired the adult function.

The three pairs of maxillipeds (Plate XLVII, fig. 4, 6, 7) have assumed so nearly the adult form that detailed descriptions of them are unnecessary.

As in the adult, the anterior lobe of the protognath of the first maxillipeds (Plate XLVII, fig. 4, *a*) is very much elongated; the straight inner margin is thickly beset with plumose setæ throughout, and with an additional series of much longer setæ on the posterior half, and also at the extremity. The endognath seems to be represented only by the long, slender, and soft appendage (*b*), apparently arising from near the base of the inner side of the stout, two-jointed exognath (*c*). The basal segment of the exognath is nearly as long as the distal lobe of the protognath, while the terminal segment is somewhat shorter, but fully as broad, and thickly marginal throughout with plumose setæ, which become very long at its extremity. In the adult, the terminal segment is more triangular in outline and the tip is more acute.

The endognath in the second pair of maxillipeds (Plate XLVII, fig. 6, *a*) differs in form slightly from that of the adult, being proportionally stouter and less flattened; the terminal segment, in particular, is proportionally, considerably shorter and consequently more tapering. The basal segment of the exognath (*b*, fig. 6) is not so much narrowed distally, and the oval, terminal segment is a little narrower than in the adult.

The broad, opercular, external maxilliped (Plate XLVII, fig. 7) is transversely truncated at the distal extremity where the palpus articulates, wanting almost wholly the conspicuous, rounded prominence of the anterior angle, just within the articulation, in the adult; and the posterior angle of the inner margin is less prominent and more broadly rounded. The palpus is much less slender, less compressed, and the terminal segment is proportionally shorter than in the adult.

The thoracic legs, like the maxillipeds, are so much like those of the adult, both in form and function, that detailed descriptions of them seem needless here. The anterior pair (Plate XLV, fig. 5) are a little more slender than in the adult, and the terminal segments, or dactyli, are not quite as thickly margined with plumose setæ. The second and third pairs are almost exactly alike and, together with the fourth pair, are specially adapted for burrowing. The fourth pair differ from the second and third principally in having the two

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terminal segments more simple in form, the propodus being nearly square, while the dactylus is small and acutely triangular, with the tip obtuse.

The posterior thoracic legs are exceedingly slender and, as in the adult, are in life, usually at least, held concealed within the branchial cavities. They are armed with comparatively few of the very slender spinulose setæ with which they are so abundantly furnished, especially toward the distal extremity, in the adult; and the series of spinulose teeth upon the prehensile edges of the short fingers of the chelæ are very short and composed of only a few individuals upon each finger. There is no doubt that the peculiar position, structure, and armament, of these slender posterior thoracic legs, in *Hippa* and allied genera, specially adapt them for cleaning the branchiæ and branchial chambers of parasites and other foreign bodies. It is worthy of note, in connection with this, that none of the maxillipeds or thoracic legs possess any traces of either exipodal or epipodal branches, some of the last of which perform the office of branchiæ-cleaners in the majority of the Brachyura and Macrura.

The abdomen (Plate XLVIII, fig. 4), when fully extended, is about as long as the carapax and resembles that of the adult in the form and proportions of the segments, but differs essentially in the structure of the appendages, especially those of the second to the fifth segment, as is usual in the megalops-stage. The first segment,* as seen from above (*a*, fig. 4), is a small plate, with the outline of the

* Latreille and Milne-Edwards have mistaken this small first segment of the abdomen of *Remipes* and *Hippa* for the last segment of the thorax, and I am not aware that the mistake has been corrected by any subsequent author. Latreille (Genera Crustaceorum et Insectorum, i, p. 45) simply says of *Remipes testudinarius*, "Cauda segmento baseos aliis multo latiore." But Milne-Edwards (Histoire naturelle des Crustacés, ii, pp. 203, 206, 208), after saying in the description of the genus *Albunea*, "Le premier anneau de l'abdomen est petit, et reçu dans une échancrure de la carapace; le second est au contraire très-grand et présente de chaque côté un grand prolongement lamelleux," goes on to say, under the genus *Remipes*, "Le dernier anneau thoracique, qui porte ces appendices [les pettes postérieures], est complet en dessus, mobile, et pas recouvert par la carapace, de manière qu'on pourrait facilement le prendre pour le premier segment de l'abdomen. Celui-ci est très-grand, et présente de chaque côté un prolongement lamelleux ovalaire qui chevanche sur la carapace; son bord postérieur est échancré pour loger le second anneau abdominal, qui est ovalaire; * * * * le cinquième et la sixième sont également petits, mais sont sondés entre eux. * * * * Les trois premiers anneaux portent, chez la femelle, des filits ovifères simples." Under *Hippa*, he further says, "Le dernier anneau thoracique n'est pas libre et à découvert comme chez les Remipèdes; mais le premier article de l'abdomen est à peu près de

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segment of a circle, filling an arcuate sinus in the posterior margin of the carapax; with which, however, it is not yet consolidated, as in the adult, for it is slightly movable upon the carapax and usually separates from it and remains attached to the second segment when the abdomen is torn away from the cephalo-thorax. As usual in the megalops-stage, and in the larval forms of Podophthalmia generally, it is without appendages. The second-segment is about five times as broad as long, three-fourths as broad as the carapax, and nearly twice as broad as the third segment; its great breadth being a result of a broad, lamellar expansion each side. The anterior margin is nearly straight, and the lamellar portion each side slips slightly over the posterior margin of the carapax, when the abdomen is folded beneath the sternum. The lateral margins are very oblique and converge rapidly to the posterior margin, which is no longer than the breadth of the third segment, for the reception of which it is excavated throughout nearly its whole extent; but the lateral expansions do not project so far posteriorly each side of the third segment as they do in the adult. The margins each side are armed with a few stiff setæ. The third segment is about as long as the second, the lateral margins only slightly projecting and rounded and, together with the anterior margin, armed with a few setæ. The fourth and fifth segments are nearly as long as the third, and each successively, very slightly narrower than the one in front of it; they both project very little laterally and are sparsely armed on the lateral and anterior margins with stiff setæ. The sixth segment is nearly as wide as the fifth and only a little wider than long, both the anterior and posterior margins

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même forme, et les anneaux suivans présentent aussi la disposition que nous avous déjà remarquée chez ces Crustacés."

Without any reference to the homology of the parts, so clearly shown by their structure and appendages in the megalops-stage, there need be no question in regard to the broad, winged segment in both *Hippa* and *Remipes* being the second, from the fact that it bears the first pair of large ovigerous appendages in the female, and that, if it be the first, that somewhere, two of the succeeding segments must have become completely consolidated into one, although there is no indication, in the segments themselves, of any such consolidation. Quite as conclusive also is the fact that, in the allied genus *Albunea*, while all the segments of the abdomen are perfectly distinct and movable one upon the other, and the anterior ones have nearly the same form as in *Hippa* and *Remipes*, the small first segment is entirely free from the carapax, though fitting closely in a deep sinus in its posterior margin. as described by Milne-Edwards in the passage just quoted. Although this consolidation dorsally of the first segment of the abdomen with the mandibular segment is of rare occurrence in the Malacostraca, it certainly seems no more abnormal than the appearance of the last thoracic segment in the same position, would be.

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being considerably curved outward in the middle. The telson is proportionally a little broader than in the adult,—the breadth being rather more than half the length, while in the adult it is usually less,—and much less acute posteriorly and at the anterio-lateral angles. The margin is sparsely fringed, nearly all round, with slender setæ or hairs.

The appendages of the second, third, fourth, and fifth segments are similar, though varying considerably in the length of the terminal lamellæ. The peduncles, or protopods, of the appendages of the second segment (Plate XLVIII, fig. 5) are stout and nearly as long as the breadth of the segment itself. The outer lamella, or exopod, (*a*, fig. 5) is nearly as broad as the peduncle, more than half as long, and margined with about twelve slender, plumose setæ (fig. 5*a*), of which the terminal ones are nearly as long as the peduncle, while the others decrease in length as they recede from the tip of the lamella. The inner lamella, or endopod, (*b*, fig. 5) is a small papilliform appendage, not as long as the diameter of the peduncle, and armed at the tip with a number of very minute appendages, which are apparently only extremely modified spines or setæ. These appendages, as shown in the accompanying illustrations (Plate XLVIII, fig. 6*a*, 6*b*), are composed of a cap, shaped like a concavo-convex lens, supported, on the less convex side, by a short peduncle inserted at the margin and nearly vertical to the under surface of the cap. The diameter of the cap is from ·007 to ·009^{mm} and the height from the surface of the lamella about the same. In alcoholic specimens at least, these appendages seem to be hard and chitinous, and I am at a loss to understand what purpose they serve in the economy of the animal, unless it be as scrapers for the removal of parasites and other foreign substances from some part of the under surface of the body, in which case it is difficult to see just how they could be applied.

The appendages of the third, fourth, and fifth segments differ from those of the second in the successive shortening of the peduncle, but, more particularly, in the successive increase in the size of the inner lamella, which in the appendages of the fifth segment (Plate XLVIII, fig. 6) have become more than half as long as the outer; the inner lamellæ are similarly armed at the tips, and the outer are all alike margined with long plumose setæ, of which there are usually a very few more on the posterior than on the anterior pairs.

The appendages of the sixth segment (Plate XLVIII, fig. 4) are of nearly the same form and proportion as in the adult. The peduncle, or protopod, is stout, broadest distally, and a little longer than the

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segment itself. The terminal lamellæ are nearly alike, each being almost as long as the peduncle, less than half as broad, oval, broadly rounded at the tip, and fringed with slender, plumose setæ, which are very long at the tips of the lamellæ but shorter along the sides. The outer lamella, or exopod, is connected with the peduncle by a very short basal segment.

The young in the megalops-stage were frequently taken at the surface in the towing-net from the 25th of August to the 10th of September. Most of them were taken in the evening, but single individuals were once or twice taken in the day-time. They were also several times obtained directly from specimens in the last zoea-stage kept in aquaria, though the greater part of the zoeæ so kept died during the process of molting. The color of the megalops during life is very much like that of the zoeæ, except that the megalops wants the spots of bright metallic pigment so conspicuous in the zoea.

While in aquaria, the young in the megalops-stage swam rapidly about the walls, at the surface of the water, but not quite as actively as in the zoea-stages. When placed in a shallow dish of sea-water with a little sand heaped up upon one side, they at once, on approaching it, darted for the edge of the miniature sand-beach and backed themselves into it, with an evident satisfaction and with an ease and agility that could not have been excelled by their sand-loving parents after months or years of practice. One, only a few hours after changing from the zoea-stage, evinced the same anxiety to be settled in a home of changing sand. They seemed perfectly content to remain thus buried just at the edge of the water and were never noticed to leave their tiny sand-beach, unless disturbed. If pushed out of the sand and placed on the opposite side of the dish, they swam uneasily about until they found their former retreat and at once plunged themselves backward quite out of sight into it.

On the southern side of Fire Island Beach, Long Island, in September, 1870, I noticed the alacrity with which the young hippas buried themselves in the beach, when they were brought in and left on the wet sand by the waves, and then suspected they were just relinquishing their early pelagic habits, but unfortunately I preserved no specimens at this time. A few days later, the young were found in considerable numbers burrowing in the damp sand in company with species of Orchestidæ, just above the edge of the waves; a situation in which I have never since observed either the young or adults.

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Three of the specimens, found upon the beach and preserved at this time, prove interesting as they apparently represent a stage next succeeding the megalops. In these specimens the carapax is a little over 4^{mm} in length and the whole integument is of a much firmer consistency than in the megalops-stage. The cephalo-thoracic appendages have all advanced slightly toward the fully mature form. The ocular peduncles have increased in length but are more slender, the diameter at the tips being apparently, absolutely very slightly less than in the megalops-stage. In the antennulæ (Plate XLVIII, fig. 2), the distal prominence upon the under side of the second segment of the peduncle, projects as far as the distal end of the third segment and is more densely fringed with setæ than before. The flagellum has increased only slightly in length and has three or four additional segments, but the secondary flagellum is a third as long as the primary and is composed of four segments, of which the ultimate is as long as the penultimate and antepenultimate together. The oral appendages and thoracic legs have changed comparatively little, although the slender posterior pair of thoracic legs are much more completely armed with setæ and spinules than before.

The first segment of the abdomen has become pretty firmly consolidated with the carapax, though the suture marking the union is still conspicuous and much deeper than in the adult. The expansions on the sides of the second segment are a little broader longitudinally, and the tip and the antero-lateral angles of the telson are more acute than in the megalops. The most marked change, however, has taken place in the appendages of the anterior segments. In all three of the specimens examined, there are no appendages whatever upon either one of the five anterior segments. The sudden and total disappearance of these appendages raises the question, whether the ovigerous appendages of the female are derived directly from the swimming legs of the megalops, or are developed specially, at a later period. It is of course very difficult, if not impossible, to distinguish the sexes, at so early a period, by any of the primary sexual characters; so that the three specimens examined may very likely all be young males; in which case the early disappearance of the appendages might, naturally enough, be expected, as there are no appendages on any of the five anterior segments in the adults of that sex. I have, however, examined several additional specimens of the young, in which the carapax was less than 7^{mm} in length, without, in any case, finding even rudiments of appendages on any of the anterior segments of the abdomen. The smallest individual in which I have been able to detect ovigerous appendages is a specimen, with the

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carapax about 7^{mm} in length, from Egmont Key, Florida. In this specimen, the appendages upon the second segment are very small and slender, about $\cdot 8^{mm}$ long, unbranched, and composed of three segments—as in the adult female (Plate XLVIII, fig. 16)—of which the terminal one is very short and tipped with a sparsely plumose seta, similar to a single other seta upon the side of the proximal segment; there are no other setæ or hairs discoverable. The appendages of the third segment are similar, but still smaller, being scarcely $\cdot 5^{mm}$ long, and composed of two segments only; although the newly formed appendages, as seen through the integument, show indication of division into three segments. No appendages whatever were discoverable upon the fourth segment. As the specimen was about to molt, there is a mere possibility that there may have been very rudimentary appendages upon this segment. In the adult female there are three pairs of unbranched appendages, composed of three segments each, and borne upon the second, third, and fourth segments; no appendages whatever being borne upon the fifth segment. In mature specimens of ordinary size (in which the carapax is about 20^{mm} long), the appendages of the second segment (Plate XLVIII, fig. 16) are about 10^{mm} long; those upon the fourth, about $3 \cdot 5^{mm}$, while those upon the third are intermediate between the two other pairs. Though these observations are by no means conclusive or satisfactory, it seems probable that the egg-bearing appendages of the adult *Hippa* are special developments, and not metamorphosed from the swimming legs of the megalops-stage.

The following table gives, in millimeters, the results of measurements, of the megalops and later stages, side by side for comparison. The measurements in the second column are from one of the three specimens, previously referred to, from Fire Island Beach, Long Island; the others are from specimens from Vineyard Sound.

	Megalops.	Fire I.	Male.	Male.	Female.	Female.
Length of carapax,	3·2	4·3	8·3	14·0	13·0	20·6
Breadth of "	2·0	2·8	6·2	10·5	9·0	15·4
Length of abdomen,	3·0	4·5	9·0	16·0	14·5	25·0
" ocular peduncle,	·6	·7	2·4	3·8	3·6	4·6
Diameter of ocular peduncle at tip,	·3	·25	·4	·5	·5	·6
Length of antennula,	1·0	1·3	3·8	5·6	5·4	7·0
" antenna,	3·4	4·8	10·0	19·0	16·0	25·0
" ultimate segment of anterior legs,	·8	1·3	3·0	4·1	3·8	5·8
" peduncle of appendage of sixth segment of abdomen,	·65	1·0	2·0	3·0	2·8	4·6
" lamellæ of the same,	·65	1·0	2·0	2·9	2·8	4·5
" telson,	1·7	2·5	5·3	9·2	8·5	14·0
Breadth of "	·9	1·3	2·4	4·0	3·9	7·0

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A zoea from Zanzibar described briefly by Claus, in his recent work on the development of Crustacea,* and by him doubtingly regarded as one of the early stages of some species of *Albunea*, is certainly the larva of some near ally of *Hippa*, though the figures and description are scarcely sufficiently detailed for a close comparison with the here described zoea-stages of that genus. The zoea described by Claus is apparently in a stage corresponding to that which I have called the third zoea-stage of *Hippa*, but it is already much larger than the zoeæ of the last stage in that genus. As far as can be judged from Claus' imperfect figures and description, the most important character, distinguishing his zoea from that of *Hippa*, is the presence of the exognaths of the external maxillipeds, which form a third pair of swimming appendages somewhat smaller than the two anterior pairs; all the other distinctions seem to be of minor importance. The exognaths of the external maxillipeds are entirely wanting in the adult Hippidæ but are present in the Albunidæ, and their presence in the supposed *Albunea* zoea, coupled with their absence in the latest zoea-stages of *Hippa*, tends strongly to confirm Claus' supposition that his zoea really belongs to the Albunidæ, and most likely to the genus *Albunea*.

From his examination of this so-called zoea of *Albunea*, together with Fritz Müller's note upon the zoea of *Hippa emerita*, previously referred to, Claus concludes that the larval stages of the *Hippa* and *Albunea* group of crustaceans agree much more closely with the zoea-stages of the Brachyura than with the larval stages of the Paguroids or any of the Macrura. The additional evidence, derived from the nearly complete post-embryonal history of *Hippa*, here presented, abundantly confirms this view. There is nothing in the zoea- or megalops-stages of *Hippa* to recall particularly any of the known early stages of the Paguroids, while there is much that forcibly recalls the Brachyuran zoea and megalops. The only conspicuous character distinguishing the *Hippa* and *Albunea* zoea from the ordinary zoeæ of Brachyura, is the absence of a dorsal spine upon the carapax, and this is apparently a distinction of minor importance. Claus has suggested a resemblance between these Hippoid zoeæ and the *Erichthus* larval stage; the motions and color of the living zoeæ of *Hippa* do certainly remind one of the early stages of some of the Stomatopods; but these distant resemblances do not appear to be

* Untersuchungen zur Erforschung der Genealogischen Grundlage des Crustaceen-Systems, Wein. 1876, p. 59. plate ix, figs. 1–10.

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the result of any structural similarity. In fact the full series of the early stages of *Hippa* shows conclusively that the young undergo an essentially Brachyural, zoea and megalops, post-embryonal development, and it furnishes an important addition to the evidence tending to confirm the view that the Anomoura are a heterogeneous group, made up, probably, of outlying Brachyural and Macrural families.

Note on the Structure of the Mandibles and Maxillæ in Hippa and Remipes.

In the description of the megalops-stage, I have already alluded to the remarkable structure of the mandibles and the first pair of maxillæ in the adult *Hippa*, but these appendages in *Hippa* and the allied genera seem worthy of more special attention, since they furnish important characters for distinguishing the Hippidæ and Albunidæ. Although the oral appendages of *Albunea*, and apparently also of the allied genera, *Blepharopoda* Randall and *Lepidopa* Stimpson, have been correctly described by all the later authorities, the structure and homology of the parts of the mandibles and the first pair of maxillæ in *Hippa* and *Remipes*, though very important in a taxonomic point of view, seem to have been misunderstood by all carcinologists who have written upon these genera.

Milne-Edwards, in his great general work on crustaceans, appears to have been the first to describe, with care, the oral appendages of any of the Hippidæ or Albunidæ, and the errors, into which he fell in describing *Hippa* and *Remipes*, have been perpetuated by succeeding writers. To be sure, J. C. Fabricius had before described the parts about the mouth, in his original description of the genus *Hippa* (Supplementum Entomologiæ Systematicæ, p. 329), but his account of the inner appendages is so confused that I am quite unable to understand what parts he had in view as the mandible and its palpus. He says, "*Mandibula* brevis, cornea, fornicata, obtusa, dorso palpigera. *Palpus* setaceus, tenuis, uti videtur multiarticulatus," which could scarcely be made to apply to the parts of the mandible as they really exist, either in *Hippa* or *Remipes*. Milne-Edwards, in his description of the genus *Remipes* (Histoire naturelle des Crustacés, ii, p. 205), says, "Les mâchoires de la seconde paire ne présentent rien de bien remarquable; celles de la première paire sont très-petites, et refoulées en avant, entre la mandibule et la lèvre supérieure, qui est très-grande et fort saillante. Enfin la mandibule, qui est fortement dentelée, porte un palpe composé de deux petits articles lamelleux, séparés du corps de la mandibule par un grand sillon membraneux." This strongly dentate "mandible"

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is evidently the terminal portion of the inner lobe of the first maxilla (Plate XLVII, fig. 10, *a*); but its "palpus composed of two small, lamellar segments" is not so easily interpreted, though it may possibly be the real mandible itself. In the figures of *Remipes testudinarius* given in Milne-Edwards' volume on the Crustacea in the third edition of Le Régne animal de Cuvier, figure 1[d], plate 42, is simply the inner lobe of the first maxilla, which is entirely wanting in the otherwise correct figure of the maxilla itself; but the only allusion to the mandibular palpus is in the explanation of figure, "1[d], Mandibules (ces organes sont dépourvus d'appendice palpiforme)." In Milne-Edwards' description of the genus *Hippa* (Histoire naturelle des Crustacés, ii, p. 207) no reference is made to the maxillæ or mandibles, and no part of either of these appendages is given in the figures of *H. emerita* in Le Régne animal de Cuvier.

De Haan's figures of mouth appendages of *Remipes* (Fauna Japonica, Plate Q) are like those of Milne-Edwards just referred to, except that he has left the basal portion of the inner lobe of the first maxilla properly attached in place to answer for the inner lobe which he describes as the "lacina interna brevissima" (op. cit., p. 201). De Haan however failed to find the mandibular palpus, for he distinctly says in the description of these maxillary mandibles, "palpis nullis." In regard to *Hippa*, De Haan seems to have been more fortunate, for, in his description of the genus (op. cit., p. 201), after correctly describing the maxillæ, he says, "Mandibulæ minutissimæ, cartilagineæ, corona integra, palpis membranaceis triarticularis." His figure of the mandible of *Hippa Asiatica* (op. cit., plate Q; copied as figure 14, Plate XLVI, of the accompanying illustrations), although evidently drawn from the real mandible, is apparently very incorrect.

The following descriptions and figures of the mandibles and maxillæ in *Hippa* and *Remipes* are taken from *H. talpoida* Say, and *R. Pacificus* Dana.

Hippa.—The form and position of the labrum, labium, and mandibles, as seen from beneath, are shown in figure 10 on Plate XLVI. The mouth is a narrow, longitudinal opening bounded, each side and posteriorly, by a margin raised above the general surface of the buccal area, and in front, by the labrum (*a*, figure 10), which projects anteriorly as a triangular, lamelliform plate above the epistome, from which, however, it is separated by a considerable space. Beneath the labrum, the margin of the epistome is reflected and projects forward each side into a slender, lamelliform process (*b*) tipped with

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very long, plumose setæ extending along each side of the labrum. At the sides of the oral opening are the mandibles (*c*, *d*, *e*), the tips of their palpi (*e*) approaching each other just in front of it. The mesial side of the basal portion of the mandible is consolidated with and forms a part of the raised lateral margin of the oral opening, while the outer side is continuous with and consolidated to the buccal walls each side. The palpus arises from this basal portion of the mandible and inwraps the posterior part of the margin of the labrum by means of an oblique fold, which distinctly separates it into an outer (*d*) and an inner (*e*) portion. This oblique fold is most marked at the anterior extremity of the palpus, where it presents a sigmoid curve, the thickening of the palpus posteriorly rendering it less distinct though well seen in a transverse section, as in figure 10*a*. The free margin (*d*, figure 10) of this outer portion is thickly armed with a series of stout spinules, while the inner portion projects above the posterior part of the labrum, as a broad, oval-pointed, lamelliform lobe, the edges of which are thickly beset with slender setæ. The coronal portion (*c*) of each mandible is a broad, obtusely rounded, thin lamella, the free terminal part of which lies just above the palpus. Each lamella of the deeply bifid labium (*f*) lies just above the mandible each side and closely infolds the margin of the oral opening.

The protognathal portion of the first maxilla (Plate XLVII, fig. 8) is divided into two deeply separated lobes and fits closely upon the labium and mandible. The somewhat oval, terminal portion of the posterior, or inner, lobe (*a*, fig. 8) lies just over the lobes of the labium each side, and the whole length of its nearly straight, inner margin is bent downward into the posterior part of the oral opening, and is edged with a double series of long, serrate setæ; while the outer convex surface is armed with a scattered, double line of minute spinules. This terminal portion of the lobe is free all round, except near the outer, posterior angle where it is connected with the buccal walls, and where a thickened, rod-like portion connects it, by an imperfect articulation, with the base of the anterior lobe. The anterior lobe (*b*) is very long and narrow, and its oblique, terminal margin lies just over the inner portion of the mandibular palpus, and is thickly armed with short, setiform spinules, while the lateral margins are thickly clothed with slender, plumose setæ, of which those at the anterior extremity of the outer margin are very much the longest. The palpus (*c*) is soft and sack-like, arises from near the base of the anterior lobe, and projects laterally into the efferent passages of the branchial chamber.

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The second pair of maxillæ offer no marked peculiarity and are essentially of the same form as in the megalops-stage (Plate XLVII, fig. 3). The two principal lobes of the protognath are very similar in form and structure to, lie directly over, and doubtless act in direct conjunction with, the two corresponding lobes of the first maxilla.

Remipes.—The general structure and arrangement of the parts of the mandibles and both pairs of maxillæ is the same as in *Hippa*, but there are important differences in the details of the form and structure of special parts, particularly in the posterior lobe of the protognath in the first pair of maxillæ.

The mandibles (Plate XLVII, fig. 9) are consolidated with and closely invest the walls of the oral opening as in *Hippa*, but the protognathal portion (*a*) is farther removed from the opening, is even more rudimentary, its margins are unarmed, and it apparently serves only as a base for the attachment of the palpus. The free margin (*b*) of the outer portion of the palpus projects in an obtuse angle in front and is armed with a few, very stout, short, and obtuse spines. The inner portion (*c*), lying just above the labrum, is somewhat orbicular, and its terminal margin is armed with numerous, exceedingly long setæ, of which the longest are plumose.

The terminal portion of the posterior lobe in the first pair of maxillæ (Plate XLVII, fig. 10, *a*) is a thick, heavily calcified, rectilinear plate united, by a broad articulation, with the basal, rod-like portion and at the posterior angle, directly with the buccal walls. Its terminal margin is armed externally with a series of five, very large, short, stout, conical, dentiform spines, and at the angles at each end of the series with a similar, but very much smaller spine. Upon the short posterior margin there are several still smaller spines of the same character. The terminal margin beneath is thickly beset with stout, plumose setæ projecting inward. The anterior lobe (*b*) is long, very narrow to near the expanded, spatulate extremity, the convex terminal margin of which is densely armed with setæ which are stout and spiniform, except anteriorly where they are long and more slender. The palpus (*c*) is nearly the same as in *Hippa*, but a little shorter and thicker.

The different parts of the second pair of maxillæ are arranged in the same way as in the *Hippa*, but the posterior lobe of the protognath is proportionally much smaller and its edge is armed with very long, plumose setæ, which are of nearly uniform thickness to very near the tips; while the anterior lobe is spatulate in form and thickly armed with similar, but somewhat shorter, setæ.

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EXPLANATION OF PLATES.

Plate XLV.

Figure 1.—Zoea in the second stage, ventral view, with the abdomen bent forward beneath the cephalo-thorax so as to show the dorsal side of the terminal portion of the abdomen, enlarged twenty-four diameters.

Figure 2.—Zoea in the last stage, dorsal view, enlarged twelve diameters.

Figure 3.—The same, in the same position as in figure 1, ventral view, the natatory limbs omitted, enlarged twelve diameters.

Figure 4.—The same, lateral view, enlarged twelve diameters.

Figure 5.—Megalops just changed from the last zoea-stage; dorsal view of anterior portion of the animal showing the eyes, antennulæ, antennæ, and the extremities of the first pair of thoracic legs, enlarged twenty-four diameters. The antennæ are shown fully extruded and the whole animal under slight compression.

Figure 6.—One of the segments of the flagellum of an antenna, megalops-stage, seen in transverse section to show the arrangement of the setæ, enlarged forty diameters.

Plate XLVI.

Figure 1.—Antennula, second zoea-stage, enlarged seventy-five diameters.

Figure 2.—Antenna of the right side seen from beneath, second zoea-stage, enlarged seventy-five diameters; *a*, *b*, dentiform processes; *c*, flagellum.

Figure 3.—The same, third zoea-stage, enlarged seventy-five diameters; *a*, *b*, dentiform processes; *c*, rudimentary flagellum.

Figure 4.—The same, last zoea-stage, enlarged forty diameters; *a*, *b*, dentiform processes; *c*, rudimentary flagellum showing the multiarticulate flagellum of the megalops-stage already formed within.

Figure 5.—Antennulæ and inner mouth appendages seen from beneath, last zoea-stage, enlarged thirty-six diameters; *a*, *a*, antennulæ; *b*, labrum; *c*, mandible; *d*, labium; *e*, first maxilla.

Figure 6.—Tips of the mandibles, last zoea-stage, enlarged 150 diameters.

Figure 7.—Peduncle of the antenna seen from beneath, megalops-stage, enlarged twelve diameters; *a*, *b*, *c*, *d*, *e*, segments of peduncts; *f*, supplementary rod or plate.

Figure 8.—Labrum, labium and mandibles seen in place from beneath, megalops-stage, enlarged seventy-five diameters; *a*, labrum; *b*, coronal portion of mandible; *c*, *d*, mandibular palpus; *e*, labium.

Figure 9.—Coronal portion of the mandible of another specimen in the same stage, enlarged 150 diameters.

Figure 10.—Labrum, labium, and mandibles seen in place from beneath, adult female, enlarged twenty-four diameters; *a*, labrum; *b*, setæ-bearing tubercle above the margin of the labrum; *c*, coronal portion of the mandible; *d*, lateral spinous margin of the basal portion of the mandibular palpus; *e*, terminal portion of the palpus; *f*, labium; *g*, *h*, line of the section shown in figure 10*a*. The letter *h*, is directly over the anterior extremity of the elongated mouth opening. 10*a*, outline of a transverse section of the mandible through the line *gh* in figure 10; the dotted lines representing the anterior edges of the appendage projected upon the section; *c*, tip of the coronal portion of the mandible; *g*, lateral spinous margin of the palpus; *h*, inner extremity of the terminal portion of the palpus.

Figure 11.—First maxilla of the left side seen from beneath, second zoea-stage, enlarged seventy-two diameters; *a*, inner lobe; *b*, outer lobe; *c*, palpus.

Figure 12.—Same, last zoea-stage, enlarged seventy-two diameters.

Figure 13.—Same, megalops-stage, enlarged seventy-five diameters.

Figure 14.—Mandible of *Hippa Asiatica*, copied from DeHaan, Fauna Japonica, Crustacea, Plate Q.

Plate XLVII.

Figure 1.—Second maxilla of the left side seen from beneath, second zoea-stage, enlarged thirty-six diameters; *a* protognath; *b*, scaphognath.

Figure 2.—Same, last zoea-stage, enlarged thirty-six diameters: *a*, *b*, protognath; *c*, scaphognath.

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Figure 3.—Same, megalops-stage, enlarged forty diameters; *a*, *a'*, *b*, lobes of the protognath; *d*, endognath; *c*, scaphognath.
Figure 4.—First maxilliped of the left side seen from beneath, megalops-stage, enlarged forty diameters; *a*, protognath; *b*, endognath; *c*, exognath.
Figure 5.—Second maxilliped of the left side seen from behind, last zoea-stage, enlarged forty diameters; *a*, protognath; *b*, endognath, *c*, natatory exognath.
Figure 6.—Second maxilliped of the left side seen from beneath, megalops-stage, enlarged forty diameters; *a*, endognath; *b*, exognath.
Figure 7.—Third (external) maxilliped of the left side seen from beneath, megalops-stage, enlarged twenty-four diameters
Figure 8.—First maxilla of the right side seen from beneath, adult female *Hippa*, enlarged twelve diameters; *a*, inner lobe; *b*, outer lobe; *c*, palpus.
Figure 9.—*Remipes Pacificus.* Mandible of the right side seen from beneath, adult female, enlarged twenty-four diameters; *a*, basal part of the mandible itself (protognath); *b*, lateral spinous margin of the basal portion of the palpus; *c*, terminal portion of the palpus.
Figure 10.—First maxilla of the right side, from the same specimen as the last figure, enlarged twelve diameters; *a*, inner lobe; *b*, outer lobe; *c*, palpus.

Plate XLVIII.

Figure 1.—Antennula, seen from the side, megalops-stage, enlarged twenty-four diameters; *a*, *b*, *c*, first, second, and third segments of the peduncle; *d*, primary flagellum; *e*, rudimentary secondary flagellum.
Figure 2.—Same, from a specimen about 5^{mm} in length, enlarged twenty-four diameters.
Figure 3.—Same, adult female, enlarged six diameters.
Figure 4.—Abdomen, megalops-stage, dorsal view, enlarged twelve diameters; *a*, first segment.
Figure 5.—One of the swimming legs of the second segment of the abdomen, megalops-stage, enlarged twenty-four diameters; *a*, outer lamella, or exopod; *b*, inner lamella, or endopod. 5*a*, portion of one of the plumose setæ, more enlarged.
Figure 6.—One of the swimming legs of the fifth segment of the abdomen of the same specimen, enlarged twenty-four diameters; *a*, outer lamella; *b*, inner lamella. 6*a*, 6*b*, side and front view of one of the peculiar appendages with which the tip of the inner lamella is armed, enlarged 500 diameters.
Figure 7.—Telson of adult male, dorsal view, enlarged two diameters.
Figure 8.—Same, adult female, enlarged two diameters.
Figure 9.—Rudimentary thoracic legs and third maxiliped of the left side, last zoea-stage, enlarged twelve diameters; *a*, base of second maxillipeds; *b*, third maxilliped; *c*, *d*, *e*, *f*, *g*, thoracic legs; *h*, branchiæ. The appendages are represented as seen from the inner (mesial) side and under slight compression so as to show the extremities which are curled in and hidden from the outside; in consequence, the branchiæ are represented diagramatically.
Figure 10.—Appendage of the right side of the sixth segment of the abdomen, second zoea-stage, dorsal view, enlarged forty diameters; *a*, basal portion; *b*, outer lamella. 10*a*, tip of lamella with the bases of the terminal setæ, enlarged 175 diameters.
Figure 11.—Corresponding appendage, third zoea-stage, enlarged forty diameters; *a*, base; *b*, outer lamella; *c*, inner lamella.
Figure 12.—Corresponding appendage, last zoea-stage, enlarged forty diameters.
Figure 13.—Posterior margin of telson, second zoea-stage, enlarged 100 diameters. In this and the next two figures the arrow indicates the median line of the telson, the margin on the left of it being exactly the reverse of that shown upon the right.
Figure 14.—The same, from another individual in the same stage, showing a single median spine in place of the two median spines and denticle in other specimens.
Figure 15.—Posterior margin of telson, last zoea-stage, enlarged 100 diameters. 15*a*, two of the denticles from the margin between the spines, enlarged 300 diameters.
Figure 16.—Ovigerous appendage of the second abdominal segment of an adult female, enlarged four diameters.

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IX. On the Equilibrium of Heterogeneous Substances. By J. Willard Gibbs.

(*Continued from page* 248).

THE CONDITIONS OF INTERNAL AND EXTERNAL EQUILIBRIUM FOR SOLIDS IN CONTACT WITH FLUIDS WITH REGARD TO ALL POSSIBLE STATES OF STRAIN OF THE SOLIDS.

In treating of the physical properties of a solid, it is necessary to consider its *state of strain*. A body is said to be *strained* when the relative position of its parts is altered, and by its *state of strain* is meant its state in respect to the relative position of its parts. We have hitherto considered the equilibrium of solids only in the case in which their state of strain is determined by pressures having the same values in all directions about any point. Let us now consider the subject without this limitation.

If x', y', z' are the rectangular co-ordinates of a point of a solid body in any completely determined state of strain, which we shall call the *state of reference*, and x, y, z, the rectangular co-ordinates of the same point of the body in the state in which its properties are the subject of discussion, we may regard x, y, z as functions of x', y', z', the form of the functions determining the second state of strain. For brevity, we may sometimes distinguish the variable state, to which x, y, z relate, and the constant state (state of reference), to which x', y', z' relate, as the *strained* and the *unstrained* states; but it must be remembered that these terms have reference merely to the change of form or *strain* determined by the functions which express the relations of x, y, z and x', y', z', and do not imply any particular physical properties in either of the two states, nor prevent their possible coincidence. The axes to which the co-ordinates x, y, z, and x', y', z' relate will be distinguished as the axes of X, Y, Z, and X', Y', Z'. It is not necessary, nor always convenient, to regard these systems of axes as identical, but they should be similar, i. e., capable of superposition.

The state of strain of any element of the body is determined by the values of the differential coefficients of x, y, and z with respect to x', y', and z'; for changes in the values of x, y, z, when the differential coefficients remain the same, only cause motions of translation of the

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body. When the differential coefficients of the first order do not vary sensibly except for distances greater than the radius of sensible molecular action, we may regard them as completely determining the state of strain of any element. There are nine of these differential coefficients, viz.,

$$\left.\begin{array}{lll} \dfrac{dx}{dx'}, & \dfrac{dx}{dy'}, & \dfrac{dx}{dz'} \\ \dfrac{dy}{dx'}, & \dfrac{dy}{dy'}, & \dfrac{dy}{dz'} \\ \dfrac{dz}{dx'}, & \dfrac{dz}{dy'}, & \dfrac{dz}{dz'} \end{array}\right\} \tag{354}$$

It will be observed that these quantities determine the orientation of the element as well as its strain, and both these particulars must be given in order to determine the nine differential coefficients. Therefore, since the orientation is capable of three independent variations, which do not affect the strain, the strain of the element, considered without regard to directions in space, must be capable of six independent variations.

The physical state of any given element of a solid in any unvarying state of strain is capable of one variation, which is produced by addition or subtraction of heat. If we write $\varepsilon_{V'}$ and $\eta_{V'}$ for the energy and entropy of the element divided by its volume in the state of reference, we shall have for any constant state of strain

$$\delta\varepsilon_{V'} = t\,\delta\eta_{V'}.$$

But if the strain varies, we may consider $\varepsilon_{V'}$ as a function of $\eta_{V'}$ and the nine quantities in (354), and may write

$$\begin{aligned} \delta\varepsilon_{V'} = t\,\delta\eta_{V'} &+ X_{X'}\,\delta\frac{dx}{dx'} + X_{Y'}\,\delta\frac{dx}{dy'} + X_{Z'}\,\delta\frac{dx}{dz'} \\ &+ Y_{X'}\,\delta\frac{dy}{dx'} + Y_{Y'}\,\delta\frac{dy}{dy'} + Y_{Z'}\,\delta\frac{dy}{dz'} \\ &+ Z_{X'}\,\delta\frac{dz}{dx'} + Z_{Y'}\,\delta\frac{dz}{dy'} + Z_{Z'}\,\delta\frac{dz}{dz'}, \end{aligned} \tag{355}$$

where $X_{X'}, \ldots Z_{Z'}$ denote the differential coefficients of $\varepsilon_{V'}$ taken with respect to $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$. The physical signification of these quantities will be apparent, if we apply the formula to an element which in the state of reference is a right parallelopiped having the edges dx', dy', dz', and suppose that in the strained state the face in which x' has the smaller constant value remains fixed, while the opposite face is moved parallel to the axis of X. If we also suppose

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no heat to be imparted to the element, we shall have, on multiplying by $dx'\,dy'\,dz'$,

$$\delta\varepsilon_{v,}\,dx'\,dy'\,dz' = X_{x,}\,\delta\frac{dx}{dx'}\,dx'\,dy'\,dz'.$$

Now the first member of this equation evidently represents the work done upon the element by the surrounding elements; the second member must therefore have the same value. Since we must regard the forces acting on opposite faces of the elementary parallelopiped as equal and opposite, the whole work done will be zero except for the face which moves parallel to X. And since $\delta\frac{dx}{dx'}\,dx'$ represents the distance moved by this face, $X_{x,}\,dy'\,dz'$ must be equal to the component parallel to X of the force acting upon this face. In general, therefore, if by the positive side of a surface for which x' is constant we understand the side on which x' has the greater value, we may say that $X_{x,}$ denotes the component parallel to X of the force exerted by the matter on the positive side of a surface for which x' is constant upon the matter on the negative side of that surface per unit of the surface measured in the state of reference. The same may be said, *mutatis mutandis*, of the other symbols of the same type.

It will be convenient to use Σ and Σ' to denote summation with respect to quantities relating to the axes X, Y, Z, and to the axes X', Y', Z', respectively. With this understanding we may write

$$\delta\varepsilon_{v,} = t\,\delta\eta_{v,} + \Sigma\,\Sigma'\left(X_{x,}\,\delta\frac{dx}{dx'}\right). \tag{356}$$

This is the complete value of the variation of $\varepsilon_{v,}$ for a given element of the solid. If we multiply by $dx'\,dy'\,dz'$, and take the integral for the whole body, we shall obtain the value of the variation of the total energy of the body, when this is supposed invariable in substance. But if we suppose the body to be increased or diminished in substance at its surface (the increment being continuous in nature and state with the part of the body to which it is joined), to obtain the complete value of the variation of the energy of the body, we must add the integral

$$\int\varepsilon_{v,}\,\delta N'\,Ds'$$

in which Ds' denotes an element of the surface measured in the state of reference, and $\delta N'$ the change in position of this surface (due to the substance added or taken away) measured normally and outward in the state of reference. The complete value of the variation of the intrinsic energy of the solid is therefore

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$$\int\int\int t\delta\eta_{\nu}dx'dy'dz' + \int\int\int \Sigma\Sigma'\left(X_{\chi}\delta\frac{dx}{dx'}\right)dx'dy'dz' + \int \varepsilon_{\nu}\delta N' Ds'. \quad (357)$$

This is entirely independent of any supposition in regard to the homogeneity of the solid.

To obtain the conditions of equilibrium for solid and fluid masses in contact, we should make the variation of the energy of the whole equal to or greater than zero. But since we have already examined the conditions of equilibrium for fluids, we need here only seek the conditions of equilibrium for the interior of a solid mass and for the surfaces where it comes in contact with fluids. For this it will be necessary to consider the variations of the energy of the fluids only so far as they are immediately connected with the changes in the solid. We may suppose the solid with so much of the fluid as is in close proximity to it to be enclosed in a fixed envelop, which is impermeable to matter and to heat, and to which the solid is firmly attached wherever they meet. We may also suppose that in the narrow space or spaces between the solid and the envelop, which are filled with fluid, there is no motion of matter or transmission of heat across any surfaces which can be generated by moving normals to the surface of the solid, since the terms in the condition of equilibrium relating to such processes may be cancelled on account of the internal equilibrium of the fluids. It will be observed that this method is perfectly applicable to the case in which a fluid mass is entirely enclosed in a solid. A detached portion of the envelop will then be necessary to separate the great mass of the fluid from the small portion adjacent to the solid, which alone we have to consider. Now the variation of the energy of the fluid mass will be, by equation (13),

$$\int^{F} t\,\delta D\eta - \int^{F} p\,\delta Dv + \Sigma_1 \int^{F} \mu_1\,\delta Dm_1, \quad (358)$$

where $\int^{F}$ denotes an integration extending over all the elements of the fluid (within the envelop), and Σ_1 denotes a summation with regard to those independently variable components of the fluid of which the solid is composed. Where the solid does not consist of substances which are components, actual or possible (see page 117), of the fluid, this term is of course to be cancelled.

If we wish to take account of gravity, we may suppose that it acts in the negative direction of the axis of Z. It is evident that the variation of the energy due to gravity for the whole mass considered is simply

$$\int\int\int g\,\Gamma'\,\delta z\,dx'\,dy'\,dz', \quad (359)$$

where g denotes the force of gravity, and Γ' the density of the

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element in the state of reference, and the triple integration, as before, extends throughout the solid.

We have, then, for the general condition of equilibrium,

$$\begin{aligned}&\int\!\!\int\!\!\int t\,\delta\eta_{\mathrm{v}'}\,dx'\,dy'\,dz' + \int\!\!\int\!\!\int \Sigma\,\Sigma'\left(X_{\mathrm{x}'}\,\delta\frac{dx}{dx'}\right)dx'\,dy'\,dz'\\&+\int\!\!\int\!\!\int g\,\Gamma'\,\delta z\,dx'\,dy'\,dz' + \int \varepsilon_{\mathrm{v}'}\,\delta N'\,Ds'\\&+\int^{\mathrm{F}} t\,\delta D\eta - \int^{\mathrm{F}} p\,\delta Dv + \Sigma_1 \int^{\mathrm{F}} \mu_1\,\delta Dm_1 \geqq 0. \qquad (360)\end{aligned}$$

The equations of condition to which these variations are subject are:
(1) that which expresses the constancy of the total entropy,

$$\int\!\!\int\!\!\int \delta\eta_{\mathrm{v}'}\,dx'\,dy'\,dz' + \int \eta_{\mathrm{v}'}\,\delta N'\,Ds' + \int^{\mathrm{F}} \delta D\eta = 0\,; \qquad (361)$$

(2) that which expresses how the value of δDv for any element of the fluid is determined by changes in the solid,

$$\delta Dv = -\,(\alpha\,\delta x + \beta\,\delta y + \gamma\,\delta z)\,Ds - v_{\mathrm{v}'}\,\delta N'\,Ds', \qquad (362)$$

where α, β, γ denote the direction cosines of the normal to the surface of the body in the state to which x, y, z relate, Ds the element of the surface in this state corresponding to Ds' in the state of reference, and $v_{\mathrm{v}'}$ the volume of an element of the solid divided by its volume in the state of reference;
(3) those which express how the values of δDm_1, δDm_2, etc. for any element of the fluid are determined by the changes in the solid,

$$\left.\begin{aligned}\delta Dm_1 &= -\,\Gamma_1'\,\delta N'\,Ds',\\ \delta Dm_2 &= -\,\Gamma_2'\,\delta N'\,Ds',\\ &\text{etc.,}\end{aligned}\right\} \qquad (363)$$

where Γ_1', Γ_2', etc. denote the separate densities of the several components in the solid in the state of reference.

Now, since the variations of entropy are independent of all the other variations, the condition of equilibrium (360), considered with regard to the equation of condition (361), evidently requires that throughout the whole system

$$t = \text{const.} \qquad (364)$$

We may therefore use (361) to eliminate the first and fifth integrals from (360). If we multiply (362) by p, and take the integrals for the whole surface of the solid and for the fluid in contact with it, we obtain the equation

$$\int^{\mathrm{F}} p\,\delta Dv = -\int p\,(\alpha\,\delta x + \beta\,\delta y + \gamma\,\delta z)\,Ds - \int p\,v_{\mathrm{v}'}\,\delta N'\,Ds', \qquad (365)$$

by means of which we may eliminate the sixth integral from (360). If we add equations (363) multiplied respectively by μ_1, μ_2, etc., and take the integrals, we obtain the equation

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$$\Sigma_1 \int^F \mu_1 \delta D m_1 = -\int \Sigma_1 (\mu_1 \Gamma_1') \, \delta N' \, Ds', \tag{366}$$

by means of which we may eliminate the last integral from (360).

The condition of equilibrium is thus reduced to the form

$$\begin{aligned} &\int\int\int \Sigma \Sigma' \left(X_{x'} \, \delta \frac{dx}{dx'} \right) dx' \, dy' \, dz' + \int\int\int g \, \Gamma' \, \delta z \, dx' \, dy' \, dz' \\ &\quad + \int \varepsilon_{v'} \, \delta N' \, Ds' - \int t \, \eta_{v'} \, \delta N' \, Ds' + \int p \, (\alpha \, \delta x + \beta \, \delta y + \gamma \, \delta z) \, Ds \\ &\qquad + \int p \, v_{v'} \, \delta N' \, Ds' - \int \Sigma_1 \, (\mu_1 \Gamma_1') \, \delta N' \, Ds' \geqq 0, \end{aligned} \tag{367}$$

in which the variations are independent of the equations of condition, and in which the only quantities relating to the fluids are p and μ_1, μ_2, etc.

Now by the ordinary method of the calculus of variations, if we write α', β', γ' for the direction-cosines of the normal to the surface of the solid in the state of reference, we have

$$\begin{aligned} &\int\int\int X_{x'} \, \delta \frac{dx}{dx'} \, dx' \, dy' \, dz' \\ &\quad = \int \alpha' \, X_{x'} \, \delta x \, Ds' - \int\int\int \frac{dX_{x'}}{dx'} \, \delta x \, dx' \, dy' \, dz', \end{aligned} \tag{368}$$

with similar expressions for the other parts into which the first integral in (367) may be divided. The condition of equilibrium is thus reduced to the form

$$\begin{aligned} &-\int\int\int \Sigma \Sigma' \left(\frac{dX_{x'}}{dx'} \, \delta x \right) dx' \, dy' \, dz' + \int\int\int g \, \Gamma' \, \delta z \, dx' \, dy' \, dz' \\ &\quad + \int \Sigma \Sigma' \, (\alpha' \, X_{x'} \, \delta x) \, Ds' + \int p \, \Sigma \, (\alpha \, \delta x) \, Ds \\ &\qquad + \int [\varepsilon_{v'} - t \, \eta_{v'} + p \, v_{v'} - \Sigma_1 \, (\mu_1 \Gamma_1')] \, \delta N' \, Ds' \geqq 0. \end{aligned} \tag{369}$$

It must be observed that if the solid mass is not continuous throughout in nature and state, the surface-integral in (368), and therefore the first surface-integral in (369), must be taken to apply not only to the external surface of the solid, but also to every surface of discontinuity within it, and that with reference to each of the two masses separated by the surface. To satisfy the condition of equilibrium, as thus understood, it is necessary and sufficient that throughout the solid mass

$$\Sigma \Sigma' \left(\frac{dX_{x'}}{dx'} \, \delta x \right) - g \, \Gamma' \, \delta z = 0; \tag{370}$$

that throughout the surfaces where the solid meets the fluid

$$Ds' \, \Sigma \Sigma' \, (\alpha' \, X_{x'} \, \delta x) + Ds \, p \, \Sigma \, (\alpha \, \delta x) = 0, \tag{371}$$

and

$$[\varepsilon_{v'} - t \, \eta_{v'} + p \, v_{v'} - \Sigma_1 (\mu_1 \Gamma_1')] \, \delta N' \geqq 0; \tag{372}$$

and that throughout the internal surfaces of discontinuity

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$$\Sigma\,\Sigma'\,(\alpha' X_{\mathrm{X}'}\,\delta x)_1 + \Sigma\,\Sigma'\,(\alpha'\,X_{\mathrm{X}'}\,\delta x)_2 = 0, \tag{373}$$

where the suffixed numerals distinguish the expressions relating to the masses on opposite sides of a surface of discontinuity.

Equation (370) expresses the mechanical conditions of internal equilibrium for a continuous solid under the influence of gravity. If we expand the first term, and set the coefficients of δx, δy, and δz separately equal to zero, we obtain

$$\left.\begin{aligned} \frac{dX_{\mathrm{X}'}}{dx'} + \frac{dX_{\mathrm{Y}'}}{dy'} + \frac{dX_{\mathrm{Z}'}}{dz'} &= 0, \\ \frac{dY_{\mathrm{X}'}}{dx'} + \frac{dY_{\mathrm{Y}'}}{dy'} + \frac{dY_{\mathrm{Z}'}}{dz'} &= 0, \\ \frac{dZ_{\mathrm{X}'}}{dx'} + \frac{dZ_{\mathrm{Y}'}}{dy'} + \frac{dZ_{\mathrm{Z}'}}{dz'} &= g\,l''. \end{aligned}\right\} \tag{374}$$

The first member of any one of these equations multiplied by $dx'\,dy'\,dz'$ evidently represents the sum of the components parallel to one of the axes X, Y, Z of the forces exerted on the six faces of the element $dx'\,dy'\,dz'$ by the neighboring elements.

As the state which we have called the state of reference is arbitrary, it may be convenient for some purposes to make it coincide with the state to which x, y, z relate, and the axes X', Y', Z' with the axes X, Y, Z. The values of $X_{\mathrm{X}'}$, . . . $Z_{\mathrm{Z}'}$ on this particular supposition may be represented by the symbols X_{X}, . . . Z_{Z}. Since

$$X_{\mathrm{Y}'} = \frac{d\varepsilon_{\mathrm{Y}'}}{d\dfrac{dx}{dy'}}, \quad \text{and} \quad Y_{\mathrm{X}'} = \frac{d\varepsilon_{\mathrm{Y}'}}{d\dfrac{dy}{dx'}},$$

and since, when the states x, y, z and x', y', z' coincide, and the axes X, Y, Z, and X', Y', Z', $d\dfrac{dx}{dy'}$ and $d\dfrac{dy}{dx'}$ represent displacements which differ only by a rotation, we must have

$$X_{\mathrm{Y}} = Y_{\mathrm{X}}, \tag{375}$$

and for similar reasons,

$$Y_{\mathrm{Z}} = Z_{\mathrm{Y}}, \qquad Z_{\mathrm{X}} = X_{\mathrm{Z}}. \tag{376}$$

The six quantities X_{X}, Y_{Y}, Z_{Z}, X_{Y} or Y_{X}, Y_{Z} or Z_{Y}, and Z_{X} or X_{Z} are called the *rectangular components of stress*, the three first being the *longitudinal stresses* and the three last the *shearing stresses*. The mechanical conditions of internal equilibrium for a solid under the influence of gravity may therefore be expressed by the equations

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$$\left.\begin{aligned}\frac{dX_X}{dx}+\frac{dX_Y}{dy}+\frac{dX_Z}{dz}&=0,\\ \frac{dY_X}{dx}+\frac{dY_Y}{dy}+\frac{dY_Z}{dz}&=0,\\ \frac{dZ_X}{dx}+\frac{dZ_Y}{dy}+\frac{dZ_Z}{dz}&=g\,\Gamma,\end{aligned}\right\}\tag{377}$$

where Γ denotes the density of the element to which the other symbols relate. Equations (375), (376) are rather to be regarded as expressing necessary relations (when $X_X, \ldots Z_Z$ are regarded as internal forces determined by the state of strain of the solid) than as expressing conditions of equilibrium. They will hold true of a solid which is not in equilibrium,—of one, for example, through which vibrations are propagated,—which is not the case with equations (377).

Equation (373) expresses the mechanical conditions of equilibrium for a surface of discontinuity within the solid. If we set the coefficients of δx, δy, δz, separately equal to zero we obtain

$$\left.\begin{aligned}(\alpha' X_{X'}+\beta' X_{Y'}+\gamma' X_{Z'})_1+(\alpha' X_{X'}+\beta' X_{Y'}+\gamma' X_{Z'})_2&=0,\\ (\alpha' Y_{X'}+\beta' Y_{Y'}+\gamma' Y_{Z'})_1+(\alpha' Y_{X'}+\beta' Y_{Y'}+\gamma' Y_{Z'})_2&=0,\\ (\alpha' Z_{X'}+\beta' Z_{Y'}+\gamma' Z_{Z'})_1+(\alpha' Z_{X'}+\beta' Z_{Y'}+\gamma' Z_{Z'})_2&=0.\end{aligned}\right\}\tag{378}$$

Now when the α', β', γ' represent the direction-cosines of the normal in the state of reference on the positive side of any surface within the solid, an expression of the form

$$\alpha' X_{X'}+\beta' X_{Y'}+\gamma' X_{Z'}\tag{379}$$

represents the component parallel to X of the force exerted upon the surface in the strained state by the matter on the positive side per unit of area measured in the state of reference. This is evident from the consideration that in estimating the force upon any surface we may substitute for the given surface a broken one consisting of elements for each of which either x' or y' or z' is constant. Applied to a surface bounding a solid, or any portion of a solid which may not be continuous with the rest, when the normal is drawn outward as usual, the same expression taken negatively represents the component parallel to X of the force exerted upon the surface (per unit of surface measured in the state of reference) by the interior of the solid, or of the portion considered. Equations (378) therefore express the condition that the force exerted upon the surface of discontinuity by the matter on one side and determined by its state of strain shall be equal and opposite to that exerted by the matter on the other side. Since

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$$(\alpha')_1 = -(\alpha')_2, \qquad (\beta')_1 = -(\beta')_2, \qquad (\gamma')_1 = -(\gamma')_2,$$

we may also write

$$\left.\begin{array}{l}\alpha'(X_{X'})_1 + \beta'(X_{Y'})_1 + \gamma'(X_{Z'})_1 = \alpha'(X_{X'})_2 + \beta'(X_{Y'})_2 + \gamma'(X_{Z'})_2, \\ \text{etc.,}\end{array}\right\} \quad (380)$$

where the signs of α', β', γ' may be determined by the normal on either side of the surface of discontinuity.

Equation (371) expresses the mechanical condition of equilibrium for a surface where the solid meets a fluid. It involves the separate equations

$$\left.\begin{array}{l}\alpha' X_{X'} + \beta' X_{Y'} + \gamma' X_{Z'} = -\alpha p \dfrac{Ds}{Ds'}, \\ \alpha' Y_{X'} + \beta' Y_{Y'} + \gamma' Y_{Z'} = -\beta p \dfrac{Ds}{Ds'}, \\ \alpha' Z_{X'} + \beta' Z_{Y'} + \gamma' Z_{Z'} = -\gamma p \dfrac{Ds}{Ds'},\end{array}\right\} \quad (381)$$

the fraction $\dfrac{Ds}{Ds'}$ denoting the ratio of the areas of the same element of the surface in the strained and unstrained states of the solid. These equations evidently express that the force exerted by the interior of the solid upon an element of its surface, and determined by the strain of the solid, must be normal to the surface and equal (but acting in the opposite direction) to the pressure exerted by the fluid upon the same element of surface.

If we wish to replace α and Ds by α', β', γ', and the quantities which express the strain of the element, we may make use of the following considerations. The product $\alpha\, Ds$ is the projection of the element Ds on the Y-Z plane. Now since the ratio $\dfrac{Ds}{Ds'}$ is independent of the form of the element, we may suppose that it has any convenient form. Let it be bounded by the three surfaces $x' =$ const., $y' =$ const., $z' =$ const., and let the parts of each of these surfaces included by the two others with the surface of the body be denoted by L, M, and N, or by L', M' and N', according as we have reference to the strained or unstrained state of the body. The areas of L', M', and N' are evidently $\alpha' Ds'$, $\beta' Ds'$, and $\gamma' Ds'$; and the sum of the projections of L, M and N upon any plane is equal to the projection of Ds upon that plane, since L, M, and N with Ds include a solid figure. (In propositions of this kind the *sides* of surfaces must be distinguished. If the normal to Ds falls outward from the small solid figure, the normals to L, M, and N must fall inward, and *vice*

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versa). Now L' is a right-angled triangle of which the perpendicular sides may be called dy' and dz'. The projection of L on the Y-Z plane will be a triangle, the angular points of which are determined by the co-ordinates

$$y,\quad z;\quad y+\frac{dy}{dy'}dy',\quad z+\frac{dz}{dy'}dy';\quad y+\frac{dy}{dz'}dz',\quad z+\frac{dz}{dz'}dz';$$

the area of such a triangle is

$$\tfrac{1}{2}\left(\frac{dy}{dy'}\frac{dz}{dz'}-\frac{dz}{dy'}\frac{dy}{dz'}\right)dy'\,dz',$$

or, since $\frac{1}{2}\,dy'\,dz'$ represents the area of L',

$$\left(\frac{dy}{dy'}\frac{dz}{dz'}-\frac{dz}{dy'}\frac{dy}{dz'}\right)\alpha'\,Ds'.$$

(That this expression has the proper sign will appear if we suppose for the moment that the strain vanishes.) The areas of the projections of M and N upon the same plane will be obtained by changing y', z' and α' in this expression into z', x', and β', and into x', y', and γ'. The sum of the three expressions may be substituted for $\alpha\,Ds$ in (381).

We shall hereafter use Σ' to denote the sum of the three terms obtained by rotary substitutions of quantities relating to the axes X', Y', Z', (i. e., by changing x', y', z' into y', z', x', and into z', x', y', with similar changes in regard to α', β', γ', and other quantities relating to these axes,) and Σ to denote the sum of the three terms obtained by similar rotary changes of quantities relating to the axes X, Y, Z. This is only an extension of our previous use of these symbols.

With this understanding, equations (381) may be reduced to the form

$$\left.\begin{array}{l}\Sigma'(\alpha'\,X_{x'})+p\,\Sigma'\left\{\alpha'\left(\dfrac{dy}{dy'}\dfrac{dz}{dz'}-\dfrac{dz}{dy'}\dfrac{dy}{dz'}\right)\right\}=0,\\ \text{etc.}\end{array}\right\}\qquad(382)$$

The formula (372) expresses the additional condition of equilibrium which relates to the dissolving of the solid, or its growth without discontinuity. If the solid consists entirely of substances which are actual components of the fluid, and there are no passive resistances which impede the formation or dissolving of the solid, $\delta N'$ may have either positive or negative values, and we must have

$$\varepsilon_{v'}-t\,\eta_{v'}+p\,v_{v'}=\Sigma_1(\mu_1\Gamma'_1).\qquad(383)$$

But if some of the components of the solid are only possible com-

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ponents (see page 117) of the fluid, $\delta N'$ is incapable of positive values, as the quantity of the solid cannot be increased, and it is sufficient for equilibrium that

$$\varepsilon_{v'} - t\eta_{v'} + p_{v'} \leqq \Sigma_1(\mu_1 \Gamma_1'). \tag{384}$$

To express condition (383) in a form independent of the state of reference, we may use ε_v, η_v, Γ_1, etc., to denote the densities of energy, of entropy, and of the several component substances in the *variable* state of the solid. We shall obtain, on dividing the equation by $v_{v'}$,

$$\varepsilon_v - t\eta_v + p = \Sigma_1(\mu_1 \Gamma_1). \tag{385}$$

It will be remembered that the summation relates to the several components of the solid. If the solid is of uniform composition throughout, or if we only care to consider the contact of the solid and the fluid at a single point, we may treat the solid as composed of a single substance. If we use μ_1 to denote the potential for this substance in the fluid, and Γ to denote the density of the solid in the variable state, (Γ', as before denoting its density in the state of reference,) we shall have

$$\varepsilon_{v'} - t\eta_{v'} + p\, v_{v'} = \mu_1 \Gamma', \tag{386}$$

and

$$\varepsilon_v - t\eta_v + p = \mu_1 \Gamma. \tag{387}$$

To fix our ideas in discussing this condition, let us apply it to the case of a solid body which is homogeneous in nature and in state of strain. If we denote by ε, η, v, and m, its energy, entropy, volume, and mass, we have

$$\varepsilon - t\eta + p\, v = \mu_1 m. \tag{388}$$

Now the mechanical conditions of equilibrium for the surface where a solid meets a fluid require that the traction upon the surface determined by the state of strain of the solid shall be normal to the surface. This condition is always satisfied with respect to three surfaces at right angles to one another. In proving this well known proposition, we shall lose nothing in generality, if we make the state of reference, which is arbitrary, coincident with the state under discussion, the axes to which these states are referred being also coincident. We shall then have, for the normal component of the traction per unit of surface across any surface for which the direction-cosines of the normal are α, β, γ, [compare (379), and for the notation X_x, etc., page 349,]

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$$S = \alpha(\alpha X_X + \beta X_Y + \gamma X_Z)$$
$$+ \beta(\alpha Y_X + \beta Y_Y + \gamma Y_Z)$$
$$+ \gamma(\alpha Z_X + \beta Z_Y + \gamma Z_Z),$$

or, by (375), (376),

$$S = \alpha^2 X_X + \beta^2 Y_Y + \gamma^2 Z_Z$$
$$+ 2\alpha\beta X_Y + 2\beta\gamma Y_Z + 2\gamma\alpha Z_X. \qquad (389)$$

We may also choose any convenient directions for the co-ordinate axes. Let us suppose that the direction of the axis of X is so chosen that the value of S for the surface perpendicular to this axis is as great as for any other surface, and that the direction of the axis of Y (supposed at right angles to X) is such that the value of S for the surface perpendicular to it is as great as for any other surface passing through the axis of X. Then, if we write $\frac{dS}{d\alpha}, \frac{dS}{d\beta}, \frac{dS}{d\gamma}$ for the differential coefficients derived from the last equation by treating α, β, and γ as *independent* variables,

$$\frac{dS}{d\alpha} d\alpha + \frac{dS}{d\beta} d\beta + \frac{dS}{d\gamma} d\gamma = 0,$$

when $\qquad \alpha\, d\alpha + \beta\, d\beta + \gamma\, d\gamma = 0,$

and $\qquad \alpha = 1, \quad \beta = 0, \quad \gamma = 0.$

That is, $\qquad \frac{dS}{d\beta} = 0, \quad \text{and} \quad \frac{dS}{d\gamma} = 0,$

when $\qquad \alpha = 1, \quad \beta = 0, \quad \gamma = 0.$

Hence, $\qquad X_Y = 0, \quad \text{and} \quad Z_X = 0. \qquad (390)$

Moreover, $\qquad \frac{dS}{d\beta} d\beta + \frac{dS}{d\gamma} d\gamma = 0,$

when $\qquad \alpha = 0, \quad d\alpha = 0,$

$$\beta\, d\beta + \gamma\, d\gamma = 0,$$

and $\qquad \beta = 1, \quad \gamma = 0.$

Hence $\qquad Y_Z = 0. \qquad (391)$

Therefore, when the co-ordinate axes have the supposed directions, which are called the *principal axes of stress*, the rectangular components of the traction across any surface (α, β, γ) are by (379)

$$\alpha X_X, \qquad \beta Y_Y, \qquad \gamma Z_Z. \qquad (392)$$

Hence, the traction across any surface will be normal to that surface,—

(1), when the surface is perpendicular to a principal axis of stress;

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(2), if two of the *principal tractions* X_X, Y_Y, Z_Z are equal, when the surface is perpendicular to the plane containing the two corresponding axes, (in this case the traction across any such surface is equal to the common value of the two principal tractions);

(3), if the principal tractions are all equal, the traction is normal and constant for all surfaces.

It will be observed that in the second and third cases the position of the principal axes of stress are partially or wholly indeterminate, (so that these cases may be regarded as included in the first,) but the values of the principal tractions are always determinate, although not always different.

If, therefore, a solid which is homogeneous in nature and in state of strain is bounded by six surfaces perpendicular to the principal axes of strain, the mechanical conditions of equilibrium for these surfaces may be satisfied by the contact of fluids having the proper pressures, [see (381),] which will in general be different for the different pairs of opposite sides, and may be denoted by p', p'', p'''. (These pressures are equal to the principal tractions of the solid taken negatively.) It will then be necessary for equilibrium with respect to the tendency of the solid to dissolve that the potential for the substance of the solid in the fluids shall have values μ_1', μ_1'', μ_1''' determined by the equations

$$\varepsilon - t\eta + p'v = \mu_1' m, \tag{393}$$

$$\varepsilon - t\eta + p''v = \mu_1'' m, \tag{394}$$

$$\varepsilon - t\eta + p'''v = \mu_1''' m. \tag{395}$$

These values, it will be observed, are entirely determined by the nature and state of the solid, and their differences are equal to the differences of the corresponding pressures divided by the density of the solid.

It may be interesting to compare one of these potentials, as μ_1', with the potential (for the same substance) in a fluid of the same temperature t and pressure p' which would be in equilibrium with the same solid subjected on all sides to the uniform pressure p'. If we write $[\varepsilon]_{p'}$, $[\eta]_{p'}$, $[v]_{p'}$, and $[\mu_1]_{p'}$ for the values which ε, η, v, and μ_1 would receive on this supposition, we shall have

$$[\varepsilon]_{p'} - t[\eta]_{p'} + p'[v]_{p'} = [\mu_1]_{p'} m. \tag{396}$$

Subtracting this from (393), we obtain

$$\varepsilon - [\varepsilon]_{p'} - t\eta + t[\eta]_{p'} + p'v - p'[v]_{p'} = \mu_1 m - [\mu_1]_{p'} m. \tag{397}$$

Now it follows immediately from the definitions of energy and entropy

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that the first four terms of this equation represent the work spent upon the solid in bringing it from the state of hydrostatic stress to the other state without change of temperature, and $p' v - p' [v]_{p'}$ evidently denotes the work done in displacing a fluid of pressure p' surrounding the solid during the operation. Therefore, the first number of the equation represents the total work done in bringing the solid *when surrounded by a fluid of pressure* p' from the state of hydrostatic stress p' to the state of stress p', p'', p'''. This quantity is necessarily positive, except of course in the limiting case when $p' = p'' = p'''$. If the quantity of matter of the solid body be unity, the increase of the potential in the fluid on the side of the solid on which the pressure remains constant, which will be necessary to maintain equilibrium, is equal to the work done as above described. Hence, μ_1' is greater than $[\mu_1]_{p'}$, and for similar reasons, μ_1'' is greater than the value of the potential which would be necessary for equilibrium if the solid were subjected to the uniform pressure p'', and μ_1''' greater than that which would be necessary for equilibrium if the solid were subjected to the uniform pressure p'''. That is, (if we adapt our language to what we may regard as the most general case, viz., that in which the fluids contain the substance of the solid but are not wholly composed of that substance,) the fluids in equilibrium with the solid are all supersaturated with respect to the substance of the solid, except when the solid is in a state of hydrostatic stress; so that if there were present in any one of these fluids any small fragment of the same kind of solid subject to the hydrostatic pressure of the fluid, such a fragment would tend to increase. Even when no such fragment is present, although there must be perfect equilibrium so far as concerns the tendency of the solid to dissolve or to increase by the accretion of similarly strained matter, yet the presence of the solid which is subject to the distorting stresses, will doubtless facilitate the commencement of the formation of a solid of hydrostatic stress upon its surface, to the same extent, perhaps, in the case of an amorphous body, as if it were itself subject only to hydrostatic stress. This may sometimes, or perhaps generally, make it a necessary condition of equilibrium in cases of contact between a fluid and an amorphous solid which can be formed out of it that the solid at the surface where it meets the fluid shall be sensibly in a state of hydrostatic stress.

But in the case of a crystalline solid, subjected to distorting stresses and in contact with solutions satisfying the conditions deduced above, although crystals of hydrostatic stress would doubtless commence to

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form upon its surface (if the distorting stresses and consequent supersaturation of the fluid should be carried too far), before they would commence to be formed within the fluid or on the surface of most other bodies, yet within certain limits the relations expressed by equations (393)–(395) must admit of realization, especially when the solutions are such as can be easily supersaturated.*

It may be interesting to compare the variations of p, the pressure in the fluid which determines in part the stresses and the state of strain of the solid, with other variations of the stresses or strains in the solid, with respect to the relation expressed by equation (388). To examine this point with complete generality, we may proceed in the following manner.

Let us consider so much of the solid as has in the state of reference the form of a cube, the edges of which are equal to unity, and parallel to the co-ordinate axes. We may suppose this body to be homogeneous in nature and in state of strain both in its state of reference and in its variable state. (This involves no loss of generality, since we may make the unit of length as small as we choose.) Let the fluid meet the solid on one or both of the surfaces for which Z' is constant. We may suppose these surfaces to remain perpendicular to the axis of Z in the variable state of the solid, and the edges in which y' and z' are both constant to remain parallel to the axis of X. It will be observed that these suppositions only fix the position of the strained body relatively to the co-ordinate axes, and do not in any way limit its state of strain.

It follows from the suppositions which we have made that

$$\frac{dz}{dx'} = \text{const.} = 0, \quad \frac{dz}{dy'} = \text{const.} = 0, \quad \frac{dy}{dx'} = \text{const.} = 0; \tag{398}$$

and

$$X_{Z'} = 0, \quad Y_{Z'} = 0, \quad Z_{Z'} = -p\,\frac{dx}{dx'}\,\frac{dy}{dy'}. \tag{399}$$

Hence, by (355),

$$d\varepsilon_{V'} = t\,d\eta_{V'} + X_{X'}\,d\frac{dx}{dx'} + X_{Y'}\,d\frac{dx}{dy'} + Y_{Y'}\,d\frac{dy}{dy'} - p\,\frac{dx}{dx'}\,\frac{dy}{dy'}\,d\,\frac{dz}{dz'}. \tag{400}$$

Again, by (388),

* The effect of distorting stresses in a solid on the phenomena of crystallization and liquefaction, as well as the effect of change of hydrostatic pressure common to the solid and liquid, was first described by Professor James Thomson. See *Trans. R. S. Edin.*, vol. xvi, p. 575; and *Proc. Roy. Soc.*, vol. xi, p. 473, or *Phil. Mag.*, S. 4, vol. xxiv, p. 395.

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$$d\varepsilon = t\,d\eta + \eta\,dt - p\,dv - v\,dp + m\,d\mu_1. \tag{401}$$

Now the suppositions which have been made require that

$$v = \frac{dx}{dx'}\,\frac{dy}{dy'}\,\frac{dz}{dz'}, \tag{402}$$

and

$$dv = \frac{dy}{dy'}\,\frac{dz}{dz'}\,d\frac{dx}{dx'} + \frac{dz}{dz'}\,\frac{dx}{dx'}\,d\frac{dy}{dy'} + \frac{dx}{dx'}\,\frac{dy}{dy'}\,d\frac{dz}{dz'}. \tag{403}$$

Combining equations (400), (401), and (403), and observing that ε_{V}, and η_{V}, are equivalent to ε and η, we obtain

$$\eta\,dt - v\,dp + m\,d\mu_1$$
$$= \left(X_{\mathrm{X}'} + p\frac{dy}{dy'}\,\frac{dz}{dz'}\right)d\frac{dx}{dx'} + X_{\mathrm{Y}'}d\frac{dx}{dy'} + \left(Y_{\mathrm{Y}'} + p\,\frac{dz}{dz'}\,\frac{dx}{dx'}\right)d\frac{dy}{dy'}. \tag{404}$$

The reader will observe that when the solid is subjected on all sides to the uniform normal pressure p, the coefficients of the differentials in the second member of this equation will vanish. For the expression $\frac{dy}{dy'}\,\frac{dz}{dz'}$ represents the projection on the Y-Z plane of a side of the parallelopiped for which x' is constant, and multiplied by p it will be equal to the component parallel to the axis of X of the total pressure across this side, i. e., it will be equal to $X_{\mathrm{X}'}$ taken negatively. The case is similar with respect to the coefficient of $d\frac{dy}{dy'}$; and $X_{\mathrm{Y}'}$ evidently denotes a force tangential to the surface on which it acts. It will also be observed, that if we regard the forces acting upon the sides of the solid parallelopiped as composed of the hydrostatic pressure p together with addition forces, the work done in any infinitesimal variation of the state of strain of the solid by these additional forces will be represented by the second member of the equation.

We will first consider the case in which the fluid is identical in substance with the solid. We have then, by equation (97), for a mass of the fluid equal to that of the solid,

$$\eta_{\mathrm{F}}\,dt - v_{\mathrm{F}}\,dp + m\,d\mu_1 = 0, \tag{405}$$

η_{F} and v_{F} denoting the entropy and volume of the fluid. By subtraction we obtain

$$-(\eta_{\mathrm{F}} - \eta)\,dt + (v_{\mathrm{F}} - v)\,dp$$
$$= \left(X_{\mathrm{X}'} + p\,\frac{dy}{dy'}\,\frac{dz}{dz'}\right)d\frac{dx}{dx'} + X_{\mathrm{Y}'}\,d\frac{dx}{dy'} + \left(Y_{\mathrm{Y}'} + p\,\frac{dz}{dz'}\,\frac{dx}{dx'}\right)d\frac{dy}{dy'}. \tag{406}$$

Now if the quantities $\frac{dx}{dx'}$, $\frac{dx}{dy'}$, $\frac{dy}{dy'}$ remain constant, we shall have for the relation between the variations of temperature and pressure which is necessary for the preservation of equilibrium

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$$\frac{dt}{dp} = \frac{v_F - v}{\eta_F - \eta} = t\frac{v_F - v}{Q}, \tag{407}$$

where Q denotes the heat which would be absorbed if the solid body should pass into the fluid state without change of temperature or pressure. This equation is similar to (131), which applies to bodies subject to hydrostatic pressure. But the value of $\frac{dt}{dp}$ will not generally be the same as if the solid were subject on all sides to the uniform normal pressure p; for the quantities v and η (and therefore Q) will in general have different values. But when the pressures on all sides are normal and equal, the value of $\frac{dt}{dp}$ will be the same, whether we consider the pressure when varied as still normal and equal on all sides, or consider the quantities $\frac{dx}{dx'}, \frac{dx}{dy'}, \frac{dy}{dy'}$ as constant.

But if we wish to know how the temperature is affected if the pressure between the solid and fluid remains constant, but the strain of the solid is varied in any way consistent with this supposition, the differential coefficients of t with respect to the quantities which express the strain are indicated by equation (406). These differential coefficients all vanish, when the pressures on all sides are normal and equal, but the differential coefficient $\frac{dt}{dp}$, when $\frac{dx}{dx'}, \frac{dx}{dy'}, \frac{dy}{dy'}$ are constant, or when the pressures on all sides are normal and equal, vanishes only when the density of the fluid is equal to that of the solid.

The case is nearly the same when the fluid is not identical in substance with the solid, if we suppose the composition of the fluid to remain unchanged. We have necessarily with respect to the fluid

$$d\mu_1 = \left(\frac{d\mu_1}{dt}\right)^{(F)}_{p,\,m} dt + \left(\frac{d\mu_1}{dp}\right)^{(F)}_{t,\,m} dp,^* \tag{408}$$

where the index (F) is used to indicate that the expression to which it is affixed relates to the fluid. But by equation (92)

$$\left(\frac{d\mu_1}{dt}\right)^{(F)}_{p,\,m} = -\left(\frac{d\eta}{dm_1}\right)^{(F)}_{t,\,p,\,m}, \text{ and } \left(\frac{d\mu_1}{dp}\right)^{(F)}_{t,\,m} = \left(\frac{dv}{dm_1}\right)^{(F)}_{t,\,p,\,m}. \tag{409}$$

Substituting these values in the preceding equation, transposing terms, and multiplying by m, we obtain

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* A suffixed m stands here, as elsewhere in this paper, for all the symbols m_1, m_2, etc., except such as may occur in the differential coefficient.

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$$m\left(\frac{d\eta}{dm_1}\right)^{(F)}_{t,\,p,\,m} dt - m\left(\frac{dv}{dm_1}\right)^{(F)}_{t,\,p,\,m} dp + m\,d\mu_1 = 0. \qquad (410)$$

By subtracting this equation from (404) we may obtain an equation similar to (406), except that in place of η_F and v_F we shall have the expressions

$$m\left(\frac{d\eta}{dm_1}\right)^{(F)}_{t,\,p,\,m} \quad \text{and} \quad m\left(\frac{dv}{dm_1}\right)^{(F)}_{t,\,p,\,m}.$$

The discussion of equation (406) will therefore apply *mutatis mutandis* to this case.

We may also wish to find the variations in the composition of the fluid which will be necessary for equilibrium when the pressure p or the quantities $\frac{dx}{dx'}, \frac{dx}{dy'}, \frac{dy}{dy'}$ are varied, the temperature remaining constant. If we know the value for the fluid of the quantity represented by ζ on page 142 in terms of t, p, and the quantities of the several components m_1, m_2, m_3, etc., the first of which relates to the substance of which the solid is formed, we can easily find the value of μ_1 in terms of the same variables. Now in considering variations in the composition of the solid, it will be sufficient if we make all but one of the components variable. We may therefore give to m_1 a constant value, and making t also constant, we shall have

$$d\mu_1 = \left(\frac{d\mu_1}{dp}\right)^{(F)}_{t,\,m} dp + \left(\frac{d\mu_1}{dm_2}\right)^{(F)}_{t,\,p,\,m} dm_2 + \left(\frac{d\mu_1}{dm_3}\right)^{(F)}_{t,\,p,\,m} dm_3 + \text{etc.}$$

Substituting this value in equation (404), and cancelling the term containing dt, we obtain

$$\left\{ m\left(\frac{d\mu_1}{dp}\right)^{(F)}_{t,\,m} - v \right\} dp + m\left(\frac{d\mu_1}{dm_2}\right)^{(F)}_{t,\,p,\,m} dm_2$$
$$+ m\left(\frac{d\mu_1}{dm_3}\right)^{(F)}_{t,\,p,\,m} dm_3 + \text{etc.} = \left(X_{X'} + p\frac{dy}{dy'}\frac{dz}{dz'}\right) d\frac{dx}{dx'}$$
$$+ X_{Y'}\, d\frac{dx}{dy'} + \left(Y_{Y'} + p\frac{dz}{dz'}\frac{dx}{dx'}\right) d\frac{dy}{dy'}. \qquad (411)$$

This equation shows the variation in the quantity of any one of the components of the fluid (other than the substance which forms the solid) which will balance a variation of p, or of $\frac{dx}{dx'}, \frac{dx}{dy'}, \frac{dy}{dy'}$, with respect to the tendency of the solid to dissolve.

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Fundamental Equations for Solids.

The principles developed in the preceding pages show that the solution of problems relating to the equilibrium of a solid, or at least their reduction to purely analytical processes, may be made to depend upon our knowledge of the composition and density of the solid at every point in some particular state, which we have called the state of reference, and of the relation existing between the quantities which have been represented by $\varepsilon_{v'}$, $\eta_{v'}$, $\frac{dx}{dx'}$, $\frac{dx}{dy'}$, . . . $\frac{dz}{dz'}$, x', y', and z'. When the solid is in contact with fluids, a certain knowledge of the properties of the fluids is also requisite, but only such as is necessary for the solution of problems relating to the equilibrium of fluids among themselves.

If in any state of which a solid is capable, it is homogeneous in its nature and in its state of strain, we may choose this state as the state of reference, and the relation between $\varepsilon_{v'}$, $\eta_{v'}$, $\frac{dx}{dx'}$, . . $\frac{dz}{dz'}$, will be independent of x', y', z'. But it is not always possible, even in the case of bodies which are homogeneous in nature, to bring all the elements simultaneously into the same state of strain. It would not be possible, for example, in the case of a Prince Rupert's drop.

If, however, we know the relation between $\varepsilon_{v'}$, $\eta_{v'}$, $\frac{dx}{dx'}$, . . . $\frac{dz}{dz'}$, for any kind of homogeneous solid, with respect to any given state of reference, we may derive from it a similar relation with respect to any other state as a state of reference. For if x', y', z' denote the co-ordinates of points of the solid in the first state of reference, and x'', y'', z'' the co-ordinates of the same points in the second state of reference, we shall have necessarily

$$\frac{dx}{dx'} = \frac{dx}{dx''}\frac{dx''}{dx'} + \frac{dx}{dy''}\frac{dy''}{dx'} + \frac{dx}{dz''}\frac{dz''}{dx'}, \text{ etc. (nine equations)}, \tag{412}$$

and if we write R for the volume of an element in the state (x'', y'', z'') divided by its volume in the state (x', y', z'), we shall have

$$R = \begin{vmatrix} \frac{dx''}{dx'} & \frac{dx''}{dy'} & \frac{dx''}{dz'} \\ \frac{dy''}{dx'} & \frac{dy''}{dy'} & \frac{dy''}{dz'} \\ \frac{dz''}{dx'} & \frac{dz''}{dy'} & \frac{dz''}{dz'} \end{vmatrix}, \tag{413}$$

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$$\varepsilon_{v'} = R\,\varepsilon_{v''}, \qquad \eta_{v'} = R\,\eta_{v''}. \tag{414}$$

If, then, we have ascertained by experiment the value of $\varepsilon_{v'}$ in terms of $\eta_{v'}$, $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$, and the quantities which express the composition of the body, by the substitution of the values given in (412)–(414), we shall obtain $\varepsilon_{v''}$ in terms of $\eta_{v''}$, $\frac{dx}{dx''}, \ldots \frac{dz}{dz''}, \frac{dx''}{dx'}, \ldots \frac{dz''}{dz'}$, and the quantities which express the composition of the body.

We may apply this to the elements of a body which may be variable from point to point in composition and state of strain in a given state of reference (x'', y'', z''), and if the body is fully described in that state of reference, both in respect to its composition and to the displacement which it would be necessary to give to a homogeneous solid of the same composition, for which $\varepsilon_{v'}$ is known in terms of $\eta_{v'}$, $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$, and the quantities which express its composition, to bring it from the state of reference (x', y', z') into a similar and similarly situated state of strain with that of the element of the non-homogeneous body, we may evidently regard $\frac{dx''}{dx'}, \ldots \frac{dz''}{dz'}$ as known for each element of the body, that is, as known in terms of x'', y'', z''. We shall then have $\varepsilon_{v''}$ in terms of $\eta_{v''}$, $\frac{dx}{dx''}, \ldots \frac{dz}{dz''}, x'', y'', z''$; and since the composition of the body is known in terms of x'', y'', z'', and the density, if not given directly, can be determined from the density of the homogeneous body in its state of reference (x', y', z'), this is sufficient for determining the equilibrium of any given state of the non-homogeneous solid.

An equation, therefore, which expresses for any kind of solid, and with reference to any determined state of reference, the relation between the quantities denoted by $\varepsilon_{v'}$, $\eta_{v'}$, $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$, involving also the quantities which express the composition of the body, when that is capable of continuous variation, or any other equation from which the same relations may be deduced, may be called a *fundamental equation* for that kind of solid. It will be observed that the sense in which this term is here used, is entirely analogous to that in which we have already applied the term to fluids and solids which are subject only to hydrostatic pressure.

When the fundamental equation between $\varepsilon_{v'}$, $\eta_{v'}$, $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$ is

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known, we may obtain by differentiation the values of t, $X_{x'}$, . . . $Z_{z'}$ in terms of the former quantities, which will give eleven independent relations between the twenty-one quantities

$$\varepsilon_{v'},\ \eta_{v'},\ \frac{dx}{dx'},\ \ldots\ \frac{dz}{dz'},\ t,\ X_{x'},\ \ldots\ Z_{z'}, \tag{415}$$

which are all that exist, since ten of these quantities are independent. All these equations may also involve variables which express the composition of the body, when that is capable of continuous variation.

If we use the symbol $\psi_{v'}$ to denote the value of ψ (as defined on pages 144, 145) for any element of a solid divided by the volume of the element in the state of reference, we shall have

$$\psi_{v'} = \varepsilon_{v'} - t\,\eta_{v'}. \tag{416}$$

The equation (356) may be reduced to the form

$$\delta\psi_{v'} = -\,\eta_{v'}\,\delta t + \Sigma\,\Sigma'\left(X_{x'}\,\delta\frac{dx}{dx'}\right). \tag{417}$$

Therefore, if we know the value of $\psi_{v'}$ in terms of the variables t, $\frac{dx}{dx'}$, . . . $\frac{dz}{dz'}$, together with those which express the composition of the body, we may obtain by differentiation the values of $\eta_{v'}$, $X_{x'}$, . . . $Z_{z'}$ in terms of the same variables. This will make eleven independent relations between the same quantities as before, except that we shall have $\psi_{v'}$ instead of $\varepsilon_{v'}$. Or if we eliminate $\psi_{v'}$ by means of equation (416), we shall obtain eleven independent equations between the quantities in (415) and those which express the composition of the body. An equation, therefore, which determines the value of $\psi_{v'}$, as a function of the quantities t, $\frac{dx}{dx'}$, . . . $\frac{dz}{dz'}$, and the quantities which express the composition of the body when it is capable of continuous variation, is a fundamental equation for the kind of solid to which it relates.

In the discussion of the conditions of equilibrium of a solid, we might have started with the principle that it is necessary and sufficient for equilibrium that the temperature shall be uniform throughout the whole mass in question, and that the variation of the force-function $(-\psi)$ of the same mass shall be null or negative for any variation in the state of the mass not affecting its temperature. We might have assumed that the value of ψ for any same element of the solid is a

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function of the temperature and the state of strain, so that for constant temperature we might write

$$\delta\psi_{v'} = \Sigma\,\Sigma'\left(X_{x'}\,\delta\frac{dx}{dx'}\right),$$

the quantities $X_{x'}, \ldots Z_{z'}$ being defined by this equation. This would be only a formal change in the definition of $X_{x'}, \ldots Z_{z'}$ and would not affect their values, for this equation holds true of $X_{x'}, \ldots Z_{z'}$ as defined by equation (355). With such data, by transformations similar to those which we have employed, we might obtain similar results.* It is evident that the only difference in the equations would be that $\psi_{v'}$ would take the place of $\varepsilon_{v'}$, and that the terms relating to entropy would be wanting. Such a method is evidently preferable with respect to the directness with which the results are obtained. The method of this paper shows more distinctly the *rôle* of *energy* and *entropy* in the theory of equilibrium, and can be extended more naturally to those dynamical problems in which motions take place under the condition of constancy of entropy of the elements of a solid (as when vibrations are propagated through a solid), just as the other method can be more naturally extended to dynamical problems in which the temperature is constant. (See note on page 145.)

We have already had occasion to remark that the state of strain of any element considered without reference to directions in space is capable of only six independent variations. Hence, it must be possible to express the state of strain of an element by six functions of $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$, which are independent of the position of the element. For these quantities we may choose the squares of the ratios of elongation of lines parallel to the three co-ordinate axes in the state of reference, and the products of the ratios of elongation for each pair of these lines multiplied by the cosine of the angle which they include in the variable state of the solid. If we denote these quantities by A, B, C, a, b, c, we shall have

* For an example of this method, see Thomson and Tait's *Natural Philosophy*, vol. i, p. 705. With regard to the general theory of elastic solids, compare also Thomson's Memoir "On the Thermo-elastic and Thermo-magnetic Properties of Matter" in the *Quarterly Journal of Mathematics*, vol. i, p. 57 (1855), and Green's memoirs on the propagation, reflection, and refraction of light in the *Transactions of the Cambridge Philosophical Society*, vol. vii.

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$$A = \Sigma\left(\frac{dx}{dx'}\right)^2, \quad B = \Sigma\left(\frac{dx}{dy'}\right)^2, \quad C = \Sigma\left(\frac{dx}{dz'}\right)^2, \tag{418}$$

$$a = \Sigma\left(\frac{dx}{dy'}\frac{dx}{dz'}\right), \quad b = \Sigma\left(\frac{dx}{dz'}\frac{dx}{dx'}\right), \quad c = \Sigma\left(\frac{dx}{dx'}\frac{dx}{dy'}\right). \tag{419}$$

The determination of the fundamental equation for a solid is thus reduced to the determination of the relation between $\varepsilon_{v'}$, $\eta_{v'}$, A, B, C, a, b, c, or of the relation between $\psi_{v'}$, t, A, B, C, a, b, c.

In the case of isotropic solids, the state of strain of an element, so far as it can affect the relation of $\varepsilon_{v'}$ and $\eta_{v'}$, or of $\psi_{v'}$ and t, is capable of only three independent variations. This appears most distinctly as a consequence of the proposition that for any given strain of an element there are three lines in the element which are at right angles to one another both in its unstrained and in its strained state. If the unstrained element is isotropic, the ratios of elongation for these three lines must with $\eta_{v'}$ determine the value $\varepsilon_{v'}$, or with t determine the value of $\psi_{v'}$.

To demonstrate the existence of such lines, which are called the *principal axes of strain*, and to find the relations of the elongations of such lines to the quantities $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$, we may proceed as follows. The ratio of elongation r of any line of which α', β', γ' are the direction-cosines in the state of reference is evidently given by the equation

$$\begin{aligned} r^2 = &\left(\frac{dx}{dx'}\alpha' + \frac{dx}{dy'}\beta' + \frac{dx}{dz'}\gamma'\right)^2 \\ &+ \left(\frac{dy}{dx'}\alpha' + \frac{dy}{dy'}\beta' + \frac{dy}{dz'}\gamma'\right)^2 \\ &+ \left(\frac{dz}{dx'}\alpha' + \frac{dz}{dy'}\beta' + \frac{dz}{dz'}\gamma'\right)^2. \end{aligned} \tag{420}$$

Now the proposition to be established is evidently equivalent to this —that it is always possible to give such directions to the two systems of rectangular axes X', Y', Z', and X, Y, Z, that

$$\left.\begin{aligned} \frac{dx}{dy'} = 0, \quad \frac{dx}{dz'} = 0, \quad \frac{dy}{dz'} = 0, \\ \frac{dy}{dx'} = 0, \quad \frac{dz}{dx'} = 0, \quad \frac{dz}{dy'} = 0. \end{aligned}\right\} \tag{421}$$

We may choose a line in the element for which the value of r is at least as great as for any other, and make the axes of X and X' parallel to this line in the strained and unstrained states respectively.

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Then
$$\frac{dy}{dx'} = 0, \quad \frac{dz}{dx'} = 0. \tag{422}$$

Moreover, if we write $\frac{d(r^2)}{d\alpha'}, \frac{d(r^2)}{d\beta'}, \frac{d(r^2)}{d\gamma'}$ for the differential coefficients obtained from (420) by treating α', β', γ' as *independent* variables,

$$\frac{d(r^2)}{d\alpha'}\,d\alpha' + \frac{d(r^2)}{d\beta'}\,d\beta' + \frac{d(r^2)}{d\gamma'}\,d\gamma' = 0,$$

when
$$\alpha'\,d\alpha' + \beta'\,d\beta' + \gamma'\,d\gamma' = 0,$$

and
$$\alpha' = 1, \quad \beta' = 0, \quad \gamma' = 0.$$

That is,
$$\frac{d(r^2)}{d\beta'} = 0, \quad \text{and} \quad \frac{d(r^2)}{d\gamma'} = 0,$$

when
$$\alpha' = 1, \quad \beta' = 0, \quad \gamma' = 0.$$

Hence,
$$\frac{dx}{dy'} = 0, \quad \frac{dx}{dz'} = 0. \tag{423}$$

Therefore a line of the element which in the unstrained state is perpendicular to X' is perpendicular to X in the strained state. Of all such lines we may choose one for which the value of r is at least as great as for any other, and make the axes of Y' and Y parallel to this line in the unstrained and in the strained state respectively. Then

$$\frac{dz}{dy'} = 0; \tag{424}$$

and it may easily be shown by reasoning similar to that which has just been employed that

$$\frac{dy}{dz'} = 0. \tag{425}$$

Lines parallel to the axes of X', Y', and Z' in the unstrained body will therefore be parallel to X, Y, and Z in the strained body, and the ratios of elongation for such lines will be

$$\frac{dx}{dx'}, \quad \frac{dy}{dy'}, \quad \frac{dz}{dz'}.$$

These lines have the common property of a stationary value of the ratio of elongation for varying directions of the line. This appears from the form to which the general value of r^2 is reduced by the positions of the co-ordinate axes, viz.,

$$r^2 = \left(\frac{dx}{dx'}\right)^2 \alpha'^2 + \left(\frac{dy}{dy'}\right)^2 \beta'^2 + \left(\frac{dz}{dz'}\right)^2 \gamma'^2.$$

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Having thus proved the existence of lines, with reference to any particular strain, which have the properties mentioned, let us proceed to find the relations between the ratios of elongation for these lines (the *principal axes of strain*) and the quantities $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$ under the most general supposition with respect to the position of the co-ordinate axes.

For any principal axis of strain we have

$$\frac{d(r^2)}{d\alpha'}\,d\alpha' + \frac{d(r^2)}{d\beta'}\,d\beta' + \frac{d(r^2)}{d\gamma'}\,d\gamma' = 0,$$

when

$$\alpha'\,d\alpha' + \beta'\,d\beta' + \gamma'\,d\gamma' = 0,$$

the differential coefficients in the first of these equations being determined from (420) as before. Therefore,

$$\frac{1}{\alpha'}\frac{d(r^2)}{d\alpha'} = \frac{1}{\beta'}\frac{d(r^2)}{d\beta'} = \frac{1}{\gamma'}\frac{d(r^2)}{d\gamma'}. \tag{426}$$

From (420) we obtain directly

$$\frac{\alpha'}{2}\frac{d(r^2)}{d\alpha'} + \frac{\beta'}{2}\frac{d(r^2)}{d\beta'} + \frac{\gamma'}{2}\frac{d(r^2)}{d\gamma'} = r^2. \tag{427}$$

From the two last equations, in virtue of the necessary relation $\alpha'^2 + \beta'^2 + \gamma'^2 = 1$, we obtain

$$\tfrac{1}{2}\frac{d(r^2)}{d\alpha'} = \alpha' r^2, \quad \tfrac{1}{2}\frac{d(r^2)}{d\beta'} = \beta' r^2, \quad \tfrac{1}{2}\frac{d(r^2)}{d\gamma'} = \gamma' r^2, \tag{428}$$

or, if we substitute the values of the differential coefficients taken from (420),

$$\left.\begin{aligned}
\alpha' \Sigma\left(\frac{dx}{dx'}\right)^2 + \beta' \Sigma\left(\frac{dx}{dx'}\frac{dx}{dy'}\right) + \gamma' \Sigma\left(\frac{dx}{dx'}\frac{dx}{dz'}\right) &= \alpha' r^2,\\
\alpha' \Sigma\left(\frac{dx}{dy'}\frac{dx}{dx'}\right) + \beta' \Sigma\left(\frac{dx}{dy'}\right)^2 + \gamma' \Sigma\left(\frac{dx}{dy'}\frac{dx}{dz'}\right) &= \beta' r^2,\\
\alpha' \Sigma\left(\frac{dx}{dz'}\frac{dx}{dx'}\right) + \beta' \Sigma\left(\frac{dx}{dz'}\frac{dx}{dy'}\right) + \gamma' \Sigma\left(\frac{dx}{dz'}\right)^2 &= \gamma' r^2.
\end{aligned}\right\} \tag{429}$$

If we eliminate α', β', γ' from these equations, we may write the result in the form,

$$\begin{vmatrix}
\Sigma\left(\frac{dx}{dx'}\right)^2 - r^2 & \Sigma\left(\frac{dx}{dx'}\frac{dx}{dy'}\right) & \Sigma\left(\frac{dx}{dx'}\frac{dx}{dz'}\right)\\
\Sigma\left(\frac{dx}{dy'}\frac{dx}{dx'}\right) & \Sigma\left(\frac{dx}{dy'}\right)^2 - r^2 & \Sigma\left(\frac{dx}{dy'}\frac{dx}{dz'}\right)\\
\Sigma\left(\frac{dx}{dz'}\frac{dx}{dx'}\right) & \Sigma\left(\frac{dx}{dz'}\frac{dx}{dy'}\right) & \Sigma\left(\frac{dx}{dz'}\right)^2 - r^2
\end{vmatrix} = 0. \tag{430}$$

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We may write

$$-r^6 + Er^4 - Fr^2 + G = 0. \tag{431}$$

Then

$$E = \Sigma' \Sigma \left(\frac{dx}{dx'}\right)^2. \tag{432}$$

Also*

$$F = \Sigma' \left\{ \Sigma \left(\frac{dx}{dx'}\right)^2 \Sigma \left(\frac{dx}{dy'}\right)^2 - \Sigma \left(\frac{dx}{dx'}\frac{dx}{dy'}\right) \Sigma \left(\frac{dx}{dx'}\frac{dx}{dy'}\right) \right\}$$

$$= \Sigma' \Sigma \left\{ \left(\frac{dx}{dx'}\right)^2 \Sigma \left(\frac{dx}{dy'}\right)^2 - \frac{dx}{dx'}\frac{dx}{dy'} \Sigma \left(\frac{dx}{dx'}\frac{dx}{dy'}\right) \right\} =$$

$$\Sigma' \Sigma \left\{ \left(\frac{dx}{dx'}\right)^2 \left(\frac{dy}{dy'}\right)^2 + \left(\frac{dx}{dx'}\right)^2 \left(\frac{dz}{dy'}\right)^2 - \frac{dx}{dx'}\frac{dx}{dy'}\frac{dy}{dx'}\frac{dy}{dy'} - \frac{dx}{dx'}\frac{dx}{dy'}\frac{dz}{dx'}\frac{dz}{dy'} \right\}$$

$$= \Sigma' \Sigma \left\{ \left(\frac{dx}{dx'}\right)^2 \left(\frac{dy}{dy'}\right)^2 + \left(\frac{dy}{dx'}\right)^2 \left(\frac{dx}{dy'}\right)^2 - 2\frac{dx}{dx'}\frac{dx}{dy'}\frac{dy}{dx'}\frac{dy}{dy'} \right\}$$

$$= \Sigma' \Sigma \left(\frac{dx}{dx'}\frac{dy}{dy'} - \frac{dy}{dx'}\frac{dx}{dy'}\right)^2. \tag{433}$$

This may also be written

$$F = \Sigma' \Sigma \begin{vmatrix} \frac{dx}{dx'} & \frac{dx}{dy'} \\ \frac{dy}{dx'} & \frac{dy}{dy'} \end{vmatrix}^2. \tag{434}$$

In the reduction of the value of G, it will be convenient to use the symbol $\sum\limits_{3+3}$ to denote the sum of the *six* terms formed by changing x, y, z, into y, z, x; z, x, y; x, z, y; y, x, z; and z, y, x; and the symbol $\sum\limits_{3-3}$ in the same sense except that the last three terms are to be taken negatively; also to use $\sum\limits'_{3-3}$ in a similar sense with respect to x', y', z'; and to use x', y', z' as equivalent to x', y', z', except that they are not to be affected by the sign of summation. With this understanding we may write

$$G = \sum_{3-3}' \left\{ \Sigma \left(\frac{dx}{dx'}\frac{dx}{dx'}\right) \Sigma \left(\frac{dx}{dy'}\frac{dx}{dy'}\right) \Sigma \left(\frac{dx}{dz'}\frac{dx}{dz'}\right) \right\}. \tag{435}$$

---

* The values of F and G given in equations (434) and (438), which are here deduced at length, may be derived from inspection of equation (430) by means of the usual theorems relating to the multiplication of determinants. See Salmon's *Lessons Introductory to the Modern Higher Algebra*, 2d Ed., Lesson III; or Baltzer's *Theorie und Anwendung der Determinanten*, § 5.

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In expanding the product of the three sums, we may cancel on account of the sign $\sum'_{3-3}$ the terms which do not contain all the three expressions dx, dy, and dz. Hence we may write

$$\begin{aligned} G &= \sum'_{3-3} \sum_{3+3} \left(\frac{dx}{dx'}\frac{dx}{dx'}\frac{dy}{dy'}\frac{dy}{dy'}\frac{dz}{dz'}\frac{dz}{dz'}\right) \\ &= \sum_{3+3} \left\{ \frac{dx}{dx'}\frac{dy}{dy'}\frac{dz}{dz'} \sum'_{3-3} \left(\frac{dx}{dx'}\frac{dy}{dy'}\frac{dz}{dz'}\right) \right\} \\ &= \sum_{3-3} \left(\frac{dx}{dx'}\frac{dy}{dy'}\frac{dz}{dz'}\right) \sum'_{3-3} \left(\frac{dx}{dx'}\frac{dy}{dy'}\frac{dz}{dz'}\right). \end{aligned} \tag{436}$$

Or, if we set

$$H = \begin{vmatrix} \frac{dx}{dx'} & \frac{dx}{dy'} & \frac{dx}{dz'} \\ \frac{dy}{dx'} & \frac{dy}{dy'} & \frac{dy}{dz'} \\ \frac{dz}{dx'} & \frac{dz}{dy'} & \frac{dz}{dz'} \end{vmatrix}, \tag{437}$$

we shall have

$$G = H^2. \tag{438}$$

It will be observed that F represents the sum of the squares of the nine minors which can be formed from the determinant in (437), and that E represents the sum of the squares of the nine constituents of the same determinant.

Now we know by the theory of equations that equation (431) will be satisfied in general by three different values of r^2, which we may denote by r_1^2, r_2^2, r_3^2, and which must represent the squares of the ratios of elongation for the three principal axes of strain; also that E, F, G, are symmetrical functions of r_1^2, r_2^2, r_3^2, viz.,

$$\left.\begin{aligned} E = r_1^2 + r_2^2 + r_3^2, \quad F = r_1^2 r_2^2 + r_2^2 r_3^2 + r_3^2 r_1^2, \\ G = r_1^2 r_2^2 r_3^2. \end{aligned}\right\} \tag{439}$$

Hence, although it is possible to solve equation (431) by the use of trigonometrical functions, it will be more simple to regard ε_v, as a function of η_v, and the quantities E, F, G (or H), which we have expressed in terms of $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$. Since ε_v, is a single-valued function of η_v, and r_1^2, r_2^2, r_3^2 (with respect to all the changes of which the body is capable), and a symmetrical function with respect to r_1^2, r_2^2, r_3^2, and since r_1^2, r_2^2, r_3^2 are *collectively* determined without ambiguity by the values of E, F, and H, the quantity ε_v, must be a

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single-valued function of $\eta_{v'}$, E, F, and H. The determination of the fundamental equation for isotropic bodies is therefore reduced to the determination of this function, or (as appears from similar considerations) the determination of $\psi_{v'}$ as a function of t, E, F, and H.

It appears from equations (439) that E represents the sum of the squares of the ratios of elongation for the principal axes of strain, that F represents the sum of the squares of the ratios of enlargement for the three surfaces determined by these axes, and that G represents the square of the ratio of enlargement of volume. Again, equation (432) shows that E represents the sum of the squares of the ratios of elongation for lines parallel to X', Y', and Z'; equation (434) shows that F represents the sum of the squares of the ratios of enlargement for surfaces parallel to the planes X'-Y', Y'-Z', Z'-X'; and equation (438), like (439), shows that G represents the square of the ratio of enlargement of volume. Since the position of the co-ordinate axes is arbitrary, it follows that the sum of the squares of the ratios of elongation or enlargement of three lines or surfaces which in the unstrained state are at right angles to one another, is otherwise independent of the direction of the lines or surfaces. Hence, $\frac{1}{3}E$ and $\frac{1}{3}F$ are the mean squares of the ratios of linear elongation and of superficial enlargement, for all possible directions in the unstrained solid.

There is not only a practical advantage in regarding the strain as determined by E, F, and H, instead of E, F, and G, because H is more simply expressed in terms of $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$, but there is also a certain theoretical advantage on the side of E, F, H. If the systems of co-ordinate axes X, Y, Z, and X', Y', Z', are either identical or such as are capable of superposition, which it will always be convenient to suppose, the determinant H will always have a positive value for any strain of which a body can be capable. But it is possible to give to x, y, z such values as functions of x', y', z' that H shall have a negative value. For example, we may make

$$x = x', \quad y = y', \quad z = -z'. \tag{440}$$

This will give $H = -1$, while

$$x = x', \quad y = y', \quad z = z' \tag{441}$$

will give $H = 1$. Both (440) and (441) give $G = 1$. Now although such a change in the position of the particles of a body as is represented by (440) cannot take place while the body remains solid, yet

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a method of representing strains may be considered incomplete, which confuses the cases represented by (440) and (441).

We may avoid all such confusion by using E, F, and H to represent a strain. Let us consider an element of the body strained which in the state (x', y', z') is a cube with its edges parallel to the axes of X', Y', Z', and call the edges dx', dy', dz according to the axes to which they are parallel, and consider the ends of the edges as positive for which the values of x', y', or z' are the greater. Whatever may be the nature of the parallelopiped in the state (x, y, z) which corresponds to the cube dx', dy', dz' and is determined by the quantities $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$, it may always be brought by continuous changes to the form of a cube and to a position in which the edges dx', dy' shall be parallel to the axes of X and Y, the positive ends of the edges toward the positive directions of the axes, and this may be done without giving the volume of the parallelopiped the value zero, and therefore without changing the sign of H. Now two cases are possible;—the positive end of the edge dz' may be turned toward the positive or toward the negative direction of the axis of Z. In the first case, H is evidently positive; in the second, negative. The determinant H will therefore be positive or negative,—we may say, if we choose, that the volume will be positive or negative,—according as the element can or cannot be brought from the state (x, y, z) to the state (x', y', z') by continuous changes without giving its volume the value zero.

If we now recur to the consideration of the principal axes of strain and the principal ratios of elongation r_1, r_2, r_3, and denote by U_1, U_2, U_3 and U_1', U_2', U_3' the principal axes of strain in the strained and unstrained element respectively, it is evident that the sign of r_1, for example, depends upon the direction in U_1 which we regard as corresponding to a given direction in U_1'. If we choose to associate directions in these axes so that r_1, r_2, r_3 shall all be positive, the positive or negative value of H will determine whether the system of axes U_1, U_2, U_3 is or is not capable of superposition upon the system U_1', U_2', U_3' so that corresponding directions in the axes shall coincide. Or, if we prefer to associate directions in the two systems of axes, so that they shall be capable of superposition, corresponding directions coinciding, the positive or negative value of H will determine whether an even or an odd number of the quantities r_1, r_2, r_3 are negative. In this case we may write

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$$r_1\, r_2\, r_3 = H = \begin{vmatrix} \frac{dx}{dx'} & \frac{dx}{dy'} & \frac{dx}{dz'} \\ \frac{dy}{dx'} & \frac{dy}{dy'} & \frac{dy}{dz'} \\ \frac{dz}{dx'} & \frac{dz}{dy'} & \frac{dz}{dz'} \end{vmatrix}. \tag{442}$$

It will be observed that to change the signs of two of the quantities r_1, r_2, r_3 is simply to give a certain rotation to the body without changing its state of strain.

Whichever supposition we make with respect to the axes U_1, U_2, U_3, it is evident that the state of strain is completely determined by the values E, F, and H, not only when we limit ourselves to the consideration of such strains as are consistent with the idea of solidity, but also when we regard any values of $\frac{dx}{dx'}, \ldots \frac{dz}{dz'}$ as possible.

Approximative Formulæ.—For many purposes the value of $\varepsilon_{v'}$ for an isotropic solid may be represented with sufficient accuracy by the formula

$$\varepsilon_{v'} = i' + e'\,E + f'\,F + h'\,H, \tag{443}$$

where i', e', f', and h' denote functions of $\eta_{v'}$; or the value of $\psi_{v'}$ by the formula

$$\psi_{v'} = i + e\,E + f\,F + h\,H, \tag{444}$$

where i, e, f, and h denote functions of t. Let us first consider the second of these formulæ. Since E, F, and H are symmetrical functions of r_1, r_2, r_3, if $\psi_{v'}$ is any function of t, E, F, H, we must have

$$\left.\begin{aligned} \frac{d\psi_{v'}}{dr_1} &= \frac{d\psi_{v'}}{dr_2} = \frac{d\psi_{v'}}{dr_3}, \\ \frac{d^2\psi_{v'}}{dr_1^{\,2}} &= \frac{d^2\psi_{v'}}{dr_2^{\,2}} = \frac{d^2\psi_{v'}}{dr_3^{\,2}}, \\ \frac{d^2\psi_{v'}}{dr_1 dr_2} &= \frac{d^2\psi_{v'}}{dr_2 dr_3} = \frac{d^2\psi_{v'}}{dr_3 dr_1}, \end{aligned}\right\} \tag{445}$$

whenever $r_1 = r_2 = r_3$. Now i, e, f, and h may be determined (as functions of t) so as to give to

$$\psi_{v'}, \quad \frac{d\psi_{v'}}{dr_1}, \quad \frac{d^2\psi_{v'}}{dr_1^{\,2}}, \quad \frac{d^2\psi_{v'}}{dr_1 dr_2}$$

their proper values at every temperature for some isotropic state of strain, which may be determined by any desired condition. We shall suppose that they are determined so as to give the proper

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values to $\psi_{V'}$, etc., when the stresses in the solid vanish. If we denote by r_0 the common value of r_1, r_2, r_3 which will make the stresses vanish at any given temperature, and imagine the true value of $\psi_{V'}$, and also the value given by equation (444) to be expressed in terms of the ascending powers of

$$r_1 - r_0, \quad r_2 - r_0, \quad r_3 - r_0, \tag{446}$$

it is evident that the expressions will coincide as far as the terms of the second degree *inclusive.* That is, the errors of the values of $\psi_{V'}$ given by equation (444) are of the same order of magnitude as the cubes of the above differences. The errors of the values of

$$\frac{d\psi_{V'}}{dr_1}, \quad \frac{d\psi_{V'}}{dr_2}, \quad \frac{d\psi_{V'}}{dr_3}$$

will be of the same order of magnitude as the squares of the same differences. Therefore, since

$$\frac{d\psi_{V'}}{d\frac{dx}{dx'}} = \frac{d\psi_{V'}}{dr_1}\frac{dr_1}{d\frac{dx}{dx'}} + \frac{d\psi_{V'}}{dr_2}\frac{dr_2}{d\frac{dx}{dx'}} + \frac{d\psi_{V'}}{dr_3}\frac{dr_3}{d\frac{dx}{dx'}} \tag{447}$$

whether we regard the true value of $\psi_{V'}$ or the value given by equation (444), and since the error in (444) does not affect the values of

$$\frac{dr_1}{d\frac{dx}{dx'}}, \quad \frac{dr_2}{d\frac{dx}{dx'}}, \quad \frac{dr_3}{d\frac{dx}{dx'}},$$

which we may regard as determined by equations (431), (432), (434), (437) and (438), the errors in the values of $X_{X'}$ derived from (444) will be of the same order of magnitude as the squares of the differences in (446). The same will be true with respect to $X_{Y'}$, $X_{Z'}$, $Y_{X'}$, etc., etc.

It will be interesting to see how the quantities e, f, and h are related to those which most simply represent the elastic properties of isotropic solids. If we denote by V and R the *elasticity of volume* and the *rigidity** (both determined under the condition of constant temperature and for states of vanishing stress), we shall have as definitions:

$$V = -v\left(\frac{dp}{dv}\right)_t, \quad \text{when} \quad v = r_0{}^3 v', \tag{448}$$

* See Thomson and Tait's *Natural Philosophy*, vol. i, p. 711.

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where p denotes a uniform pressure to which the solid is subjected, v its volume, and v' its volume in the state of reference; and

$$\left.\begin{aligned} & R = \frac{dX_{Y'}}{d\frac{dx}{dy'}} = \frac{d^2\psi_{V'}}{\left(d\frac{dx}{dy'}\right)^2}, \\ \text{when} \quad & \frac{dx}{dx'} = \frac{dy}{dy'} = \frac{dz}{dz'} = r_0, \\ \text{and} \quad & \frac{dx}{dy'} = \frac{dx}{dz'} = \frac{dy}{dz'} = \frac{dy}{dx'} = \frac{dz}{dx'} = \frac{dz}{dy'} = 0. \end{aligned}\right\} \tag{449}$$

Now when the solid is subject to uniform pressure on all sides, if we consider so much of it as has the volume unity in the state of reference, we shall have

$$r_1 = r_2 = r_3 = v^{\frac{1}{3}}, \tag{450}$$

and by (444) and (439),

$$\psi_{V'} = i + 3\,e\,v^{\frac{2}{3}} + 3\,f\,v^{\frac{4}{3}} + h\,v. \tag{451}$$

Hence, by equation (88), since $\psi_{V'}$ is equivalent to ψ,

$$-p = \left(\frac{d\psi}{dv}\right)_t = 2\,e\,v^{-\frac{1}{3}} + 4\,f\,v^{\frac{1}{3}} + h, \tag{452}$$

$$-v\left(\frac{dp}{dv}\right)_t = -\tfrac{2}{3}\,e\,v^{-\frac{1}{3}} + \tfrac{4}{3}\,f\,v^{\frac{1}{3}}; \tag{453}$$

and by (448),

$$V = -\tfrac{2}{3}\frac{e}{r_0} + \tfrac{4}{3}f\,r_0. \tag{454}$$

To obtain the value of R in accordance with the definition (449), we may suppose the values of E, F, and H given by equations (432), (434), and (437) to be substituted in equation (444). This will give for the value of R

$$R = 2\,e + 4\,f\,r_0^{\,2}. \tag{455}$$

Moreover, since p must vanish in (452) when $v = r_0^{\,3}$, we have

$$2\,e + 4\,f\,r_0^{\,2} + h\,r_0 = 0. \tag{456}$$

From the three last equations may be obtained the values of e, f, h, in terms of r_0, V, and R; viz.,

$$e = \frac{R - 3\,r_0\,V}{4}, \quad f = \frac{R + 3\,r_0\,V}{8\,r_0^{\,2}}, \quad h = -\frac{R}{r_0}. \tag{457}$$

The quantity r_0, like R and V, is a function of the temperature, the differential coefficient $\frac{d\log r_0}{dt}$ representing the rate of linear expansion of the solid when without stress.

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It will not be necessary to discuss equation (443) at length, as the case is entirely analogous to that which has just been treated. [It must be remembered that $\eta_{v'}$, in the discussion of (443) will take the place everywhere of the temperature in the discussion of (444).] If we denote by V' and R' the *elasticity of volume* and the *rigidity*, both determined under the condition of *constant entropy*, (i. e., of *no transmission of heat*,) and for states of vanishing stress, we shall have the equations:

$$V' = -\frac{2e'}{3r_0} + \tfrac{4}{3}f' r_0, \tag{458}$$

$$R' = 2e' + 4f' r_0^2, \tag{459}$$

$$2e' + 4f' r_0^2 + h' r_0 = 0. \tag{460}$$

Whence

$$e' = \frac{R' - 3r_0 V'}{4}, \quad f' = \frac{R' + 3r_0 V'}{8r_0^2}, \quad h' = -\frac{R'}{r_0}. \tag{461}$$

In these equations r_0, R', and V' are to be regarded as functions of the quantity $\eta_{v'}$.

If we wish to change from one state of reference to another (also isotropic), the changes required in the fundamental equation are easily made. If a denotes the length of any line of the solid in the second state of reference divided by its length in the first, it is evident that when we change from the first state of reference to the second the values of the symbols $\varepsilon_{v'}$, $\eta_{v'}$, $\psi_{v'}$, H are divided by a^3, that of E by a^2, and that of F by a^4. In making the change of the state of reference, we must therefore substitute in the fundamental equation of the form (444) $a^3\psi_{v'}$, a^2E, a^4F, a^3H for $\psi_{v'}$, E, F, and H, respectively. In the fundamental equation of the form (443), we must make the analogous substitutions, and also substitute $a^3\eta_{v'}$ for $\eta_{v'}$. [It will be remembered that i', e', f', and h' represent functions of $\eta_{v'}$, and that it is only when their values in terms of $\eta_{v'}$ are substituted, that equation (443) becomes a fundamental equation.]

Concerning Solids which absorb Fluids.

There are certain bodies which are solid with respect to some of their components, while they have other components which are fluid. In the following discussion, we shall suppose both the solidity and the fluidity to be perfect, so far as any properties are concerned which can affect the conditions of equilibrium,—i. e., we shall suppose that the solid matter of the body is entirely free from plasticity, and that there are no passive resistances to the motion of the fluid

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components except such as vanish with the velocity of the motion,—leaving it to be determined by experiment how far and in what cases these suppositions are realized.

It is evident that equation (356) must hold true with regard to such a body, when the quantities of the fluid components contained in a given element of the solid remain constant. Let Γ_a', Γ_b', etc., denote the quantities of the several fluid components contained in an element of the body divided by the volume of the element in the state of reference, or, in other words, let these symbols denote the densities which the several fluid components would have, if the body should be brought to the state of reference while the matter contained in each element remained unchanged. We may then say that equation (356) will hold true, when Γ_a', Γ_b', etc., are constant. The complete value of the differential of $\varepsilon_{v'}$ will therefore be given by an equation of the form

$$d\varepsilon_{v'} = t\, d\eta_{v'} + \Sigma\,\Sigma'\left(X_{x'}\, d\frac{dx}{dx'}\right) + L_a\, d\Gamma_a' + L_b\, d\Gamma_b' + \text{etc.} \quad (462)$$

Now when the body is in a state of hydrostatic stress, the term in this equation containing the signs of summation will reduce to $-p\, dv_{v'}$ ($v_{v'}$ denoting, as elsewhere, the volume of the element divided by its volume in the state of reference). For in this case

$$X_{x'} = -p\left(\frac{dy}{dy'}\frac{dz}{dz'} - \frac{dz}{dy'}\frac{dy}{dz'}\right), \quad (463)$$

$$\Sigma\,\Sigma'\left(X_{x'}\, d\frac{dx}{dx'}\right) = -p\,\Sigma\,\Sigma'\left\{\left(\frac{dy}{dy'}\frac{dz}{dz'} - \frac{dz}{dy'}\frac{dy}{dz'}\right) d\frac{dx}{dx'}\right\}$$

$$= -p\, d\begin{vmatrix} \frac{dx}{dx'} & \frac{dx}{dy'} & \frac{dx}{dz'} \\ \frac{dy}{dx'} & \frac{dy}{dy'} & \frac{dy}{dz'} \\ \frac{dz}{dx'} & \frac{dz}{dy'} & \frac{dz}{dz'} \end{vmatrix}$$

$$= -p\, dv_{v'}. \quad (464)$$

We have, therefore, for a state of hydrostatic stress,

$$d\varepsilon_{v'} = t\, d\eta_{v'} - p\, dv_{v'} + L_a\, d\Gamma_a' + L_b\, d\Gamma_b' + \text{etc.}, \quad (465)$$

and multiplying by the volume of the element in the state of reference, which we may regard as constant,

$$d\varepsilon = t\, d\eta - p\, dv + L_a\, dm_a + L_b\, dm_b + \text{etc.}, \quad (466)$$

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where ε, η, v, m_a, m_b, etc., denote the energy, entropy, and volume of the element, and the quantities of its several fluid components. It is evident that the equation will also hold true, if these symbols are understood as relating to a homogeneous body of finite size. The only limitation with respect to the variations is that the element or body to which the symbols relate shall always contain the same solid matter. The varied state may be one of hydrostatic stress or otherwise.

But when the body is in a state of hydrostatic stress, and the solid matter is considered invariable, we have by equation (12)

$$d\varepsilon = t\,d\eta - p\,dv + \mu_a\,dm_a + \mu_b\,dm_b + \text{etc.} \tag{467}$$

It should be remembered that the equation cited occurs in a discussion which relates only to bodies of hydrostatic stress, so that the varied state as well as the initial is there regarded as one of hydrostatic stress. But a comparison of the two last equations shows that the last will hold true without any such limitation, and moreover, that the quantities L_a, L_b, etc., when determined for a state of hydrostatic stress, are equal to the potentials μ_a, μ_b, etc.

Since we have hitherto used the term *potential* solely with reference to bodies of hydrostatic stress, we may apply this term as we choose with regard to other bodies. We may therefore call the quantities L_a, L_b, etc., the *potentials* for the several fluid components in the body considered, whether the state of the body is one of hydrostatic stress or not, since this use of the term involves only an extension of its former definition. It will also be convenient to use our ordinary symbol for a potential to represent these quantities. Equation (462) may then be written

$$d\varepsilon_{\mathrm{v}'} = t\,d\eta_{\mathrm{v}'} + \Sigma\,\Sigma'\left(X_{\mathrm{x}'}\,d\frac{dx}{dx'}\right) + \mu_a\,d\Gamma_a' + \mu_b\,d\Gamma_b' + \text{etc.} \tag{468}$$

This equation holds true of solids having fluid components without any limitation with respect to the initial state or to the variations, except that the solid matter to which the symbols relate shall remain the same.

In regard to the conditions of equilibrium for a body of this kind, it is evident in the first place that if we make Γ_a', Γ_b', etc., constant, we shall obtain from the general criterion of equilibrium all the conditions which we have obtained for ordinary solids, and which are expressed by the formulæ (364), (374), (380), (382)–(384). The quantities Γ_1', Γ_2', etc., in the last two formulæ include of course

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those which have just been represented by Γ_a', Γ_b', etc., and which relate to the fluid components of the body, as well as the corresponding quantities relating to its solid components. Again, if we suppose the solid matter of the body to remain without variation in quantity or position, it will easily appear that the potentials for the substances which form the fluid components of the solid body must satisfy the same conditions in the solid body and in the fluids in contact with it, as in the case of entirely fluid masses. See eqs. (22).

The above conditions must however be slightly modified in order to make them sufficient for equilibrium. It is evident that if the solid is dissolved at its surface, the fluid components which are set free may be absorbed by the solid as well as by the fluid mass, and in like manner if the quantity of the solid is increased, the fluid components of the new portion may be taken from the previously existing solid mass. Hence, whenever the *solid* components of the solid body are actual components of the fluid mass, (whether the case is the same with the *fluid* components of the solid body or not,) an equation of the form (383) must be satisfied, in which the potentials μ_a, μ_b, etc., contained implicitly in the second member of the equation are determined from the solid body. Also if the *solid* components of the solid body are all possible but not all actual components of the fluid mass, a condition of the form (384) must be satisfied, the values of the potentials in the second member being determined as in the preceding case.

The quantities

$$t, \quad X_{X'}, \ldots Z_{Z'}, \quad \mu_a, \quad \mu_b, \quad \text{etc.}, \tag{469}$$

being differential coefficients of $\varepsilon_{V'}$ with respect to the variables

$$\eta_{V'}, \quad \frac{dx}{dx'}, \ldots \frac{dz}{dz'}, \quad \Gamma_a', \quad \Gamma_b', \quad \text{etc.}, \tag{470}$$

will of course satisfy the necessary relations

$$\frac{dt}{d\frac{dx}{dx'}} = \frac{dX_{X'}}{d\eta_{V'}}, \quad \text{etc.} \tag{471}$$

This result may be generalized as follows. Not only is the second member of equation (468) a complete differential in its present form, but it will remain such if we transfer the sign of differentiation (d) from one factor to the other of any term (the sum indicated by the symbol $\Sigma\,\Sigma'$ is here supposed to be expanded into nine terms), and at the same time change the sign of the term from $+$ to $-$. For to

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substitute $-\eta_{V'}dt$ for $t\,d\eta_{V'}$, for example, is equivalent to subtracting the complete differential $d(t\,\eta_{V'})$. Therefore, if we consider the quantities in (469) and (470) which occur in any same term in equation (468) as forming a pair, we may choose as independent variables either quantity of each pair, and the differential coefficient of the remaining quantity of any pair with respect to the independent variable of another pair will be equal to the differential coefficient of the remaining quantity of the second pair with respect to the independent variable of the first, taken positively, if the independent variables of these pairs are both affected by the sign d in equation (468), or are neither thus affected, but otherwise taken negatively. Thus

$$\left(\frac{dX_{X'}}{d\Gamma_a'}\right)_{\frac{dx}{dx'}} = \left(\frac{d\mu_a}{d\frac{dx}{dx'}}\right)_{\Gamma_a'}, \quad \left(\frac{dX_{X'}}{d\mu_a}\right)_{\frac{dx}{dx'}} = -\left(\frac{d\Gamma_a'}{d\frac{dx}{dx'}}\right)_{\mu_a}, \tag{472}$$

$$\left(\frac{d\frac{dx}{dx'}}{d\mu_a}\right)_{X_{X'}} = \left(\frac{d\Gamma_a'}{dX_{X'}}\right)_{\mu_a}, \quad \left(\frac{d\frac{dx}{dx'}}{d\Gamma_a'}\right)_{X_{X'}} = -\left(\frac{d\mu_a}{dX_{X'}}\right)_{\Gamma_a'}, \tag{473}$$

where in addition to the quantities indicated by the suffixes, the following are to be considered as constant: either t or $\eta_{V'}$, either $X_{Y'}$ or $\frac{dx}{dy'}$, . . . either $Z_{Z'}$ or $\frac{dz}{dz'}$, either μ_b or Γ_b', etc.

It will be observed that when the temperature is constant the conditions $\mu_a =$ const., $\mu_b =$ const. represent the physical condition of a body in contact with a fluid of which the phase does not vary, and which contains the components to which the potentials relate. Also that when Γ_a', Γ_b', etc., are constant, the heat absorbed by the body in any infinitesimal change of condition per unit of volume measured in the state of reference is represented by $t\,d\eta_{V'}$. If we denote this quantity by $dQ_{V'}$, and use the suffix $_Q$ to denote the condition of no transmission of heat, we may write

$$\left(\frac{d\log t}{d\frac{dx}{dx'}}\right)_Q = \left(\frac{dX_{X'}}{dQ_{V'}}\right)_{\frac{dx}{dx'}}, \quad \left(\frac{d\log t}{dX_{X'}}\right)_Q = -\left(\frac{d\frac{dx}{dx'}}{dQ_{V'}}\right)_{X_{X'}}, \tag{474}$$

$$\left(\frac{dQ_{V'}}{dX_{X'}}\right)_t = \left(\frac{d\frac{dx}{dx'}}{d\log t}\right)_{X_{X'}}, \quad \left(\frac{dQ_{V'}}{d\frac{dx}{dx'}}\right)_t = -\left(\frac{dX_{X'}}{d\log t}\right)_{\frac{dx}{dx'}}, \tag{475}$$

where Γ_a', Γ_b', etc., must be regarded as constant in all the equations, and either $X_{Y'}$ or $\frac{dx}{dy'}$, . . . either $Z_{Z'}$ or $\frac{dz}{dz'}$, in each equation.

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Influence of surfaces of discontinuity upon the equilibrium of heterogeneous masses.—Theory of capillarity.

We have hitherto supposed, in treating of heterogeneous masses in contact, that they might be considered as separated by mathematical surfaces, each mass being unaffected by the vicinity of the others, so that it might be homogeneous quite up to the separating surfaces both with respect to the density of each of its various components and also with respect to the densities of energy and entropy. That such is not rigorously the case is evident from the consideration that if it were so with respect to the densities of the components it could not be so in general with respect to the density of energy, as the sphere of molecular action is not infinitely small. But we know from observation that it is only within very small distances of such a surface that any mass is sensibly affected by its vicinity,—a natural consequence of the exceedingly small sphere of sensible molecular action,—and this fact renders possible a simple method of taking account of the variations in the densities of the component substances and of energy and entropy, which occur in the vicinity of surfaces of discontinuity. We may use this term, for the sake of brevity, without implying that the discontinuity is absolute, or that the term distinguishes any surface with mathematical precision. It may be taken to denote the non-homogeneous film which separates homogeneous or nearly homogeneous masses.

Let us consider such a surface of discontinuity in a fluid mass which is in equilibrium and uninfluenced by gravity. For the precise measurement of the quantities with which we have to do, it will be convenient to be able to refer to a geometrical surface, which shall be sensibly coincident with the physical surface of discontinuity, but shall have a precisely determined position. For this end, let us take some point in or very near to the physical surface of discontinuity, and imagine a geometrical surface to pass through this point and all other points which are similarly situated with respect to the condition of the adjacent matter. Let this geometrical surface be called the *dividing surface*, and designated by the symbol S. It will be observed that the position of this surface is as yet to a certain extent arbitrary, but that the directions of its normals are already everywhere determined, since all the surfaces which can be formed in the manner described are evidently parallel to one another. Let us also imagine a closed surface cutting the surface S and including a part of the homogeneous mass on each side. We will so far limit the

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form of this closed surface as to suppose that on each side of S, as far as there is any want of perfect homogeneity in the fluid masses, the closed surface is such as may be generated by a moving normal to S. Let the portion of S which is included by the closed surface be denoted by **s**, and the area of this portion by s. Moreover, let the mass contained within the closed surface be divided into three parts by two surfaces, one on each side of S, and very near to that surface, although at such distance as to lie entirely beyond the influence of the discontinuity in its vicinity. Let us call the part which contains the surface **s** (with the physical surface of discontinuity) M, and the homogeneous parts M′ and M″, and distinguish by ε, ε', ε'', η, η', η'', m_1, m_1', m_1'', m_2, m_2', m_2'', etc., the energies and entropies of these masses, and the quantities which they contain of their various components.

It is necessary, however, to define more precisely what is to be understood in cases like the present by the energy of masses which are only separated from other masses by imaginary surfaces. A part of the total energy which belongs to the matter in the vicinity of the separating surface, relates to pairs of particles which are on different sides of the surface, and such energy is not in the nature of things referable to either mass by itself. Yet, to avoid the necessity of taking separate account of such energy, it will often be convenient to include it in the energies which we refer to the separate masses. When there is no break in the homogeneity at the surface, it is natural to treat the energy as distributed with a uniform density. This is essentially the case with the initial state of the system which we are considering, for it has been divided by surfaces passing in general through homogeneous masses. The only exception—that of the surface which cuts at right angles the non-homogeneous film—(apart from the consideration that without any important loss of generality we may regard the part of this surface within the film as very small compared with the other surfaces) is rather apparent than real, as there is no change in the state of the matter *in the direction perpendicular to this surface.* But in the variations to be considered in the state of the system, it will not be convenient to limit ourselves to such as do not create any discontinuity at the surfaces bounding the masses M, M′, M″: we must therefore determine how we will estimate the energies of the masses in case of such infinitesimal discontinuities as may be supposed to arise. Now the energy of each mass will be most easily estimated by neglecting the discontinuity, i. e., if we estimate the energy on the supposition that

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beyond the bounding surface the phase is identical with that within the surface. This will evidently be allowable, if it does not affect the total amount of energy. To show that it does not affect this quantity, we have only to observe that, if the energy of the mass on one side of a surface where there is an infinitesimal discontinuity of phase is greater as determined by this rule than if determined by any other (suitable) rule, the energy of the mass on the other side must be less by the same amount when determined by the first rule than when determined by the second, since the discontinuity relative to the second mass is equal but opposite in character to the discontinuity relative to the first.

If the entropy of the mass which occupies any one of the spaces considered is not in the nature of things determined without reference to the surrounding masses, we may suppose a similar method to be applied to the estimation of entropy.

With this understanding, let us return to the consideration of the equilibrium of the three masses M, M′, and M″. We shall suppose that there are no limitations to the possible variations of the system due to any want of perfect mobility of the components by means of which we express the composition of the masses, and that these components are independent, i. e., that no one of them can be formed out of the others.

With regard to the mass M, which includes the surface of discontinuity, it is necessary for its internal equilibrium that when its boundaries are considered constant, and when we consider only *reversible* variations (i. e., those of which the opposite are also possible), the variation of its energy should vanish with the variations of its entropy and of the quantities of its various components. For changes within this mass will not affect the energy or the entropy of the surrounding masses (when these quantities are estimated on the principle which we have adopted), and it may therefore be treated as an isolated system. For fixed boundaries of the mass M, and for reversible variations, we may therefore write

$$\delta\varepsilon = A_0\,\delta\eta + A_1\,\delta m_1 + A_2\,\delta m_2 + \text{etc.}, \tag{476}$$

where A_0, A_1, A_2, etc., are quantities determined by the initial (unvaried) condition of the system. It is evident that A_0 is the temperature of the lamelliform mass to which the equation relates, or the *temperature at the surface of discontinuity.* By comparison of this equation with (12) it will be seen that the definition of A_1, A_2, etc., is entirely analogous to that of the potentials in homo-

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geneous masses, although the mass to which the former quantities relate is not homogeneous, while in our previous definition of potentials, only homogeneous masses were considered. By a natural extension of the term *potential*, we may call the quantities A_1, A_2, etc., the *potentials at the surface of discontinuity*. This designation will be farther justified by the fact, which will appear hereafter, that the value of these quantities is independent of the thickness of the lamina (M) to which they relate. If we employ our ordinary symbols for temperature and potentials, we may write

$$\delta\varepsilon = t\,\delta\eta + \mu_1\,\delta m_1 + \mu_2\,\delta m_2 + \text{etc.} \tag{477}$$

If we substitue $\geqq$ for $=$ in this equation, the formula will hold true of all variations whether reversible or not;* for if the variation of energy could have a value less than that of the second member of the equation, there must be variation in the condition of M in which its energy is diminished without change of its entropy or of the quantities of its various components.

It is important, however, to observe that for any given values of $\delta\eta$, δm_1, δm_2, etc., while there *may* be possible variations of the nature and state of M for which the value of $\delta\varepsilon$ is greater than that of the second member of (477), there *must* always be possible variations for which the value of $\delta\varepsilon$ is equal to that of the second member.

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* To illustrate the difference between variations which are reversible, and those which are not, we may conceive of two entirely different substances meeting in equilibrium at a mathematical surface without being at all mixed. We may also conceive of them as mixed in a thin film about the surface where they meet, and then the amount of mixture is capable of variation both by increase and by diminution. But when they are absolutely unmixed, the amount of mixture can be increased, but is incapable of diminution, and it is then consistent with equilibrium that the value of $\delta\varepsilon$ (for a variation of the system in which the substances commence to mix) should be greater than the second member of (477). It is not necessary to determine whether precisely such cases actually occur; but it would not be legitimate to overlook the possible occurrence of cases in which variations may be possible while the opposite variations are not.

It will be observed that the sense in which the term *reversible* is here used is entirely different from that in which it is frequently used in treatises on thermodynamics, where a process by which a system is brought from a state A to a state B is called reversible, to signify that the system may also be brought from the state B to the state A through the same series of intermediate states taken in the reverse order by means of external agencies of the opposite character. The variation of a system from a state A to a state B (supposed to differ infinitely little from the first) is here called reversible when the system is capable of another state B′ which bears the same relation to the state A that A bears to B.

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It will be convenient to have a notation which will enable us to express this by an equation. Let $\mathfrak{d}\varepsilon$ denote the smallest value (i. e., the value nearest to $-\infty$) of $\delta\varepsilon$ consistent with given values of the other variations, then

$$\mathfrak{d}\varepsilon = t\,\delta\eta + \mu_1\,\delta m_1 + \mu_2\,\delta m_2 + \text{etc.} \tag{478}$$

For the internal equilibrium of the whole mass which consists of the parts M, M′, M″, it is necessary that

$$\delta\varepsilon + \delta\varepsilon' + \delta\varepsilon'' \geqq 0 \tag{479}$$

for all variations which do not affect the enclosing surface or the total entropy or the total quantity of any of the various components. If we also regard the surfaces separating M, M′, and M″ as invariable, we may derive from this condition, by equations (478) and (12), the following as a *necessary* condition of equilibrium:

$$\begin{aligned} &t\,\delta\eta + \mu_1\,\delta m_1 + \mu_2\,\delta m_2 + \text{etc.} \\ &+ t'\,\delta\eta' + \mu_1'\,\delta m_1' + \mu_2'\,\delta m_2' + \text{etc.} \\ &+ t''\,\delta\eta'' + \mu_1''\,\delta m_1'' + \mu_2''\,\delta m_2'' + \text{etc.} \geqq 0, \end{aligned} \tag{480}$$

the variations being subject to the equations of conditions

$$\left.\begin{aligned} &\delta\eta + \delta\eta' + \delta\eta'' = 0, \\ &\delta m_1 + \delta m_1' + \delta m_1'' = 0, \\ &\delta m_2 + \delta m_2' + \delta m_2'' = 0, \\ &\text{etc.} \end{aligned}\right\} \tag{481}$$

It may also be the case that some of the quantities $\delta m_1'$, $\delta m_1''$, $\delta m_2'$, $\delta m_2''$, etc., are incapable of negative values or can only have the value zero. This will be the case when the substances to which these quantities relate are not actual or possible components of M′ or M″. (See page 117.) To satisfy the above condition it is necessary and sufficient that

$$t = t' = t'', \tag{482}$$

$$\mu_1'\,\delta m_1' \geqq \mu_1\,\delta m_1', \quad \mu_2'\,\delta m_2' \geqq \mu_2\,\delta m_2', \quad \text{etc.}, \tag{483}$$

$$\mu_1''\,\delta m_1'' \geqq \mu_1\,\delta m_1'', \quad \mu_2''\,\delta m_2'' \geqq \mu_2\,\delta m_2'', \quad \text{etc.} \tag{484}$$

It will be observed that, if the substance to which μ_1, for instance, relates is an actual component of each of the homogeneous masses, we shall have $\mu_1 = \mu_1' = \mu_1''$. If it is an actual component of the first only of these masses, we shall have $\mu_1 = \mu_1'$. If it is also a possible component of the second homogeneous mass, we shall also have $\mu_1 \geqq \mu_1''$. If this substance occurs only at the surface of dis-

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continuity, the value of the potential μ_1 will not be determined by any equation, but cannot be greater than the potential for the same substance in either of the homogeneous masses in which it may be a possible component.

It appears, therefore, that the particular conditions of equilibrium *relating to temperature and the potentials* which we have before obtained by neglecting the influence of the surfaces of discontinuity (pp. 119, 120, 128) are not invalidated by the influence of such discontinuity in their application to homogeneous parts of the system bounded like M′ and M″ by imaginary surfaces lying within the limits of homogeneity,—a condition which may be fulfilled by surfaces very near to the surfaces of discontinuity. It appears also that similar conditions will apply to the non-homogeneous films like M′, which separate such homogeneous masses. The properties of such films, which are of course different from those of homogeneous masses, require our farther attention.

The volume occupied by the mass M is divided by the surface s into two parts, which we will call v''' and v'''', v''' lying next to M′, and v'''' to M″. Let us imagine these volumes filled by masses having throughout the same temperature, pressure and potentials, and the same densities of energy and entropy, and of the various components, as the masses M′ and M″ respectively. We shall then have, by equation (12), if we regard the volumes as constant,

$$\delta\varepsilon''' = t'\,\delta\eta''' + \mu_1'\,\delta m_1''' + \mu_2'\,\delta m_2''' + \text{etc.}, \tag{485}$$

$$\delta\varepsilon'''' = t''\,\delta\eta'''' + \mu_1''\,\delta m_1'''' + \mu_2''\,\delta m_2'''' + \text{etc.}\,; \tag{486}$$

whence, by (482)–(484), we have for reversible variations

$$\delta\varepsilon''' = t\,\delta\eta''' + \mu_1\,\delta m_1''' + \mu_2\,\delta m_2''' + \text{etc.}, \tag{487}$$

$$\delta\varepsilon'''' = t\,\delta\eta'''' + \mu_1\,\delta m_1'''' + \mu_2\,\delta m_2'''' + \text{etc.} \tag{488}$$

From these equations and (477), we have for reversible variations

$$\delta(\varepsilon - \varepsilon''' - \varepsilon'''') = t\,\delta(\eta - \eta''' - \eta'''')$$
$$+ \mu_1\,\delta(m_1 - m_1''' - m_1'''') + \mu_2\,\delta(m_2 - m_2''' - m_2'''') + \text{etc.} \tag{489}$$

Or, if we set*

$$\varepsilon^{\mathrm{s}} = \varepsilon - \varepsilon''' - \varepsilon'''', \quad \eta^{\mathrm{s}} = \eta - \eta''' - \eta'''', \tag{490}$$

$$m_1^{\mathrm{s}} = m_1 - m_1''' - m_1'''', \quad m_2^{\mathrm{s}} = m_2 - m_2''' - m_2'''', \quad \text{etc.}, \tag{491}$$

* It will be understood that the s here used is not an algebraic exponent, but is only intended as a distinguishing mark. The Roman letter S has not been used to denote any *quantity*.

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we may write

$$\delta\varepsilon^{s} = t\,\delta\eta^{s} + \mu_1\,\delta m_1^{s} + \mu_2\,\delta m_2^{s} + \text{etc.} \tag{492}$$

This is true of reversible variations in which the surfaces which have been considered are fixed. It will be observed that ε^s denotes the excess of the energy of the actual mass which occupies the total volume which we have considered over that energy which it would have, if on each side of the surface S the density of energy had the same uniform value quite up to that surface which it has at a sensible distance from it; and that η^s, m_1^s, m_2^s, etc., have analogous significations. It will be convenient, and need not be a source of any misconception, to call ε^s and η^s the energy and entropy *of the surface* (or the *superficial* energy and entropy), $\frac{\varepsilon^s}{s}$ and $\frac{\eta^s}{s}$ the *superficial densities* of energy and entropy, $\frac{m_1^s}{s}$, $\frac{m_2^s}{s}$, etc., the *superficial densities* of the several components.

Now these quantities (ε^s, η^s, m_1^s, etc.) are determined partly by the state of the physical system which we are considering, and partly by the various imaginary surfaces by means of which these quantities have been defined. The position of these surfaces, it will be remembered, has been regarded as fixed in the variation of the system. It is evident, however, that the form of that portion of these surfaces, which lies in the region of homogeneity on either side of the surface of discontinuity cannot affect the values of these quantities. To obtain the complete value of $\delta\varepsilon^s$ for reversible variations, we have therefore only to regard variations in the position and form of the limited surface s, as this determines all of the surfaces in question lying within the region of non-homogeneity. Let us first suppose the form of s to remain unvaried and only its position in space to vary, either by translation or rotation. No change in (492) will be necessary to make it valid in this case. For the equation is valid if s remains fixed and the material system is varied in position; also, if the material system and s are both varied in position, while their relative position remains unchanged. Therefore, it will be valid if the surface alone varies its position.

But if the form of s be varied, we must add to the second member (492) terms which shall represent the value of

$$\delta\varepsilon^{s} - t\,\delta\eta^{s} - \mu_1\,\delta m_1^{s} - \mu_2\,\delta m_2^{s} - \text{etc.}$$

due to such variation in the form of s. If we suppose s to be suffi-

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ciently small to be considered uniform throughout in its curvatures and in respect to the state of the surrounding matter, the value of the above expression will be determined by the variation of its area δs and the variations of its principal curvatures δc_1 and δc_2, and we may write

$$\delta\varepsilon^s = t\,\delta\eta^s + \mu_1\,\delta m_1^s + \mu_2\,\delta m_2^s + \text{etc.} + \sigma\,\delta s + C_1\,\delta c_1 + C_2\,\delta c_2, \tag{493}$$

or

$$\delta\varepsilon^s = t\,\delta\eta^s + \mu_1\,\delta m_1^s + \mu_2\,\delta m_2^s + \text{etc.} + \sigma\,\delta s + \tfrac{1}{2}(C_1 + C_2)\,\delta(c_1 + c_2) + \tfrac{1}{2}(C_1 - C_2)\,\delta(c_1 - c_2), \tag{494}$$

σ, C_1, and C_2 denoting quantities which are determined by the initial state of the system and position and form of s. The above is the complete value of the variation of ε^s for reversible variations of the system. But it is always possible to give such a position to the surface s that $C_1 + C_2$ shall vanish.

To show this, it will be convenient to write the equation in the longer form [see (490), (491)]

$$\begin{aligned} &\delta\varepsilon - t\,\delta\eta - \mu_1\,\delta m_1 - \mu_2\,\delta m_2 - \text{etc.} \\ &- \delta\varepsilon''' + t\,\delta\eta''' + \mu_1\,\delta m_1''' + \mu_2\,\delta m_1''' + \text{etc.} \\ &- \delta\varepsilon'''' + t\,\delta\eta'''' + \mu_1\,\delta m_1'''' + \mu_2\,\delta m_2'''' + \text{etc.} \\ &= \sigma\,\delta s + \tfrac{1}{2}(C_1 + C_2)\,\delta(c_1 + c_2) + \tfrac{1}{2}(C_1 - C_2)\,\delta(c_1 - c_2), \end{aligned} \tag{495}$$

i. e., by (482)–(484) and (12),

$$\begin{aligned} &\delta\varepsilon - t\,\delta\eta - \mu_1\,\delta m_1 - \mu_2\,\delta m_2 - \text{etc.} + p'\,\delta v''' + p''\,\delta v'''' \\ &= \sigma\,\delta s + \tfrac{1}{2}(C_1 + C_2)\,\delta(c_1 + c_2) + \tfrac{1}{2}(C_3 - C_2)\,\delta(c_1 - c_2). \end{aligned} \tag{496}$$

From this equation it appears in the first place that the pressure is the same in the two homogeneous masses separated by a plane surface of discontinuity. For let us imagine the material system to remain unchanged, while the plane surface s without change of area or of form moves in the direction of its normal. As this does not affect the boundaries of the mass M,

$$\delta\varepsilon - t\,\delta\eta - \mu_1\,\delta m_1 - \mu_2\,\delta m_2 - \text{etc.} = 0.$$

Also $\delta s = 0$, $\delta(c_1 + c_2) = 0$, $\delta(c_1 - c_2) = 0$, and $\delta v''' = -\delta v''''$. Hence $p' = p''$, when the surface of discontinuity is plane.

Let us now examine the effect of different positions of the surface s in the same material system upon the value of $C_1 + C_2$, supposing at first that in the initial state of the system the surface of discontinuity is plane. Let us give the surface s some particular position. In the

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initial state of the system this surface will of course be plane like the physical surface of discontinuity, to which it is parallel. In the varied state of the system, let it become a portion of a spherical surface having positive curvature; and at sensible distances from this surface let the matter be homogeneous and with the same phases as in the initial state of the system; also at and about the surface let the state of the matter so far as possible be the same as at and about the plane surface in the initial state of the system. (Such a variation in the system may evidently take place negatively as well as positively, as the surface may be curved toward either side. But whether such a variation is consistent with the maintenance of equilibrium is of no consequence, since in the preceding equations only the initial state is supposed to be one of equilibrium.) Let the surface $\mathfrak{s}$, placed as supposed, whether in the initial or the varied state of the surface, be distinguished by the symbol $\mathfrak{s}'$. Without changing either the initial or the varied state of the material system, let us make another supposition with respect to the imaginary surface $\mathfrak{s}$. In the unvaried system let it be parallel to its former position but removed from it a distance λ on the side on which lie the centers of positive curvature. In the varied state of the system, let it be spherical and concentric with $\mathfrak{s}'$, and separated from it by the same distance λ. It will of course lie on the same side of $\mathfrak{s}'$ as in the unvaried system. Let the surface $\mathfrak{s}$, placed in accordance with this second supposition, be distinguished by the symbol $\mathfrak{s}''$. Both in the initial and the varied state, let the perimeters of $\mathfrak{s}'$ and $\mathfrak{s}''$ be traced by a common normal. Now the value of

$$\delta\varepsilon - t\,\delta\eta - \mu_1\,\delta m_1 - \mu_2\,\delta m_2 - \text{etc.}$$

in equation (496) is not affected by the position of $\mathfrak{s}$, being determined simply by the body M: the same is true $p'\,\delta v''' + p''\,\delta v''''$ or $p'\delta(v''' + v'''')$, $v''' + v''''$ being the volume of M. Therefore the second member of (496) will have the same value whether the expressions relate to $\mathfrak{s}'$ or $\mathfrak{s}''$. Moreover, $\delta(c_1 - c_2) = 0$ both for $\mathfrak{s}'$ and $\mathfrak{s}''$. If we distinguish the quantities determined for $\mathfrak{s}'$ and for $\mathfrak{s}''$ by the marks $'$ and $''$, we may therefore write

$$\sigma'\,\delta s' + \tfrac{1}{2}(C_1' + C_2')\,\delta(c_1' + c_2') = \sigma''\,\delta s'' + \tfrac{1}{2}(C_1'' + C_2'')\,\delta(c_1'' + c_2'').$$

Now if we make $\qquad \delta s'' = 0,$

we shall have by geometrical necessity

$$\delta s' = s\,\lambda\,\delta(c_1'' + c_2'').$$

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Hence

$$\sigma' s \lambda \delta(c_1'' + c_2'') + \tfrac{1}{2}(C_1' + C_2')\,\delta(c_1' + c_2') = \tfrac{1}{2}(C_2'' + C_2'')\,\delta(c_1'' + c_2'').$$

But $$\delta(c_1' + c_2') = \delta(c_1'' + c_2'').$$

Therefore, $$C_1' + C_2' + 2\,\sigma' s\,\lambda = C_1'' + C_2''.$$

This equation shows that we may give a positive or negative value to $C_1'' + C_2''$ by placing $\mathbf{s}''$ a sufficient distance on one or on the other side of $\mathbf{s}'$. Since this is true when the (unvaried) surface is plane, it must also be true when the surface is nearly plane. And for this purpose a surface may be regarded as nearly plane, when the radii of curvature are very large in proportion to the thickness of the non-homogeneous film. This is the case when the radii of curvature have any sensible size. In general, therefore, whether the surface of discontinuity is plane or curved it is possible to place the surface $\mathbf{s}$ so that $C_1 + C_2$ in equation (494) shall vanish.

Now we may easily convince ourselves by equation (493) that if $\mathbf{s}$ is placed within the non-homogeneous film, and $s = 1$, the quantity σ is of the same order of magnitude as the values of ε^s, η^s, m_1^s, m_2^s, etc., while the values of C_1 and C_2 are of the same order of magnitude as the changes in the values of the former quantities caused by increasing the curvature of $\mathbf{s}$ by unity. Hence, on account of the thinness of the non-homogeneous film, since it can be very little affected by such a change of curvature in $\mathbf{s}$, the values of C_1 and C_2 must in general be very small relatively to σ. And hence, if $\mathbf{s}'$ be placed within the non-homogeneous film, the value of λ which will make $C_1'' + C_2''$ vanish must be very small (of the same order of magnitude as the thickness of the non-homogeneous film). The position of $\mathbf{s}$, therefore, which will make $C_1 + C_2$ in (494) vanish, will in general be sensibly coincident with the physical surface of discontinuity.

We shall hereafter suppose, when the contrary is not distinctly indicated that the surface $\mathbf{s}$, in the unvaried state of the system, has such a position as to make $C_1 + C_2 = 0$. It will be remembered that the surface $\mathbf{s}$ is a part of a larger surface S, which we have called the *dividing surface*, and which is coextensive with the physical surface of discontinuity. We may suppose that the position of the dividing surface is everywhere determined by similar considerations. This is evidently consistent with the suppositions made on page 380 with regard to this surface.

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We may therefore cancel the term

$$\tfrac{1}{2}(C_1 + C_2)\,\delta\,(c_1 + c_2)$$

in (494). In regard to the following term, it will be observed that C_1 must necessarily be equal to C_2, when $c_1 = c_2$, which is the case when the surface of discontinuity is plane. Now on account of the thinness of the non-homogeneous film, we may always regard it as composed of parts which are approximately plane. Therefore, without danger of sensible error, we may also cancel the term

$$\tfrac{1}{2}(C_1 - C_2)\,\delta\,(c_1 - c_2).$$

Equation (494) is thus reduced to the form

$$\delta\varepsilon^s = t\,\delta\eta^s + \sigma\,\delta s + \mu_1\,\delta m_1^s + \mu_2\,\delta m_2^s + \text{etc.} \tag{497}$$

We may regard this as the complete value of $\delta\varepsilon^s$, for all reversible variations in the state of the system supposed initially in equilibrium, when the dividing surface has its initial position determined in the manner described.

The above equation is of fundamental importance in the theory of capillarity. It expresses a relation with regard to surfaces of discontinuity analogous to that expressed by equation (12) with regard to homogeneous masses. From the two equations may be directly deduced the conditions of equilibrium of heterogeneous masses in contact, subject or not to the action of gravity, without disregard of the influence of the surfaces of discontinuity. The general problem, including the action of gravity, we shall take up hereafter: at present we shall only consider, as hitherto, a small part of a surface of discontinuity with a part of the homogeneous mass on either side, in order to deduce the additional condition which may be found when we take account of the motion of the dividing surface.

We suppose as before that the mass especially considered is bounded by a surface of which all that lies in the region of non-homogeneity is such as may be traced by a moving normal to the dividing surface. But instead of dividing the mass as before into four parts, it will be sufficient to regard it as divided into two parts by the dividing surface. The energy, entropy, etc., of these parts, estimated on the supposition that its nature (including density of energy, etc.) is uniform quite up to the dividing surface, will be denoted by ε', η', etc., ε'', η'', etc. Then the total energy will be $\varepsilon^s + \varepsilon' + \varepsilon''$, and the general condition of internal equilibrium will be that

$$\delta\varepsilon^s + \delta\varepsilon' + \delta\varepsilon'' \geqq 0, \tag{498}$$

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when the bounding surface is fixed, and the total entropy and total quantities of the various components are constant. We may suppose η^s, η', η'', m_1^s, m_1', m_1'', m_2^s, m_2', m_2'', etc., to be all constant. Then by (497) and (12) the condition reduces to

$$\sigma\,\delta s - p'\,\delta v' - p''\,\delta v'' = 0. \tag{499}$$

(We may set $=$ for $\geqq$, since changes in the position of the dividing surface can evidently take place in either of two opposite directions.) This equation has evidently the same form as if a membrane without rigidity and having a tension σ, uniform in all directions, existed at the dividing surface. Hence, the particular position which we have chosen for this surface may be called the surface of tension, and σ the superficial tension. If all parts of the dividing surface move a uniform normal distance δN, we shall have

$$\delta s = (c_1 + c_2)\,s\,\delta N, \qquad \delta v' = s\,\delta N, \qquad \delta v'' = -\,s\,\delta N;$$

whence

$$\sigma\,(c_1 + c_2) = p' - p'', \tag{500}$$

the curvatures being positive when their centers lie on the side to which p' relates. This is the condition which takes the place of that of equality of pressure (see pp. 119, 128) for heterogeneous fluid masses in contact, when we take account of the influence of the surfaces of discontinuity. We have already seen that the conditions relating to temperature and the potentials are not affected by these surfaces.

Fundamental Equations for Surfaces of Discontinuity.

In equation (497) the initial state of the system is supposed to be one of equilibrium. The only limitation with respect to the varied state is that the variation shall be reversible, i. e., that an opposite variation shall be possible. Let us now confine our attention to variations in which the system remains in equilibrium. To distinguish this case, we may use the character d instead δ, and write

$$d\varepsilon^s = t\,d\eta^s + \sigma\,ds + \mu_1\,dm_1^s + \mu_2\,dm_2^s + \text{etc.} \tag{501}$$

Both the states considered being states of equilibrium, the limitation with respect to the reversibility of the variations may be neglected, since the variations will always be reversible in at least one of the states considered.

If we integrate this equation, supposing the area s to increase from zero to any finite value s, while the material system to a part of which the equation relates remains without change, we obtain

$$\varepsilon^s = t\,\eta^s + \sigma\,s + \mu_1\,m_1^s + \mu_2\,m_2^s + \text{etc.}, \tag{502}$$

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which may be applied to any portion of any surface of discontinuity (in equilibrium) which is of the same nature throughout, or throughout which the values of t, σ, μ_1, μ_2, etc. are constant.

If we differentiate this equation, regarding all the quantities as variable, and compare the result with (501), we obtain

$$\eta^s\, dt + s\, d\sigma + m_1^s\, d\mu_1 + m_2^s\, d\mu_2 + \text{etc.} = 0. \tag{503}$$

If we denote the *superficial densities* of energy, of entropy, and of the several component substances (see page 386) by ε_s, η_s, Γ_1, Γ_2, etc., we have

$$\varepsilon_s = \frac{\varepsilon^s}{s}, \qquad \eta_s = \frac{\eta^s}{s}, \tag{504}$$

$$\Gamma_1 = \frac{m_1^s}{s}, \qquad \Gamma_2 = \frac{m_2^s}{s}, \qquad \text{etc.}, \tag{505}$$

and the preceding equations may be reduced to the form:—

$$d\varepsilon_s = t\, d\eta_s + \mu_1\, d\Gamma_1 + \mu_2\, d\Gamma_2 + \text{etc.}, \tag{506}$$

$$\varepsilon_s = t\, \eta_s + \sigma + \mu_1\, \Gamma_1 + \mu_2\, \Gamma_2 + \text{etc.}, \tag{507}$$

$$d\sigma = -\eta_s\, dt - \Gamma_1\, d\mu_1 - \Gamma_2\, d\mu_2 - \text{etc.} \tag{508}$$

Now the contact of the two homogeneous masses does not impose any restriction upon the variations of phase of either, except that the temperature and the potentials for actual components shall have the same value in both. [See (482)–(484) and (500).] For however the values of the pressures in the homogeneous masses may vary (on account of arbitrary variations of the temperature and potentials), and however the superficial tension may vary, equation (500) may always be satisfied by giving the proper curvature to the surface of tension, so long, at least, as the difference of pressures is not great. Moreover, if any of the potentials μ_1, μ_2, etc. relate to substances which are found only at the surface of discontinuity, their values may be varied by varying the superficial densities of those substances. The values of t, μ_1, μ_2, etc. are therefore independently variable, and it appears from equation (508) that σ is a function of these quantities. If the form of this function is known, we may derive from it by differentiation $n+1$ equations (n denoting the total number of component substances) giving the values of η_s, Γ_1, Γ_2, etc. in terms of the variables just mentioned. This will give us, with (507), $n+3$ independent equations between the $2n+4$ quantities which occur in that equation. These are all that exist, since $n+1$

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of these quantities are independently variable. Or, we may consider that we have $n+3$ independent equations between the $2n+5$ quantities occurring in equation (502), of which $n+2$ are independently variable.

An equation, therefore, between

$$\sigma, \quad t, \quad \mu_1, \quad \mu_2, \quad \text{etc.}, \tag{509}$$

may be called a fundamental equation for the surface of discontinuity. An equation between

$$\varepsilon^s, \quad \eta^s, \quad s, \quad m_1^s \quad m_2^s, \quad \text{etc.}, \tag{510}$$

or between

$$\varepsilon_s, \quad \eta_s, \quad \Gamma_1, \quad \Gamma_2, \quad \text{etc.}, \tag{511}$$

may also be called a fundamental equation in the same sense. For it is evident from (501) that an equation may be regarded as subsisting between the variables (510), and if this equation be known, since $n+2$ of the variables may be regarded as independent (viz., $n+1$ for the $n+1$ variations in the nature of the surface of discontinuity, and one for the area of the surface considered), we may obtain by differentiation and comparison with (501), $n+2$ additional equations between the $2n+5$ quantities occurring in (502). Equation (506) shows that equivalent relations can be deduced from an equation between the variables (511). It is moreover quite evident that an equation between the variables (510) must be reducible to the form of an equation between the ratios of these variables, and therefore to an equation between the variables (511).

The same designation may be applied to any equation from which, by differentiation and the aid only of general principles and relations, $n+3$ independent relations between the same $2n+5$ quantities may be obtained.

If we set

$$\psi^s = \varepsilon^s, - t\,\eta^s, \tag{512}$$

we obtain by differentiation and comparison with (501)

$$d\psi^s = -\eta^s\,dt + \sigma\,ds + \mu_1\,dm_1^s + \mu_2\,dm_2^s + \text{etc.} \tag{513}$$

An equation, therefore, between $\psi^s, t, s, m_1^s, m_2^s$, etc., is a fundamental equation, and is to be regarded as entirely equivalent to either of the other fundamental equations which have been mentioned.

The reader will not fail to notice the analogy between these fundamental equations, which relate to surfaces of discontinuity, and those relating to homogeneous masses, which have been described on pages 140–144.

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On the Experimental Determination of Fundamental Equations for Surfaces of Discontinuity.

When all the substances which are found at a surface of discontinuity are components of one or the other of the homogeneous masses, the potentials μ_1, μ_2, etc., as well as the temperature, may be determined from these homogeneous masses.* The tension σ may be determined by means of the relation (500). But our measurements are practically confined to cases in which the difference of the pressures in the homogeneous masses is small; for with increasing differences of pressure the radii of curvature soon become too small for measurement. Therefore, although the equation $p' = p''$ (which is equivalent to an equation between t, μ_1, μ_2, etc., since p' and p'' are both functions of these variables) may not be exactly satisfied in cases in which it is convenient to measure the tension, yet this equation is so nearly satisfied in all the measurements of tension which we can make, that we must regard such measurements as simply establishing the values of σ for values of t, μ_1, μ_2, etc., which satisfy the equation $p' = p''$, but not as sufficient to establish the rate of change in the value of σ for variations of t, μ_1, μ_2, etc., which are inconsistent with the equation $p' = p''$.

To show this more distinctly, let t, μ_2, m_3, etc. remain constant, then by (508) and (98)

$$d\sigma = -\Gamma_1\, d\mu_1,$$
$$dp' = \gamma_1'\, d\mu_1,$$
$$dp'' = \gamma_1''\, d\mu_1,$$

γ_1' and γ_1'' denoting the densities $\frac{m_1'}{v'}$ and $\frac{m_1''}{v''}$. Hence,

$$dp' - dp'' = (\gamma_1' - \gamma_1'')\, d\mu_1,$$

and
$$\Gamma_1\, d(p' - p'') = (\gamma_1'' - \gamma_1')\, d\sigma.$$

But by (500)

$$(c_1 + c_2)\, d\sigma + \sigma\, d(c_1 + c_2) = d(p' - p'').$$

Therefore,

$$\Gamma_1 (c_1 + c_2)\, d\sigma + \Gamma_1\, \sigma\, d(c_1 + c_2) = (\gamma_1'' - \gamma_1')\, d\sigma,$$

or
$$\{\gamma_1'' - \gamma_1' - \Gamma_1 (c_1 + c_2)\}\, d\sigma = \Gamma_1\, \sigma\, d(c_1 + c_2).$$

* It is here supposed that the thermodynamic properties of the homogeneous masses have already been investigated, and that the fundamental equations of these masses may be regarded as known.

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Now $\Gamma_1 (c_1 + c_2)$ will generally be very small compared with $\gamma_1'' - \gamma_1'$. Neglecting the former term, we have

$$\frac{d\sigma}{\sigma} = \frac{\Gamma_1}{\gamma_1'' - \gamma_1'} d(c_1 + c_2).$$

To integrate this equation, we may regard Γ_1, γ_1', γ_1'' as constant. This will give, as an approximate value,

$$\log \frac{\sigma}{\sigma'} = \frac{\Gamma_1}{\gamma_1'' - \gamma_1'} (c_1 + c_2),$$

σ' denoting the value of σ when the surface is plane. From this it appears that when the radii of curvature have any sensible magnitude, the value of σ will be sensibly the same as when the surface is plane and the temperature and all the potentials except one have the same values, unless the component for which the potential has not the same value has very nearly the same density in the two homogeneous masses, in which case, the condition under which the variations take place is nearly equivalent to the condition that the pressures shall remain equal.

Accordingly, we cannot in general expect to determine the superficial density Γ_1 from its value $-\left(\frac{d\sigma}{d\mu_1}\right)_{t,\mu}$* by measurements of superficial tensions. The case will be the same with Γ_2, Γ_3, etc., and also with η_s, the superficial density of entropy.

The quantities ε_s, η_s, Γ_1, Γ_2, etc. are evidently too small in general to admit of direct measurement. When one of the components, however, is found only at the surface of discontinuity, it may be more easy to measure its superficial density than its potential. But except in this case, which is of secondary interest, it will generally be easy to determine σ in terms of t, μ_1, μ_2, etc., with considerable accuracy for plane surfaces, and extremely difficult or impossible to determine the fundamental equation more completely.

Fundamental Equations for Plane Surfaces of Discontinuity.

An equation giving σ in terms of t, μ_1, μ_2, etc., which will hold true only so long as the surface of discontinuity is plane, may be called a fundamental equation for a plane surface of discontinuity. It will be interesting to see precisely what results can be obtained from such an equation, especially with respect to the energy and entropy

* The suffixed μ is used to denote that all the potentials except that occurring in the denominator of the differential coefficient are to be regarded as constant.

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and the quantities of the component substances in the vicinity of the surface of discontinuity.

These results can be exhibited in a more simple form, if we deviate to a certain extent from the method which we have been following. The particular position adopted for the dividing surface (which determines the superficial densities) was chosen in order to make the term $\frac{1}{2}(C_1+C_2)\,\delta\,(c_1+c_2)$ in (494) vanish. But when the curvature of the surface is not supposed to vary, such a position of the dividing surface is not necessary for the simplification of the formula. It is evident that equation (501) will hold true for plane surfaces (supposed to remain such) without reference to the position of the dividing surfaces, except that it shall be parallel to the surface of discontinuity. We are therefore at liberty to choose such a position for the dividing surface as may for any purpose be convenient.

None of the equations (502)–(513), which are either derived from (501), or serve to define new symbols, will be affected by such a change in the position of the dividing surface. But the expressions ε^s, η^s, m_1^s, m_2^s, etc., as also ε_s, η_s, Γ_1, Γ_2, etc. and ψ^s, will of course have different values when the position of that surface is changed. The quantity σ, however, which we may regard as defined by equations (501), or, if we choose, by (502) or (507), will not be affected in value by such a change. For if the dividing surface be moved a distance λ measured normally and toward the side to which v'' relates, the quantities

$$\varepsilon_s,\quad \eta_s,\quad \Gamma_1,\quad \Gamma_2,\quad \text{etc.},$$

will evidently receive the respective increments

$$\lambda\,(\varepsilon_v''-\varepsilon_v'),\quad \lambda\,(\eta_v''-\eta_v'),\quad \lambda\,(\gamma_1''-\gamma_1'),\quad \lambda\,(\gamma_2''-\gamma_2'),\quad \text{etc.},$$

ε_v', ε_v'', η_v', η_v'' denoting the densities of energy and entropy in the two homogeneous masses. Hence, by equation (507), σ will receive the increment

$$\lambda(\varepsilon_v''-\varepsilon_v')-t\,\lambda(\eta_v''-\eta_v')-\mu_1\lambda(\gamma_1''-\gamma_1')-\mu_2\lambda(\gamma_2''-\gamma_2')-\text{etc.}$$

But by (93)

$$-p''=\varepsilon_v''-t\,\eta_v''-\mu_1\,\gamma_1''-\mu_2\,\gamma_2''-\text{etc.},$$
$$-p'=\varepsilon_v'-t\,\eta_v'-\mu_1\,\gamma_1'-\mu_2\,\gamma_2'-\text{etc.}$$

Therefore, since $p'=p''$, the increment in the value of σ is zero. The value of σ is therefore independent of the position of the dividing surface, when this surface is plane. But when we call this quantity the superficial tension, we must remember that it will not have

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its characteristic properties as a tension with reference to any arbitrary surface. Considered as a tension, its position is in the surface which we have called the surface of tension, and, strictly speaking, nowhere else. The positions of the dividing surface, however, which we shall consider, will not vary from the surface of tension sufficiently to make this distinction of any practical importance.

It is generally possible to place the dividing surface so that the total quantity of any desired component in the vicinity of the surface of discontinuity shall be the same as if the density of that component were uniform on each side quite up to the dividing surface. In other words, we may place the dividing surface so as to make any one of the quantities Γ_1, Γ_2, etc., vanish. The only exception is with regard to a component which has the same density in the two homogeneous masses. With regard to a component which has very nearly the same density in the two masses such a location of the dividing surface might be objectionable, as the dividing surface might fail to coincide sensibly with the physical surface of discontinuity. Let us suppose that γ_1' is not equal (nor very nearly equal) to γ_1'', and that the dividing surface is so placed as to make $\Gamma_1 = 0$. Then equation (508) reduces to

$$d\sigma = -\eta_{s(1)}dt - \Gamma_{2(1)}d\mu_2 - \Gamma_{3(1)}d\mu_3 - \text{etc.}, \qquad (514)$$

where the symbols $\eta_{s(1)}$, $\Gamma_{2(1)}$, etc., are used for greater distinctness to denote the values of η_s, Γ_2, etc., as determined by a dividing surface placed so that $\Gamma_1 = 0$. Now we may consider all the differentials in the second member of this equation as independent, without violating the condition that the surface shall remain plane, i. e., that $dp' = dp''$. This appears at once from the values of dp' and dp'' given by equation (98). Moreover, as has already been observed, when the fundamental equations of the two homogeneous masses are known, the equation $p' = p''$ affords a relation between the quantities t, μ_1, μ_2, etc. Hence, when the value of σ is also known for plane surfaces in terms of t, μ_1, μ_2, etc., we can eliminate μ_1 from this expression by means of the relation derived from the equality of pressures, and obtain the value of σ for plane surfaces in terms of t, μ_2, μ_3, etc. From this, by differentiation, we may obtain directly the values of $\eta_{s(1)}$, $\Gamma_{2(1)}$, $\Gamma_{3(1)}$, etc., in terms of t, μ_2, μ_3, etc. This would be a convenient form of the fundamental equation. But, if the elimination of p', p'', and μ_1 from the finite equations presents algebraic difficulties, we can in all cases easily eliminate dp', dp'', $d\mu_1$ from the corresponding differential equations and thus obtain a

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differential equation from which the values of $\eta_{s(1)}$, $\Gamma_{2(1)}$, $\Gamma_{3(1)}$, etc. in terms of t, μ_1, μ_2, etc., may be at once obtained by comparison with (514).*

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* If liquid mercury meets the mixed vapors of water and mercury in a plane surface, and we use μ_1 and μ_2 to denote the potentials of mercury and water respectively, and place the dividing surface so that $\Gamma_1 = 0$, i. e., so that the total quantity of mercury is the same as if the liquid mercury reached this surface on one side and the mercury vapor on the other without change of density on either side, then $\Gamma_{2(1)}$ will represent the amount of water in the vicinity of this surface, per unit of surface, above that which there would be, if the water-vapor just reached the surface without change of density, and this quantity (which we may call the quantity of water condensed upon the surface of the mercury) will be determined by the equation

$$\Gamma_{2(1)} = -\frac{d\sigma}{d\mu_2}.$$

(In this differential coefficient as well as the following, the temperature is supposed to remain constant and the surface of discontinuity plane. Practically, the latter condition may be regarded as fulfilled in the case of any ordinary curvatures.)

If the pressure in the mixed vapors conforms to the law of Dalton (see pp. 215, 218), we shall have for constant temperature

$$dp_2 = \gamma_2\, d\mu_2,$$

where p_2 denotes the part of the pressure in the vapor due to the water-vapor, and γ_2 the density of the water-vapor. Hence we obtain

$$\Gamma_{2(1)} = -\gamma_2 \frac{d\sigma}{dp_2}.$$

For temperatures below 100° centigrade, this will certainly be accurate, since the pressure due to the vapor of mercury may be neglected.

The value of σ for $p_2 = 0$ and the temperature of 20° centigrade must be nearly the same as the superficial tension of mercury in contact with air, or 55.03 grammes per linear metre according to Quincke (Pogg. Ann., Bd. 139, p. 27). The value of σ at the same temperature, when the condensed water begins to have the properties of water in mass, will be equal to the sum of the superficial tensions of mercury in contact with water and of water in contact with its own vapor. This will be, according to the same authority, 42.58 + 8.25, or 50.83 grammes per metre, if we neglect the difference of the tensions of water with its vapor and water with air. As p_2, therefore, increases from zero to 236400 grammes per square metre (when water begins to be condensed *in mass*), σ diminishes from about 55.03 to about 50.83 grammes per linear metre. If the general course of the values of σ for intermediate values of p_2 were determined by experiment, we could easily form an approximate estimate of the values of the superficial density $\Gamma_{2(1)}$ for different pressures less than that of saturated vapor. It will be observed that the determination of the superficial density does not by any means depend upon inappreciable differences of superficial tension. The greatest difficulty in the determination would doubtless be that of distinguishing between the diminution of superficial tension due to the water and that due to other substances which might accidentally be present. Such determinations are of considerable practical importance on account of the use of mercury in measurements of the specific gravity of vapors.

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The same physical relations may of course be deduced without giving up the use of the surface of tension as a dividing surface, but the formulæ which express them will be less simple. If we make t, μ_3, μ_4, etc. constant, we have by (98) and (508)

$$dp' = \gamma_1' \, d\mu_1 + \gamma_2' \, d\mu_2,$$
$$dp'' = \gamma_1'' \, d\mu_1 + \gamma_2'' \, d\mu_2,$$
$$d\sigma = -\Gamma_1 \, d\mu_1 - \Gamma_2 \, d\mu_2,$$

where we may suppose Γ_1 and Γ_2 to be determined with reference to the surface of tension. Then, if $dp' = dp''$,

$$(\gamma_1' - \gamma_1'') \, d\mu_1 + (\gamma_2' - \gamma_2'') \, d\mu_2 = 0,$$

and

$$d\sigma = \Gamma_1 \frac{\gamma_2' - \gamma_2''}{\gamma_1' - \gamma_1''} d\mu_2 - \Gamma_2 \, d\mu_2.$$

That is,

$$\left(\frac{d\sigma}{d\mu_2}\right)_{p'-p'', t, \mu_3, \mu_4, \text{etc.}} = -\Gamma_2 + \Gamma_1 \frac{\gamma_2' - \gamma_2''}{\gamma_1' - \gamma_1''}. \qquad (515)$$

The reader will observe that $\frac{\Gamma_1}{\gamma_1' - \gamma_1''}$ represents the distance between the surface of tension and that dividing surface which would make $\Gamma_1 = 0$; the second number of the last equation is therefore equivalent to $-\Gamma_{2(1)}$.

If any component substance has the same density in the two homogeneous masses separated by a plane surface of discontinuity, the value of the superficial density for that component is independent of the position of the dividing surface. In this case alone we may derive the value of the superficial density of a component with reference to the surface of tension from the fundamental equation for plane surfaces alone. Thus in the last equation, when $\gamma_2' = \gamma_2''$, the second member will reduce to $-\Gamma_2$. It will be observed that to make $p' - p''$, t, μ_3, μ_4, etc. constant is in this case equivalent to making t, μ_1, μ_3, μ_4, etc. constant.

Substantially the same is true of the superficial density of entropy or of energy, when either of these has the same density in the two homogeneous masses.*

* With respect to questions which concern only the *form* of surfaces of discontinuity, such precision as we have employed in regard to the position of the dividing surface is evidently quite unnecessary. This precision has not been used for the sake of the mechanical part of the problem, which does not require the surface to be defined with greater nicety than we can employ in our observations, but in order to give

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Concerning the Stability of Surfaces of Discontinuity.

We shall first consider the stability of a film separating homogeneous masses with respect to changes in its nature, while its position and the nature of the homogeneous masses are not altered. For this purpose, it will be convenient to suppose that the homogeneous masses are very large, and thoroughly stable with respect to the possible formation of any different homogeneous masses out of their components, and that the surface of discontinuity is plane and uniform.

Let us distinguish the quantities which relate to the actual components of one or both of the homogeneous masses by the suffixes $_a$, $_b$, etc., and those which relate to components which are found only at the surface of discontinuity by the suffixes $_g$, $_h$, etc., and consider the variation of the energy of the whole system in consequence of a given change in the nature of a small part of the surface of discontinuity, while the entropy of the whole system and the total quantities of the several components remain constant, as well as the volume of each of the homogeneous masses, as determined by the surface of tension. This small part of the surface of discontinuity in its changed state is supposed to be still uniform in nature, and such as may subsist in equilibrium between the given homogeneous masses, which will evidently not be sensibly altered in nature or thermodynamic state. The remainder of the surface of discontinuity is also supposed to remain uniform, and on account of its infinitely greater size to be infinitely less altered in its nature than the first part. Let $\Delta\varepsilon^s$ denote the increment of the superficial energy of this first part, $\Delta\eta^s$, Δm_a^s, Δm_b^s, etc., Δm_g^s, Δm_h^s, etc., the increments of its superficial

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determinate values to the superficial densities of energy, entropy, and the component substances, which quantities, as has been seen, play an important part in the relations between the tension of a surface of discontinuity, and the composition of the masses which it separates.

The product σs of the superficial tension and the area of the surface, may be regarded as the *available energy* due to the surface in a system in which the temperature and the potentials μ_1, μ_2, etc.—or the differences of these potentials and the gravitational potential (see page 208) when the system is subject to gravity—are maintained sensibly constant. The value of σ, as well as that of s, is sensibly independent of the precise position which we may assign to the dividing surface (so long as this is sensibly coincident with the surface of discontinuity), but ε_s, the *superficial density of energy*, as the term is used in this paper, like the superficial densities of entropy and of the component substances, requires a more precise localization of the dividing surface.

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entropy and of the quantities of the components which we regard as belonging to the surface. The increments of entropy and of the various components which the rest of the system receive will be expressed by

$$-\Delta\eta^s, \quad -\Delta m_a^s, \quad -\Delta m_b^s, \quad \text{etc.}, \quad -\Delta m_g^s, \quad -\Delta m_h^s, \quad \text{etc.},$$

and the consequent increment of energy will be by (12) and (501)

$$-t\,\Delta\eta^s - \mu_a\,\Delta m_a^s - \mu_b\,\Delta m_b^s - \text{etc.} - \mu_g\,\Delta m_g^s - \mu_h\,\Delta m_h^s - \text{etc.}$$

Hence the total increment of energy in the whole system will be

$$\left.\begin{aligned}\Delta\varepsilon^s - t\,\Delta\eta^s - \mu_a\,\Delta m_a^s - \mu_b\,\Delta m_b^s - \text{etc.}\\ - \mu_g\,\Delta m_g^s - \mu_h\,\Delta m_h^s - \text{etc.}\end{aligned}\right\} \qquad (516)$$

If the value of this expression is necessarily positive, for finite changes as well as infinitesimal in the nature of the part of the film to which $\Delta\varepsilon^s$, etc. relate,* the increment of energy of the whole system will be positive for any possible changes in the nature of the film, and the film will be stable, at least with respect to changes in its nature, as distinguished from its position. For, if we write

$$D\varepsilon^s, \quad D\eta^s, \quad Dm_a^s, \quad Dm_b^s, \quad \text{etc.}, \quad Dm_g^s, \quad Dm_h^s, \quad \text{etc.}$$

for the energy, etc. of any element of the surface of discontinuity, we have from the supposition just made

$$\begin{aligned}\Delta D\varepsilon^s - t\,\Delta D\eta^s - \mu_a\,\Delta Dm_a^s - \mu_b\,\Delta Dm_b^s - \text{etc.}\\ - \mu_g\,\Delta Dm_g^s - \mu_h\,\Delta Dm_h^s - \text{etc.} > 0;\end{aligned} \qquad (517)$$

and integrating for the whole surface, since

$$\Delta\int Dm_g^s = 0, \quad \Delta\int Dm_h^s = 0, \quad \text{etc.},$$

we have

$$\Delta\int D\varepsilon^s - t\,\Delta\int D\eta^s - \mu_a\,\Delta\int Dm_a^s - \mu_b\,\Delta\int Dm_b^s - \text{etc.} > 0. \quad (518)$$

Now $\Delta\int D\eta^s$ is the increment of the entropy of the whole surface, and $-\Delta\int D\eta^s$ is therefore the increment of the entropy of the two homogeneous masses. In like manner, $-\Delta\int Dm_a^s$, $-\Delta\int Dm_b^s$, etc. are the increments of the quantities of the components in these masses. The expression

$$-t\,\Delta\int D\eta^s - \mu_a\,\Delta\int Dm_a^s - \mu_b\,\Delta\int Dm_b^s - \text{etc.}$$

* In the case of infinitesimal changes in the nature of the film, the sign Δ must be interpreted, as elsewhere in this paper, without neglect of infinitesimals of the higher orders. Otherwise, by equation (501), the above expression would have the value zero.

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denotes therefore, according to equation (12), the increment of energy of the two homogeneous masses, and since $\Delta \int D\varepsilon^{s}$ denotes the increment of energy of the surface, the above condition expresses that the increment of the total energy of the system is positive. That we have only considered the possible formation of such films as are capable of existing in equilibrium between the given homogeneous masses can not invalidate the conclusion in regard to the stability of the film, for in considering whether any state of the system will have less energy than the given state, we need only consider the state of least energy, which is necessarily one of equilibrium.

If the expression (516) is capable of a negative value for an infinitesimal change in the nature of the part of the film to which the symbols relate, the film is obviously unstable.

If the expression is capable of a negative value, but only for finite and not for infinitesimal changes in the nature of this part of the film, the film is *practically unstable*,* i. e., if such a change were made in a small part of the film, the disturbance would tend to increase. But it might be necessary that the initial disturbance should also have a finite magnitude in respect to the extent of surface in which it occurs; for we cannot suppose that the thermodynamic relations of an infinitesimal part of a surface of discontinuity are independent of the adjacent parts. On the other hand, the changes which we have been considering are such that every part of the film remains in equilibrium with the homogeneous masses on each side; and if the energy of the system can be diminished by a finite change satisfying this condition, it may perhaps be capable of diminution by an infinitesimal change which does not satisfy the same condition. We must therefore leave it undetermined whether the film, which in this case is practically unstable, is or is not unstable in the strict mathematical sense of the term.

Let us consider more particularly the condition of practical stability, in which we need not distinguish between finite and infinitesimal changes. To determine whether the expression (516) is capable of a negative value, we need only consider the least value of which it is capable. Let us write it in the fuller form

$$\left.\begin{aligned} \varepsilon^{s''} - \varepsilon^{s'} - t(\eta^{s''} - \eta^{s'}) - \mu_a(m_a^{s''} - m_a^{s'}) - \mu_b(m_b^{s''} - m_b^{s'}) - \text{etc.} \\ - \mu_g'(m_g^{s''} - m_g^{s'}) - \mu_h'(m_h^{s''} - m_h^{s'}) - \text{etc.,} \end{aligned}\right\} \quad (519)$$

where the single and double accents distinguish the quantities which

* With respect to the sense in which this term is used, compare page 133.

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relate to the first and second states of the film, the letters without accents denoting those quantities which have the same value in both states. The differential of this expression when the quantities distin guished by double accents are alone considered variable, and the area of the surface is constant, will reduce by (501) to the form

$$(\mu_g'' - \mu_g')\, dm_g^{s''} + (\mu_h'' - \mu_h')\, dm_h^{s''} + \text{etc.}$$

To make this incapable of a negative value, we must have

$$\mu_g'' = \mu_g', \quad \text{unless} \quad m_g^{s''} = 0,$$

$$\mu_h'' = \mu_h', \quad \text{unless} \quad m_h^{s''} = 0.$$

In virtue of these relations and by equation (502), the expression (519), i. e., (516), will reduce to

$$\sigma'' s - \sigma' s,$$

which will be positive or negative according as

$$\sigma'' - \sigma' \tag{520}$$

is positive or negative.

That is, if the tension of the film is less than that of any other film which can exist between the same homogeneous masses (which has therefore the same values of t, μ_a, μ_b, etc.), and which moreover has the same values of the potentials μ_g, μ_h, etc., so far as it contains the substances to which these relate, then the first film will be stable. But the film will be practically unstable, if any other such film has a less tension. [Compare the expression (141), by which the practical stability of homogeneous masses is tested.]

It is, however, evidently necessary for the stability of the surface of discontinuity with respect to *deformation*, that the value of the superficial tension should be positive. Moreover, since we have by (502) for the surface of discontinuity

$$\varepsilon^s - t\eta^s - \mu_a m_a^s - \mu_b m_b^s - \text{etc.} - \mu_g m_g^s - \mu_h m_h^s - \text{etc.} = \sigma s,$$

and by (93) for the two homogeneous masses

$$\varepsilon' - t\eta' + pv' - \mu_a m_a' - \mu_b m_b' - \text{etc.} = 0,$$

$$\varepsilon'' - t\eta'' + pv'' - \mu_a m_a'' - \mu_b m_b'' - \text{etc.} = 0,$$

if we denote by

$$\varepsilon, \quad \eta, \quad v, \quad m_a, \quad m_b, \quad \text{etc.}, \quad m_g, \quad m_h, \quad \text{etc.},$$

the total energy, etc. of a composite mass consisting of two such homogeneous masses divided by such a surface of discontinuity, we shall have by addition of these equations

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$$\varepsilon - t\eta + pv - \mu_a m_a - \mu_b m_b - \text{etc.} - \mu_g m_g - \mu_h m_h - \text{etc.} = \sigma s.$$

Now if the value of σ is negative, the value of the first member of this equation will decrease as s increases, and may therefore be decreased by making the mass to consist of thin alternate strata of the two kinds of homogeneous masses which we are considering. There will be no limit to the decrease which is thus possible with a given value of v, so long as the equation is applicable, i. e., so long as the strata have the properties of similar bodies in mass. But it may easily be shown (as in a similar case on pages 131, 132) that when the values of

$$t, \quad p, \quad \mu_a, \quad \mu_b, \quad \text{etc.}, \quad \mu_g, \quad \mu_h, \quad \text{etc.}$$

are regarded as fixed, being determined by the surface of discontinuity in question, and the values of

$$\varepsilon, \quad \eta, \quad m_a, \quad m_b, \quad \text{etc.}, \quad m_g, \quad m_h, \quad \text{etc.}$$

are variable and may be determined by any body having the given volume v, the first member of this equation cannot have an infinite negative value, and must therefore have a least possible value, which will be negative, if any value is negative, that is, if σ is negative.

The body determining ε, η, etc. which will give this least value to this expression will evidently be sensibly homogeneous. With respect to the formation of such a body, the system consisting of the two homogeneous masses and the surface of discontinuity with the negative tension is by (53) (see also page 133) at least practically unstable, if the surface of discontinuity is very large, so that it can afford the requisite material without sensible alteration of the values of the potentials. (This limitation disappears, if all the component substances are found in the homogeneous masses.) Therefore, in a system satisfying the conditions of practical stability with respect to the possible formation of all kinds of homogeneous masses, negative tensions of the surfaces of discontinuity are necessarily excluded.

Let us now consider the condition which we obtain by applying (516) to infinitesimal changes. The expression may be expanded as before to the form (519), and then reduced by equation (502) to the form

$$s(\sigma'' - \sigma') + m_g^{s''}(\mu_g'' - \mu_g') + m_h^{s''}(\mu_h'' - \mu_h') + \text{etc.}$$

That the value of this expression shall be positive when the quantities are determined by two films which differ infinitely little is a necessary condition of the stability of the film to which the single

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accents relate. But if one film is stable, the other will in general be so too, and the distinction between the films with respect to stability is of importance only at the limits of stability. If all films for all values of μ_g, μ_h, etc. are stable, or all within certain limits, it is evident that the value of the expression must be positive when the quantities are determined by any two infinitesimally different films within the same limits. For such collective determinations of stability the condition may be written

$$-s\,\Delta\sigma - m_g^s\,\Delta\mu_g - m_h^s\,\Delta\mu_h - \text{etc.} > 0,$$

or

$$\Delta\sigma < -\Gamma_g\,\Delta\mu_g - \Gamma_h\,\Delta\mu_h - \text{etc.} \tag{521}$$

On comparison of this formula with (508), it appears that within the limits of stability the second and higher differential coefficients of the tension considered as a function of the potentials for the substances which are found only at the surface of discontinuity (the potentials for the substances found in the homogeneous masses and the temperature being regarded as constant) satisfy the conditions which would make the tension a maximum if the necessary conditions relative to the first differential coefficients were fulfilled.

In the foregoing discussion of stability, the surface of discontinuity is supposed plane. In this case, as the tension is supposed positive, there can be no tendency to a change of form of the surface. We now pass to the consideration of changes consisting in or connected with motion and change of form of the surface of tension, which we shall at first suppose to be and to remain spherical and uniform throughout.

In order that the equilibrium of a spherical mass entirely surrounded by an indefinitely large mass of different nature shall be neutral with respect to changes in the value of r, the radius of the sphere, it is evidently necessary that equation (500), which in this may be written

$$2\,\sigma = r\,(p' - p''), \tag{522}$$

as well as the other conditions of equilibrium, shall continue to hold true for varying values of r. Hence, for a state of equilibrium which is on the limit between stability and instability, it is necessary that the equation

$$2\,d\sigma = (p' - p'')\,dr + r\,dp'$$

shall be satisfied, when the relations between $d\sigma$, dp', and dr are determined from the fundamental equations on the supposition that

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the conditions of equilibrium relating to temperature and the potentials remain satisfied. (The differential coefficients in the equations which follow are to be determined on this supposition.) Moreover, if

$$r\frac{dp'}{dr} < 2\frac{d\sigma}{dr} - p' + p'', \tag{523}$$

i. e., if the pressure of the interior mass increases less rapidly (or decreases more rapidly) with increasing radius than is necessary to preserve neutral equilibrium, the equilibrium is stable. But if

$$r\frac{dp'}{dr} > 2\frac{d\sigma}{dr} - p' + p'', \tag{524}$$

the equilibrium is unstable. In the remaining case, when

$$r\frac{dp'}{dr} = 2\frac{d\sigma}{dr} - p' + p'', \tag{525}$$

farther conditions are of course necessary to determine absolutely whether the equilibrium is stable or unstable, but in general the equilibrium will be stable in respect to change in one direction and unstable in respect to change in the opposite direction, and is therefore to be considered unstable. In general, therefore, we may call (523) the condition of stability.

When the interior mass and the surface of discontinuity are formed entirely of substances which are components of the external mass, p' and σ cannot vary and condition (524) being satisfied the equilibrium is unstable.

But if either the interior homogeneous mass or the surface of discontinuity contains substances which are not components of the enveloping mass, the equilibrium may be stable. If there is but one such substance, and we denote its densities and potential by γ'_1, Γ_1, and μ_1, the condition of stability (523) will reduce to the form

$$\left(r\frac{dp'}{d\mu_1} - 2\frac{d\sigma}{d\mu_1}\right)\frac{d\mu_1}{dr} < p'' - p',$$

or, by (98) and (508),

$$(r\gamma_1' + 2\Gamma_1)\frac{d\mu_1}{dr} < p'' - p'. \tag{526}$$

In these equations and in all which follow in the discussion of this case, the temperature and the potentials μ_2, μ_3, etc. are to be regarded as constant. But

$$\gamma_1' v' + \Gamma_1 s,$$

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which represents the total quantity of the component specified by the suffix, must be constant. It is evidently equal to

$$\tfrac{4}{3}\pi r^3 \gamma_1' + 4\pi r^2 \Gamma_1.$$

Dividing by 4π and differentiating, we obtain

$$(r^2 \gamma_1' + 2 r \Gamma_1)\, dr + \tfrac{1}{3} r^3\, d\gamma_1' + r^2\, d\Gamma_1 = 0,$$

or, since γ_1' and Γ_1 are functions of μ_1,

$$(r \gamma_1' + 2 \Gamma_1)\, dr + \left(\frac{r^2}{3}\frac{d\gamma_1'}{d\mu_1} + r\frac{d\Gamma_1}{d\mu_1}\right) d\mu_1 = 0. \tag{527}$$

By means of this equation, the condition of stability is brought to the form

$$\frac{(r\gamma_1' + 2\Gamma_1)^2}{\frac{r^2}{3}\frac{d\gamma_1'}{d\mu_1} + r\frac{d\Gamma_1}{d\mu_1}} > p' - p''. \tag{528}$$

If we eliminate r by equation (522), we have

$$\frac{\left(\frac{\gamma_1'}{p'-p''} + \frac{\Gamma_1}{\sigma}\right)^2}{\frac{1}{3(p'-p'')}\frac{d\gamma_1'}{d\mu_1} + \frac{1}{2\sigma}\frac{d\Gamma_1}{d\mu_1}} > 1. \tag{529}$$

If p' and σ are known in terms of t, μ_1, μ_2, etc., we may express the first member of this condition in terms of the same variables and p''. This will enable us to determine, for any given state of the external mass, the values of μ_1 which will make the equilibrium stable or unstable.

If the component to which γ_1' and Γ_1 relate is found only at the surface of discontinuity, the condition of stability reduces to

$$\frac{\Gamma_1^2}{\sigma}\frac{d\mu_1}{d\Gamma_1} > \frac{1}{2}. \tag{530}$$

Since

$$\Gamma_1 = -\frac{d\sigma}{d\mu_1},$$

we may also write

$$\frac{\Gamma_1}{\sigma}\frac{d\sigma}{d\Gamma_1} < -\frac{1}{2}, \quad \text{or} \quad \frac{d\log\sigma}{d\log\Gamma_1} < -\frac{1}{2}. \tag{531}$$

Again, if $\Gamma_1 = 0$ and $\frac{d\Gamma_1}{d\mu_1} = 0$, the condition of stability reduces to

$$\frac{3\gamma_1'^2}{p'-p''}\frac{d\mu_1}{d\gamma_1'} > 1. \tag{532}$$

Since

$$\gamma_1' = \frac{dp'}{d\mu_1},$$

we may also write

$$\frac{\gamma_1'}{p'-p''}\frac{dp'}{d\gamma_1'} > \frac{1}{3}, \quad \text{or} \quad \frac{d\log(p'-p'')}{d\log\gamma_1'} > \frac{1}{3}. \tag{533}$$

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When r is large, this will be a close approximation for any values of Γ_1, unless γ_1' is very small. The two special conditions (531) and (533) might be derived from very elementary considerations.

Similar conditions of stability may be found when there are more substances than one in the inner mass or the surface of discontinuity, which are not components of the enveloping mass. In this case, we have instead of (526) a condition of the form

$$(r\,\gamma_1' + 2\,\Gamma_1)\frac{d\mu_1}{dr} + (r\,\gamma_2' + 2\,\Gamma_2)\frac{d\mu_2}{dr} + \text{etc.} < p'' - p', \quad (534)$$

from which $\frac{d\mu_1}{dr}$, $\frac{d\mu_2}{dr}$, etc. may be eliminated by means of equations derived from the conditions that

$$\gamma_1'\,v' + \Gamma_1\,s, \quad \gamma_2'\,v' + \Gamma_2\,s, \quad \text{etc.}$$

must be constant.

Nearly the same method may be applied to the following problem. Two different homogeneous fluids are separated by a diaphragm having a circular orifice, their volumes being invariable except by the motion of the surface of discontinuity, which adheres to the edge of the orifice:—to determine the stability or instability of this surface when in equilibrium.

The condition of stability derived from (522) may in this case be written

$$r\,\frac{d(p'-p'')}{dv'} < 2\,\frac{d\sigma}{dv'} - (p' - p'')\,\frac{dr}{dv'}, \quad (535)$$

where the quantities relating to the concave side of the surface of tension are distinguished by a single accent.

If both the masses are infinitely large, or if one which contains all the components of the system is infinitely large, $p'-p''$ and σ will be constant, and the condition reduces to

$$\frac{dr}{dv'} < 0.$$

The equilibrium will therefore be stable or unstable according as the surface of tension is less or greater than a hemisphere.

To return to the general problem:—if we denote by x the part of the axis of the circular orifice intercepted between the center of the orifice and the surface of tension, by R the radius of the orifice, and by V' the value of v' when the surface of tension is plane, we shall have the geometrical relations

$$R^2 = 2\,r\,x - x^2,$$

and

$$v' = V' + \tfrac{2}{3}\,\pi\,r^2\,x - \tfrac{1}{3}\,\pi\,R^2\,(r - x)$$
$$= V' + \pi\,r\,x^2 - \tfrac{1}{3}\,\pi\,x^3.$$

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By differentiation we obtain

$$(r - x)\, dx + x\, dr = 0,$$

and $$dv' = \pi x^2\, dr + (2\pi r x - \pi x^2)\, dx;$$

whence $$(r - x)\, dv' = -\pi r x^2\, dr. \tag{536}$$

By means of this relation, the condition of stability may be reduced to the form

$$\frac{dp'}{dv'} - \frac{dp''}{dv'} - \frac{2}{r}\frac{d\sigma}{dv'} < (p' - p'')\frac{r - x}{\pi r^2 x^2}. \tag{537}$$

Let us now suppose that the temperature and all the potentials except one, μ_1, are to be regarded as constant. This will be the case when one of the homogeneous masses is very large and contains all the components of the system except one, or when both these masses are very large and there is a single substance at the surface of discontinuity which is not a component of either; also when the whole system contains but a single component, and is exposed to a constant temperature at its surface. Condition (537) will reduce by (98) and (508) to the form

$$\left(\gamma_1' - \gamma_1'' + \frac{2\Gamma_1}{r}\right)\frac{d\mu_1}{dv'} < (p' - p'')\frac{r - x}{\pi r^2 x^2}. \tag{538}$$

But $$\gamma_1' v' + \gamma_1'' v'' + \Gamma_1 s$$

(the total quantity of the component specified by the suffix) must be constant; therefore, since

$$dv'' = -dv', \quad \text{and} \quad ds = \frac{2}{r}\, dv',$$

$$\left(v'\frac{d\gamma_1'}{d\mu_1} + v''\frac{d\gamma_1''}{d\mu_1} + s\frac{d\Gamma_1}{d\mu_1}\right) d\mu_1 + \left(\gamma_1' - \gamma_1'' + \frac{2\Gamma_1}{r}\right) dv' = 0. \tag{539}$$

By this equation, the condition of stability is brought to the form

$$\frac{\left(\gamma_1' - \gamma_1'' + \frac{2\Gamma_1}{r}\right)^2}{v'\frac{d\gamma_1'}{d\mu_1} + v''\frac{d\gamma_1''}{d\mu_1} + s\frac{d\Gamma_1}{d\mu_1}} > (p' - p'')\frac{x - r}{\pi x^2 r^2}. \tag{540}$$

When the substance specified by the suffix is a component of either of the homogeneous masses, the terms $\frac{2\Gamma_1}{r}$ and $s\frac{d\Gamma_1}{d\mu_1}$ may generally be neglected. When it is not a component of either, the terms γ_1', γ_1'', $v'\frac{d\gamma_1'}{d\mu_1}$, $v''\frac{d\gamma_1''}{d\mu_1}$ may of course be cancelled, but we must not apply the formula to cases in which the substance spreads over the diaphragm separating the homogeneous masses.

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In the cases just discussed, the problem of the stability of certain surfaces of tension has been solved by considering the case of neutral equilibrium,—a condition of neutral equilibrium affording the equation of the limit of stability. This method probably leads as directly as any to the result, when that consists in the determination of the value of a certain quantity at the limit of stability, or of the relation which exists at that limit between certain quantities specifying the state of the system. But problems of a more general character may require a more general treatment.

Let it be required to ascertain the stability or instability of a fluid system in a given state of equilibrium with respect to motion of the surfaces of tension and accompanying changes. It is supposed that the conditions of internal stability for the separate homogeneous masses are satisfied, as well as those conditions of stability for the surfaces of discontinuity which relate to small portions of these surfaces with the adjacent masses. (The conditions of stability which are here supposed to be satisfied have been already discussed in part and will be farther discussed hereafter.) The fundamental equations for all the masses and surfaces occurring in the system are supposed to be known. In applying the general criteria of stability which are given on page 110, we encounter the following difficulty.

The question of the stability of the system is to be determined by the consideration of states of the system which are slightly varied from that of which the stability is in question. These varied states of the system are not in general states of equilibrium, and the relations expressed by the fundamental equations may not hold true of them. More than this,—if we attempt to describe a varied state of the system by varied values of the quantities which describe the initial state, if these varied values are such as are inconsistent with equilibrium, they may fail to determine with precision any state of the system. Thus, when the phases of two contiguous homogeneous masses are specified, if these phases are such as satisfy all the conditions of equilibrium, the nature of the surface of discontinuity (if without additional components) is entirely determined; but if the phases do not satisfy all the conditions of equilibrium, the nature of the surface of discontinuity is not only undetermined, but incapable of determination by specified values of such quantities as we have employed to express the nature of surfaces of discontinuity in equilibrium. For example, if the temperatures in contiguous homogeneous masses are different, we cannot specify the thermal state of the surface of discontinuity by assigning to it any particular temperature. It would be

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necessary to give the law by which the temperature passes over from one value to the other. And if this were given, we could make no use of it in the determination of other quantities, unless the rate of change of the temperature were so gradual, that at every point we could regard the thermodynamic state as unaffected by the change of temperature in its vicinity. It is true that we are also ignorant in respect to surfaces of discontinuity *in equilibrium* of the law of change of those quantities which are different in the two phases in contact, such as the densities of the components, but this, although unknown to us, is entirely determined by the nature of the phases in contact, so that no vagueness is occasioned in the definition of any of the quantities which we have occasion to use with reference to such surfaces of discontinuity.

It may be observed that we have established certain differential equations, especially (497), in which only the initial state is necessarily one of equilibrium. Such equations may be regarded as establishing certain properties of states bordering upon those of equilibrium. But these are properties which hold true only when we disregard quantities proportional to the square of those which express the degree of variation of the system from equilibrium. Such equations are therefore sufficient for the determination of the conditions of equilibrium, but not sufficient for the determination of the conditions of stability

We may, however, use the following method to decide the question of stability in such a case as has been described.

Beside the real system of which the stability is in question, it will be convenient to conceive of another system, to which we shall attribute in its initial state the same homogeneous masses and surfaces of discontinuity which belong to the real system. We shall also suppose that the homogeneous masses and surfaces of discontinuity of this system, which we may call the imaginary system, have the same fundamental equations as those of the real system. But the imaginary system is to differ from the real in that the variations of its state are limited to such as do not violate the conditions of equilibrium relating to temperature and the potentials, and that the fundamental equations of the surfaces of discontinuity hold true for these varied states, although the condition of equilibrium expressed by equation (500) may not be satisfied.

Before proceeding farther, we must decide whether we are to examine the question of stability under the condition of a constant external temperature, or under the condition of no transmission of

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heat to or from external bodies, and in general, to what external influences we are to regard the system as subject. It will be convenient to suppose that the exterior of the system is fixed, and that neither matter nor heat can be transmitted through it. Other cases may easily be reduced to this, or treated in a manner entirely analogous.

Now if the real system in the given state is unstable, there must be some slightly varied state in which the energy is less, but the entropy and the quantities of the components the same as in the given state, and the exterior of the system unvaried. But it may easily be shown that the given state of the system may be made stable by constraining the surfaces of discontinuity to pass through certain fixed lines situated in the unvaried surfaces. Hence, if the surfaces of discontinuity are constrained to pass through corresponding fixed lines in the surfaces of discontinuity belonging to the varied state just mentioned, there must be a state of stable equilibrium for the system thus constrained which will differ infinitely little from the given state of the system, the stability of which is in question, and will have the same entropy, quantities of components and exterior, but less energy. The imaginary system will have a similar state, since the real and imaginary systems do not differ in respect to those states which satisfy all the conditions of equilibrium for each surface of discontinuity. That is, the imaginary system has a state, differing infinitely little from the given state, and with the same entropy, quantities of components, and exterior, but with less energy.

Conversely, if the imaginary system has such a state as that just described, the real system will also have such a state. This may be shown by fixing certain lines in the surfaces of discontinuity of the imaginary system in its state of less energy and then making the energy a minimum under the conditions. The state thus determined will satisfy all the conditions of equilibrium for each surface of discontinuity, and the real system will therefore have a corresponding state, in which the entropy, quantities of components, and exterior will be the same as in the given state, but the energy less.

We may therefore determine whether the given system is or is not unstable, by applying the general criterion of instability (7) to the imaginary system.

If the system is not unstable, the equilibrium is either neutral or stable. Of course we can determine which of these is the case by reference to the imaginary system, since this determination depends upon states of equilibrium, in regard to which the real and imaginary

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systems do not differ. We may therefore determine whether the equilibrium of the given system is stable, neutral, or unstable, by applying the criteria (3)–(7) to the imaginary system.

The result which we have obtained may be expressed as follows:—In applying to a fluid system which is in equilibrium, and of which all the small parts taken separately are stable, the criteria of stable, neutral, and unstable equilibrium, we may regard the system as under constraint to satisfy the conditions of equilibrium relating to temperature and the potentials, and as satisfying the relations expressed by the fundamental equations for masses and surfaces, even when the condition of equilibrium relating to pressure [equation (500)] is not satisfied.

It follows immediately from this principle, in connection with equations (501) and (86), that in a stable system each surface of tension must be a surface of minimum area for constant values of the volumes which it divides, when the other surfaces bounding these volumes and the perimeter of the surface of tension are regarded as fixed; that in a system in neutral equilibrium each surface of tension will have as small an area as it can receive by any slight variations under the same limitations; and that in seeking the remaining conditions of stable or neutral equilibrium, when these are satisfied, it is only necessary to consider such varied surfaces of tension as have similar properties with reference to the varied volumes and perimeters.

We may illustrate the method which has been described by applying it to a problem but slightly different from one already (pp. 408, 409) discussed by a different method. It is required to determine the conditions of stability for a system in equilibrium, consisting of two different homogeneous masses meeting at a surface of discontinuity, the perimeter of which is invariable, as well as the exterior of the whole system, which is also impermeable to heat.

To determine what is necessary for stability in addition to the condition of minimum area for the surface of tension, we need only consider those varied surfaces of tension which satisfy the same condition. We may therefore regard the surface of tension as determined by v', the volume of one of the homogeneous masses. But the state of the system would evidently be completely determined by the position of the surface of tension and the temperature and potentials, if the entropy and the quantities of the components were variable; and therefore, since the entropy and the quantities of the components are constant, the state of the system must be completely determined by the position of the surface of tension. We may therefore regard

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all the quantities relating to the system as functions of v', and the condition of stability may be written

$$\frac{d\varepsilon}{dv'}dv' + \frac{1}{2}\frac{d^2\varepsilon}{dv'^2}dv'^2 + \text{etc.} > 0,$$

where ε denotes the total energy of the system. Now the conditions of equilibrium require that

$$\frac{d\varepsilon}{dv'} = 0,$$

Hence, the general condition of stability is that

$$\frac{d^2\varepsilon}{dv'^2} > 0. \tag{541}$$

Now if we write ε', ε'', ε^s for the energies of the two masses and of the surface, we have by (86) and (501), since the total entropy and the total quantities of the several components are constant,

$$d\varepsilon = d\varepsilon' + d\varepsilon'' + d\varepsilon^s = -p'\,dv' - p''\,dv'' + \sigma\,ds,$$

or, since $dv'' = -dv'$,

$$\frac{d\varepsilon}{dv'} = -p' + p'' + \sigma\frac{ds}{dv'}. \tag{542}$$

Hence,

$$\frac{d^2\varepsilon}{dv'^2} = -\frac{dp'}{dv'} + \frac{dp''}{dv'} + \frac{d\sigma}{dv'}\frac{ds}{dv'} + \sigma\frac{d^2s}{dv'^2}, \tag{543}$$

and the condition of stability may be written

$$\sigma\frac{d^2s}{dv'^2} > \frac{dp'}{dv'} - \frac{dp''}{dv'} - \frac{d\sigma}{dv'}\frac{ds}{dv'}. \tag{544}$$

If we now simplify the problem by supposing, as in the similar case on page 409, that we may disregard the variations of the temperature and of all the potentials except one, the condition will reduce to

$$\sigma\frac{d^2s}{dv'^2} > \left(\gamma_1' - \gamma_1'' + \Gamma_1\frac{ds}{dv'}\right)\frac{d\mu_1}{dv'}. \tag{545}$$

The total quantity of the substance indicated by the suffix $_1$ is

$$\gamma_1' v' + \gamma_1'' v'' + \Gamma_1 s.$$

Making this constant, we have

$$\left(\gamma_1' - \gamma_1'' + \Gamma_1\frac{ds}{dv'}\right)dv' + \left(v'\frac{d\gamma_1'}{d\mu_1} + v''\frac{d\gamma_1''}{d\mu_1} + s\frac{d\Gamma_1}{d\mu_1}\right)d\mu_1 = 0. \tag{546}$$

The condition of equilibrium is thus reduced to the form

$$\sigma\frac{d^2s}{dv'^2} > -\frac{\left(\gamma_1' - \gamma_1'' + \Gamma_1\frac{ds}{dv'}\right)^2}{v'\frac{d\gamma_1'}{d\mu_1} + v''\frac{d\gamma_1''}{d\mu_1} + s\frac{d\Gamma_1}{d\mu_1}}, \tag{547}$$

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where $\frac{ds}{dv'}$ and $\frac{d^2s}{dv'}$ are to be determined from the form of the surface of tension by purely geometrical considerations, and the other differential coefficients are to be determined from the fundamental equations of the homogeneous masses and the surface of discontinuity. Condition (540) may be easily deduced from this as a particular case.

The condition of stability with reference to motion of surfaces of discontinuity admits of a very simple expression when we can treat the temperature and potentials as constant. This will be the case when one or more of the homogeneous masses, containing together all the component substances, may be considered as indefinitely large, the surfaces of discontinuity being finite. For if we write $\Sigma\Delta\varepsilon$ for the sum of the variations of the energies of the several homogeneous masses, and $\Sigma\Delta\varepsilon^s$ for the sum of the variations of the energies of the several surfaces of discontinuity, the condition of stability may be written

$$\Sigma\Delta\varepsilon + \Sigma\Delta\varepsilon^s > 0, \tag{548}$$

the total entropy and the total quantities of the several components being constant. The variations to be considered are infinitesimal, but the character Δ signifies, as elsewhere in this paper, that the expression is to be interpreted without neglect of infinitesimals of the higher orders. Since the temperature and potentials are sensibly constant, the same will be true of the pressures and surface-tensions, and by integration of (86) and (501) we may obtain for any homogeneous mass

$$\Delta\varepsilon = t\,\Delta\eta - p\,\Delta v + \mu_1\,\Delta m_1 + \mu_2\,\Delta m_2 + \text{etc.},$$

and for any surface of discontinuity

$$\Delta\varepsilon^s = t\,\Delta\eta^s + \sigma\,\Delta s + \mu_1\,\Delta m_1^s + \mu_2^s\,\Delta m_2 + \text{etc.}$$

These equations will hold true of finite differences, when t, p, σ, μ_1, μ_2, etc. are constant, and will therefore hold true of infinitesimal differences, under the same limitations, without neglect of the infinitesimals of the higher orders. By substitution of these values, the condition of stability will reduce to the form

$$-\Sigma(p\Delta v) + \Sigma(\sigma\Delta s) > 0,$$

or

$$\Sigma(p\Delta v) - \Sigma(\sigma\Delta s) < 0. \tag{549}$$

That is, the sum of the products of the volumes of the masses by their pressures diminished by the sum of the products of the areas of the surfaces of discontinuity by their tensions must be a maximum. This is a purely geometrical condition, since the pressures and ten-

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sions are constant. This condition is of interest, because it is always *sufficient* for stability with reference to motion of surfaces of discontinuity. For any system may be reduced to the kind described by putting certain parts of the system in communication (by means of fine tubes if necessary) with large masses of the proper temperatures and potentials. This may be done without introducing any new movable surfaces of discontinuity. The condition (549) when applied to the altered system is therefore the same as when applied to the original system. But it is sufficient for the stability of the altered system, and therefore sufficient for its stability if we diminish its freedom by breaking the connection between the original system and the additional parts, and therefore sufficient for the stability of the original system.

On the Possibility of the Formation of a Fluid of different Phase within any Homogeneous Fluid.

The study of surfaces of discontinuity throws considerable light upon the subject of the stability of such homogeneous fluid masses as have a less pressure than others formed of the same components (or some of them) and having the same temperature and the same potentials for their actual components.*

In considering this subject, we must first of all inquire how far our method of treating surfaces of discontinuity is applicable to cases in which the radii of curvature of the surfaces are of insensible magnitude. That it should not be applied to such cases without limitation is evident from the consideration that we have neglected the term $\frac{1}{2}(C_1 - C_2)\delta(c_1 - c_2)$ in equation (494) on account of the magnitude of the radii of curvature compared with the thickness of the non-homogeneous film. (See page 390). When, however, only spherical masses are considered, this term will always disappear, since C_1 and C_2 will necessarily be equal.

Again, the surfaces of discontinuity have been regarded as separating homogeneous masses. But we may easily conceive that a globular mass (surrounded by a large homogeneous mass of different nature) may be so small that no part of it will be homogeneous, and that even at its center the matter cannot be regarded as having any phase of matter *in mass*. This, however, will cause no difficulty, if we regard the phase of the interior mass as determined by the same

* See page 161, where the term stable is used (as indicated on page 159) in a less strict sense than in the discussion which here follows.

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relations to the exterior mass as in other cases. Beside the phase of the exterior mass, there will always be another phase having the same temperature and potentials, but of the general nature of the small globule which is surrounded by that mass and in equilibrium with it. This phase is completely determined by the system considered, and in general entirely stable and perfectly capable of realization in mass, although not such that the exterior mass could exist in contact with it at a plane surface. This is the phase which we are to attribute to the mass which we conceive as existing within the dividing surface.*

With this understanding with regard to the phase of the fictitious interior mass, there will be no ambiguity in the meaning of any of the symbols which we have employed, when applied to cases in which the surface of discontinuity is spherical, however small the radius may be. Nor will the demonstration of the general theorems require any material modification. The dividing surface, which determines the value of ε^s, η^s, m_1^s, m_2^s, etc., is as in other cases to be placed so as to make the term $\frac{1}{2}(C_1 + C_1)\delta(c_1 + c_2)$ in equation (494) vanish, i. e., so as to make equation (497) valid. It has been shown on pages 387–389 that when thus placed it will sensibly coincide with the physical surface of discontinuity, when this consists of a non-homogeneous film separating homogeneous masses, and having radii of curvature which are large compared with its thickness. But in regard to globular masses too small for this theorem to have any application, it will be worth while to examine how far we may be certain that the radius of the dividing surface will have a real and positive value, since it is only then that our method will have any natural application.

The value of the radius of the dividing surface, supposed spherical, of any globule in equilibrium with a surrounding homogeneous fluid may be most easily obtained by eliminating σ from equations (500) and (502), which have been derived from (497), and contain the radius implicitly. If we write r for this radius, equation (500) may be written

$$2\,\sigma = (p' - p'')\,r, \tag{550}$$

the single and double accents referring respectively to the interior and exterior masses. If we write $[\varepsilon]$, $[\eta]$, $[m_1]$, $[m_2]$, etc. for the

* For example, in applying our formulæ to a microscopic globule of water in steam, by the density or pressure of the interior mass we should understand, not the actual density or pressure at the center of the globule, but the density of liquid water (in large quantities) which has the temperature and potential of the steam.

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excess of the total energy, entropy, etc. in and about the globular mass above what would be in the same space if it were uniformly filled with matter of the phase of the exterior mass, we shall have necessarily with reference to the whole dividing surface

$$\varepsilon^s = [\varepsilon] - v'(\varepsilon_v' - \varepsilon_v''), \quad \eta^s = [\eta] - v'(\eta_v' - \eta_v''),$$
$$m_1^s = [m_1] - v'(\gamma_1' - \gamma_1''), \quad m_2^s = [m_2] - v'(\gamma_2' - \gamma_2''), \text{ etc.},$$

where ε_v', ε_v'', η_v', η_v'', γ_1', γ_1'', etc. denote, in accordance with our usage elsewhere, the volume-densities of energy, of entropy, and of the various components, in the two homogeneous masses. We may thus obtain from equation (502)

$$\sigma s = [\varepsilon] - v'(\varepsilon_v' - \varepsilon_v'') - t[\eta] + t v'(\eta_v' - \eta_v'')$$
$$- \mu_1[m_1] + \mu_1 v'(\gamma_1' - \gamma_1'') - \mu_2[m_2] + \mu_2 v'(\gamma_2' - \gamma_2'') - \text{etc.} \quad (551)$$

But by (93),

$$p' = -\varepsilon_v' + t\eta_v' + \mu_1\gamma_1' + \mu_2\gamma_2' + \text{etc.},$$
$$p'' = -\varepsilon_v'' + t\eta_v'' + \mu_1\gamma_1'' + \mu_2\gamma_2'' + \text{etc.}$$

Let us also write for brevity

$$W = [\varepsilon] - t[\eta] - \mu_1[m_1] - \mu_2[m_2] - \text{etc.} \quad (552)$$

(It will be observed that the value of W is entirely determined by the nature of the physical system considered, and that the notion of the dividing surface does not in any way enter into its definition.) We shall then have

$$\sigma s = W + v'(p' - p''), \quad (553)$$

or, substituting for s and v' their values in terms of r,

$$4\pi r^2 \sigma = W + \tfrac{4}{3}\pi r^3 (p' - p''), \quad (554)$$

and eliminating σ by (550),

$$\tfrac{2}{3}\pi r^3 (p' - p'') = W, \quad (555)$$

$$r = \left(\frac{3W}{2\pi(p' - p'')}\right)^{\frac{1}{3}}. \quad (556)$$

If we eliminate r instead σ, we have

$$\frac{16\pi\sigma^3}{3(p' - p'')^2} = W, \quad (557)$$

$$\sigma = \left(\frac{3W(p' - p'')^2}{16\pi}\right)^{\frac{1}{3}}. \quad (558)$$

Now, if we first suppose the difference of the pressures in the homogeneous masses to be very small, so that the surface of discontinuity is nearly plane, since without any important loss of generality

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we may regard σ as positive (for if σ is not positive when $p'=p''$, the surface when plane would not be stable in regard to position, as it certainly is, in every actual case, when the proper conditions are fulfilled with respect to its perimeter), we see by (550) that the pressure in the interior mass must be the greater; i. e., we may regard σ, $p'-p''$, and r as all positive. By (555), the value of W will also be positive. But it is evident from equation (552), which defines W, that the value of this quantity is necessarily real, in any possible case of equilibrium, and can only become infinite when r becomes infinite and $p'=p''$. Hence, by (556) and (558), as $p'-p''$ increases from very small values, W, r, and σ have single, real, and positive values until they simultaneously reach the value zero. Within this limit, our method is evidently applicable; beyond this limit, if such exist, it will hardly be profitable to seek to interpret the equations. But it must be remembered that the vanishing of the radius of the somewhat arbitrarily determined *dividing surface* may not necessarily involve the vanishing of the physical heterogeneity. It is evident, however, (see pp. 387–389,) that the globule must become insensible in magnitude before r can vanish.

It may easily be shown that the quantity denoted by W is the work which would be required to form (by a reversible process) the heterogeneous globule in the interior of a very large mass having initially the uniform phase of the exterior mass. For this work is equal to the increment of energy of the system when the globule is formed without change of the entropy or volume of the whole system or of the quantities of the several components. Now $[\eta]$, $[m_1]$, $[m_2]$, etc. denote the increments of entropy and of the components in the space where the globule is formed. Hence these quantities with the negative sign will be equal to the increments of entropy and of the components in the rest of the system. And hence, by equation (86),

$$-t[\eta]-\mu_1[m_1]-\mu_2[m_2]-\text{etc.}$$

will denote the increment of energy in all the system except where the globule is formed. But $[\varepsilon]$ denotes the increment of energy in that part of the system. Therefore, by (552), W denotes the total increment of energy in the circumstances supposed, or the work required for the formation of the globule.

The conclusions which may be drawn from these considerations with respect to the stability of the homogeneous mass of the pressure p'' (supposed less than p', the pressure belonging to a different phase of the same temperature and potentials) are very obvious.

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Within those limits within which the method used has been justified, the mass in question must be regarded as in strictness stable with respect to the growth of a globule of the kind considered, since W, the work required for the formation of such a globule of a certain size (viz., that which would be in equilibrium with the surrounding mass), will always be positive. Nor can smaller globules be formed, for they can neither be in equilibrium with the surrounding mass, being too small, nor grow to the size of that to which W relates. If, however, by any external agency such a globular mass (of the size necessary for equilibrium) were formed, the equilibrium has already (page 406) been shown to be unstable, and with the least excess in size, the interior mass would tend to increase without limit except that depending on the magnitude of the exterior mass. We may therefore regard the quantity W as affording a kind of measure of the *stability* of the phase to which p'' relates. In equation (557) the value of W is given in terms of σ and $p'-p''$. If the three fundamental equations which give σ, p', and p'' in terms of the temperature and the potentials were known, we might regard the stability (W) as known in terms of the same variables. It will be observed that when $p'=p''$ the value of W is infinite. If $p'-p''$ increases without greater changes of the phases than are necessary for such increase, W will vary at first very nearly inversely as the square of $p'-p''$. If $p'-p''$ continues to increase, it may perhaps occur that W reaches the value zero; but until this occurs the phase is certainly stable with respect to the kind of change considered. Another kind of change is conceivable, which initially is small in degree but may be great in its extent in space. Stability in this respect or *stability in respect to continuous changes of phase* has already been discussed (see page 162), and its limits determined. These limits depend entirely upon the fundamental equation of the homogeneous mass of which the stability is in question. But with respect to the kind of changes here considered, which are initially small in extent but great in degree, it does not appear how we can fix the limits of stability with the same precision. But it is safe to say that if there is such a limit it must be at or beyond the limit at which σ vanishes. This latter limit is determined entirely by the fundamental equation of the surface of discontinuity between the phase of which the stability is in question and that of which the possible formation is in question. We have already seen that when σ vanishes, the radius of the dividing surface and the work W vanish with it. If the fault in the homogeneity of the mass vanishes at the same time, (it evidently

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cannot vanish sooner,) the phase becomes unstable at this limit. But if the fault in the homogeneity of the physical mass does not vanish with r, σ and W,—and no sufficient reason appears why this should not be considered as the general case,—although the amount of work necessary to upset the equilibrium of the phase is infinitesimal, this is not enough to make the phase unstable. It appears therefore that W is a somewhat one-sided measure of stability.

It must be remembered in this connection that the fundamental equation of a surface of discontinuity can hardly be regarded as capable of experimental determination, except for plane surfaces, (see pp. 394, 395,) although the relation for spherical surfaces is in the nature of things entirely determined, at least so far as the phases are separately capable of existence. Yet the foregoing discussion yields the following practical results. It has been shown that the real stability of a phase extends in general beyond that limit (discussed on pages 160, 161), which may be called the limit of practical stability, at which the phase can exist in contact with another at a plane surface, and a formula has been deduced to express the degree of stability in such cases as measured by the amount of work necessary to upset the equilibrium of the phase when supposed to extend indefinitely in space. It has also been shown to be entirely consistent with the principles established that this stability should have limits, and the manner in which the general equations would accommodate themselves to this case has been pointed out.

By equation (553), which may be written

$$W = \sigma s - (p' - p'') v', \tag{559}$$

we see that the work W consists of two parts, of which one is always positive, and is expressed by the product of the superficial tension and the area of the surface of tension, and the other is always negative, and is numerically equal to the product of the difference of pressure by the volume of the interior mass. We may regard the first part as expressing the work spent in forming the surface of tension, and the second part the work gained in forming the interior mass.*

* To make the physical significance of the above more clear, we may suppose the two processes to be performed separately in the following manner. We may suppose a large mass of the same phase as that which has the volume v' to exist initially in the interior of the other. Of course, it must be surrounded by a resisting envelop, on account of the difference of the pressures. We may, however, suppose this envelop permeable to all the component substances, although not of such properties that a mass can form on the exterior like that within. We may allow the

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Moreover, the second of these quantities, if we neglect its sign, is always equal to two-thirds of the first, as appears from equation (550) and the geometrical relation $v' = \frac{1}{3} rs$. We may therefore write

$$W = \tfrac{1}{3}\,\sigma\, s = \tfrac{1}{2}\,(p' - p'')\, v'. \tag{560}$$

On the Possible Formation at the Surface where two different Homogeneous Fluids meet of a Fluid of different Phase from either.

Let A, B, and C be three different fluid phases of matter, which satisfy all the conditions necessary for equilibrium when they meet at plane surfaces. The components of A and B may be the same or different, but C must have no components except such as belong to A or B. Let us suppose masses of the phases A and B to be separated by a very thin sheet of the phase C. This sheet will not necessarily be plane, but the sum of its principal curvatures must be zero. We may treat such a system as consisting simply of masses of the phases A and B with a certain surface of discontinuity, for in our previous discussion there has been nothing to limit the thickness or the nature of the film separating homogeneous masses, except that its thickness has generally been supposed to be small in comparison with its radii of curvature. The value of the superficial tension for such a film will be $\sigma_{AC} + \sigma_{BC}$, if we denote by these symbols the tensions of the surfaces of contact of the phases A and C, and B and C, respectively. This not only appears from evident mechanical considerations, but may also be easily verified by equations (502) and (93), the first of which may be regarded as defining the quantity σ. This value will not be affected by diminishing the thickness of the film, until the

envelop to yield to the internal pressure until its contents are increased by v' without materially affecting its superficial area. If this be done sufficiently slowly, the phase of the mass within will remain constant. (See page 139.) A homogeneous mass of the volume v' and of the desired phase has thus been produced, and the work gained is evidently $(p'-p'')v'$.

Let us suppose that a small aperture is now opened and closed in the envelop so as to let out exactly the volume v' of the mass within, the envelop being pressed inwards in another place so as to diminish its contents by this amount. During the extrusion of the drop and until the orifice is entirely closed, the surface of the drop must adhere to the edge of the orifice, but not elsewhere to the outside surface of the envelop. The work done in forming the surface of the drop will evidently be σs or $\frac{3}{2}(p'-p'')v'$. Of this work, the amount $(p'-p'')v'$ will be expended in pressing the envelop inward, and the rest in opening and closing the orifice. Both the opening and the closing will be resisted by the capillary tension. If the orifice is circular, it must have, when widest open, the radius determined by equation (550).

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limit is reached at which the interior of the film ceases to have the properties of matter in mass. Now if $\sigma_{AC}+\sigma_{BC}$ is greater than σ_{AB}, the tension of the ordinary surface between A and B, such a film will be at least practically unstable. (See page 403.) We cannot suppose that $\sigma_{AB}>\sigma_{AC}+\sigma_{BC}$, for this would make the ordinary surface between A and B unstable and difficult to realize. If $\sigma_{AB}=\sigma_{AC}+\sigma_{BC}$, we may assume, in general, that this relation is not accidental, and that the ordinary surface of contact for A and B is of the kind which we have described.

Let us now suppose the phases A and B to vary, so as still to satisfy the conditions of equilibrium at plane contact, but so that the pressure of the phase C determined by the temperature and potentials of A and B shall become less than the pressure of A and B. A system consisting of the phases A and B will be entirely stable with respect to the formation of any phase like C. (The case is not quite identical with that considered on page 161, since the system in question contains two different phases, but the principles involved are entirely the same.)

With respect to variations of the phases A and B in the opposite direction we must consider two cases separately. It will be convenient to denote the pressures of the three phases by p_A, p_B, p_C, and to regard these quantities as functions of the temperature and potentials.

If $\sigma_{AB}=\sigma_{AC}+\sigma_{BC}$ for values of the temperature and potentials which make $p_A=p_B=p_C$, it will not be possible to alter the temperature and potentials at the surface of contact of the phases A and B so that $p_A=p_B$, and $p_C>p_A$, for the relation of the temperature and potentials necessary for the equality of the three pressures will be preserved by the increase of the mass of the phase C. Such variations of the phases A and B might be brought about in separate masses, but if these were brought into contact, there would be an immediate formation of a mass of the phase C, with reduction of the phases of the adjacent masses to such as satisfy the conditions of equilibrium with that phase.

But if $\sigma_{AB}<\sigma_{AC}+\sigma_{BC}$, we can vary the temperature and potentials so that $p_A=p_B$, and $p_C>p_A$, and it will not be possible for a sheet of the phase of C to form *immediately*, i. e., while the pressure of C is sensibly equal to that of A and B; for mechanical work equal to $\sigma_{AC}+\sigma_{BC}-\sigma_{AB}$ per unit of surface might be obtained by bringing the system into its original condition, and therefore produced without any external expenditure, unless it be that of heat at the temperature of the system, which is evidently incapable of producing the work.

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The stability of the system in respect to such a change must therefore extend beyond the point where the pressure of C commences to be less than that of A and B. We arrive at the same result if we use the expression (520) as a test of stability. Since this expression has a finite positive value when the pressures of the phases are all equal, the ordinary surface of discontinuity must be stable, and it must require a finite change in the circumstances of the case to make it become unstable.*

In the preceding paragraph it is shown that the surface of contact of phases A and B is stable under certain circumstances, with respect to the formation of a thin sheet of the phase C. To complete the demonstration of the stability of the surface with respect to the formation of the phase C, it is necessary to show that this phase cannot be formed at the surface in lentiform masses. This is the more necessary, since it is in this manner, if at all, that the phase is likely to be formed, for an incipient sheet of phase C would evidently be unstable when $\sigma_{AB} < \sigma_{AC} + \sigma_{BC}$, and would immediately break up into lentiform masses.

It will be convenient to consider first a lentiform mass of phase C in equilibrium between masses of phases A and B which meet in a plane surface. Let figure 10 represent a section of such a system through the centers of the spherical surfaces, the mass of phase A lying on the left of D E H′ F G, and that of phase B on the right of D E H″ F G. Let the line joining the centers cut the spherical surfaces in H′ and H″, and the plane of the surface of contact of A and B in I. Let the radii of E H′ F and E H″ F be denoted by r', r'', and the segments I H′, I H″ by x', x''. Also let I E, the radius of the circle in which the spherical surfaces intersect, be denoted by R. By a suitable application of the general condition of equilibrium we may easily obtain the equation

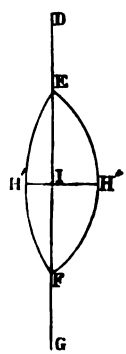

FIG. 10.

$$\sigma_{AC} \frac{r' - x'}{r'} + \sigma_{BC} \frac{r'' - x''}{r''} = \sigma_{AB}, \tag{561}$$

* It is true that such a case as we are now considering is formally excluded in the discussion referred to, which relates to a plane surface, and in which the system is supposed thoroughly stable with respect to the possible formation of any different homogeneous masses. Yet the reader will easily convince himself that the criterion (520) is perfectly valid in this case with respect to the possible formation of a thin sheet of the phase C, which, as we have seen, may be treated simply as a different kind of surface of discontinuity.

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which signifies that the components parallel to EF of the tension σ_{AC} and σ_{BC} are together equal to σ_{AB}. If we denote by W the amount of work which must be expended in order to form such a lentiform mass as we are considering between masses of indefinite extent having the phases A and B, we may write

$$W = M - N, \tag{562}$$

where M denotes the work expended in replacing the surface between A and B by the surfaces between A and C and B and C, and N denotes the work gained in replacing the masses of phases A and B by the mass of phase C. Then

$$M = \sigma_{AC}\, s_{AC} + \sigma_{BC}\, s_{BC} - \sigma_{AB}\, s_{AB}, \tag{563}$$

where s_{AC}, s_{BC}, s_{AB} denote the areas of the three surfaces concerned; and

$$N = V'(p_C - p_A) + V''(p_C - p_B), \tag{564}$$

where V' and V'' denote the volumes of the masses of the phases A and B which are replaced. Now by (500),

$$p_C - p_A = \frac{2\sigma_{AC}}{r'}, \quad \text{and} \quad p_C - p_B = \frac{2\sigma_{BC}}{r''}. \tag{565}$$

We have also the geometrical relations

$$\left.\begin{aligned} V' &= \tfrac{2}{3}\pi r'^2 x' - \tfrac{1}{3}\pi R^2 (r' - x'), \\ V'' &= \tfrac{2}{3}\pi r''^2 x'' - \tfrac{1}{3}\pi R^2 (r'' - x''). \end{aligned}\right\} \tag{566}$$

By substitution we obtain

$$\begin{aligned} N = {} & \tfrac{4}{3}\pi \sigma_{AC} r' x' - \tfrac{2}{3}\pi R^2 \sigma_{AC} \frac{r' - x'}{r'} \\ & + \tfrac{4}{3}\pi \sigma_{BC} r'' x'' - \tfrac{2}{3}\pi R^2 \sigma_{BC} \frac{r'' - x''}{r''}, \end{aligned} \tag{567}$$

and by (561),

$$N = \tfrac{4}{3}\pi \sigma_{AC} r' x' + \tfrac{4}{3}\pi \sigma_{BC} r'' x'' - \tfrac{2}{3}\pi R^2 \sigma_{AB}. \tag{568}$$

Since

$$2\pi r' x' = s_{AC}, \quad 2\pi r'' x'' = s_{BC}, \quad \pi R^2 = s_{AB},$$

we may write

$$N = \tfrac{2}{3}(\sigma_{AC}\, s_{AC} + \sigma_{BC}\, s_{BC} - \sigma_{AB}\, s_{AB}). \tag{569}$$

(The reader will observe that the ratio of M and N is the same as that of the corresponding quantities in the case of the spherical mass treated on pages 416–422.) We have therefore

$$W = \tfrac{1}{3}(\sigma_{AC}\, s_{AC} + \sigma_{BC}\, s_{BC} - \sigma_{AB}\, s_{AB}). \tag{570}$$

This value is positive so long as

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$$\sigma_{AC} + \sigma_{BC} > \sigma_{AB},$$

since $$s_{AC} > s_{AB}, \quad \text{and} \quad s_{BC} > s_{AB}.$$

But at the limit, when

$$\sigma_{AC} + \sigma_{BC} = \sigma_{AB},$$

we see by (561) that

$$s_{AC} = s_{AB}, \quad \text{and} \quad s_{BC} = s_{AB},$$

and therefore $$W = 0.$$

It should however be observed that in the immediate vicinity of the circle in which the three surfaces of discontinuity intersect, the physical state of each of these surfaces must be affected by the vicinity of the others. We cannot, therefore, rely upon the formula (570) except when the dimensions of the lentiform mass are of sensible magnitude.

We may conclude that after we pass the limit at which p_C becomes greater than p_A and p_B (supposed equal) lentiform masses of phase C will not be formed until either $\sigma_{AB}=\sigma_{AC}+\sigma_{BC}$, or p_C-p_A becomes so great that the lentiform mass which would be in equilibrium is one of insensible magnitude. [The diminution of the radii with increasing values of p_C-p_A is indicated by equation (565).] Hence, no mass of phase C will be formed until one of these limits is reached. Although the demonstration relates to a *plane* surface between A and B, the result must be applicable whenever the radii of curvature have a sensible magnitude, since the effect of such curvature may be disregarded when the lentiform mass is of sufficiently small.

The equilibrium of the lentiform mass of phase C is easily proved to be unstable, so that the quantity W affords a kind of measure of the stability of plane surfaces of contact of the phases A and B.*

---

* If we represent phases by the position of points in such a manner that coexistent phases (in the sense in which the term is used on page 152) are represented by the same point, and allow ourselves, for brevity, to speak of the phases as having the positions of the points by which they are represented, we may say that three coexistent phases are situated where three series of pairs of coexistent phases meet or intersect. If the three phases are all fluid, or when the effects of solidity may be disregarded, two cases are to be distinguished. Either the three series of coexistent phases all intersect,—this is when each of the three surface-tensions is less than the sum of the two others,—or one of the series terminates where the two others intersect,—this is where one surface tension is equal to the sum of the others. The series of coexistent phases will be represented by lines or surfaces, according as the phases have one or two independently variable components. Similar relations exist when the number of components is greater, except that they are not capable of geometrical representation without some limitation, as that of constant temperature or pressure or certain constant potentials.

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Essentially the same principles apply to the more general problem in which the phases A and B have moderately different pressures, so that their surfaces of contact must be curved, but the radii of curvature have a sensible magnitude.

In order that a thin film of the phase C may be in equilibrium between masses of the phases A and B, the following equations must be satisfied—

$$\sigma_{AC}(c_1 + c_2) = p_A - p_C,$$

$$\sigma_{BC}(c_1 + c_2) = p_C - p_B,$$

where c_1 and c_2 denote the principal curvatures of the film, the centers of positive curvature lying in the mass having the phase A. Eliminating $c_1 + c_2$, we have

$$\sigma_{BC}(p_A - p_C) = \sigma_{AC}(p_C - p_B),$$

or

$$p_C = \frac{\sigma_{BC} p_A + \sigma_{AC} p_B}{\sigma_{BC} + \sigma_{AC}}. \tag{571}$$

It is evident that if p_C has a value greater than that determined by this equation, such a film will develop into a larger mass; if p_C has a less value, such a film will tend to diminish. Hence, when

$$p_C < \frac{\sigma_{BC} p_A + \sigma_{AC} p_B}{\sigma_{BC} + \sigma_{AC}}, \tag{572}$$

the phases A and B have a stable surface of contact.

Again, if more than one kind of surface of discontinuity is possible between A and B, for any given values of the temperature and potentials, it will be impossible for that having the greater tension to displace the other, at the temperature and with the potentials considered. Hence, when p_C has the value determined by equation (571), and consequently $\sigma_{AC} + \sigma_{BC}$ is one value of the tension for the surface between A and B, it is impossible that the ordinary tension of the surface σ_{AB} should be greater than this. If $\sigma_{AB} = \sigma_{AC} + \sigma_{BC}$, when equation (571) is satisfied, we may presume that a thin film of the phase C actually exists at the surface between A and B, and that a variation of the phases such as would make p_C greater than the second number of (571) cannot be brought about at that surface, as it would be prevented by the formation of a larger mass of the phase C. But if $\sigma_{AB} < \sigma_{AC} + \sigma_{BC}$ when equation (571) is satisfied, this equation does not mark the limit of the stability of the surface between A and B, for the temperature or potentials must receive a finite

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change before the film of phase C, or (as we shall see in the following paragraph) a lentiform mass of that phase, can be formed.

The work which must be expended in order to form on the surface between indefinitely large masses of phases A and B a lentiform mass of phase C in equilibrium, may evidently be represented by the formula

$$W = \sigma_{AC} S_{AC} + \sigma_{BC} S_{BC} - \sigma_{AB} S_{AB} - p_C V_C + p_A V_A + p_B V_B, \tag{573}$$

where S_{AC}, S_{BC} denote the areas of the surfaces formed between A and C, and B and C, S_{AB} the diminution of the area of the surface between A and B, V_C the volume formed of the phase C, and V_A, V_B the diminution of the volumes of the phases A and B. Let us now suppose σ_{AC}, σ_{BC}, σ_{AB}, p_A, p_B to remain constant and the external boundary of the surface between A and B to remain fixed, while p_C increases and the surfaces of tension receive such alterations as are necessary for equilibrium. It is not necessary that this should be physically possible in the actual system; we may suppose the changes to take place, for the sake of argument, although involving changes in the fundamental equations of the masses and surfaces considered. Then, regarding W simply as an abbreviation for the second member of the preceding equation, we have

$$dW = \sigma_{AC}\, dS_{AC} + \sigma_{BC}\, dS_{BC} - \sigma_{AB}\, dS_{AB} - p_C\, dV_C + p_A\, dV_A + p_B\, dV_B - V_C\, dp_C. \tag{574}$$

But the conditions of equilibrium require that

$$\sigma_{AC}\, dS_{AC} + \sigma_{BC}\, dS_{BC} - \sigma_{AB}\, dS_{AB} - p_C\, dV_C + p_A\, dV_A + p_B\, dV_B = 0. \tag{575}$$

Hence,

$$dW = -V_C\, dp_C. \tag{576}$$

Now it is evident that V_C will diminish as p_C increases. Let us integrate the last equation supposing p_C to increase from its original value until V_C vanishes. This will give

$$W'' - W' = \text{a negative quantity}, \tag{577}$$

where W' and W'' denote the initial and final values of W. But $W''=0$. Hence W' is positive. But this is the value of W in the original system containing the lentiform mass, and expresses the work necessary to form the mass between the phases A and B. It is therefore impossible that such a mass should form on a surface be-

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tween these phases. We must however observe the same limitation as in the less general case already discussed,—that $p_C - p_A$, $p_C - p_B$ must not be so great that the dimensions of the lentiform mass are of insensible magnitude. It may also be observed that the value of these differences may be so small that there will not be room on the surface between the masses of phases A and B for a mass of phase C sufficiently large for equilibrium. In this case we may consider a mass of phase C which is in equilibrium upon the surface between A and B in virtue of a *constraint* applied to the line in which the three surfaces of discontinuity intersect, which will not allow this line to become longer, although not preventing it from becoming shorter. We may prove that the value of W is positive by such an integration as we have used before.

Substitution of Pressures for Potentials in Fundamental Equations for Surfaces.

The fundamental equation of a surface which gives the value of the tension in terms of the temperature and potentials seems best adapted to the purposes of theoretical discussion, especially when the number of components is large or undetermined. But the experimental determination of the fundamental equations, or the application of any result indicated by theory to actual cases, will be facilitated by the use of other quantities in place of the potentials, which shall be capable of more direct measurement, and of which the numerical expression (when the necessary measurements have been made) shall depend upon less complex considerations. The numerical value of a potential depends not only upon the system of units employed, but also upon the arbitrary constants involved in the definition of the energy and entropy of the substance to which the potential relates, or, it may be, of the elementary substances of which that substance is formed. (See page 152.) This fact and the want of means of direct measurement may give a certain vagueness to the idea of the potentials, and render the equations which involve them less fitted to give a clear idea of physical relations.

Now the fundamental equation of each of the homogeneous masses which are separated by any surface of discontinuity affords a relation between the pressure in that mass and the temperature and potentials. We are therefore able to eliminate one or two potentials from the fundamental equation of a surface by introducing the pressures in the adjacent masses. Again, when one of these masses is a gas-

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mixture which satisfies Dalton's law as given on page 215, the potential for each simple gas may be expressed in terms of the temperature and the partial pressure belonging to that gas. By the introduction of these partial pressures we may eliminate as many potentials from the fundamental equation of the surface as there are simple gases in the gas-mixture.

An equation obtained by such substitutions may be regarded as a fundamental equation for the surface of discontinuity to which it relates, for when the fundamental equations of the adjacent masses are known, the equation in question is evidently equivalent to an equation between the tension, temperature, and potentials, and we must regard the knowledge of the properties of the adjacent masses as an indispensable preliminary, or an essential part, of a complete knowledge of any surface of discontinuity. It is evident, however, that from these fundamental equations involving pressures instead of potentials we cannot obtain by differentiation (without the use of the fundamental equations of the homogeneous masses) precisely the same relations as by the differentiation of the equations between the tensions, temperatures, and potentials. It will be interesting to inquire, at least in the more important cases, what relations may be obtained by differentiation from the fundamental equations just described alone.

If there is but one component, the fundamental equations of the two homogeneous masses afford one relation more than is necessary for the elimination of the potential. It may be convenient to regard the tension as a function of the temperature and the difference of the pressures. Now we have by (508) and (98)

$$d\sigma = -\eta_s\, dt - \Gamma\, d\mu_1,$$

$$d(p'-p'') = (\eta_v'-\eta_v'')\, dt + (\gamma'-\gamma'')\, d\mu_1.$$

Hence we derive the equation

$$d\sigma = -\left(\eta_s - \frac{\Gamma}{\gamma'-\gamma''}(\eta_v'-\eta_v'')\right) dt - \frac{\Gamma}{\gamma'-\gamma''}\, d(p'-p''), \quad (578)$$

which indicates the differential coefficients of σ with respect to t and $p'-p''$. For surfaces which may be regarded as nearly plane, it is evident that $\frac{\Gamma}{\gamma'-\gamma''}$ represents the distance from the surface of tension to a dividing surface located so as to make the superficial density of the single component vanish, (being positive, when the

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latter surface is on the side specified by the double accents,) and that the coefficient of dt (without the negative sign) represents the superficial density of entropy as determined by the latter dividing surface, i. e., the quantity denoted by $\eta_{s(1)}$ on page 397.

When there are two components, neither of which is confined to the surface of discontinuity, we may regard the tension as a function of the temperature and the pressures in the two homogeneous masses. The values of the differential coefficients of the tension with respect to these variables may be represented in a simple form if we choose such substances for the components that in the particular state considered each mass shall consist of a single component. This will always be possible when the composition of the two masses is not identical, and will evidently not affect the values of the differential coefficients. We then have

$$d\sigma = -\eta_s\, dt - \Gamma_{,}\, d\mu_{,} - \Gamma_{,,}\, d\mu_{,,},$$
$$dp' = \eta_v'\, dt + \gamma'\, d\mu_{,},$$
$$dp'' = \eta_v''\, dt + \gamma''\, d\mu_{,,},$$

where the marks $_{,}$ and $_{,,}$ are used instead of the usual $_1$ and $_2$ to indicate the identity of the component specified with the substance of the homogeneous masses specified by $'$ and $''$. Eliminating $d\mu_{,}$ and $d\mu_{,,}$ we obtain

$$d\sigma = -\left(\eta_s - \frac{\Gamma_{,}}{\gamma'}\eta_v' - \frac{\Gamma_{,,}}{\gamma''}\eta_v''\right) dt - \frac{\Gamma_{,}}{\gamma'}\, dp' - \frac{\Gamma_{,,}}{\gamma''}\, dp''. \qquad (579)$$

We may generally neglect the difference of p' and p'', and write

$$d\sigma = -\left(\eta_s - \frac{\Gamma_{,}}{\gamma'}\eta_v' - \frac{\Gamma_{,,}}{\gamma''}\eta_v''\right) dt - \left(\frac{\Gamma_{,}}{\gamma'} + \frac{\Gamma_{,,}}{\gamma''}\right) dp. \qquad (580)$$

The equation thus modified is strictly to be regarded as the equation for a plane surface. It is evident that $\frac{\Gamma_{,}}{\gamma'}$ and $\frac{\Gamma_{,}}{\gamma'}$ represent the distances from the surface of tension of the two surfaces of which one would make $\Gamma_{,}$ vanish, and the other $\Gamma_{,,}$, that $\frac{\Gamma_{,}}{\gamma'} + \frac{\Gamma_{,,}}{\gamma''}$ represents the distance between these two surfaces, or the *diminution of volume* due to a unit of the surface of discontinuity, and that the coefficient of dt (without the negative sign) represents the excess of entropy in a system consisting of a unit of the surface of discontinuity with a part of each of the adjacent masses above that which the same matter would have if it existed in two homogeneous masses of the same phases but without any surface of discontinuity.

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(A mass thus existing without any surface of discontinuity must of course be entirely surrounded by matter of the same phase.)*

The form in which the values of $\left(\frac{d\sigma}{dt}\right)_p$ and $\left(\frac{d\sigma}{dp}\right)_t$ are given in equation (580) is adapted to give a clear idea of the relations of these quantities to the particular state of the system for which they are to be determined, but not to show how they vary with the state of the system. For this purpose it will be convenient to have the values of these differential coefficients expressed with reference to ordinary components. Let these be specified as usual by $_1$ and $_2$. If we eliminate $d\mu_1$ and $d\mu_2$ from the equations

$$-d\sigma = \eta_s\, dt + \Gamma_1\, d\mu_1 + \Gamma_2\, d\mu_2,$$
$$dp = \eta_v'\, dt + \gamma_1'\, d\mu_1 + \gamma_2'\, d\mu_2,$$
$$dp = \eta_v''\, dt + \gamma_1''\, d\mu_1 + \gamma_2'', d\mu_2,$$

---

* If we set

$$V = -\frac{\Gamma_1}{\gamma'} - \frac{\Gamma_2}{\gamma''}, \tag{a}$$

$$\mathrm{H}_s = \eta_s - \frac{\Gamma_1}{\gamma'}\eta_v' - \frac{\Gamma_2}{\gamma''}\eta_v'', \tag{b}$$

and in like manner

$$\mathrm{E}_s = \varepsilon_s - \frac{\Gamma_1}{\gamma'}\varepsilon_v' - \frac{\Gamma_2}{\gamma''}\varepsilon_v'', \tag{c}$$

we may easily obtain, by means of equations (93) and (507),

$$\mathrm{E}_s = t\,\mathrm{H}_s + \sigma - pV. \tag{d}$$

Now equation (580) may be written

$$d\sigma = -\mathrm{H}_s\, dt + V\, dp. \tag{e}$$

Differentiating (*d*), and comparing the result with (*e*), we obtain

$$d\mathrm{E}_s = t\, d\mathrm{H}_s - p\, dV. \tag{f}$$

The quantities E_s and H_s might be called the superficial densities of energy and entropy quite as properly as those which we denote by ε_s and η_s. In fact, when the composition of both of the homogeneous masses is invariable, the quantities E_s and H are much more simple in their definition than ε_s and η_s, and would probably be more naturally suggested by the terms *superficial density of energy* and *of entropy*. It would also be natural in this case to regard the quantities of the homogeneous masses as determined by the total quantities of matter, and not by the surface of tension or any other dividing surface. But such a nomenclature and method could not readily be extended so as to treat cases of more than two components with entire generality.

In the treatment of surfaces of discontinuity in this paper, the definitions and nomenclature which have been adopted will be strictly adhered to. The object of this note is to suggest to the reader how a different method might be used in some cases with advantage, and to show the precise relations between the quantities which are used in this paper and others which might be confounded with them, and which may be made more prominent when the subject is treated differently.

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we obtain

$$d\sigma = \frac{B}{A}\,dt + \frac{C}{A}\,dp, \tag{581}$$

where

$$A = \gamma_1''\,\gamma_2' - \gamma_1'\,\gamma_2'', \tag{582}$$

$$B = \begin{vmatrix} \eta_S & \Gamma_1 & \Gamma_2 \\ \eta_V' & \gamma_1' & \gamma_2' \\ \eta_V'' & \gamma_1'' & \gamma_2'' \end{vmatrix}, \tag{583}$$

$$C = \Gamma_1\,(\gamma_2'' - \gamma_2') + \Gamma_2\,(\gamma_1' - \gamma_1''). \tag{584}$$

It will be observed that A vanishes when the composition of the two homogeneous masses is identical, while B and C do not, in general, and that the value of A is negative or positive according as the mass specified by $'$ contains the component specified by $_1$ in a greater or less proportion than the other mass. Hence, the values both of $\left(\frac{d\sigma}{dt}\right)_p$ and of $\left(\frac{d\sigma}{dp}\right)_t$ become infinite when the difference in the composition of the masses vanishes, and change sign when the greater proportion of a component passes from one mass to the other. This might be inferred from the statements on page 155 respecting coexistent phases which are identical in composition, from which it appears that when two coexistent phases have nearly the same composition, a small variation of the temperature or pressure of the coexistent phases will cause a relatively very great variation in the composition of the phases. The same relations are indicated by the graphical method represented in figure 6 on page 184.

With regard to gas-mixtures which conform to Dalton's law, we shall only consider the fundamental equation for plane surfaces, and shall suppose that there is not more than one component in the liquid which does not appear in the gas-mixture. We have already seen that in limiting the fundamental equation to plane surfaces we can get rid of one potential by choosing such a dividing surface that the superficial density of one of the components vanishes. Let this be done with respect to the component peculiar to the liquid, if such there is; if there is no such component, let it be done with respect to one of the gaseous components. Let the remaining potentials be eliminated by means of the fundamental equations of the simple gases. We may thus obtain an equation between the superficial tension, the temperature, and the several pressures of the simple gases in the gas-mixture or all but one of these pressures. Now, if we eliminate $d\mu_2$, $d\mu_3$, etc. from the equations

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$$d\sigma = -\eta_{s(1)}\,dt - \Gamma_{2(1)}\,d\mu_2 + \Gamma_{3(1)}\,d\mu_3 + \text{etc.},$$
$$dp_2 = \eta_{v2}\,dt + \gamma_2\,d\mu_2,$$
$$dp_3 = \eta_{v3}\,dt + \gamma_3\,d\mu_3,$$
$$\text{etc.},$$

where the suffix $_1$ relates to the component of which the surface-density has been made to vanish, and γ_2, γ_3, etc. denote the densities of the gases specified in the gas mixture, and p_2, p_3, etc., η_{v2}, η_{v3}, etc. the pressures and the densities of entropy due to these several gases, we obtain

$$d\sigma = -\left(\eta_{s(1)} - \frac{\Gamma_{2(1)}}{\gamma_2}\eta_{v2} - \frac{\Gamma_{3(1)}}{\gamma_3}\eta_{v3} - \text{etc.}\right)dt$$
$$- \frac{\Gamma_{2(1)}}{\gamma_2}\,dp_2 - \frac{\Gamma_{3(1)}}{\gamma_3}\,dp_3 - \text{etc.} \qquad (585)$$

This equation affords values of the differential coefficients of σ with respect to t, p_2, p_3, etc., which may be set equal to those obtained by differentiating the equation between these variables.

Thermal and Mechanical Relations pertaining to the Extension of a Surface of Discontinuity.

The fundamental equation of a surface of discontinuity with one or two component substances, beside its statical applications, is of use to determine the heat absorbed when the surface is extended under certain conditions.

Let us first consider the case in which there is only a single component substance. We may treat the surface as plane, and place the dividing surface so that the surface density of the single component vanishes. (See page 397.) If we suppose the area of the surface to be increased by unity without change of temperature or of the quantities of liquid and vapor, the entropy of the whole will be increased by $\eta_{s(1)}$. Therefore, if we denote by Q the quantity of heat which must be added to satisfy the conditions, we shall have

$$Q = t\,\eta_{s(1)}, \qquad (586)$$

and by (514),

$$Q = -t\frac{d\sigma}{dt} = -\frac{d\sigma}{d\log t}. \qquad (587)$$

It will be observed that the condition of constant quantities of liquid and vapor as determined by the dividing surface which we have adopted is equivalent to the condition that the total volume shall remain constant.

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Again, if the surface is extended without application of heat, while the pressure in the liquid and vapor remains constant, the temperature will evidently be maintained constant by condensation of the vapor. If we denote by M the mass of vapor condensed per unit of surface formed, and by η_M' and η_M'' the entropies of the liquid and vapor per unit of mass, the condition of no addition of heat will require that

$$M(\eta_M'' - \eta_M') = \eta_{S(1)}. \tag{588}$$

The increase of the volume of liquid will be

$$\frac{\eta_{S(1)}}{\gamma'(\eta_M'' - \eta_M')}, \tag{589}$$

and the diminution of the volume of vapor

$$\frac{\eta_{S(1)}}{\gamma''(\eta_M'' - \eta_M')}. \tag{590}$$

Hence, for the work done (per unit of surface formed) by the external bodies which maintain the pressure, we shall have

$$W = \frac{p\,\eta_{S(1)}}{\eta_M'' - \eta_M'}\left(\frac{1}{\gamma''} - \frac{1}{\gamma'}\right). \tag{591}$$

and, by (514) and (131),

$$W = -p\frac{d\sigma}{dt}\frac{dt}{dp} = -p\frac{d\sigma}{dp} = -\frac{d\sigma}{d\log p}. \tag{592}$$

The work expended directly in extending the film will of course be equal to σ.

Let us now consider the case in which there are two component substances, neither of which is confined to the surface. Since we cannot make the superficial density of both these substances vanish by any dividing surface, it will be best to regard the surface of tension as the dividing surface. We may, however, simplify the formula by choosing such substances for components that each homogeneous mass shall consist of a single component. Quantities relating to these components will be distinguished as on page 431. If the surface is extended until its area is increased by unity, while heat is added at the surface so as to keep the temperature constant, and the pressure of the homogeneous masses is also kept constant, the phase of these masses will necessarily remain unchanged, but the quantity of one will be diminished by $\Gamma_{,}$, and that of the other by $\Gamma_{,,}$. Their entropies will therefore be diminished by $\frac{\Gamma_{,}}{\gamma'}\eta_V'$ and $\frac{\Gamma_{,,}}{\gamma''}\eta_V''$, respect-

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ively. Hence, since the surface receives the increment of entropy η_s, the total quantity of entropy will be increased by

$$\eta_s - \frac{\Gamma'}{\gamma'}\eta_v' - \frac{\Gamma''}{\gamma''}\eta_v'',$$

which by equation (580) is equal to

$$-\left(\frac{d\sigma}{dt}\right)_p.$$

Therefore, for the quantity of heat Q imparted to the surface, we shall have

$$Q = -t\left(\frac{d\sigma}{dt}\right)_p = -\left(\frac{d\sigma}{d\log t}\right)_p. \tag{593}$$

We must notice the difference between this formula and (587). In (593) the quantity of heat Q is determined by the condition that the temperature and pressures shall remain constant. In (587) these conditions are equivalent and insufficient to determine the quantity of heat. The additional condition by which Q is determined may be most simply expressed by saying that the total volume must remain constant. Again, the differential coefficient in (593) is defined by considering p as constant; in the differential coefficient in (587) p cannot be considered as constant, and no condition is necessary to give the expression a definite value. Yet, notwithstanding the difference of the two cases, it is quite possible to give a single demonstration which shall be applicable to both. This may be done by considering a cycle of operations after the method employed by Sir William Thomson, who first pointed out these relations.*

The diminution of volume (per unit of surface formed) will be

$$V = \frac{\Gamma'}{\gamma'} + \frac{\Gamma''}{\gamma''} = -\left(\frac{d\sigma}{dp}\right)_t; \tag{594}$$

and the work done (per unit of surface formed) by the external bodies which maintain the pressure constant will be

$$W = -p\left(\frac{d\sigma}{dp}\right)_t = -\left(\frac{d\sigma}{d\log p}\right)_t. \tag{595}$$

Compare equation (592).

The values of Q and W may also be expressed in terms of quantities relating to the ordinary components. By substitution in (593) and (595) of the values of the differential coefficients which are given by (581), we obtain

* See *Proc. Roy. Soc.*, vol. ix, p. 255, (June, 1858); or *Phil. Mag.*, Ser. 4, vol. xvii, p. 61.

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$$Q = -t\frac{B}{A}, \qquad W = -p\frac{C}{A}. \tag{596}$$

where A, B, and C represent the expressions indicated by (582)–(584). It will be observed that the values of Q and W are in general infinite for the surface of discontinuity between coexistent phases which differ infinitesimally in composition, and change sign with the quantity A. When the phases are absolutely identical in composition, it is not in general possible to counteract the effect of extension of the surface of discontinuity by any supply of heat. For the matter at the surface will not in general have the same composition as the homogeneous masses, and the matter required for the increased surface cannot be obtained from these masses without altering their phase. The infinite values of Q and W are explained by the fact that when the phases are nearly identical in composition, the extension of the surface of discontinuity is accompanied by the vaporization or condensation of a very large mass, according as the liquid or the vapor is the richer in that component which is necessary for the formation of the surface of discontinuity.

If, instead of considering the amount of heat necessary to keep the phases from altering while the surface of discontinuity is extended, we consider the variation of temperature caused by the extension of the surface while the pressures remain constant, it appears that this variation of temperature changes sign with $\gamma_1''\gamma_2' - \gamma_1'\gamma_2''$, but vanishes with this quantity, i. e., vanishes when the composition of the phases becomes the same. This may be inferred from the statements on page 155, or from a consideration of the figure on page 184. When the composition of the homogeneous masses is initially absolutely identical, the effect on the temperature of a finite extension or contraction of the surface of discontinuity will be the same,—either of the two will lower or raise the temperature according as the temperature is a maximum or minimum for constant pressure.

The effect of the extension of a surface of discontinuity which is most easily verified by experiment is the effect upon the tension before complete equilibrium has been reëstablished throughout the adjacent masses. A fresh surface between coexistent phases may be regarded in this connection as an extreme case of a recently extended surface. When sufficient time has elapsed after the extension of a surface originally in equilibrium between coexistent phases, the superficial tension will evidently have sensibly its original value, unless there are substances at the surface which are either not found

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at all in the adjacent masses, or are found only in quantities comparable to those in which they exist at the surface. But a surface newly formed or extended may have a very different tension.

This will not be the case, however, when there is only a single component substance, since all the processes necessary for equilibrium are confined to a film of insensible thickness, and will require no appreciable time for their completion.

When there are two components, neither of which is confined to the surface of discontinuity, the reëstablishment of equilibrium after the extension of the surface does not necessitate any processes reaching into the interior of the masses except the transmission of heat between the surface of discontinuity and the interior of the masses. It appears from equation (593) that if the tension of the surface diminishes with a rise of temperature, heat must be supplied to the surface to maintain the temperature uniform when the surface is extended, i. e., the effect of extending the surface is to cool it; but if the tension of any surface increases with the temperature, the effect of extending the surface will be to raise its temperature. In either case, it will be observed, the immediate effect of extending the surface is to increase its tension. A contraction of the surface will of course have the opposite effect. But the time necessary for the reestablishment of sensible thermal equilibrium after extension or contraction of the surface must in most cases be very short.

In regard to the formation or extension of a surface between two coexistent phases of more than two components, there are two extreme cases which it is desirable to notice. When the superficial density of each of the components is exceeding small compared with its density in either of the homogeneous masses, the matter (as well as the heat) necessary for the formation or extension of the normal surface can be taken from the immediate vicinity of the surface without sensibly changing the properties of the masses from which it is taken. But if any one of these superficial densities has a considerable value, while the density of the same component is very small in each of the homogeneous masses, both absolutely and relatively to the densities of the other components, the matter necessary for the formation or extension of the normal surface must come from a considerable distance. Especially if we consider that a small difference of density of such a component in one of the homogeneous masses will probably make a considerable difference in the value of the corresponding potential [see eq. (217)], and that a small difference in the value of the potential will make a considerable difference in the ten-

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sion [see eq. (508)], it will be evident that in this case a considerable time will be necessary after the formation of a fresh surface or the extension of an old one for the reëstablishment of the normal value of the superficial tension. In intermediate cases, the reëstablishment of the normal tension will take place with different degrees of rapidity.

But whatever the number of component substances, provided that it is greater than one, and whether the reëstablishment of equilibrium is slow or rapid, extension of the surface will generally produce increase and contraction decrease of the tension. It would evidently be inconsistent with stability that the opposite effects should be produced. In general, therefore, a fresh surface between coexistent phases has a greater tension than an old one.* By the use of fresh surfaces, in experiments in capillarity, we may sometimes avoid the effect of minute quantities of foreign substances, which may be present without our knowledge or desire, in the fluids which meet at the surface investigated.

When the establishment of equilibrium is rapid, the variation of the tension from its normal value will be manifested especially during the extension or contraction of the surface, the phenomenon resembling that of viscosity, except that the variations of tension arising from variations in the densities at and about the surface will be the same in all directions, while the variations of tension due to any property of the surface really analogous to viscosity would be greatest in the direction of the most rapid extension.

We may here notice the different action of traces in the homogeneous masses of those substances which increase the tension and of those which diminish it. When the volume-densities of a component are very small, its surface-density may have a considerable positive value, but can only have a very minute negative one.† For the value when negative cannot exceed (numerically) the product of the greater volume-density by the thickness of the non-homogeneous

* When, however, homogeneous masses which have not coexistent phases are brought into contact, the superficial tension may increase with the course of time. The superficial tension of a drop of alcohol and water placed in a large room will increase as the potential for alcohol is equalized throughout the room, and is diminished in the vicinity of the surface of discontinuity.

† It is here supposed that we have chosen for components such substances as are incapable of resolution into other components which are independently variable in the homogeneous masses. In a mixture of alcohol and water, for example, the components must be pure alcohol and pure water.

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film. Each of these quantities is exceedingly small. The surface-density when positive is of the same order of magnitude as the thickness of the non-homogeneous film, but is not necessarily small compared with other surface-densities because the volume-densities of the same substance in the adjacent masses are small. Now the potential of a substance which forms a very small part of a homogeneous mass certainly increases, and probably very rapidly, as the proportion of that component is increased. [See (171) and (217).] The pressure, temperature, and the other potentials, will not be sensibly affected. [See (98).] But the effect on the tension of this increase of the potential will be proportional to the surface-density, and will be to diminish the tension when the surface-density is positive. [See (508).] It is therefore quite possible that a very small trace of a substance in the homogeneous masses should greatly diminish the tension, but not possible that such a trace should greatly increase it.*

Impermeable Films.

We have so far supposed, in treating of surfaces of discontinuity, that they afford no obstacle to the passage of any of the component substances from either of the homogeneous masses to the other. The case, however, must be considered, in which there is a film of matter at the surface of discontinuity which is impermeable to some or all of

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* From the experiments of M. E. Duclaux, (*Annales de Chimie et de Physique*, Ser. 4, vol. xxi, p. 383,) it appears that one per cent. of alcohol in water will diminish the superficial tension to .933, the value for pure water being unity. The experiments do not extend to pure alcohol, but the difference of the tensions for mixtures of alcohol and water containing 10 and 20 per cent. water is comparatively small, the tensions being .322 and .336 respectively.

According to the same authority (page 427 of the volume cited), one 3200th part of Castile soap will reduce the superficial tension of water by one-fourth; one 800th part of soap by one-half. These determinations, as well as those relating to alcohol and water, are made by the method of drops, the weight of the drops of different liquids (from the same pipette) being regarded as proportional to their superficial tensions.

M. Athanase Dupré has determined the superficial tensions of solutions of soap by different methods. A statical method gives for one part of common soap in 5000 of water a superficial tension about one-half as great as for pure water, but if the tension be measured on a jet close to the orifice, the value (for the same solution) is sensibly identical with that of pure water. He explains these different values of the superficial tension of the same solution as well as the great effect on the superficial tension which a very small quantity of soap or other trifling impurity may produce, by the tendency of the soap or other substance to form a film on the surface of the liquid. (See *Annales de Chimie et de Physique*, Ser. 4, vol. vii, p. 409, and vol. ix, p. 379.)

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the components of the contiguous masses. Such may be the case, for example, when a film of oil is spread on a surface of water, even when the film is too thin to exhibit the properties of the oil in mass. In such cases, if there is communication between the contiguous masses through other parts of the system to which they belong, such that the components in question can pass freely from one mass to the other, the impossibility of a direct passage through the film may be regarded as an immaterial circumstance, so far as states of equilibrium are concerned, and our formulæ will require no change. But when there is no such indirect communication, the potential for any component for which the film is impermeable may have entirely different values on opposite sides of the film, and the case evidently requires a modification of our usual method.

A single consideration will suggest the proper treatment of such cases. If a certain component which is found on both sides of a film cannot pass from either side to the other, the fact that the part of the component which is on one side is the same kind of matter with the part on the other side may be disregarded. All the general relations must hold true, which would hold if they were really different substances. We may therefore write μ_1 for the potential of the component on one side of the film, and μ_2 for the potential of the same substance (to be treated as if it were a different substance) on the other side; m_1^s for the excess of the quantity of the substance on the first side of the film above the quantity which would be on that side of the dividing surface (whether this is determined by the surface of tension or otherwise) if the density of the substance were the same near the dividing surface as at a distance, and m_2^s for a similar quantity relating to the other side of the film and dividing surface. On the same principle, we may use Γ_1 and Γ_2 to denote the values of m_1^s and m_2^s per unit of surface, and m_1', m_2'', γ_1', γ_2'' to denote the quantities of the substance and its densities in the two homogeneous masses.

With such a notation, which may be extended to cases in which the film is impermeable to any number of components, the equations relating to the surface and the contiguous masses will evidently have the same form as if the substances specified by the different suffixes were all really different. The superficial tension will be a function of μ_1 and μ_2, with the temperature and the potentials for the other components, and $-\Gamma_1$, $-\Gamma_2$ will be equal to its differential coefficients with respect to μ_1 and μ_2. In a word, all the general relations which have been demonstrated may be applied to this case, if

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we remember always to treat the component as a different substance according as it is found on one side or the other of the impermeable film.

When there is free passage for the component specified by the suffixes $_1$ and $_2$ through other parts of the system, (or through any flaws in the film,) we shall have in case of equilibrium $\mu_1 = \mu_2$. If we wish to obtain the fundamental equation for the surface when satisfying this condition, without reference to other possible states of the surface, we may set a single symbol for μ_1 and μ_2 in the more general form of the fundamental equation. Cases may occur of an impermeability which is not absolute, but which renders the transmission of some of the components exceedingly slow. In such cases, it may be necessary to distinguish at least two different fundamental equations, one relating to a state of approximate equilibrium which may be quickly established, and another relating to the ultimate state of complete equilibrium. The former may be derived from the latter by such substitutions as that just indicated.

The Conditions of Internal Equilibrium for a System of Heterogeneous Fluid Masses without neglect of the Influence of the Surfaces of Discontinuity or of Gravity.

Let us now seek the complete value of the variation of the energy of a system of heterogeneous fluid masses, in which the influence of gravity and of the surfaces of discontinuity shall be included, and deduce from it the conditions of internal equilibrium for such a system. In accordance with the method which has been developed, the intrinsic energy, (*i. e.*, the part of the energy which is independent of of gravity,) the entropy, and the quantities of the several components must each be divided into two parts, one of which we regard as belonging to the surfaces which divide approximately homogeneous masses, and the other as belonging to these masses. The elements of intrinsic energy, entropy, etc., relating to an element of surface Ds will be denoted by $D\varepsilon^{\mathrm{s}}$, $D\eta^{\mathrm{s}}$, Dm_1^{s}, Dm_2^{s}, etc., and those relating to an element of volume Dv, by $D\varepsilon^{\mathrm{v}}$, $D\eta^{\mathrm{v}}$, Dm_1^{v}, Dm_2^{v}, etc. We shall also use Dm^{s} or ΓDs and Dm^{v} or γDv to denote the total quantities of matter relating to the elements Ds and Dv respectively. That is,

$$Dm^{\mathrm{s}} = \Gamma Ds = Dm_1^{\mathrm{s}} + Dm_2^{\mathrm{s}} + \text{etc.}, \tag{597}$$

$$Dm^{\mathrm{v}} = \gamma Dv = Dm_1^{\mathrm{v}} + Dm_2^{\mathrm{v}} + \text{etc.} \tag{598}$$

The part of the energy which is due to gravity must also be divided

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into two parts, one of which relates to the elements Dm^{s}, and the other to the elements Dm^{v}. The complete value of the variation of the energy of the system will be represented by the expression

$$\delta \int D\varepsilon^{\mathrm{v}} + \delta \int D\varepsilon^{\mathrm{s}} + \delta \int g\, z\, Dm^{\mathrm{v}} + \delta \int g\, z\, Dm^{\mathrm{s}}, \qquad (599)$$

in which g denotes the force of gravity, and z the height of the element above a fixed horizontal plane.

It will be convenient to limit ourselves at first to the consideration of reversible variations. This will exclude the formation of new masses or surfaces. We may therefore regard any infinitesimal variation in the state of the system as consisting of infinitesimal variations of the quantities relating to its several elements, and bring the sign of variation in the preceding formula after the sign of integration. If we then substitute for $\delta D\varepsilon^{\mathrm{v}}$, $\delta D\varepsilon^{\mathrm{s}}$, δDm^{v}, δDm^{s}, the values given by equations (13), (497), (597), (598), we shall have for the condition of equilibrium with respect to reversible variations of the internal state of the system

$$\begin{aligned} &\int t\, \delta D\eta^{\mathrm{v}} - \int p\, \delta Dv + \int \mu_1\, \delta Dm_1^{\mathrm{v}} + \int \mu_2\, \delta Dm_2^{\mathrm{v}} + \text{etc.} \\ &+ \int t\, \delta D\eta^{\mathrm{s}} + \int \sigma\, \delta Ds + \int \mu_1\, \delta Dm_1^{\mathrm{s}} + \int \mu_2\, \delta Dm_2^{\mathrm{s}} + \text{etc.} \\ &\qquad + \int g\, \delta z\, Dm^{\mathrm{v}} + \int g\, z\, \delta Dm_1^{\mathrm{v}} + \int g\, z\, \delta Dm_2^{\mathrm{v}} + \text{etc.} \\ &\qquad + \int g\, \delta z\, Dm^{\mathrm{s}} + \int g\, z\, \delta Dm_1^{\mathrm{s}} + \int g\, z\, \delta Dm_2^{\mathrm{s}} + \text{etc.} = 0, \qquad (600) \end{aligned}$$

Since equation (497) relates to surfaces of discontinuity which are initially in equilibrium, it might seem that this condition, although always necessary for equilibrium, may not always be sufficient. It is evident, however, from the form of the condition, that it includes the particular conditions of equilibrium relating to every possible deformation of the system, or reversible variation in the distribution of entropy or of the several components. It therefore includes all the relations between the different parts of the system which are necessary for equilibrium, so far as reversible variations are concerned. (The necessary relations between the various quantities relating to each element of the masses and surfaces are expressed by the fundamental equation of the mass or surface concerned, or may be immediately derived from it. See pp. 140–144 and 391–393.)

The variations in (600) are subject to the conditions which arise from the nature of the system and from the supposition that the changes in the system are not such as to affect external bodies. This supposition is necessary, unless we are to consider the variations in the state of the external bodies, and is evidently allowable in seeking the conditions of equilibrium which relate to the interior of the sys-

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tem.* But before we consider the equations of condition in detail, we may divide the condition of equilibrium (600) into the three conditions

$$\int t\,\delta D\eta^{\mathrm{v}} + \int t\,\delta D\eta^{\mathrm{s}} = 0, \tag{601}$$

$$-\int p\,\delta Dv + \int \sigma\,\delta Ds + \int g\,\delta z\,Dm^{\mathrm{v}} + \int g\,\delta z\,Dm^{\mathrm{s}} = 0, \tag{602}$$

$$\begin{aligned}&\int \mu_1\,\delta Dm_1^{\mathrm{v}} + \int \mu_1\,\delta Dm_1^{\mathrm{s}} + \int g\,z\,\delta Dm_1^{\mathrm{v}} + \int gz\,\delta Dm_1^{\mathrm{s}}\\ &+ \int \mu_2\,\delta Dm_2^{\mathrm{v}} + \int \mu_2\,\delta Dm_2^{\mathrm{s}} + \int g\,z\,\delta Dm_2^{\mathrm{v}} + \int g\,z\,\delta Dm_2^{\mathrm{s}}\\ &+ \text{etc.} = 0.\end{aligned} \tag{603}$$

For the variations which occur in any one of the three are evidently independent of those which occur in the other two, and the equations of condition will relate to one or another of these conditions separately.

The variations in condition (601) are subject to the condition that the entropy of the whole system shall remain constant. This may be expressed by the equation

$$\int \delta D\eta^{\mathrm{v}} + \int \delta D\eta^{\mathrm{s}} = 0. \tag{604}$$

To satisfy the condition thus limited it is necessary and sufficient that

$$t = \text{const.} \tag{605}$$

throughout the whole system, which is the condition of thermal equilibrium.

The conditions of mechanical equilibrium, or those that relate to the possible deformation of the system, are contained in (602), which may also be written

$$-\int p\,\delta Dv + \int \sigma\,\delta Ds + \int g\,\gamma\,\delta z\,Dv + \int g\,\Gamma\,\delta z\,Ds = 0. \tag{606}$$

It will be observed that this condition has the same form as if the different fluids were separated by heavy and elastic membranes without rigidity and having at every point a tension uniform in all directions in the plane of the surface. The variations in this formula,

* We have sometimes given a physical expression to a supposition of this kind, in problems in which the peculiar condition of matter in the vicinity of surfaces of discontinuity was to be neglected, by regarding the system as surrounded by a rigid and impermeable envelop. But the more exact treatment which we are now to give the problem of equilibrium would require us to take account of the influence of the envelop on the immediately adjacent matter. Since this involves the consideration of surfaces of discontinuity between solids and fluids, and we wish to limit ourselves at present to the consideration of the equilibrium of fluid masses, we shall give up the conception of an impermeable envelop, and regard the system as bounded simply by a imaginary surface, which is not a surface of discontinuity. The variations of the system must be such as do not deform the surface, nor affect the matter external to it.

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beside their necessary geometrical relations, are subject to the conditions that the external surface of the system, and the lines in which the surfaces of discontinuity meet it, are fixed. The formula may be reduced by any of the usual methods, so as to give the particular conditions of mechanical equilibrium. Perhaps the following method will lead as directly as any to the desired result.

It will be observed the quantities affected by δ in (606) relate exclusively to the position and size of the elements of volume and surface into which the system is divided, and that the variations δp and $\delta\sigma$ do not enter into the formula either explicitly or implicitly. The equations of condition which concern this formula also relate exclusively to the variations of the system of geometrical elements, and do not contain either δp or $\delta\sigma$. Hence, in determining whether the first member of the formula has the value zero for every possible variation of the system of geometrical elements, we may assign to δp and $\delta\sigma$ any values whatever, which may simplify the solution of the problem, without inquiring whether such values are physically possible.

Now when the system is in its initial state, the pressure p, in each of the parts into which the system is divided by the surfaces of tension, is a function of the co-ordinates which determine the position of the element Dv, to which the pressure relates. In the varied state of the system, the element Dv will in general have a different position. Let the variation δp be determined solely by the change in position of the element Dv. This may be expressed by the equation

$$\delta p = \frac{dp}{dx}\,\delta x + \frac{dp}{dy}\,\delta y + \frac{dp}{dz}\,\delta z, \tag{607}$$

in which $\frac{dp}{dx}$, $\frac{dp}{dy}$, $\frac{dp}{dz}$ are determined by the function mentioned, and δx, δy, δz by the variation of the position of the element Dv.

Again, in the initial state of the system the tension σ, in each of the different surfaces of discontinuity, is a function of two co-ordinates, ω_1, ω_2, which determine the position of the element Ds. In the varied state of the system, this element will in general have a different position. The change of position may be resolved into a component lying in the surface and another normal to it. Let the variation $\delta\sigma$ be determined solely by the first of these components of the motion of Ds. This may be expressed by the equation

$$\delta\sigma = \frac{d\sigma}{d\omega_1}\,\delta\omega_1 + \frac{d\sigma}{d\omega_2}\,\delta\omega_2, \tag{608}$$

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in which $\frac{d\sigma}{d\omega_1}$, $\frac{d\sigma}{d\omega_2}$ are determined by the function mentioned, and $\delta\omega_1$, $\delta\omega_2$, by the component of the motion of Ds which lies in the plane of the surface.

With this understanding, which is also to apply to δp and $\delta\sigma$ when contained implicitly in any expression, we shall proceed to the reduction of the condition (606).

With respect to any one of the volumes into which the system is divided by the surfaces of discontinuity, we may write

$$\int p\,\delta Dv = \delta \int p\,Dv - \int \delta p\,Dv.$$

But it is evident that

$$\delta \int p\,Dv = \int p\,\delta N\,Ds,$$

where the second integral relates to the surfaces of discontinuity bounding the volume considered, and δN denotes the normal component of the motion of an element of the surface, measured outward. Hence,

$$\int p\,\delta Dv = \int p\,\delta N\,Ds - \int \delta p\,Dv.$$

Since this equation is true of each separate volume into which the system is divided, we may write for the whole system

$$\int p\,\delta Dv = \int (p' - p'')\,\delta N\,Ds - \int \delta p\,Dv, \tag{609}$$

where p' and p'' denote the pressures on opposite sides of the element Ds, and δN is measured toward the side specified by double accents.

Again, for each of the surfaces of discontinuity, taken separately,

$$\int \sigma\,\delta Ds = \delta \int \sigma\,Ds - \int \delta\sigma\,Ds,$$

and

$$\delta \int \sigma\,Ds = \int \sigma\,(c_1 + c_2)\,\delta N\,Ds + \int \sigma\,\delta T\,Dl,$$

where c_1 and c_2 denote the principal curvatures of the surface, (positive, when the centers are on the side opposite to that toward which δN is measured,) Dl an element of the perimeter of the surface, and δT the component of the motion of this element which lies in the plane of the surface and is perpendicular to the perimeter, (positive, when it extends the surface). Hence we have for the whole system

$$\int \sigma\,\delta Ds = \int \sigma\,(c_1 + c_2)\,\delta N\,Ds + \int \Sigma\,(\sigma\,\delta T)\,Dl - \int \delta\sigma\,Ds, \tag{610}$$

where the integration of the elements Dl extends to all the lines in which the surfaces of discontinuity meet, and the symbol Σ denotes a summation with respect to the several surfaces which meet in such a line.

By equations (609) and (610), the general condition of mechanical equilibrium is reduced to the form

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$$-\int(p'-p'')\,\delta N\,Ds+\int\delta p\,Dv+\int\sigma\,(c_1+c_2)\,\delta N\,Ds$$
$$+\int\Sigma(\sigma\,\delta T)\,Dl-\int\delta\sigma\,Ds+\int g\,\gamma\,\delta z\,Dv+\int g\,\Gamma\,\delta z\,Ds=0.$$

Arranging and combining terms, we have

$$\int(g\,\gamma\,\delta z+\delta p)\,Dv$$
$$+\int[\,(p''-p')\,\delta N+\sigma\,(c_1+c_2)\,\delta N+g\,\Gamma\,\delta z-\delta\sigma\,]\,Ds$$
$$+\int\Sigma(\sigma\,\delta T)\,Dl=0. \quad (611)$$

To satisfy this condition, it is evidently necessary that the coefficients of Dv, Ds, and Dl shall vanish throughout the system.

In order that the coefficient of Dv shall vanish, it is necessary and sufficient that, in each of the masses into which the system is divided by the surfaces of tension, p shall be a function of z alone, such that

$$\frac{dp}{dz}=-g\,\gamma. \quad (612)$$

In order that the coefficient of Ds shall vanish in all cases, it is necessary and sufficient that it shall vanish for normal and for tangential movements of the surface. For normal movements we may write

$$\delta\sigma=0, \quad \text{and} \quad \delta z=\cos\vartheta\,\delta N,$$

where ϑ denotes the angle which the normal makes with a vertical line. The first condition therefore gives the equation

$$p'-p''=\sigma\,(c_1+c_2)+g\,\Gamma\cos\vartheta, \quad (613)$$

which must hold true at every point in every surface of discontinuity. The condition with respect to tangential movements shows that in each surface of tension σ is a function of z alone, such that

$$\frac{d\sigma}{dz}=g\,\Gamma. \quad (614)$$

In order that the coefficient of Dl in (611) shall vanish, we must have, for every point in every line in which surfaces of discontinuity meet, and for any infinitesimal displacement of the line,

$$\Sigma(\sigma\,\delta T)=0. \quad (615)$$

This condition evidently expresses the same relations between the tensions of the surfaces meeting in the line and the directions of perpendiculars to the line drawn in the planes of the various surfaces, which hold for the magnitudes and directions of forces in equilibrium in a plane.

In condition (603), the variations which relate to any component are to be regarded as having the value zero in any part of the system in

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which that substance is not an actual component.* The same is true with respect to the equations of condition, which are of the form

$$\left.\begin{array}{c} \int \delta D m_1^{\mathrm{v}} + \int \delta D m_1^{\mathrm{s}} = 0, \\ \int \delta D m_2^{\mathrm{v}} + \int \delta D m_2^{\mathrm{s}} = 0, \\ \text{etc.} \end{array}\right\} \tag{616}$$

(It is here supposed that the various components are independent, *i. e.*, that none can be formed out of others, and that the parts of the system in which any component actually occurs are not entirely separated by parts in which it does not occur.) To satisfy the condition (603), subject to these equations of condition, it is necessary and sufficient that the conditions

$$\left.\begin{array}{c} \mu_1 + g z = M_1, \\ \mu_2 + g z = M_2, \\ \text{etc.,} \end{array}\right\} \tag{617}$$

(M_1, M_2, etc. denoting constants,) shall each hold true in those parts of the system in which the substance specified is an actual component. We may here add the condition of equilibrium relative to the possible absorption of any substance (to be specified by the suffix $_a$) by parts of the system of which it is not an actual component, viz., that the expression $\mu_a + gz$ must not have a less value in such parts of the system than in a contiguous part in which the substance is an actual component.

From equation (613) with (605) and (617) we may easily obtain the differential equation of a surface of tension (in the geometrical sense of the term), when p', p'', and σ are known in terms of the temperature and potentials. For $c_1 + c_2$ and $\mathcal{D}$ may be expressed in terms of the first and second differential coefficients of z with respect to the horizontal co-ordinates, and p', p'', σ, and Γ in terms of the temperature and potentials. But the temperature is constant, and for each of the potentials we may substitute $-gz$ increased by a constant. We thus obtain an equation in which the only variables are z and its first and second differential coefficients with respect to the horizontal co-ordinates. But it will rarely be necessary to use so exact a method. Within moderate differences of level, we may regard γ', γ'', and σ as constant. We may then integrate the equation [derived from (612)]

$$d(p' - p'') = g(\gamma'' - \gamma')\, dz,$$

* The term *actual component* has been defined for homogeneous masses on page 117, and the definition may be extended to surfaces of discontinuity. It will be observed that if a substance is an actual component of either of the masses separated by a surface of discontinuity, it must be regarded as an actual component for that surface, as well as when it occurs at the surface but not in either of the contiguous masses.

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which will give

$$p'-p''=g(\gamma''-\gamma')z, \tag{618}$$

where z is to be measured from the horizontal plane for which $p'=p''$. Substituting this value in (613), and neglecting the term containing Γ, we have

$$c_1+c_2=\frac{g(\gamma''-\gamma')}{\sigma}z, \tag{619}$$

where the coefficient of z is to be regarded as constant. Now the value of z cannot be very large, in any surface of sensible dimensions, unless $\gamma''-\gamma'$ is very small. We may therefore consider this equation as practically exact, unless the densities of the contiguous masses are very nearly equal. If we substitute for the sum of the curvatures its value in terms of the differential coefficients of z with respect to the horizontal rectangular co-ordinates, x and y, we have

$$\frac{\left(1+\frac{dz^2}{dy^2}\right)\frac{d^2z}{dx^2}-2\frac{dz}{dx}\frac{dz}{dy}\frac{d^2z}{dxdy}+\left(1+\frac{dz^2}{dx^2}\right)\frac{d^2z}{dy^2}}{\left(1+\frac{dz^2}{dx^2}+\frac{dz^2}{dy^2}\right)^{\frac{3}{2}}}=\frac{g(\gamma''-\gamma')}{\sigma}z. \tag{620}$$

With regard to the sign of the root in the denominator of the fraction, it is to be observed that, if we always take the positive value of the root, the value of the whole fraction will be positive or negative according as the greater concavity is turned upward or downward. But we wish the value of the fraction to be positive when the greater concavity is turned toward the mass specified by a single accent. We should therefore take the positive or negative value of the root according as this mass is above or below the surface.

The particular conditions of equilibrium which are given in the last paragraph but one may be regarded in general as the conditions of chemical equilibrium between the different parts of the system, since they relate to the separate components.* But such a designation is not entirely appropriate unless the number of components is greater than one. In no case are the conditions of mechanical equilibrium entirely independent of those which relate to temperature and the potentials. For the conditions (612) and (614) may be regarded as consequences of (605) and (617) in virtue of the necessary relations (98) and (508).†

* Concerning another kind of conditions of chemical equilibrium, which relate to the molecular arrangement of the components, and not to their sensible distribution in space, see pages 197–203.

† Compare page 206, where a similar problem is treated without regard to the influence of the surfaces of discontinuity.

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The mechanical conditions of equilibrium, however, have an especial importance, since we may always regard them as satisfied in any liquid (and not decidedly viscous) mass in which no sensible motions are observable. In such a mass, when isolated, the attainment of mechanical equilibrium will take place very soon; thermal and chemical equilibrium will follow more slowly. The thermal equilibrium will generally require less time for its approximate attainment than the chemical; but the processes by which the latter is produced will generally cause certain inequalities of temperature until a state of complete equilibrium is reached.

When a surface of discontinuity has more components than one which do not occur in the contiguous masses, the adjustment of the potentials for these components in accordance with equations (617) may take place very slowly, or not at all, for want of sufficient mobility in the components of the surface. But when this surface has only one component which does not occur in the contiguous masses, and the temperature and potentials in these masses satisfy the conditions of equilibrium, the potential for the component peculiar to the surface will very quickly conform to the law expressed in (617), since this is a necessary consequence of the condition of mechanical equilibrium (614) in connection with the conditions relating to temperature and the potentials which we have supposed to be satisfied. The necessary distribution of the substance peculiar to the surface will be brought about by expansions and contractions of the surface. If the surface meets a third mass containing this component and no other which is foreign to the masses divided by the surface, the potential for this component in the surface will of course be determined by that in the mass which it meets.

The particular conditions of mechanical equilibrium (612)–(615), which may be regarded as expressing the relations which must subsist between contiguous portions of a fluid system in a state of mechanical equilibrium, are serviceable in determining whether a given system is or is not in such a state. But the mechanical theorems which relate to finite parts of the system, although they may be deduced from these conditions by integration, may generally be more easily obtained by a suitable application of the general condition of mechanical equilibrium (606), or by the application of ordinary mechanical principles to the system regarded as subject to the forces indicated by this equation.

It will be observed that the conditions of equilibrium relating to temperature and the potentials are not affected by the surfaces of

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discontinuity. [Compare (228) and (234).] * Since a phase cannot vary continuously without variations of the temperature or the potentials, it follows from these conditions that the phase at any point in a fluid system which has the same independently variable components throughout, and is in equilibrium under the influence of gravity, must be one of a certain number of phases which are completely determined by the phase at any given point and the difference of level of the two points considered. If the phases throughout the fluid system satisfy the general condition of practical stability for phases existing in large masses (viz., that the pressure shall be the least consistent with the temperature and potentials), they will be entirely determined by the phase at any given point and the differences of level. (Compare page 210, where the subject is treated without regard to the influence of the surfaces of discontinuity.)

Conditions of equilibrium relating to irreversible changes.—The conditions of equilibrium relating to the absorption by any part of the system of substances which are not actual components of that part have been given on page 448. Those relating to the formation of new masses and surfaces are included in the conditions of stability relating to such changes, and are not always distinguishable from them. They are evidently independent of the action of gravity. We have already discussed the conditions of stability with respect to the formation of new fluid masses within a homogeneous fluid and at the surface when two such masses meet (see pages 416–429), as well as the condition relating to the possibility of a change in the nature of a surface of discontinuity. (See pages 400–403, where the surface considered is plane, but the result may easily be extended to curved surfaces.) We shall hereafter consider, in some of the more important cases, the conditions of stability with respect to the formation of new masses and surfaces which are peculiar to lines in which several surfaces of discontinuity meet, and points in which several such lines meet.

Conditions of stability relating to the whole system.—Beside the conditions of stability relating to very small parts of a system, which are substantially independent of the action of gravity, and are discussed elsewhere, there are other conditions, which relate to the

* If the fluid system is divided into separate masses by solid diaphragms which are permeable to all the components of the fluids independently, the conditions of equilibrium of the fluids relating to temperature and the potentials will not be affected. (Compare page 139.) The propositions which follow in the above paragraph may be extended to this case.

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whole system or to considerable parts of it. To determine the question of the stability of a given fluid system under the influence of gravity, when all the conditions of equilibrium are satisfied as well as those conditions of stability which relate to small parts of the system taken separately, we may use the method described on page 413, the demonstration of which (pages 411, 412) will not require any essential modification on account of gravity.

When the variations of temperature and of the quantities M_1, M_2, etc. [see (617)] involved in the changes considered are so small that they may be neglected, the condition of stability takes a very simple form, as we have already seen to be the case with respect to a system uninfluenced by gravity. (See page 415.)

We have to consider a varied state of the system in which the total entropy and the total quantities of the various components are unchanged, and all variations vanish at the exterior of the system,—in which, moreover, the conditions of equilibrium relating to temperature and the potentials are satisfied, and the relations expressed by the fundamental equations of the masses and surfaces are to be regarded as satisfied, although the state of the system is not one of complete equilibrium. Let us imagine the state of the system to vary continuously in the course of time in accordance with these conditions and use the symbol d to denote the simultaneous changes which take place at any instant. If we denote the total energy of the system by E, the value of dE may be expanded like that of δE in (599) and (600), and then reduced (since the values of t, μ_1+gz, μ_2+gz, etc. are uniform throughout the system, and the total entropy and total quantities of the several components are constant) to the form

$$\begin{aligned} dE &= -\int p\,dDv + \int g\,dz\,Dm^{\mathrm{v}} + \int \sigma\,dDs + \int g\,dz\,Dm^{\mathrm{s}} \\ &= -\int p\,dDv + \int g\,\gamma\,dz\,Dv + \int \sigma\,dDs + \int g\,\Gamma\,dz\,Ds, \end{aligned} \tag{621}$$

where the integrations relate to the elements expressed by the symbol D. The value of p at any point in any of the various masses, and that of σ at any point in any of the various surfaces of discontinuity are entirely determined by the temperature and potentials at the point considered. If the variations of t and M_1, M_2, etc. are to be neglected, the variations of p and σ will be determined solely by the change in position of the point considered. Therefore, by (612) and (614),

$$dp = -g\,\gamma\,dz, \quad d\sigma = g\,\Gamma\,dz;$$

and

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$$dE = -\int p\, dDv - \int dp\, Dv + \int \sigma\, dDs + \int d\sigma\, Ds$$
$$= -d\int p\, Dv + d\int \sigma\, Ds. \tag{622}$$

If we now integrate with respect to d, commencing at the given state of the system, we obtain

$$\Delta E = -\Delta \int p\, Dv + \Delta \int \sigma\, Ds, \tag{623}$$

where Δ denotes the value of a quantity in a varied state of the system diminished by its value in the given state. This is true for finite variations, and is therefore true for infinitesimal variations without neglect of the infinitesimals of the higher orders. The condition of stability is therefore that

$$\Delta \int p\, Dv - \Delta \int \sigma\, Ds < 0, \tag{624}$$

or that the quantity

$$\int p\, Dv - \int \sigma\, Ds \tag{625}$$

has a maximum value, the values of p and σ, for each different mass or surface, being regarded as determined functions of z. (In ordinary cases σ may be regarded as constant in each surface of discontinuity, and p as a linear function of z in each different mass.) It may easily be shown (compare page 416) that this condition is always *sufficient* for stability with reference to motion of surfaces of discontinuity, even when the variations of t, M_1, M_2, etc. cannot be neglected in the determination of the *necessary* condition of stability with respect to such changes.

On the Possibility of the Formation of a New Surface of Discontinuity where several Surfaces of Discontinuity meet.

When more than three surfaces of discontinuity between homogeneous masses meet along a line, we may conceive of a new surface being formed between any two of the masses which do not meet in a surface in the original state of the system. The condition of stability with respect to the formation of such a surface may be easily obtained by the consideration of the limit between stability and instability, as exemplified by a system which is in equilibrium when a very small surface of the kind is formed.

To fix our ideas, let us suppose that there are four homogeneous masses A, B, C, and D, which meet one another in four surfaces, which we may call A-B, B-C, C-D, and D-A, these surfaces all meeting along a line L. This is indicated in figure 11 by a section of the sur-

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faces cutting the line L at right angles at a point O. In an infinitesimal variation of the state of the system, we may conceive of a small surface being formed between A and C (to be called A-C), so that the section of the surfaces of discontinuity by the same plane takes the form indicated in figure 12. Let us suppose that

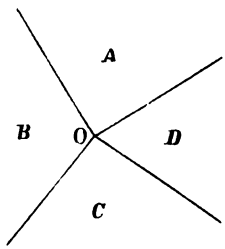

FIG. 11.

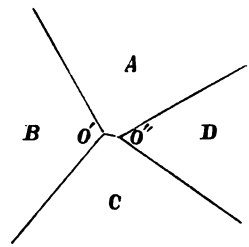

FIG. 12.

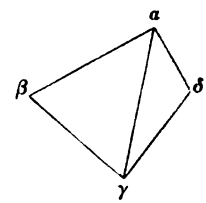

FIG. 13.

the condition of equilibrium (615) is satisfied both for the line L in which the surfaces of discontinuity meet in the original state of the system, and for the two such lines (which we may call L′ and L″) in the varied state of the system, at least at the points O′ and O″ where they are cut by the plane of section. We may therefore form a quadrilateral of which the sides $\alpha\beta$, $\beta\gamma$, $\gamma\delta$, $\delta\alpha$ are equal in numerical value to the tensions of the several surfaces A-B, B-C, C-D, D-A, and are parallel to the normals to these surfaces at the point O in the original state of the system. In like manner, for the varied state of the system we can construct two triangles having similar relations to the surfaces of discontinuity meeting at O′ and O″. But the directions of the normals to the surfaces A-B and B-C at O′ and to C-D and D-A at O″ in the varied state of the system differ infinitely little from the directions of the corresponding normals at O in the initial state. We may therefore regard $\alpha\beta$, $\beta\gamma$ as two sides of the triangle representing the surfaces meeting at O′, and $\gamma\delta$, $\delta\alpha$ as two sides of the triangle representing the surfaces meeting at O″. Therefore, if we join $\alpha\gamma$, this line will represent the direction of the normal to the surface A-C, and the value of its tension. If the tension of a surface between such masses as A and C had been greater than that represented by $\alpha\gamma$, it is evident that the initial state of the system of surfaces (represented in figure 11) would have been stable with respect to the possible formation of any such surface. If the tension had been less, the state of the system would have been at least practically unstable. To determine whether it is unstable in the strict sense of the term, or whether or not it is prop-

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erly to be regarded as in equilibrium, would require a more refined analysis than we have used.*

The result which we have obtained may be generalized as follows. When more than three surfaces of discontinuity in a fluid system meet in equilibrium along a line, with respect to the surfaces and masses immediately adjacent to any point of this line we may form a polygon of which the angular points shall correspond in order to the different masses separated by the surfaces of discontinuity, and the sides to these surfaces, each side being perpendicular to the corresponding surface, and equal to its tension. With respect to the formation of new surfaces of discontinuity in the vicinity of the point especially considered, the system is stable, if every diagonal of the polygon is less, and practically unstable, if any diagonal is greater, than the tension which would belong to the surface of discontinuity between the corresponding masses. In the limiting case, when the diagonal is exactly equal to the tension of the corresponding surface, the system may often be determined to be unstable by the application of the principle enunciated to an adjacent point of the line in which the surfaces of discontinuity meet. But when, in the polygons constructed for all points of the line, no diagonal is in any case greater

* We may here remark that a nearer approximation in the theory of equilibrium and stability might be attained, by taking special account, in our general equations, of the lines in which surfaces of discontinuity meet. These lines might be treated in a manner entirely analogous to that in which we have treated surfaces of discontinuity. We might recognize linear densities of energy, of entropy, and of the several substances which occur about the line, also a certain linear tension. With respect to these quantities and the temperature and potentials, relations would hold analogous to those which have been demonstrated for surfaces of discontinuity. (See pp. 391–393.) If the sum of the tensions of the lines L′ and L″, mentioned above, is greater than the tension of the line L, this line will be in strictness stable (although practically unstable) with respect to the formation of a surface between A and C, when the tension of such a surface is a little less than that represented by the diagonal $a\gamma$.

The different use of the term *practically unstable* in different parts of this paper need not create confusion, since the general meaning of the term is in all cases the same. A system is called practically unstable when a very small (not necessarily indefinitely small) disturbance or variation in its condition will produce a considerable change. In the former part of this paper, in which the influence of surfaces of discontinuity was neglected, a system was regarded as practically unstable when such a result would be produced by a disturbance of the same order of magnitude as the quantities relating to surfaces of discontinuity which were neglected. But where surfaces of discontinuity are considered, a system is not regarded as practically unstable, unless the disturbance which will produce such a result is very small compared with the quantities relating to surfaces of discontinuity of any appreciable magnitude.

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than the tension of the corresponding surface, but a certain diagonal is equal to the tension in the polygons constructed for a finite portion of the line, farther investigations are necessary to determine the stability of the system. For this purpose, the method described on page 413 is evidently applicable.

A similar proposition may be enunciated in many cases with respect to a point about which the angular space is divided into solid angles by surfaces of discontinuity. If these surfaces are in equilibrium, we can always form a closed solid figure without re-entrant angles of which the angular points shall correspond to the several masses, the edges to the surfaces of discontinuity, and the sides to the lines in which these edges meet, the edges being perpendicular to the corresponding surfaces, and equal to their tensions, and the sides being perpendicular to the corresponding lines. Now if the solid angles in the physical system are such as may be subtended by the sides and bases of a triangular prism enclosing the vertical point, or can be derived from such by deformation, the figure representing the tensions will have the form of two triangular pyramids on opposite sides of the same base, and the system will be stable or practically unstable with respect to the formation of a surface between the masses which only meet in a point, according as the tension of a surface between such masses is greater or less than the diagonal joining the corresponding angular points of the solid representing the tensions. This will easily appear on consideration of the case in which a very small surface between the masses would be in equilibrium.

The Conditions of Stability for Fluids relating to the Formation of a New Phase at a Line in which Three Surfaces of Discontinuity meet.

With regard to the formation of new phases there are particular conditions of stability which relate to lines in which several surfaces of discontinuity meet. We may limit ourselves to the case in which there are three such surfaces, this being the only one of frequent occurrence, and may treat them as meeting in a straight line. It will be convenient to commence by considering the equilibrium of a system in which such a line is replaced by a filament of a different phase.

Let us suppose that three homogeneous fluid masses, A, B, and C, are separated by cylindrical (or plane) surfaces, A-B, B-C, C-A, which at first meet in a straight line, each of the surface-tensions σ_{AB}, σ_{BC}, σ_{CA} being less than the sum of the other two. Let us suppose that the

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system is then modified by the introduction of a fourth fluid mass D, which is placed between A, B, and C, and is separated from them by cylindrical surfaces D-A, D-B, D-C meeting A-B, B-C, and C-A in straight lines. The general form of the surfaces is shown by figure 14, in which the full lines represent a section perpendicular to all the surfaces. The system thus modified is to be in equilibrium, as well as the original system, the position of the surfaces A-B, B-C, C-A being unchanged. That the last condition is consistent with equilibrium will appear from the following mechanical considerations.

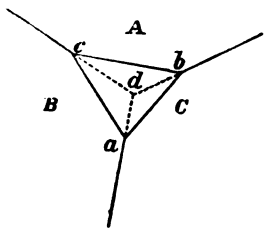

FIG. 14.

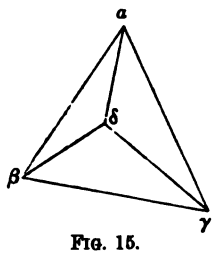

FIG. 15.

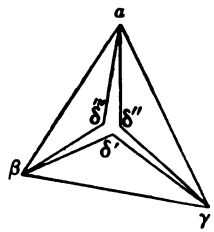

FIG. 16.

Let v_D denote the volume of the mass D per unit of length or the area of the curvilinear triangle $a\,b\,c$. Equilibrium is evidently possible for any values of the surface-tensions (if only σ_{AB}, σ_{BC}, σ_{CA} satisfy the condition mentioned above, and the tensions of the three surfaces meeting at each of the edges of D satisfy a similar condition) with any value (not too large) of v_D, if the edges of D are constrained to remain in the original surfaces A-B, B-C, and C-A, or these surfaces extended, if necessary, without change of curvature. (In certain cases one of the surfaces D-A, D-B, D-C may disappear and D will be bounded by only two cylindrical surfaces.) We may therefore regard the system as maintained in equilibrium by forces applied to the edges of D and acting at right angles to A-B, B-C, C-A. The same forces would keep the system in equilibrium if D were rigid. They must therefore have a zero resultant, since the nature of the mass D is immaterial when it is rigid, and no forces external to the system would be required to keep a corresponding part of the original system in equilibrium. But it is evident from the points of application and directions of these forces that they cannot have a zero resultant unless each force is zero. We may therefore introduce a fourth mass D without disturbing the parts which remain of the surfaces A-B, B-C, C-D.

It will be observed that all the angles at a, b, c, and d in figure 14 are entirely determined by the six surface-tensions σ_{AB}, σ_{BC}, σ_{CA}, σ_{DA}, σ_{DB}, σ_{DC}. [See (615).] The angles may be derived from the tensions

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by the following construction, which will also indicate some important properties. If we form a triangle $\alpha\,\beta\,\gamma$ (figure 15 or 16) having sides equal to σ_{AB}, σ_{BC}, σ_{CA}, the angles of the triangle will be supplements of the angles at d. To fix our ideas, we may suppose the sides of the triangle to be perpendicular to the surfaces at d. Upon $\beta\,\gamma$ we may then construct (as in figure 16) a triangle $\beta\,\gamma\,\delta'$ having sides equal to σ_{BC}, σ_{DC}, σ_{DB}, upon $\gamma\,\alpha$ a triangle $\gamma\,\alpha\,\delta''$ having sides equal to σ_{CA}, σ_{DA}, σ_{DC}, and upon $\alpha\,\beta$ a triangle $\alpha\beta\delta'''$ having sides equal to σ_{AB}, σ_{DB}, σ_{DA}. These triangles are to be on the same sides of the lines $\beta\,\gamma$, $\gamma\,\alpha$, $\alpha\,\beta$, respectively, as the triangle $\alpha\,\beta\,\gamma$. The angles of these triangles will be supplements of the angles of the surfaces of discontinuity at a, b, and c. Thus $\beta\,\gamma\,\delta'=d\,a\,b$, and $\alpha\,\gamma\,\delta''=d\,b\,a$. Now if δ' and δ'' fall together in a single point δ within the triangle $\alpha\,\beta\,\gamma$, δ''' will fall in the same point, as in figure 15. In this case we shall have $\beta\,\gamma\,\delta+\alpha\,\gamma\,\delta=\alpha\,\gamma\,\beta$, and the three angles of the curvilinear triangle $a\,d\,b$ will be together equal to two right angles. The same will be true of the three angles of each of the triangles $b\,d\,c$, $c\,d\,a$, and hence of the three angles of the triangle $a\,b\,c$. But if δ', δ'', δ''' do not fall together in the same point within the triangle $\alpha\,\beta\,\gamma$, it is either possible to bring these points to coincide within the triangle by increasing some or all of the tensions σ_{DA}, σ_{DB}, σ_{DC}, or to effect the same result by diminishing some or all of these tensions. (This will easily appear when one of the points δ', δ'', δ''' falls within the triangle, if we let the two tensions which determine this point remain constant, and the third tension vary. When all the points δ', δ'', δ''' fall without the triangle $\alpha\,\beta\,\gamma$, we may suppose the greatest of the tensions σ_{DA}, σ_{DB}, σ_{DC}—the two greatest, when these are equal, and all three when they all are equal—to diminish until one of the points δ', δ'', δ''' is brought within the triangle $\alpha\,\beta\,\gamma$.) In the first case we may say that the tensions of the new surfaces are too small to be represented by the distances of an internal point from the vertices of the triangle representing the tensions of the original surfaces (or, for brevity, that they are too small to be represented as in figure 15); in the second case we may say that they are too great to be thus represented. In the first case, the sum of the angles in each of the triangles $a\,d\,b$, $b\,d\,c$, $c\,d\,a$ is less than two right angles (compare figures 14 and 16): in the second case, each pair of the triangles $\alpha\,\beta\,\delta'''$, $\beta\,\gamma\,\delta''$, $\gamma\,\alpha\,\delta''$ will overlap, at least when the tensions σ_{DA}, σ_{DB}, σ_{DC} are only a little too great to be represented as in figure 15, and the sum of the angles of each of the triangles $a\,d\,b$, $b\,d\,c$, $c\,d\,a$ will be greater than two right angles.

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Let us denote by v_A, v_B, v_C the portions of v_D which were originally occupied by the masses A, B, C, respectively, by s_{DA}, s_{DB}, s_{DC}, the areas of the surfaces specified per unit of length of the mass D, and by s_{AB}, s_{BC}, s_{CA}, the areas of the surfaces specified which were replaced by the mass D per unit of its length. In numerical value, v_A, v_B, v_C will be equal to the areas of the curvilinear triangles $b\,c\,d$, $c\,a\,d$, $a\,b\,d$; and s_{DA}, s_{DB}, s_{DC}, s_{AB}, s_{BC}, s_{CA} to the lengths of the lines $b\,c$, $c\,a$, $a\,b$, $c\,d$, $a\,d$, $b\,d$. Also let

$$W_S = \sigma_{DA}\, s_{DA} + \sigma_{DB}\, s_{DB} + \sigma_{DC}\, s_{DC} - \sigma_{AB}\, s_{AB} - \sigma_{BC}\, s_{BC} - \sigma_{CA}\, s_{CA}, \quad (626)$$

and

$$W_V = p_D\, v_D - p_A\, v_A - p_B\, v_B - p_C\, v_C. \quad (627)$$

The general condition of mechanical equilibrium for a system of homogeneous masses not influenced by gravity, when the exterior of the whole system is fixed, may be written

$$\Sigma\,(\sigma\,\delta s) - \Sigma\,(p\,\delta v) = 0. \quad (628)$$

[See (606).] If we apply this both to the original system consisting of the masses A, B, and C, and to the system modified by the introduction of the mass D, and take the difference of the results, supposing the deformation of the system to be the same in each case, we shall have

$$\sigma_{DA}\,\delta s_{DA} + \sigma_{DB}\,\delta s_{DB} + \sigma_{DC}\,\delta s_{DC} - \sigma_{AB}\,\delta s_{AB} - \sigma_{BC}\,\delta s_{BC}$$
$$- \sigma_{CA}\,\delta s_{CA} - p_D\,\delta v_D + p_A\,\delta v_A + p_B\,\delta v_B + p_C\,\delta v_C = 0. \quad (629)$$

In view of this relation, if we differentiate (626) and (627) regarding all quantities except the pressures as variable, we obtain

$$d\,W_S - d\,W_V = s_{DA}\,d\sigma_{DA} + s_{DB}\,d\sigma_{DB} + s_{DC}\,d\sigma_{DC}$$
$$- s_{AB}\,d\sigma_{AB} - s_{BC}\,d\sigma_{BC} - s_{CA}\,d\sigma_{CA}. \quad (630)$$

Let us now suppose the system to vary in size, remaining always similar to itself in form, and that the tensions diminish in the same ratio as lines, while the pressures remain constant. Such changes will evidently not impair the equilibrium. Since all the quantities s_{DA}, σ_{DA}, s_{DB}, σ_{DB}, etc. vary in the same ratio,

$$s_{DA}\,d\sigma_{DA} = \tfrac{1}{2}\,d(\sigma_{DA}\,s_{DA}), \qquad s_{DB}\,d\sigma_{DB} = \tfrac{1}{2}\,d(\sigma_{DB}\,s_{DB}), \quad \text{etc.} \quad (631)$$

We have therefore by integration of (630)

$$W_S - W_V = \tfrac{1}{2}\,(\sigma_{DA}s_{DA} + \sigma_{DB}s_{DB} + \sigma_{DC}s_{DC} - \sigma_{AB}s_{AB} - \sigma_{BC}s_{BC} - \sigma_{CA}s_{CA}), \quad (632)$$

whence, by (626),

$$W_S = 2\,W_V, \quad (633)$$

The condition of stability for the system when the pressures and tensions are regarded as constant, and the position of the surfaces

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A-B, B-C, C-A as fixed, is that $W_S - W_V$ shall be a minimum under the same conditions. [See (549).] Now for any constant values of the tensions and of p_A, p_B, p_C, we may make v_D so small that when it varies, the system remaining in equilibrium, (which will in general require a variation of p_D,) we may neglect the curvatures of the lines $d\,a$, $d\,b$, $d\,c$, and regard the figure $a\,b\,c\,d$ as remaining similar to itself. For the *total curvature* (*i. e.*, the curvature measured in degrees) of each of the lines $a\,b$, $b\,c$, $c\,a$ may be regarded as constant, being equal to the constant difference of the sum of the angles of one of the curvilinear triangles $a\,d\,b$, $b\,d\,c$, $c\,d\,a$ and two right angles. Therefore, when v_D is very small, and the system is so deformed that equilibrium would be preserved if p_D had the proper variation, but this pressure as well as the others and all the tensions remain constant, W_S will vary as the lines in the figure $a\,b\,c\,d$, and W_V as the square of these lines. Therefore, for such deformations,

$$W_V \propto W_S^2.$$

This shows that the system cannot be stable for constant pressures and tensions when v_D is small and W_V is positive, since $W_S - W_V$ will not be a minimum. It also shows that the system is stable when W_V is negative. For, to determine whether $W_S - W_V$ is a minimum for constant values of the pressures and tensions, it will evidently be sufficient to consider such varied forms of the system as give the least value to $W_S - W_V$ for any value of v_D in connection with the constant pressures and tensions. And it may easily be shown that such forms of the system are those which would preserve equilibrium if p_D had the proper value.

These results will enable us to determine the most important questions relating to the stability of a line along which three homogeneous fluids A, B, C meet, with respect to the formation of a different fluid D. The components of D must of course be such as are found in the surrounding bodies. We shall regard p_D and σ_{DA}, σ_{DB}, σ_{DC} as determined by that phase of D which satisfies the conditions of equilibrium with the other bodies relating to temperature and the potentials. These quantities are therefore determinable, by means of the fundamental equations of the mass D and of the surfaces D-A, D-B, D-C, from the temperature and potentials of the given system.

Let us first consider the case in which the tensions, thus determined, can be represented as in figure 15, and p_D has a value consistent with the equilibrium of a small mass such as we have been considering. It appears from the preceding discussion that when v_D is

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sufficiently small the figure $a\,b\,c\,d$ may be regarded as rectilinear, and that its angles are entirely determined by its tensions. Hence the ratios of v_A, v_B, v_C, v_D, for sufficiently small values of v_D, are determined by the tensions alone, and for convenience in calculating these ratios, we may suppose p_A, p_B, p_C to be equal, which will make the figure $a\,b\,c\,d$ absolutely rectilinear, and make p_D equal to the other pressures, since it is supposed that this quantity has the value necessary for equilibrium. We may obtain a simple expression for the ratios of v_A, v_B, v_C, v_D in terms of the tensions in the following manner. We shall write [D B C], [D C A], etc., to denote the areas of triangles having sides equal to the tensions of the surfaces between the masses specified.

$$\begin{aligned} v_A : v_B &:: \text{triangle } b\,d\,c : \text{triangle } a\,d\,c \\ &:: bc \sin bcd : ac \sin acd \\ &:: \sin bac \sin bcd : \sin abc \sin acd \\ &:: \sin \gamma\delta\beta \sin \delta\alpha\beta : \sin \gamma\delta\alpha \sin \delta\beta\alpha \\ &:: \sin \gamma\delta\beta\ \delta\beta : \sin \gamma\delta\alpha\ \delta\alpha \\ &:: \text{triangle } \gamma\,\delta\,\beta : \text{triangle } \gamma\,\delta\,\alpha \\ &:: [\mathrm{D\,B\,C}] : [\mathrm{D\,C\,A}]. \end{aligned}$$

Hence,

$$v_A : v_B : v_C : v_D :: [\mathrm{D\,B\,C}] : [\mathrm{D\,C\,A}] : [\mathrm{D\,A\,B}] : [\mathrm{A\,B\,C}], \tag{634}$$

where

$$\tfrac{1}{4}\sqrt{[(\sigma_{AB}+\sigma_{BC}+\sigma_{CA})(\sigma_{AB}+\sigma_{BC}-\sigma_{CA})(\sigma_{BC}+\sigma_{CA}-\sigma_{AB})(\sigma_{CA}+\sigma_{AB}-\sigma_{BC})]}$$

may be written for [A B C], and analogous expressions for the other symbols, the sign $\sqrt{\ }$ denoting the positive root of the necessarily positive expression which follows. This proportion will hold true in any case of equilibrium, when the tensions satisfy the condition mentioned and v_D is sufficiently small. Now if $p_A = p_B = p_C$, p_D will have the same value, and we shall have by (627) $W_V = 0$, and by (633) $W_S = 0$. But when v_D is very small, the value of W_S is entirely determined by the tensions and v_D. Therefore, whenever the tensions satisfy the condition supposed, and v_D is very small (whether p_A, p_B, p_C are equal or unequal,)

$$0 = W_S = W_V = p_D\,v_D - p_A\,v_A - p_B\,v_B - p_C\,v_C, \tag{635}$$

which with (634) gives

$$p_D = \frac{[\mathrm{D\,B\,C}]\,p_A + [\mathrm{D\,C\,A}]\,p_B + [\mathrm{D\,A\,B}]\,p_C}{[\mathrm{D\,B\,C}] + [\mathrm{D\,C\,A}] + [\mathrm{D\,A\,B}]}. \tag{636}$$

Since this is the only value of p_D for which equilibrium is possible when

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the tensions satisfy the condition supposed and v_D is small, it follows that when p_D has a less value, the line where the fluids A, B, C meet is stable with respect to the formation of the fluid D. When p_D has a greater value, if such a line can exist at all, it must be at least practically unstable, *i. e.*, if only a very small mass of the fluid D should be formed it would tend to increase.

Let us next consider the case in which the tensions of the new surfaces are too small to be represented as in figure 15. If the pressures and tensions are consistent with equilibrium for any very small value of v_D, the angles of each of the curvilinear triangles $a\,d\,b$, $b\,d\,c$, $c\,d\,a$ will be together less than two right angles, and the lines $a\,b$, $b\,c$, $c\,a$, will be convex toward the mass D. For given values of the pressures and tensions, it will be easy to determine the magnitude of v_D. For the tensions will give the total curvatures (in degrees) of the lines $a\,b$, $b\,c$, $c\,a$; and the pressures will give the radii of curvature. These lines are thus completely determined. In order that v_D shall be very small it is evidently necessary that p_D shall be less than the other pressures. Yet if the tensions of the new surfaces are only a very little too small to be represented as in figure 15, v_D may be quite small when the value of p_D is only a little less than that given by equation (636). In any case, when the tensions of the new surfaces are too small to be represented as in figure 15, and v_D is small, W_V is negative, and the equilibrium of the mass D is stable. Moreover, $W_S - W_V$, which represents the work necessary to form the mass D with its surfaces in place of the other masses and surfaces, is negative.

With respect to the stability of a line in which the surfaces A-B, B-C, C-A meet, when the tensions of the new surfaces are too small to be represented as in figure 15, we first observe that when the pressures and tensions are such as to make v_D moderately small but not so small as to be neglected, [this will be when p_D is somewhat smaller than the second member of (636),—more or less smaller according as the tensions differ more or less from such as are represented in figure 15,] the equilibrium of such a line as that supposed (if it is capable of existing at all) is at least practically unstable. For greater values of p_D (with the same values of the other pressures and the tensions) the same will be true. For somewhat smaller values of p_D, the mass of the phase D which will be formed will be so small, that we may neglect this mass and regard the surfaces A-B, B-C, C-A as meeting in a line in stable equilibrium. For still smaller values of p_D, we may likewise regard the surfaces A-B, B-C, C-A as capable

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of meeting in stable equilibrium. It may be observed that when v_D, as determined by our equations, becomes quite insensible, the conception of a small mass D having the properties deducible from our equations ceases to be accurate, since the matter in the vicinity of a line where these surfaces of discontinuity meet must be in a peculiar state of equilibrium not recognized by our equations.* But this cannot affect the validity of our conclusion with respect to the stability of the line in question.

The case remains to be considered in which the tensions of the new surfaces are too great to be represented as in figure 15. Let us suppose that they are not very much too great to be thus represented. When the pressures are such as to make v_D moderately small (in case of equilibrium) but not so small that the mass D to which it relates ceases to have the properties of matter in mass, [this will be when p_D is somewhat greater than the second member of (636),—more or less greater according as the tensions differ more or less from such as are represented in figure 15,] the line where the surfaces A-B, B-C, C-A meet will be in stable equilibrium with respect to the formation of such a mass as we have considered, since $W_S - W_V$ will be positive. The same will be true for less values of p_D. For greater values of p_D, the value of $W_S - W_V$, which measures the stability with respect to the kind of change considered, diminishes. It does not vanish, according to our equations, for finite values of p_D. But these equations are not to be trusted beyond the limit at which the mass D ceases to be of sensible magnitude.

But when the tensions are such as we now suppose, we must also consider the possible formation of a mass D within a closed figure in which the surfaces D-A, D-B, D-C meet together (with the surfaces A-B, B-C, C-A) in two opposite points. If such a figure is to be in equilibrium, the six tensions must be such as can be represented by

* See note on page 455. We may here add that the linear tension there mentioned may have a negative value. This would be the case with respect to a line in which three surfaces of discontinuity are regarded as meeting, but where nevertheless there really exists in stable equilibrium a filament of different phase from the three surrounding masses. The value of the linear tension for the supposed line, would be nearly equal to the value of $W_S - W_V$ for the actually existing filament. (For the exact value of the linear tension it would be necessary to add the sum of the linear tensions of the three edges of the filament.) We may regard two soap-bubbles adhering together as an example of this case. The reader will easily convince himself that in an exact treatment of the equilibrium of such a double bubble we must recognize a certain negative tension in the line of intersection of the three surfaces of discontinuity.

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the six distances of four points in space (see page 455),—a condition which evidently agrees with the supposition which we have made. If we denote by w_V the work gained in forming the mass D (of such size and form as to be in equilibrium) in place of the other masses, and by w_S the work expended in forming the new surfaces in place of the old, it may easily be shown by a method similar to that used on page 459 that $w_S = \frac{3}{2} w_V$. From this we obtain $w_S - w_V = \frac{1}{2} w_V$. This is evidently positive when p_D is greater than the other pressures. But it diminishes with increase of p_D, as easily appears from the equivalent expression $\frac{1}{3} w_S$. Hence the line of intersection of the surfaces of discontinuity A-B, B-C, C-A is stable for values of p_D greater than the other pressures (and therefore for all values of p_D) so long as our method is to be regarded as accurate, which will be so long as the mass D which would be in equilibrium has a sensible size.

In certain cases in which the tensions of the new surfaces are much too large to be represented as in figure 15, the reasoning of the two last paragraphs will cease to be applicable. These are cases in which the six tensions cannot be represented by the sides of a tetrahedron. It is not necessary to discuss these cases, which are distinguished by the different shape which the mass D would take if it should be formed, since it is evident that they can constitute no exception to the results which we have obtained. For an increase of the values of σ_{DA}, σ_{DB}, σ_{DC} cannot favor the formation of D, and hence cannot impair the stability of the line considered, as deduced from our equations. Nor can an increase of these tensions essentially affect the fact that the stability thus demonstrated may fail to be realized when p_D is considerably greater than the other pressures, since the *a priori* demonstration of the stability of any one of the surfaces A-B, B-C, C-A, taken singly, is subject to the limitation mentioned. (See page 426.)

The Condition of Stability for Fluids relating to the Formation of a New Phase at a Point where the Vertices of Four Different Masses meet.

Let four different fluid masses A, B, C, D meet about a point, so as to form the six surfaces of discontinuity A-B, B-C, C-A, D-A, D-B, D-C, which meet in the four lines A-B-C, B-C-D, C-D-A, D-A-B, these lines meeting in the vertical point. Let us suppose the system stable in other respects, and consider the conditions of stability for the vertical point with respect to the possible formation of a different fluid mass E.

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If the system can be in equilibrium when the vertical point has been replaced by a mass E against which the four masses A, B, C, D abut, being truncated at their vertices, it is evident that E will have four vertices, at each of which six surfaces of discontinuity meet. (Thus at one vertex there will be the surfaces formed by A, B, C, and E.) The tensions of each set of six surfaces (like those of the six surfaces formed by A, B, C, and D) must therefore be such that they can be represented by the six edges of a tetrahedron. When the tensions do not satisfy these relations, there will be no particular condition of stability for the point about which A, B, C, and D meet, since if a mass E should be formed, it would distribute itself along some of the lines or surfaces which meet at the vertical point, and it is therefore sufficient to consider the stability of these lines and surfaces. We shall suppose that the relations mentioned are satisfied.

If we denote by W_V the work gained in forming the mass E (of such size and form as to be in equilibrium) in place of the portions of the other masses which are suppressed, and by W_S the work expended in forming the new surfaces in place of the old, it may easily be shown by a method similar to that used on page 459 that

$$W_S = \tfrac{3}{2} W_V, \tag{637}$$

whence

$$W_S - W_V = \tfrac{1}{2} W_V; \tag{638}$$

also, that when the volume E is small, the equilibrium of E will be stable or unstable according as W_S and W_V are negative or positive.

A critical relation for the tensions is that which makes equilibrium possible for the system of the five masses A, B, C, D, E, when all the surfaces are plane. The ten tensions may then be represented in magnitude and direction by the ten distances of five points in space α, β, γ, δ, ε, viz., the tension of A-B and the direction of its normal by the line $\alpha\beta$, etc. The point ε will lie within the tetrahedron formed by the other points. If we write v_E for the volume of E, and v_A, v_B, v_C, v_D for the volumes of the parts of the other masses which are suppressed to make room for E, we have evidently

$$W_V = p_E v_E - p_A v_A - p_B v_B - p_C v_C - p_D v_D. \tag{639}$$

Hence, when all the surfaces are plane, $W_V = 0$, and $W_S = 0$. Now equilibrium is always possible for a given small value of v_E with any given values of the tensions and of p_A, p_B, p_C, p_D. When the tensions satisfy the critical relation, $W_S = 0$, if $p_A = p_B = p_C = p_D$. But when v_E is small and constant, the value of W_S must be independent of p_A, p_B, p_C, p_D, since the angles of the surfaces are determined by the tensions and their curvatures may be neglected. Hence, $W_S = 0$, and

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$W_V = 0$, when the critical relation is satisfied and v_E small. This gives

$$p_E = \frac{v_A p_A + v_B p_B + v_C p_C + v_D p_D}{v_E}. \tag{640}$$

In calculating the ratios of v_A, v_B, v_C, v_D, v_E, we may suppose all the surfaces to be plane. Then E will have the form of a tetrahedron, the vertices of which may be called a, b, c, d, (each vertex being named after the mass which is not found there,) and v_A, v_B, v_C, v_D will be the volumes of the tetrahedra into which it may be divided by planes passing through its edges and an interior point e. The volumes of these tetrahedra are proportional to those of the five tetrahedra of the figure $\alpha\,\beta\,\gamma\,\delta\,\varepsilon$, as will easily appear if we recollect that the line a b is common to the surfaces C-D, D-E, E-C, and therefore perpendicular to the surface common to the lines $\gamma\,\delta$, $\delta\,\varepsilon$, $\varepsilon\,\gamma$, *i. e.*, to the surface $\gamma\,\delta\,\varepsilon$, and so in other cases, (it will be observed that γ, δ, and ε are the letters which do not correspond to a or b); also that the surface a b c is the surface D-E and therefore perpendicular to $\delta\,\varepsilon$, etc. Let tetr abcd, trian abc, etc. denote the volume of the tetrahedron or the area of the triangle specified, sin (ab, bc), sin (abc, dbc), sin (abc, ad), etc. the sines of the angles made by the lines and surfaces specified, and [B C D E], [C D E A], etc., the volumes of tetrahedra having edges equal to the tensions of the surfaces between the masses specified. Then, since we may express the volume of a tetrahedron either by $\frac{1}{3}$ of the product of one side, an edge leading to the opposite vertex, and the sine of the angle which these make, or by $\frac{2}{3}$ of the product of two sides divided by the common edge and multiplied by the sine of the included angle,

$$\begin{aligned}
v_A : v_B &:: \text{tetr bcde} : \text{tetr acde} \\
&:: \text{bc sin (bc, cde)} : \text{ac sin (ac, cde)} \\
&:: \sin(\text{ba, ac}) \sin(\text{bc, cde}) : \sin(\text{ab, bc}) \sin(\text{ac, cde}) \\
&:: \sin(\gamma\delta\varepsilon, \beta\delta\varepsilon) \sin(\alpha\delta\varepsilon, \alpha\beta) : \sin(\gamma\delta\varepsilon, \alpha\delta\varepsilon) \sin(\beta\delta\varepsilon, \alpha\beta) \\
&:: \frac{\text{tetr}\,\gamma\beta\delta\varepsilon}{\text{trian}\,\beta\delta\varepsilon} \frac{\text{tetr}\,\beta\alpha\delta\varepsilon}{\text{trian}\,\alpha\delta\varepsilon} : \frac{\text{tetr}\,\gamma\alpha\delta\varepsilon}{\text{trian}\,\alpha\delta\varepsilon} \frac{\text{tetr}\,\alpha\beta\delta\varepsilon}{\text{trian}\,\beta\delta\varepsilon} \\
&:: \text{tetr}\,\gamma\beta\delta\varepsilon : \text{tetr}\,\gamma\alpha\delta\varepsilon \\
&:: [\text{B C D E}] : [\text{C D E A}].
\end{aligned}$$

Hence,

$$v_A : v_B : v_C : v_D :: [\text{B C D E}] : [\text{C D E A}] : [\text{D E A B}] : \text{E A B C}], \tag{641}$$

and (640) may be written

$$p_E = \frac{[\text{B C D E}]p_A + [\text{C D E A}]p_B + [\text{D E A B}]p_C + [\text{E A B C}]p_D}{[\text{B C D E}] + [\text{C D E A}] + [\text{D E A B}] + [\text{E A B C}]}. \tag{642}$$

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If the value of p_E is less than this, when the tensions satisfy the critical relation, the point where vertices of the masses A, B, C, D meet is stable with respect to the formation of any mass of the nature of E. But if the value of p_E is greater, either the masses A, B, C, D cannot meet at a point in equilibrium, or the equilibrium will be at least practically unstable.

When the tensions of the new surfaces are too small to satisfy the critical relation with the other tensions, these surfaces will be convex toward E; when their tensions are too great for that relation, the surfaces will be concave toward E. In the first case, W_V is negative, and the equilibrium of the five masses A, B, C, D, E is stable, but the equilibrium of the four masses A, B, C, D meeting at a point is impossible or at least practically unstable. This is subject to the limitation that when p_E is sufficiently small the mass E which will form will be so small that it may be neglected. This will only be the case when p_E is smaller—in general considerably smaller—than the second number of (642). In the second case, the equilibrium of the five masses A, B, C, D, E will be unstable, but the equilibrium of the four masses A, B, C, D will be stable unless v_E (calculated for the case of the five masses) is of insensible magnitude. This will only be the case when p_E is greater—in general considerably greater—than the second member of (642).

Liquid Films.

When a fluid exists in the form of a thin film between other fluids, the great inequality of its extension in different directions will give rise to certain peculiar properties, even when its thickness is sufficient for its interior to have the properties of matter in mass. The frequent occurrence of such films, and the remarkable properties which they exhibit, entitle them to particular consideration. To fix our ideas, we shall suppose that the film is liquid and that the contiguous fluids are gaseous. The reader will observe our results are not dependent, so far as their general character is concerned, upon this supposition.

Let us imagine the film to be divided by surfaces perpendicular to its sides into small portions of which all the dimensions are of the same order of magnitude as the thickness of the film,—such portions to be called *elements of the film*,—it is evident that far less time will in general be required for the attainment of approximate equilibrium between the different parts of any such element and the other fluids which are immediately contiguous, than for the attainment of equi-

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librium between all the different elements of the film. There will accordingly be a time, commencing shortly after the formation of the film, in which its separate elements may be regarded as satisfying the conditions of internal equilibrium, and of equilibrium with the contiguous gases, while they may not satisfy all the conditions of equilibrium with each other. It is when the changes due to this want of complete equilibrium take place so slowly that the film appears to be at rest, except so far as it accommodates itself to any change in the external conditions to which it is subjected, that the characteristic properties of the film are most striking and most sharply defined.

Let us therefore consider the properties which will belong to a film sufficiently thick for its interior to have the properties of matter in mass, in virtue of the approximate equilibrium of all its elements taken separately, when the matter contained in each element is regarded as invariable, with the exception of certain substances which are components of the contiguous gas-masses and have their potentials thereby determined. The occurrence of a film which precisely satisfies these conditions may be exceptional, but the discussion of this somewhat ideal case will enable us to understand the principal laws which determine the behavior of liquid films in general.

Let us first consider the properties which will belong to each element of the film under the conditions mentioned. Let us suppose the element extended, while the temperature and the potentials which are determined by the contiguous gas-masses are unchanged. If the film has no components except those of which the potentials are maintained constant, there will be no variation of tension in its surfaces. The same will be true when the film has only one component of which the potential is not maintained constant, provided that this is a component of the interior of the film and not of its surface alone. If we regard the thickness of the film as determined by *dividing surfaces* which make the surface-density of this component vanish, the thickness will vary inversely as the area of the element of the film, but no change will be produced in the nature or the tension of its surfaces. If, however, the single component of which the potential is not maintained constant is confined to the surfaces of the film, an extension of the element will generally produce a decrease in the potential of this component, and an increase of tension. This will certainly be true in those cases in which the component shows a tendency to distribute itself with a uniform superficial density.

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When the film has two or more components of which the potentials are not maintained constant by the contiguous gas masses, they will not in general exist in the same proportion in the interior of the film as on its surfaces, but those components which diminish the tensions will be found in greater proportion on the surfaces. When the film is extended, there will therefore not be enough of these substances to keep up the same volume- and surface-densities as before, and the deficiency will cause a certain increase of tension. The value of the *elasticity of the film*, (*i. e.*, the infinitesimal increase of the united tensions of its surfaces divided by the infinitesimal increase of area in a unit of surface), may be calculated from the quantities which specify the nature of the film, when the fundamental equations of the interior mass, of the contiguous gas-masses, and of the two surfaces of discontinuity are known. We may illustrate this by a simple example.

Let us suppose that the two surfaces of a plane film are entirely alike, that the contiguous gas-masses are identical in phase, and that they determine the potentials of all the components of the film except two. Let us call these components S_1 and S_2, the latter denoting that which occurs in greater proportion on the surface than in the interior of the film. Let us denote by γ_1 and γ_2 the densities of these components in the interior of the film, by λ the thickness of the film determined by such dividing surfaces as make the surface-density of S_1 vanish (see page 397), by $\Gamma_{2(1)}$ the surface-density of the other component as determined by the same surfaces, by σ and s the tension and area of one of these surfaces, and by E the elasticity of the film when extended under the supposition that the total quantities of S_1 and S_2 in the part of the film extended are invariable, as also the temperature and the potentials of the other components. From the definition of E we have

$$2\,d\sigma = E\frac{ds}{s}, \tag{643}$$

and from the conditions of the extension of the film

$$\frac{ds}{s} = -\frac{d(\lambda\gamma_1)}{\lambda\gamma_1} = -\frac{d(\lambda\gamma_2 + 2\Gamma_{2(1)})}{\lambda\gamma_2 + 2\Gamma_{2(1)}}. \tag{644}$$

Hence we obtain

$$\lambda\gamma_1\frac{ds}{s} = -\gamma_1\,d\lambda - \lambda\,d\gamma_1,$$

$$(\lambda\gamma_2 + 2\Gamma_{2(1)})\frac{ds}{s} = -\gamma_2\,d\lambda - \lambda\,d\gamma_2 - 2\,d\Gamma_{2(1)};$$

and eliminating $d\lambda$,

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$$2\gamma_1\Gamma_{2(1)}\frac{ds}{s}=-\lambda\gamma_1\,d\gamma_2+\lambda\gamma_2\,d\gamma_1-2\gamma_1\,d\Gamma_{2(1)}. \tag{645}$$

If we set
$$r=\frac{\gamma_2}{\gamma_1}, \tag{646}$$

we have
$$dr=\frac{\gamma_1\,d\gamma_2-\gamma_2\,d\gamma_1}{\gamma_1^{\,2}}, \tag{647}$$

and
$$2\,\Gamma_{2(1)}\frac{ds}{s}=-\lambda\gamma_1\,dr-2\,d\Gamma_{2(1)}. \tag{648}$$

With this equation we may eliminate ds from (643). We may also eliminate $d\sigma$ by the necessary relation [see (514)]

$$d\sigma=-\Gamma_{2(1)}\,d\mu_2.$$

This will give
$$4\,\Gamma_{2(1)}^{\,2}\,d\mu_2=E\,(\lambda\gamma_1\,dr+2\,d\Gamma_{2(1)}), \tag{649}$$

or
$$\frac{4\,\Gamma_{2(1)}^{\,2}}{E}=\lambda\gamma_1\frac{dr}{d\mu_2}+2\frac{d\Gamma_{2(1)}}{d\mu_2}, \tag{650}$$

where the differential coefficients are to be determined on the conditions that the temperature and all the potentials except μ_1 and μ_2 are constant, and that the pressure in the interior of the film shall remain equal to that in the contiguous gas-masses. The latter condition may be expressed by the equation

$$(\gamma_1-\gamma_1')\,d\mu_1+(\gamma_2-\gamma_2')\,d\mu_2=0, \tag{651}$$

in which γ_1' and γ_2' denote the densities of S_1 and S_2 in the contiguous gas-masses. [See (98).] When the tension of the surfaces of the film and the pressures in its interior and in the contiguous gas-masses are known in terms of the temperature and potentials, equation (650) will give the value of E in terms of the same variables together with λ.

If we write G_1 and G_2 for the total quantities of S_1 and S_2 per unit of area of the film, we have

$$G_1=\lambda\gamma_1, \tag{652}$$
$$G_2=\lambda\gamma_2+2\Gamma_{2(1)}. \tag{653}$$

Therefore,

$$G_2=G_1r+2\Gamma_{2(1)},$$
$$\left(\frac{dG_2}{d\mu_2}\right)_{G_1}=\lambda\gamma_1\frac{dr}{d\mu_2}+2\frac{d\Gamma_{2(1)}}{d\mu_2}, \tag{654}$$

where the differential coefficients in the second member are to be determined as in (650), and that in the first member with the additional condition that G_1 is constant. Therefore,

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$$\frac{4\,\Gamma_{2(1)}{}^2}{E} = \left(\frac{dG_2}{d\mu_2}\right)_{G_1},$$

and

$$E = 4\,\Gamma_{2(1)}{}^2 \left(\frac{d\mu_2}{dG_2}\right)_{G_1}, \tag{655}$$

the last differential coefficient being determined by the same conditions as that in the preceding equation. It will be observed that the value of E will be positive in any ordinary case.

These equations give the elasticity of any element of the film when the temperature and the potentials for the substances which are found in the contiguous gas-masses are regarded as constant, and the potentials for the other components, μ_1 and μ_2, have had time to equalize themselves throughout the element considered. The increase of tension immediately after a rapid extension will be greater than that given by these equations.

The existence of this elasticity, which has thus been established from *a priori* considerations, is clearly indicated by the phenomena which liquid films present. Yet it is not to be demonstrated simply by comparing the tensions of films of different thickness, even when they are made from the same liquid, for difference of thickness does not necessarily involve any difference of tension. When the phases within the films as well as without are the same, and the surfaces of the films are also the same, there will be no difference of tension. Nor will the tension of the same film be altered, if a part of the interior drains away in the course of time, without affecting the surfaces. In case the thickness of the film is reduced by evaporation, the tension may be either increased or diminished. (The evaporation of the substance S_1, in the case we have just considered, would diminish the tension.) Yet it may easily be shown that extension increases the tension of a film and contraction diminishes it. When a plane film is held vertically, the tension of the upper portions must evidently be greater than that of the lower. The tensions in every part of the film may be reduced to equality by turning it into a horizontal position. By restoring the original position we may restore the original tensions, or nearly so. It is evident that the same element of the film is capable of supporting very unequal tensions. Nor can this be always attributed to viscosity of the film. For in many cases, if we hold the film nearly horizontal, and elevate first one side and then an other, the lighter portions of the film will dart from one side to the other, so as to show a very striking mobility in the film. The differences of tension which cause these rapid movements are only a very

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small fraction of the difference of tension in the upper and lower portions of the film when held vertically.

If we account for the power of an element of the film to support an increase of tension by viscosity, it will be necessary to suppose that the viscosity offers a resistance to a deformation of the film in which its surface is enlarged and its thickness diminished, which is enormously great in comparison with the resistance to a deformation in which the film is extended in the direction of one tangent and contracted in the direction of another, while its thickness and the areas of its surfaces remain constant. This is not to be readily admitted as a physical explanation, although to a certain extent the phenomena resemble those which would be caused by such a singular viscosity. (See page 439.) The only natural explanation of the phenomena is that the extension of an element of the film, which is the immediate result of an increase of external force applied to its perimeter, causes an increase of its tension, by which it is brought into true equilibrium with the external forces.

The phenomena to which we have referred are such as are apparent to a very cursory observation. In the following experiment, which is described by M. Plateau,* an increased tension is manifested in a film while contracting after a previous extension. The warmth of a finger brought near to a bubble of soap-water with glycerine, which is thin enough to show colors, causes a spot to appear indicating a diminution of thickness. When the finger is removed, the spot returns to its original color. This indicates a contraction, which would be resisted by any viscosity of the film, and can only be due to an excess of tension in the portion stretched on the return of its original temperature.

We have so far supposed that the film is thick enough for its interior to have the properties of matter in mass. Its properties are then entirely determined by those of the three phases and the two surfaces of discontinuity. From these we can also determine, in part at least, the properties of a film at the limit at which the interior ceases to have the properties of matter in mass. The elasticity of the film, which increases with its thinness, cannot of course vanish at that limit, so that the film cannot become unstable with respect to extension and contraction of its elements immediately after passing that limit.

Yet a certain kind of instability will probably arise, which we may

* "Statique expérimentale et théorique des liquides soumis aux seules forces moléculaires," vol. i, p. 294.

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here notice, although it relates to changes in which the condition of the invariability of the quantities of certain components in an element of the film is not satisfied. With respect to variations in the distribution of its components, a film will in general be stable, when its interior has the properties of matter in mass, with the single exception of variations affecting its thickness without any change of phase or of the nature of the surfaces. With respect to this kind of change, which may be brought about by a current in the interior of the film, the equilibrium is neutral. But when the interior ceases to have the properties of matter in mass, it is to be supposed that the equilibrium will generally become unstable in this respect. For it is not likely that the neutral equilibrium will be unaffected by such a change of circumstances, and since the film certainly becomes unstable when it is sufficiently reduced in thickness, it is most natural to suppose that the first effect of diminishing the thickness will be in the direction of instability rather than in that of stability. (We are here considering liquid films between gaseous masses. In certain other cases, the opposite supposition might be more natural, as in respect to a film of water between mercury and air, which would certainly become stable when sufficiently reduced in thickness.)

Let us now return to our former suppositions—that the film is thick enough for the interior to have the properties of matter in mass, and that the matter in each element is invariable, except with respect to those substances which have their potentials determined by the contiguous gas-masses—and consider what conditions are necessary for equilibrium in such a case.

In consequence of the supposed equilibrium of its several elements, such a film may be treated as a simple surface of discontinuity between the contiguous gas-masses (which may be similar or different), whenever its radius of curvature is very large in comparison with its thickness,—a condition which we shall always suppose to be fulfilled. With respect to the film considered in this light, the mechanical conditions of equilibrium will always be satisfied, or very nearly so, as soon as a state of approximate rest is attained, except in those cases in which the film exhibits a decided viscosity. That is, the relations (613), (614), (615) will hold true, when by σ we understand the tension of the film regarded as a simple surface of discontinuity (this is equivalent to the sum of the tensions of the two surfaces of the film), and by Γ its mass per unit of area diminished by the mass of gas which would occupy the same space if the film should be suppressed and the gases should meet at its surface of tension. This

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surface of tension of the film will evidently divide the distance between the surfaces of tension for the two surfaces of the film taken separately, in the inverse ratio of their tensions. For practical purposes, we may regard Γ simply as the mass of the film per unit of area. It will be observed that the terms containing Γ in (613) and (614) are not to be neglected in our present application of these equations.

But the mechanical conditions of equilibrium for the film regarded as an approximately homogeneous mass in the form of a thin sheet bounded by two surfaces of discontinuity are not necessarily satisfied when the film is in a state of apparent rest. In fact, these conditions cannot be satisfied (in any place where the force of gravity has an appreciable intensity) unless the film is horizontal. For the pressure in the interior of the film cannot satisfy simultaneously condition (612), which requires it to vary rapidly with the height z, and condition (613) applied separately to the different surfaces, which makes it a certain mean between the pressures in the adjacent gas-masses. Nor can these conditions be deduced from the general condition of mechanical equilibrium (606) or (611), without supposing that the interior of the film is free to move independently of the surfaces, which is contrary to what we have supposed.

Moreover, the potentials of the various components of the film will not in general satisfy conditions (617), and cannot (when the temperature is uniform) unless the film is horizontal. For if these conditions were satisfied, equation (612) would follow as a consequence. (See page 449.)

We may here remark that such a film as we are considering cannot form any exception to the principle indicated on page 450,—that when a surface of discontinuity which satisfies the conditions of mechanical equilibrium has only one component which is not found in the contiguous masses, and these masses satisfy all the conditions of equilibrium, the potential for the component mentioned must satisfy the law expressed in (617), as a consequence of the condition of mechanical equilibrium (614). Therefore, as we have just seen that it is impossible that all the potentials in a liquid film which is not horizontal should conform to (617) when the temperature is uniform, it follows that if a liquid film exhibits any persistence which is not due to viscosity, or to a horizontal position, or to differences of temperature, it must have more than one component of which the potential is not determined by the contiguous gas-masses in accordance with (617).

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The difficulties of the quantitative experimental verification of the properties which have been described would be very great, even in cases in which the conditions we have imagined were entirely fulfilled. Yet the general effect of any divergence from these conditions will be easily perceived, and when allowance is made for such divergence, the general behavior of liquid films will be seen to agree with the requirements of theory.

The formation of a liquid film takes place most symmetrically when a bubble of air rises to the top of a mass of the liquid. The motion of the liquid, as it is displaced by the bubble, is evidently such as to stretch the two surfaces in which the liquid meets the air, where these surfaces approach one another. This will cause an increase of tension, which will tend to restrain the extension of the surfaces. The extent to which this effect is produced will vary with the nature of the liquid. Let us suppose that the case is one in which the liquid contains one or more components which, although constituting but a very small part of its mass, greatly reduce its tension. Such components will exist in excess on the surfaces of the liquid. In this case the restraint upon the extension of the surfaces will be considerable, and as the bubble of air rises above the general level of the liquid, the motion of the latter will consist largely of a running out from between the two surfaces. But this running out of the liquid will be greatly retarded by its viscosity as soon as it is reduced to the thickness of a film, and the effect of the extension of the surfaces in increasing their tension will become greater and more permanent as the quantity of liquid diminishes which is available for supplying the substances which go to form the increased surfaces.

We may form a rough estimate of the amount of motion which is possible for the interior of a liquid film, relatively to its exterior, by calculating the descent of water between parallel vertical planes at which the motion of the water is reduced to zero. If we use the coefficient of viscosity as determined by Helmholtz and Piotrowski,* we obtain

$$V = 581\, D^2, \tag{656}$$

where V denotes the mean velocity of the water (*i. e.*, that velocity

* Sitzungsberichte der Wiener Akademie, (mathemat.-naturwiss. Classe), B. xl, S. 607. The calculation of formula (656) and that of the factor ($\frac{2}{3}$) applied to the formula of Poiseuille, to adapt it to a current between plane surfaces, have been made by means of the general equations of the motion of a viscous liquid as given in the memoir referred to.

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which, if it were uniform throughout the whole space between the fixed planes, would give the same discharge of water as the actual variable velocity) expressed in millimetres per second, and D denotes the distance in millimetres between the fixed planes, which is supposed to be very small in proportion to their other dimensions. This is for the temperature of 24.5° C. For the same temperature, the experiments of Poiseuille * give

$$V = 337\, D^2$$

for the descent of water in long capillary tubes, which is equivalent to

$$V = 899\, D^2 \tag{657}$$

for descent between parallel planes. The numerical coefficient in this equation differs considerably from that in (656), which is derived from experiments of an entirely different nature, but we may at least conclude that in a film of a liquid which has a viscosity and specific gravity not very different from those of water at the temperature mentioned the mean velocity of the interior relatively to the surfaces will not probably exceed $1000\, D^2$. This is a velocity of $.1^{mm}$ per second for a thickness of $.01^{mm}$, $.06^{mm}$ per *minute* for a thickness of .001 (which corresponds to the red of the fifth order in a film of water), and $.036^{mm}$ per *hour* for a thickness of $.0001^{mm}$ (which corresponds to the white of the first order). Such an internal current is evidently consistent with great persistence of the film, especially in those cases in which the film can exist in a state of the greatest tenuity. On the other hand, the above equations give so large a value of V for thicknesses of 1^{mm} or $.1^{mm}$, that the film can evidently be formed without carrying up any great weight of liquid, and any such thicknesses as these can have only a momentary existence.

A little consideration will show that the phenomenon is essentially of the same nature when films are formed in any other way, as by dipping a ring or the mouth of a cup in the liquid and then withdrawing it. When the film is formed in the mouth of a pipe, it may sometimes be extended so as to form a large bubble. Since the elasticity (*i. e.*, the increase of the tension with extension) is greater in the thinner parts, the thicker parts will be most extended, and the effect of this process (so far as it is not modified by gravity) will be to diminish the ratio of the greatest to the least thickness of the film. During this extension, as well as at other times, the increased elasticity due to imperfect communication of heat, etc., will serve to protect the bubble from fracture by shocks received from the air or the

* Ibid., p. 653; or Mémoires des Savants Étrangers, vol. ix, p. 532.

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pipe. If the bubble is now laid upon a suitable support, the condition (613) will be realized almost instantly. The bubble will then tend toward conformity with condition (614), the lighter portions rising to the top, more or less slowly, according to the viscosity of the film. The resulting difference of thickness between the upper and the lower parts of the bubble is due partly to the greater tension to which the upper parts are subject, and partly to a difference in the matter of which they are composed. When the film has only two components of which the potentials are not determined by the contiguous atmosphere, the laws which govern the arrangement of the elements of the film may be very simply expressed. If we call these components S_1 and S_2, the latter denoting (as on page 469) that which exists in excess at the surface, one element of the film will tend toward the same level with another, or a higher, or a lower level, according as the quantity of S_2 bears the same ratio to the quantity of S_1 in the first element as in the second, or a greater, or a less ratio.

When a film, however formed, satisfies both the conditions (613) and (614), its thickness being sufficient for its interior to have the properties of matter in mass, the interior will still be subject to the slow current which we have already described, if it is truly fluid, however great its viscosity may be. It seems probable, however, that this process is often totally arrested by a certain gelatinous consistency of the mass in question, in virtue of which, although practically fluid in its behavior with reference to ordinary stresses, it may have the properties of a solid with respect to such very small stresses as those which are caused by gravity in the interior of a very thin film which satisfies the conditions (613) and (614).

However this may be, there is another cause which is often more potent in producing changes in a film, when the conditions just mentioned are approximately satisfied, than the action of gravity on its interior. This will be seen if we turn our attention to the edge where the film is terminated. At such an edge we generally find a liquid mass, continuous in phase with the interior of the film, which is bounded by concave surfaces, and in which the pressure is therefore less than in the interior of the film. This liquid mass therefore exerts a strong suction upon the interior of the film, by which its thickness is rapidly reduced. This effect is best seen when a film which has been formed in a ring is held in a vertical position. Unless the film is very viscous, its diminished thickness near the edge causes a rapid upward current on each side, while the central portion slowly

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descends. Also at the bottom of the film, where the edge is nearly horizontal, portions which have become thinned escape from their position of unstable equilibrium beneath heavier portions, and pass upwards, traversing the central portion of the film until they find a position of stable equilibrium. By these processes, the whole film is rapidly reduced in thickness.

The energy of the suction which produces these effects may be inferred from the following considerations. The pressure in the slender liquid mass which encircles the film is of course variable, being greater in the lower portions than in the upper, but it is everywhere less than the pressure of the atmosphere. Let us take a point where the pressure is less than that of the atmosphere by an amount represented by a column of the liquid one centimetre in height. (It is probable that much greater differences of pressure occur.) At a point near by in the interior of the film the pressure is that of the atmosphere. Now if the difference of pressure of these two points were distributed uniformly through the space of one centimetre, the intensity of its action would be exactly equal to that of gravity. But since the change of pressure must take place very suddenly (in a small fraction of a millimetre), its effect in producing a current in a limited space must be enormously great compared with that of gravity.

Since the process just described is connected with the descent of the liquid in the mass encircling the film, we may regard it as another example of the downward tendency of the interior of the film. There is a third way in which this descent may take place, when the principal component of the interior is volatile, viz., through the air. Thus, in the case of a film of soap-water, if we suppose the atmosphere to be of such humidity that the potential for water at a level mid-way between the top and bottom of the film has the same value in the atmosphere as in the film, it may easily be shown that evaporation will take place in the upper portions and condensation in the lower. These processes, if the atmosphere were otherwise undisturbed, would occasion currents of diffusion and other currents, the general effect of which would be to carry the moisture downward. Such a precise adjustment would be hardly attainable, and the processes described would not be so rapid as to have a practical importance.

But when the potential for water in the atmosphere differs considerably from that in the film, as in the case of a film of soap-water in a dry atmosphere, or a film of soap-water with glycerine in a moist

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atmosphere, the effect of evaporation or condensation is not to be neglected. In the first case, the diminution of the thickness of the film will be accelerated, in the second, retarded. In the case of the film containing glycerine, it should be observed that the water condensed cannot in all respects replace the fluid carried down by the internal current but that the two processes together will tend to wash out the glycerine from the film.

But when a component which greatly diminishes the tension of the film, although forming but a small fraction of its mass, (therefore existing in excess at the surface,) is volatile, the effect of evaporation and condensation may be considerable, even when the mean value of the potential for that component is the same in the film as in the surrounding atmosphere. To illustrate this, let us take the simple case of two components S_1 and S_2, as before. (See page 469.) It appears from equation (508) that the potentials must vary in the film with the height z, since the tension does, and from (98) that these variations must (very nearly) satisfy the relation

$$\gamma_1 \frac{d\mu_1}{dz} + \gamma_2 \frac{d\mu_2}{dz} = 0, \tag{658}$$

γ_1 and γ_2 denoting the densities of S_1 and S_2 in the interior of the film. The variation of the potential of S_2 as we pass from one level to another is therefore as much more rapid than that of S_1, as its density in the interior of the film is less. If then the resistances restraining the evaporation, transmission through the atmosphere, and condensation of the two substances are the same, these processes will go on much more rapidly with respect to S_2. It will be observed that the values of $\frac{d\mu_1}{dz}$ and $\frac{d\mu_2}{dz}$ will have opposite signs, the tendency of S_1 being to pass down through the atmosphere, and that of S_2 to pass up. Moreover, it may easily be shown that the evaporation or condensation of S_2 will produce a very much greater effect than the evaporation or condensation of the same quantity of S_1. These effects are really of the same kind. For if condensation of S_2 takes place at the top of the film, it will cause a diminution of tension, and thus occasion an extension of this part of the film, by which its thickness will be reduced, as it would be by evaporation of S_1. We may infer that it is a general condition of the persistence of liquid films, that the substance which causes the diminution of tension in the upper parts of the film must not be volatile.

But apart from any action of the atmosphere, we have seen that a

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film which is truly fluid in its interior is in general subject to a continual diminution of thickness by the internal currents due to gravity and the suction at its edge. Sooner or later, the interior will somewhere cease to have the properties of matter in mass. The film will then probably become unstable with respect to a flux of the interior (see page 473), the thinnest parts tending to become still more thin (apart from any external cause) very much as if there were an attraction between the surfaces of the film, insensible at greater distances, but becoming sensible when the thickness of the film is sufficiently reduced. We should expect this to determine the rupture of the film, and such is doubtless the case with most liquids. In a film of soap-water, however, the rupture does not take place, and the processes which go on can be watched. It is apparent even to a very superficial observation that a film of which the tint is approaching the black exhibits a remarkable instability. The continuous change of tint is interrupted by the breaking out and rapid extension of black spots. That in the formation of these bright spots a separation of different substances takes place, and not simply an extension of a part of the film, is shown by the fact that the film is made thicker at the edge of these spots.

This is very distinctly seen in a plane vertical film, when a single black spot breaks out and spreads rapidly over a considerable area which was before of a nearly uniform tint approaching the black. The edge of the black spot as it spreads is marked as it were by a string of bright beads, which unite together on touching, and thus becoming larger, glide down across the bands of color below. Under favorable circumstances, there is often quite a shower of these bright spots. They are evidently small spots very much thicker—apparently many times thicker—than the part of the film out of which they are formed. Now if the formation of the black spots were due to a simple extension of the film, it is evident that no such appearance would be presented. The thickening of the edge of the film cannot be accounted for by *contraction.* For an extension of the upper portion of the film and contraction of the lower and thicker portion, with descent of the intervening portions, would be far less resisted by viscosity, and far more favored by gravity than such extensions and contractions as would produce the appearances described. But the rapid formation of a thin spot by an internal current would cause an accumulation at the edge of the spot of the material forming the interior of the film, and necessitate a thickening of the film in that place.

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That which is most difficult to account for in the formation of the black spots is the arrest of the process by which the film grows thinner. It seems most natural to account for this, *if possible*, by passive resistance to motion due to a very viscous or gelatinous condition of the film. For it does not seem likely that the film, after becoming unstable by the flux of matter from its interior, would become stable (without the support of such resistance) by a continuance of the same process. On the other hand, gelatinous properties are very marked in soap-water which contains somewhat more soap than is best for the formation of films, and it is entirely natural that, even when such properties are wanting in the interior of a mass or thick film of a liquid, they may still exist in the immediate vicinity of the surface (where we know that the soap or some of its components exists in excess), or throughout a film which is so thin that the interior has ceased to have the properties of matter in mass.* But these considerations do not amount to any *a priori* probability of an arrest of the tendency toward an internal current between adjacent elements of a black spot which may differ slightly in thickness, in time to prevent rupture of the film. For, in a thick film, the increase of the tension with the extension, which is necessary for its stability with respect to extension, is connected with an excess of the soap (or of some of its components) at the surface as compared with the interior of the film. With respect to the black spots, although the interior has ceased to have the properties of matter in mass, and any quantitative determinations derived from the surfaces of a mass of the liquid will not be applicable, it is natural to account for the stability with reference to extension by supposing that the same general difference of composition still exists. If therefore we account for the arrest of internal currents by the increasing density of soap or some of its components in the interior of the film, we must still suppose that the characteristic difference of composition in the interior and surface of the film has not been obliterated.

The preceding discussion relates to liquid films between masses of gas. Similar considerations will apply to liquid films between other liquids or between a liquid and a gas, and to films of gas between

* The experiments of M. Plateau (chapter VII of the work already cited) show that this is the case to a very remarkable degree with respect to a solution of saponine. With respect to soap-water, however, they do not indicate any greater superficial viscosity than belongs to pure water. But the resistance to an internal current, such as we are considering, is not necessarily measured by the resistance to such motions as those of the experiments referred to.

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masses of liquid. The latter may be formed by gently depositing a liquid drop upon the surface of a mass of the same or a different liquid. This may be done (with suitable liquids) so that the continuity of the air separating the liquid drop and mass is not broken, but a film of air is formed, which, if the liquids are similar, is a counterpart of the liquid film which is formed by a bubble of air rising to the top of a mass of the liquid. (If the bubble has the same volume as the drop, the films will have precisely the same form, as well as the rest of the surfaces which bound the bubble and the drop.) Sometimes, when the weight and momentum of the drop carry it through the surface of the mass on which it falls, it appears surrounded by a complete spherical film of air, which is the counterpart on a small scale of a soap-bubble hovering in air.* Since, however, the substance to which the necessary differences of tension in the film are mainly due is a component of the liquid masses on each side of the air film, the necessary differences of the potential of this substance cannot be permanently maintained, and these films have little persistence compared with films of soap-water in air. In this respect, the case of these air-films is analogous to that of liquid films in an atmosphere containing substances by which their tension is greatly reduced. Compare page 479.

Surfaces of Discontinuity between Solids and Fluids.

We have hitherto treated of surfaces of discontinuity on the supposition that the contiguous masses are fluid. This is by far the most simple case for any rigorous treatment, since the masses are necessarily isotropic both in nature and in their state of strain. In this case, moreover, the mobility of the masses allows a satisfactory experimental verification of the mechanical conditions of equilibrium. On the other hand, the rigidity of solids is in general so great, that any tendency of the surfaces of discontinuity to variation in area or form may be neglected in comparison with the forces which are produced in the interior of the solids by any sensible strains, so that it is not generally necessary to take account of the surfaces of discontinuity in determining the state of strain of solid masses. But we must take account of the nature of the surfaces of discontinuity

* These spherical air-films are easily formed in soap-water. They are distinguishable from ordinary air-bubbles by their general behavior and by their appearance. The two concentric spherical surfaces are distinctly seen, the diameter of one appearing to be about three-quarters as large as that of the other. This is of course an optical illusion, depending upon the index of refraction of the liquid.

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between solids and fluids with reference to the tendency toward solidification or dissolution at such surfaces, and also with reference to the tendencies of different fluids to spread over the surfaces of solids.

Let us therefore consider a surface of discontinuity between a fluid and a solid, the latter being either isotropic or of a continuous crystalline structure, and subject to any kind of stress compatible with a state of mechanical equilibrium with the fluid. We shall not exclude the case in which substances foreign to the contiguous masses are present in small quantities at the surface of discontinuity, but we shall suppose that the nature of this surface (*i. e.*, of the non-homogeneous film between the approximately homogeneous masses), is entirely determined by the nature and state of the masses which it separates, and the quantities of the foreign substances which may be present. The notions of the *dividing surface*, and of the *superficial densities* of energy, entropy, and the several components, which we have used with respect to surfaces of discontinuity between fluids (see pages 380 and 386), will evidently apply without modification to the present case. We shall use the suffix $_1$ with reference to the substance of the solid, and shall suppose the dividing surface to be determined so as to make the superficial density of this substance vanish. The superficial densities of energy, of entropy, and of the other component substances may then be denoted by our usual symbols (see page 397),

$$\varepsilon_{s(1)},\quad \eta_{s(1)},\quad \Gamma_{2(1)},\quad \Gamma_{3(1)},\quad \text{etc.}$$

Let the quantity σ be defined by the equation

$$\sigma = \varepsilon_{s(1)} - t\,\eta_{s(1)} - \mu_2\,\Gamma_{2(1)} - \mu_3\,\Gamma_{3(1)} - \text{etc.}, \tag{659}$$

in which t denotes the temperature, and μ_2, μ_3, etc. the potentials for the substances specified at the surface of discontinuity.

As in the case of two fluid masses, (see page 421,) we may regard σ as expressing the work spent in forming a unit of the surface of discontinuity—under certain conditions, which we need not here specify—but it cannot properly be regarded as expressing the tension of the surface. The latter quantity depends upon the work spent in *stretching* the surface, while the quantity σ depends upon the work spent in *forming* the surface. With respect to perfectly fluid masses, these processes are not distinguishable, unless the surface of discontinuity has components which are not found in the contiguous masses, and even in this case, (since the surface must be supposed to be formed out of matter supplied at the same potentials which belong to the matter in the surface,) the work spent in increasing the surface infinitesi-

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mally by stretching is identical with that which must be spent in forming an equal infinitesimal amount of new surface. But when one of the masses is solid, and its states of strain are to be distinguished, there is no such equivalence between the stretching of the surface and the forming of new surface.*

With these preliminary notions, we now proceed to discuss the condition of equilibrium which relates to the dissolving of a solid at the surface where it meets a fluid, when the thermal and mechanical conditions of equilibrium are satisfied. It will be necessary for us to consider the case of isotropic and of crystallized bodies separately, since in the former the value of σ is independent of the direction of the surface, except so far as it may be influenced by the state of strain of the solid, while in the latter the value of σ varies greatly with the direction of the surface with respect to the axes of crystallization, and in such a manner as to have a large number of sharply defined minima.† This may be inferred from the phenomena which crystalline bodies present, as will appear more distinctly in the following discussion. Accordingly, while a variation in the direction of an

* This will appear more distinctly if we consider a particular case. Let us consider a thin plane sheet of a crystal in a vacuum (which may be regarded as a limiting case of a very attenuated fluid), and let us suppose that the two surfaces of the sheet are alike. By applying the proper forces to the edges of the sheet, we can make all stress vanish in its interior. The *tensions* of the two surfaces, are in equilibrium with these forces, and are measured by them. But the tensions of the surfaces, thus determined, may evidently have different values in different directions, and are entirely different from the quantity which we denote by σ, which represents the work required to form a unit of the surface by any reversible process, and is not connected with any idea of direction.

In certain cases, however, it appears probable that the values of σ and of the superficial tension will not greatly differ. This is especially true of the numerous bodies which, although generally (and for many purposes properly) regarded as solids, are really very viscous fluids. Even when a body exhibits no fluid properties at its actual temperature, if its surface has been formed at a higher temperature, at which the body was fluid, and the change from the fluid to the solid state has been by insensible gradations, we may suppose that the value of σ coincided with the superficial tension until the body was decidedly solid, and that they will only differ so far as they may be differently affected by subsequent variations of temperature and of the stresses applied to the solid. Moreover, when an amorphous solid is in a state of equilibrium with a solvent, although it may have no fluid properties in its interior, it seems not improbable that the particles at its surface, which have a greater degree of mobility, may so arrange themselves that the value of σ will coincide with the superficial tension, as in the case of fluids.

† The differential coefficients of σ with respect to the direction-cosines of the surface appear to be discontinuous functions of the latter quantities.

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element of the surface may be neglected (with respect to its effect on the value of σ) in the case of isotropic solids, it is quite otherwise with crystals. Also, while the surfaces of equilibrium between fluids and soluble isotropic solids are without discontinuities of direction, being in general curved, a crystal in a state of equilibrium with a fluid in which it can dissolve is bounded in general by a broken surface consisting of sensibly plane portions.

For isotropic solids, the conditions of equilibrium may be deduced as follows. If we suppose that the solid is unchanged, except that an infinitesimal portion is dissolved at the surface where it meets the fluid, and that the fluid is considerable in quantity and remains homogeneous, the increment of energy in the vicinity of the surface will be represented by the expression

$$\int [\varepsilon_v' - \varepsilon_v'' + (c_1 + c_2)\, \varepsilon_{S(1)}]\, \delta N\, Ds$$

where Ds denotes an element of the surface, δN the variation in its position (measured normally, and regarded as *negative* when the solid is dissolved), c_1 and c_2 its principal curvatures (positive when their centers lie on the same side as the solid), $\varepsilon_{S(1)}$ the surface-density of energy, ε_v' and ε_v'' the volume-densities of energy in the solid and fluid respectively, and the sign of integration relates to the elements Ds. In like manner, the increments of entropy and of the quantities of the several components in the vicinity of the surface will be

$$\int [\eta_v' - \eta_v'' + (c_1 + c_2)\, \eta_{S(1)}]\, \delta N\, Ds,$$
$$\int [\gamma_1' - \gamma_1'']\, \delta N\, Ds,$$
$$\int [-\gamma_2'' + (c_1 + c_2)\, \Gamma_{2(1)}]\, \delta N\, Ds,$$
etc.

The entropy and the matter of different kinds represented by these expressions we may suppose to be derived from the fluid mass. These expressions, therefore, with a change of sign, will represent the increments of entropy and of the quantities of the components in the whole space occupied by the fluid except that which is immediately contiguous to the solid. Since this space may be regarded as constant, the increment of energy in this space may be obtained [according to equation (12)] by multiplying the above expression relating to entropy by $-t$, and those relating to the components by $-\mu_1''$, $-\mu_2$, etc.,* and taking the sum. If to this

* The potential μ_1'' is marked by double accents in order to indicate that its value is to be determined in the fluid mass, and to distinguish it from the potential μ_1'

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we add the above expression for the increment of energy near the surface, we obtain the increment of energy for the whole system. Now by (93) we have

$$p'' = -\varepsilon_v'' + t\eta_v'' + \mu_1''\gamma_1'' + \mu_2\gamma_2'' + \text{etc.}$$

By this equation and (659), our expression for the total increment of energy in the system may be reduced to the form

$$\int[\varepsilon_v' - t\eta_v' - \mu_1''\gamma_1' + p'' + (c_1+c_2)\sigma]\,\delta N Ds. \qquad (660)$$

In order that this shall vanish for any values of δN, it is necessary that the coefficient of $\delta N Ds$ shall vanish. This gives for the condition of equilibrium

$$\mu_1'' = \frac{\varepsilon_v' - t\eta_v' + p'' + (c_1+c_2)\sigma}{\gamma_1'}. \qquad (661)$$

This equation is identical with (387), with the exception of the term containing σ, which vanishes when the surface is plane.*

We may also observe that when the solid has no stresses except an isotropic pressure, if the quantity represented by σ is equal to the true tension of the surface, $p'' + (c_1 + c_2)\sigma$ will represent the pressure in the interior of the solid, and the second member of the equation will represent [see equation (93)] the value of the potential in the solid for the substance of which it consists. In this case, therefore, the equation reduces to

$$\mu_1'' = \mu_1',$$

that is, it expresses the equality of the potentials for the substance of the solid in the two masses—the same condition which would subsist if both masses were fluid.

Moreover, the compressibility of all solids is so small that, although σ may not represent the true tension of the surface, nor $p'' + (c_1 + c_2)\sigma$ the true pressure in the solid when its stresses are isotropic, the quantities ε_v' and η_v' if calculated for the pressure $p'' + (c_1 + c_2)\sigma$ with the actual temperature will have sensibly the same values as if calculated for the true pressure of the solid. Hence, the second member

relating to the solid mass (when this is in a state of isotropic stress), which, as we shall see, may not always have the same value. The other potentials μ_2, etc., have the same values as in (659), and consist of two classes, one of which relates to substances which are components of the fluid mass, (these might be marked by the double accents,) and the other relates to substances found only at the surface of discontinuity. The expressions to be multiplied by the potentials of this latter class all have the value zero.

* In equation (387), the density of the solid is denoted by Γ, which is therefore equivalent to γ_1' in (661).

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of equation (661), when the stresses of the solid are sensibly isotropic, is sensibly equal to the potential of the same body at the same temperature but with the pressure $p'' + (c_1 + c_2)\sigma$, and the condition of equilibrium with respect to dissolving for a solid of isotropic stresses may be expressed with sufficient accuracy by saying that the potential for the substance of the solid in the fluid must have this value. In like manner, when the solid is not in a state of isotropic stress, the difference of the two pressures in question will not sensibly affect the values of ε_v' and η_v', and the value of the second member of the equation may be calculated as if $p'' + (c_1 + c_2)\sigma$ represented the true pressure in the solid in the direction of the normal to the surface. Therefore, if we had taken for granted that the quantity σ represents the tension of a surface between a solid and a fluid, as it does when both masses are fluid, this assumption would not have led us into any practical error in determining the value of the potential μ_1'' which is necessary for equilibrium. On the other hand, if in the case of any amorphous body the value of σ differs notably from the true surface-tension, the latter quantity substituted for σ in (661) will make the second member of the equation equal to the true value of μ_1', when the stresses are isotropic, but this will not be equal to the value of μ_1'' in case of equilibrium, unless $c_1 + c_2 = 0$.

When the stresses in the solid are not isotropic, equation (661) may be regarded as expressing the condition of equilibrium with respect to the dissolving of the solid, and is to be distinguished from the condition of equilibrium with respect to an increase of solid matter, since the new matter would doubtless be deposited in a state of isotropic stress. (The case would of course be different with crystalline bodies, which are not considered here.) The value of μ_1'' necessary for equilibrium with respect to the formation of new matter is a little less than that necessary for equilibrium with respect to the dissolving of the solid. In regard to the actual behavior of the solid and fluid, all that the theory enables us to predict with certainty is that the solid will not dissolve if the value of the potential μ_1'' is greater than that given by the equation for the solid with its distorting stresses, and that new matter will not be formed if the value of μ_1'' is less than the same equation would give for the case of the solid with isotropic stresses.* It seems probable, however, that

* The possibility that the new solid matter might differ in composition from the original solid is here left out of account. This point has been discussed on pages 134–137, but without reference to the state of strain of the solid or the influence of the curvature of the surface of discontinuity. The statement made above may be

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if the fluid in contact with the solid is not renewed, the system will generally find a state of equilibrium in which the outermost portion of the solid will be in a state of isotropic stress. If at first the solid should dissolve, this would supersaturate the fluid, perhaps until a state is reached satisfying the condition of equilibrium with the stressed solid, and then, if not before, a deposition of solid matter in a state of isotropic stress would be likely to commence and go on until the fluid is reduced to a state of equilibrium with this new solid matter.

The action of gravity will not affect the nature of the condition of equilibrium for any single point at which the fluid meets the solid, but it will cause the values of p'' and μ_1'' in (661) to vary according to the laws expressed by (612) and (617). If we suppose that the outer part of the solid is in a state of isotropic stress, which is the most important case, since it is the only one in which the equilibrium is in every sense stable, we have seen that the condition (661) is at least sensibly equivalent to this:—that the potential for the substance of the solid which would belong to the solid mass at the temperature t and the pressure $p''+(c_1+c_2)\sigma$ must be equal to μ_1''. Or, if we denote by (p') the pressure belonging to solid with the temperature t and the potential equal to μ_1'', the condition may be expressed in the form

$$(p')=p''+(c_1+c_2)\sigma. \tag{662}$$

Now if we write γ'' for the total density of the fluid, we have by (612)

$$dp''=-g\gamma''dz.$$

By (98) $$d(p')=\gamma_1'\,d\mu_1'',$$

and by (617) $$d\mu_1''=-g\,dz;$$

whence $$d(p')=-g\gamma_1'\,dz.$$

Accordingly we have

$$d(p')-dp''=g(\gamma''-\gamma_1')\,dz,$$

and

$$(p')-p''=g(\gamma''-\gamma_1')z,$$

z being measured from the horizontal plane for which $(p')=p''$. Substituting this value in (662), we obtain

$$c_1+c_2=\frac{g(\gamma''-\gamma_1')}{\sigma}z, \tag{663}$$

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generalized so as to hold true of the formation of new solid matter of any kind on the surface as follows:—that new solid matter of any kind will not be formed upon the surface (with more than insensible thickness), if the second member of (661) calculated for such new matter is greater than the potential in the fluid for such matter.

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precisely as if both masses were fluid, and σ denoted the tension of their common surface, and (p') the true pressure in the mass specified. [Compare (619).]

The obstacles to an exact experimental realization of these relations are very great, principally from the want of absolute uniformity in the internal structure of amorphous solids, and on account of the passive resistances to the processes which are necessary to bring about a state satisfying the conditions of theoretical equilibrium, but it may be easy to verify the general tendency toward diminution of surface, which is implied in the foregoing equations.*

Let us apply the same method to the case in which the solid is a crystal. The surface between the solid and fluid will now consist of plane portions, the directions of which may be regarded as invari-

* It seems probable that a tendency of this kind plays an important part in some of the phenomena which have been observed with respect to the freezing together of pieces of ice. (See especially Professor Faraday's "Note on Regelation" in the *Proceedings of the Royal Society*, vol. x, p. 440; or in the *Philosophical Magazine*, 4th ser., vol. xxi, p. 146.) Although this is a body of crystalline structure, and the action which takes place is doubtless influenced to a certain extent by the directions of the axes of crystallization, yet, since the phenomena have not been observed to depend upon the orientation of the pieces of ice, we may conclude that the effect, so far as its general character is concerned, is such as might take place with an isotropic body. In other words, for the purposes of a general explanation of the phenomena we may neglect the differences in the values of σ_{IW} (the suffixes are used to indicate that the symbol relates to the surface between ice and water) for different orientations of the axes of crystallization, and also neglect the influence of the surface of discontinuity with respect to crystalline structure, which must be formed by the freezing together of the two masses of ice when the axes of crystallization in the two masses are not similarly directed. In reality, this surface—or the necessity of the formation of such a surface if the pieces of ice freeze together—must exert an influence adverse to their union, measured by a quantity σ_{II}, which is determined for this surface by the same principles as when one of two contiguous masses is fluid, and varies with the orientations of the two systems of crystallographic axes relatively to each other and to the surface. But under the circumstances of the experiment, since we may neglect the possibility of the two systems of axes having precisely the same directions, this influence is probably of a tolerably constant character, and is evidently not sufficient to alter the general nature of the result. In order wholly to prevent the tendency of pieces of ice to freeze together, when meeting in water with curved surfaces and without pressure, it would be necessary that $\sigma_{II} \geqq 2\sigma_{IW}$, except so far as the case is modified by passive resistances to change, and by the inequality in the values of σ_{II} and σ_{IW} for different directions of the axes of crystallization.

It will be observed that this view of the phenomena is in harmony with the opinion of Professor Faraday. With respect to the union of pieces of ice as an indirect consequence of pressure, see page 198 of volume xi of the *Proceedings of the Royal Society*; or the *Philosophical Magazine*, 4th ser., vol. xxiii, p. 407.

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able. If the crystal grows on one side a distance δN, without other change, the increment of energy in the vicinity of the surface will be

$$(\varepsilon_v' - \varepsilon_v'')\, s\, \delta N + \Sigma'(\varepsilon_{s(1)}'\, l' \operatorname{cosec} \omega' - \varepsilon_{s(1)}\, l' \cot \omega')\, \delta N,$$

where ε_v' and ε_v'' denote the volume-densities of energy in the crystal and fluid respectively, s the area of the side on which the crystal grows, $\varepsilon_{s(1)}$ the surface-density of energy on that side, $\varepsilon_{s(1)}'$ the surface-density of energy on an adjacent side, ω' the external angle of these two sides, l' their common edge, and the symbol Σ' a summation with respect to the different sides adjacent to the first. The increments of entropy and of the quantities of the several components will be represented by analogous formulæ, and if we deduce as on pages 485, 486 the expression for the increase of energy in the whole system due to the growth of the crystal without change of the total entropy or volume, and set this expression equal to zero, we shall obtain for the condition of equilibrium

$$(\varepsilon_v' - t\,\eta_v' - \mu_1''\,\gamma_1' + p'')\, s\, \delta N$$
$$+ \Sigma'(\sigma'\, l' \operatorname{cosec} \omega' - \sigma\, l' \cot \omega')\, \delta N = 0, \qquad (664)$$

where σ and σ' relate respectively to the same sides as $\varepsilon_{s(1)}$ and $\varepsilon_{s(1)}'$ in the preceding formula. This gives

$$\mu_1'' = \frac{\varepsilon_v' - t\,\eta_v' + p''}{\gamma_1'} + \frac{\Sigma'(\sigma'\, l' \operatorname{cosec} \omega' - \sigma\, l' \cot \omega')}{s\,\gamma_1'}. \qquad (665)$$

It will be observed that unless the side especially considered is small or narrow, we may neglect the second fraction in this equation, which will then give the same value of μ_1'' as equation (387), or as equation (661) applied to a plane surface.

Since a similar equation must hold true with respect to every other side of the crystal of which the equilibrium is not affected by meeting some other body, the condition of equilibrium for the crystalline form (when unaffected by gravity) is that the expression

$$\frac{\Sigma'(\sigma'\, l' \operatorname{cosec} \omega' - \sigma\, l' \cot \omega')}{s} \qquad (666)$$

shall have the same value for each side of the crystal. (By the value of this expression for any side of the crystal is meant its value when σ and s are determined by that side and the other quantities by the surrounding sides in succession in connection with the first side.) This condition will not be affected by a change in the size of a crystal while its proportions remain the same. But the tendencies of similar crystals toward the form required by this condition, as measured by the inequalities in the composition or the temperature of

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the surrounding fluid which would counterbalance them, will be inversely as the linear dimensions of the crystals, as appears from the preceding equation.

If we write v for the volume of a crystal, and $\Sigma(\sigma s)$ for the sum of the areas of all its sides multiplied each by the corresponding value of σ, the numerator and denominator of the fraction (666), multiplied each by δN, may be represented by $\delta\Sigma(\sigma s)$ and δv respectively. The value of the fraction is therefore equal to that of the differential coefficient

$$\frac{d\Sigma(\sigma s)}{dv}$$

as determined by the displacement of a particular side while the other sides are fixed. The condition of equilibrium for the form of a crystal (when the influence of gravity may be neglected) is that the value of this differential coefficient must be independent of the particular side which is supposed to be displaced. For a constant volume of the crystal, $\Sigma(\sigma s)$ has therefore a minimum value when the condition of equilibrium is satisfied, as may easily be proved more directly.

When there are no foreign substances at the surfaces of the crystal, and the surrounding fluid is indefinitely extended, the quantity $\Sigma(\sigma s)$ represents the work required to form the surfaces of the crystal, and the coefficient of $s\,\delta N$ in (664) with its sign reversed represents the work gained in forming a mass of volume unity like the crystal but regarded as without surfaces. We may denote the work required to form the crystal by

$$W_s - W_v,$$

W_s denoting the work required to form the surfaces [*i. e.*, $\Sigma(\sigma s)$], and W_v the work gained in forming the mass as distinguished from the surfaces. Equation (664) may then be written

$$-\delta W_v + \Sigma(\sigma\,\delta s) = 0. \qquad (667)$$

Now (664) would evidently continue to hold true if the crystal were diminished in size, remaining similar to itself in form and in nature, if the values of σ in all the sides were supposed to diminish in the same ratio as the linear dimensions of the crystal. The variation of W_s would then be determined by the relation

$$d\,W_s = d\Sigma(\sigma s) = \tfrac{3}{2}\,\Sigma(\sigma\,ds),$$

and that of W_v by (667). Hence,

$$d\,W_s = \tfrac{3}{2}\,d\,W_v,$$

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and, since W_s and W_v vanish together,

$$W_s = \tfrac{3}{2} W_v,$$

$$W_s - W_v = \tfrac{1}{3} W_s = \tfrac{1}{2} W_v, \tag{668}$$

—the same relation which we have before seen to subsist with respect to a spherical mass of fluid as well as in other cases. (See pages 421, 425, 465.)

The equilibrium of the crystal is unstable with respect to variations in size when the surrounding fluid is indefinitely extended, but it may be made stable by limiting the quantity of the fluid.

To take account of the influence of gravity, we must give to μ_1'' and p'' in (665) their average values in the side considered. These coincide (when the fluid is in a state of internal equilibrium) with their values at the center of gravity of the side. The values of γ_1', ε_v', η_v' may be regarded as constant, so far as the influence of gravity is concerned. Now since by (612) and (617)

$$dp'' = -g\,\gamma''\,dz,$$

and

$$d\mu_1'' = -g\,dz,$$

we have

$$d(\gamma_1'\mu_1'' - p'') = g\,(\gamma'' - \gamma_1')\,dz.$$

Comparing (664), we see that the upper or the lower faces of the crystal will have the greater tendency to grow, (other things being equal,) according as the crystal is lighter or heavier than the fluid. When the densities of the two masses are equal, the effect of gravity on the form of the crystal may be neglected.

In the preceding paragraph the fluid is regarded as in a state of internal equilibrium. If we suppose the composition and temperature of the fluid to be uniform, the condition which will make the effect of gravity vanish will be that

$$\frac{d(\gamma_1'\mu_1'' - p'')}{dz} = 0,$$

when the value of the differential coefficient is determined in accordance with this supposition. This condition reduces to

$$\left(\frac{d\mu_1}{dp}\right)''_{t,\,m} = \frac{1}{\gamma_1'},*$$

which, by equation (92), is equivalent to

$$\left(\frac{dv}{dm_1}\right)''_{t,\,p,\,m} = \frac{1}{\gamma_1'}. \tag{669}$$

* A suffixed m is used to represent all the symbols m_1, m_2, etc., except such as may occur in the differential coefficient.

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The tendency of a crystal to grow will be greater in the upper or lower parts of the fluid, according as the growth of a crystal at constant temperature and pressure will produce expansion or contraction.

Again, we may suppose the composition of the fluid and its entropy per unit of mass to be uniform. The temperature will then vary with the pressure, that is, with z. We may also suppose the temperature of different crystals or different parts of the same crystal to be determined by the fluid in contact with them. These conditions express a state which may perhaps be realized when the fluid is gently stirred. Owing to the differences of temperature we cannot regard ε_v' and η_v' in (664) as constant, but we may regard their variations as subject to the relation $d\varepsilon_v' = t\, d\eta_v'$. Therefore, if we make $\eta_v' = 0$ for the mean temperature of the fluid, (which involves no real loss of generality,) we may treat $\varepsilon_v' - t\,\eta_v'$ as constant. We shall then have for the condition that the effect of gravity shall vanish—

$$\frac{d(\gamma_1'\,\mu_1'' - p'')}{dz} = 0,$$

which signifies in the present case that

$$\left(\frac{d\mu_1}{dp}\right)''_{\eta,\,m} = \frac{1}{\gamma_1'},$$

or, by (90),

$$\left(\frac{dv}{dm_1}\right)''_{\eta,\,p,\,m} = \frac{1}{\gamma_1'}. \tag{670}$$

Since the entropy of the crystal is zero, this equation expresses that the dissolving of a small crystal in a considerable quantity of the fluid will produce neither expansion nor contraction, when the pressure is maintained constant and no heat is supplied or taken away.

The manner in which crystals actually grow or dissolve is often principally determined by other differences of phase in the surrounding fluid than those which have been considered in the preceding paragraph. This is especially the case when the crystal is growing or dissolving rapidly. When the great mass of the fluid is considerably supersaturated, the action of the crystal keeps the part immediately contiguous to it nearer the state of exact saturation. The farthest projecting parts of the crystal will therefore be most exposed to the action of the supersaturated fluid, and will grow most rapidly. The same parts of a crystal will dissolve most rapidly in a fluid considerably below saturation.*

* See O. Lehmann "Ueber das Wachsthum der Krystalle," *Zeitschrift für Krystallographie und Mineralogie*, Bd. i, S. 453; or the review of the paper in Wiedemann's *Beiblätter*, Bd. ii, S. 1.

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But even when the fluid is supersaturated only so much as is necessary in order that the crystal shall grow at all, it is not to be expected that the form in which $\Sigma(\sigma s)$ has a minimum value (or such a modification of that form as may be due to gravity or to the influence of the body supporting the crystal) will always be the ultimate result. For we cannot imagine a body of the internal structure and external form of a crystal to grow or dissolve by an entirely continuous process, or by a process in the same sense continuous as condensation or evaporation between a liquid and gas, or the corresponding processes between an amorphous solid and a fluid. The process is rather to be regarded as periodic, and the formula (664) cannot properly represent the true value of the quantities intended unless δN is equal to the distance between two successive layers of molecules in the crystal, or a multiple of that distance. Since this can hardly be treated as an infinitesimal, we can only conclude with certainty that sensible changes cannot take place for which the expression (664) would have a positive value.*

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* That it is necessary that certain relations shall be precisely satisfied in order that equilibrium may subsist between a liquid and gas with respect to evaporation, is explained (see Clausius "Ueber die Art der Bewegung, welche wir Wärme nennen," *Pogg. Ann.*, Bd. c, S. 353; or *Abhand. über die mech. Wärmetheorie*, XIV,) by supposing that a passage of individual molecules from the one mass to the other is continually taking place, so that the slightest circumstance may give the preponderance to the passage of matter in either direction. The same supposition may be applied, at least in many cases, to the equilibrium between amorphous solids and fluids. Also in the case of crystals in equilibrium with fluids, there may be a passage of individual molecules from one mass to the other, so as to cause insensible fluctuations in the mass of the solid. If these fluctuations are such as to cause the occasional deposit or removal of a whole layer of particles, the least cause would be sufficient to make the probability of one kind of change prevail over that of the other, and it would be necessary for equilibrium that the theoretical conditions deduced above should be precisely satisfied. But this supposition seems quite improbable, except with respect to a very small side.

The following view of the molecular state of a crystal when in equilibrium with respect to growth or dissolution appears as probable as any. Since the molecules at the corners and edges of a perfect crystal would be less firmly held in their places than those in the middle of a side, we may suppose that when the condition of theoretical equilibrium (665) is satisfied several of the outermost layers of molecules on each side of the crystal are incomplete toward the edges. The boundaries of these imperfect layers probably fluctuate, as individual molecules attach themselves to the crystal or detach themselves, but not so that a layer is entirely removed (on any side of considerable size), to be restored again simply by the irregularities of the motions of the individual molecules. Single molecules or small groups of molecules may indeed attach themselves to the side of the crystal but they will speedily be dislodged, and if any molecules are thrown out from the middle of a surface, these deficiencies

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Let us now examine the special condition of equilibrium which relates to a line at which three different masses meet, when one or more of these masses is solid. If we apply the method of page 685 to a system containing such a line, it is evident that we shall obtain in the expression corresponding to (660), beside the integral relating to the surfaces, a term of the form

$$\int \Sigma(\sigma\, \delta T)\, Dl$$

to be interpreted as the similar term in (611), except so far as the definition of σ has been modified in its extension to solid masses. In order that this term shall be incapable of a negative value it is neces-

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will also soon be made good; nor will the frequency of these occurrences be such as greatly to affect the general smoothness of the surfaces, except near the edges where the surfaces fall off somewhat, as before described. Now a continued growth on any side of a crystal is impossible unless new layers can be formed. This will require a value of μ_1'' which may exceed that given by equation (665) by a finite quantity. Since the difficulty in the formation of a new layer is at or near the commencement of the formation, the necessary value of μ_1'' may be independent of the area of the side, except when the side is very small. The value of μ_1'' which is necessary for the growth of the crystal will however be different for different kinds of surfaces, and probably will generally be greatest for the surfaces for which σ is least.

On the whole, it seems not improbable that the form of very minute crystals in equilibrium with solvents is principally determined by equation (665), (*i. e.*, by the condition that $\Sigma(\sigma s)$ shall be a minimum for the volume of the crystal except so far as the case is modified by gravity or the contact of other bodies,) but as they grow larger (in a solvent no more supersaturated than is necessary to make them grow at all), the deposition of new matter on the different surfaces will be determined more by the nature (orientation) of the surfaces and less by their size and relations to the surrounding surfaces. As a final result, a large crystal, thus formed, will generally be bounded by those surfaces alone on which the deposit of new matter takes place least readily, with small, perhaps insensible truncations. If one kind of surfaces satisfying this condition cannot form a closed figure, the crystal will be bounded by two or three kinds of surfaces determined by the same condition. The kinds of surface thus determined will probably generally be those for which σ has the least values. But the relative development of the different kinds of sides, even if unmodified by gravity or the contact of other bodies, will not be such as to make $\Sigma(\sigma s)$ a minimum. The growth of the crystal will finally be confined to sides of a single kind.

It does not appear that any part of the operation of removing a layer of molecules presents any especial difficulty so marked as that of commencing a new layer; yet the values of μ_1'' which will just allow the different stages of the process to go on must be slightly different, and therefore, for the continued dissolving of the crystal the value of μ_1'' must be less (by a finite quantity) than that given by equation (665). It seems probable that this would be especially true of those sides for which σ has the least values. The effect of dissolving a crystal (even when it is done as slowly as possible) is therefore to produce a form which probably differs from that of theoretical equilibrium in a direction opposite to that of a growing crystal.

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sary that at every point of the line

$$\Sigma(\sigma\,\delta T) \geqq 0 \tag{671}$$

for any *possible* displacement of the line. Those displacements are to be regarded as possible which are not prevented by the solidity of the masses, when the interior of every solid mass is regarded as incapable of motion. At the surfaces between solid and fluid masses, the processes of solidification and dissolution will be possible in some cases, and impossible in others.

The simplest case is when two masses are fluid and the third is solid and insoluble. Let us denote the solid by S, the fluids by A and B, and the angles filled by these fluids by α and β respectively. If the surface of the solid is continuous at the line where it meets the two fluids, the condition of equilibrium reduces to

$$\sigma_{AB} \cos \alpha = \sigma_{BS} - \sigma_{AS}. \tag{672}$$

If the line where these masses meet is at an edge of the solid, the condition of equilibrium is that

$$\left.\begin{aligned} &\sigma_{AB} \cos \alpha \leqq \sigma_{BS} - \sigma_{AS}, \\ \text{and} \quad &\sigma_{AB} \cos \beta \leqq \sigma_{AS} - \sigma_{BS}; \end{aligned}\right\} \tag{673}$$

which reduces to the preceding when $\alpha + \beta = \pi$. Since the displacement of the line can take place by a purely mechanical process, this condition is capable of a more satisfactory experimental verification than those conditions which relate to processes of solidification and dissolution. Yet the frictional resistance to a displacement of the line is enormously greater than in the case of three fluids, since the relative displacements of contiguous portions of matter are enormously greater. Moreover, foreign substances adhering to the solid are not easily displaced, and cannot be distributed by extensions and contractions of the surface of discontinuity, as in the case of fluid masses. Hence, the distribution of such substances is arbitrary to a greater extent than in the case of fluid masses, (in which a single foreign substance in any surface of discontinuity is uniformly distributed, and a greater number are at least so distributed as to make the tension of the surface uniform,) and the presence of these substances will modify the conditions of equilibrium in a more irregular manner.

If one or more of three surfaces of discontinuity which meet in a line divides an amorphous solid from a fluid in which it is soluble, such a surface is to be regarded as movable, and the particular conditions involved in (671) will be accordingly modified. If the soluble solid is a crystal, the case will properly be treated by the method used on page 490. The condition of equilibrium relating to the line

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will not in this case be entirely separable from those relating to the adjacent surfaces, since a displacement of the line will involve a displacement of the whole side of the crystal which is terminated at this line. But the expression for the total increment of energy in the system due to any internal changes not involving any variation in the total entropy or volume will consist of two parts, of which one relates to the properties of the masses of the system, and the other may be expressed in the form

$$\delta \Sigma(\sigma s),$$

the summation relating to all the surfaces of discontinuity. This indicates the same tendency toward changes diminishing the value of $\Sigma(\sigma s)$, which appears in other cases.*

General Relations.—For any constant state of strain of the surface of the solid, we may write

$$d\varepsilon_{s(1)} = t\, d\eta_{s(1)} + \mu_2\, d\Gamma_{2(1)} + \mu_3\, d\Gamma_{3(1)} + \text{etc.}, \tag{674}$$

since this relation is implied in the definition of the quantities involved. From this and (659) we obtain

$$d\sigma = -\eta_{s(1)}\, dt - \Gamma_{2(1)}\, d\mu_2 - \Gamma_{3(1)}\, d\mu_3 - \text{etc.}, \tag{675}$$

which is subject, in strictness, to the same limitation—that the state of strain of the surface of the solid remains the same. But this limitation may in most cases be neglected. (If the quantity σ represented the true tension of the surface, as in the case of a surface between fluids, the limitation would be wholly unnecessary.)

Another method and notation.—We have so far supposed that we have to do with a non-homogeneous film of matter between two homogeneous (or very nearly homogeneous) masses, and that the nature and state of this film is in all respects determined by the

* The freezing together of wool and ice may be mentioned here. The fact that a fiber of wool which remains in contact with a block of ice under water will become attached to it seems to be strictly analogous to the fact that if a solid body be brought into such a position that it just touches the free surface of water, the water will generally rise up about the point of contact so as to touch the solid over a surface of some extent. The condition of the latter phenomenon is

$$\sigma_{SA} + \sigma_{WA} > \sigma_{SW},$$

where the suffixes $_S$, $_A$, and $_W$ refer to the solid, to air, and to water, respectively. In like manner, the condition for the freezing of the ice to the wool, if we neglect the æolotropic properties of the ice, is

$$\sigma_{SW} + \sigma_{IW} > \sigma_{SI},$$

where the suffixes $_S$, $_W$, and $_I$ relate to wool, to water, and to ice, respectively. See *Proc. Roy. Soc.*, vol. x, p. 447; or *Phil. Mag.*, 4th ser., vol. xxi, p. 151.

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nature and state of these masses together with the quantities of the foreign substances which may be present in the film. (See page 483.) Problems relating to processes of solidification and dissolution seem hardly capable of a satisfactory solution, except on this supposition, which appears in general allowable with respect to the surfaces produced by these processes. But in considering the equilibrium of fluids at the surface of an unchangeable solid, such a limitation is neither necessary nor convenient. The following method of treating the subject will be found more simple and at the same time more general.

Let us suppose the superficial density of energy to be determined by the excess of energy in the vicinity of the surface over that which would belong to the solid, if (with the same temperature and state of strain) it were bounded by a vacuum in place of the fluid, and to the fluid, if it extended with a uniform volume-density of energy just up to the surface of the solid, or, if in any case this does not sufficiently define a surface, to a surface determined in some definite way by the exterior particles of the solid. Let us use the symbol (ε_S) to denote the superficial energy *thus defined.* Let us suppose a superficial density of entropy to be determined in a manner entirely analogous, and be denoted by (η_S). In like manner also, for all the components of the fluid, and for all foreign fluid substances which may be present at the surface, let the superficial densities be determined, and denoted by (Γ_2), (Γ_3), etc. These *superficial densities of the fluid components* relate solely to the matter which is fluid or movable. All matter which is immovably attached to the solid mass is to be regarded as a part of the same. Moreover, let ς be defined by the equation

$$\varsigma = (\varepsilon_S) - t\,(\eta_S) - \mu_2\,(\Gamma_2) - \mu_3\,(\Gamma_3) - \text{etc.} \tag{676}$$

These quantities will satisfy the following general relations—

$$d(\varepsilon_S) = t\,d(\eta_S) + \mu_2\,d(\Gamma_2) + \mu_3\,d(\Gamma_3) + \text{etc.}, \tag{677}$$

$$d\varsigma = -\,(\eta_S)\,dt - (\Gamma_2)\,d\mu_2 - (\Gamma_3)\,d\mu_3 - \text{etc.} \tag{678}$$

In strictness, these relations are subject to the same limitation as (674) and (675). But this limitation may generally be neglected. In fact, the values of ς, (ε_S), etc. must in general be much less affected by variations in the state of strain of the surface of the solid than those of σ, $\varepsilon_{S(1)}$, etc.

The quantity ς evidently represents the tendency to contraction in that portion of the surface of the fluid which is in contact with the solid. It may be called *the superficial tension of the fluid in contact with the solid.* Its value may be either positive or negative.

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It will be observed that for the same solid surface and for the same temperature but for different fluids the values of σ (in all cases to which the definition of this quantity is applicable) will differ from those of ς by a constant, viz., the value of σ for the solid surface in a vacuum.

For the condition of equilibrium of two different fluids at a line on the surface of the solid, we may easily obtain

$$\sigma_{AB} \cos \alpha = \varsigma_{BS} - \varsigma_{AS}, \tag{679}$$

the suffixes, etc., being used as in (672), and the condition being subject to the same modification when the fluids meet at an edge of the solid.

It must also be regarded as a condition of theoretical equilibrium at the line considered, [subject, like (679), to limitation on account of passive resistances to motion,] that if there are any foreign substances in the surfaces A-S and B-S, the potentials for these substances shall have the same value on both sides of the line; or, if any such substance is found only on one side of the line, that the potential for that substance must not have a less value on the other side; and that the potentials for the components of the mass A, for example, must have the same values in the surface B-C as in the mass A, or, if they are not actual components of the surface B-C, a value not less than in A. Hence, we cannot determine the difference of the surface-tensions of two fluids in contact with the same solid, by bringing them together upon the surface of the solid, unless these conditions are satisfied, as well as those which are necessary to prevent the mixing of the fluid masses.

The investigation on pages 442–448 of the conditions of equilibrium for a fluid system under the influence of gravity may easily be extended to the case in which the system is bounded by or includes solid masses, when these can be treated as rigid and incapable of dissolution. The general condition of mechanical equilibrium would be of the form

$$-\int p\,\delta Dv + \int g\,\gamma\,\delta z\,Dv + \int \sigma\,\delta Ds + \int g\,\Gamma\,\delta z\,Ds$$
$$+ \int g\,\delta z\,\mathrm{Dm} + \int \varsigma\,\delta Ds + \int g\,(\Gamma)\,\delta z\,Ds = 0, \tag{680}$$

where the first four integrals relate to the fluid masses and the surfaces which divide them, and have the same signification as in equation (606), the fifth integral relates to the movable solid masses, and the sixth and seventh to the surfaces between the solids and fluids, (Γ) denoting the sum of the quantities (Γ_2), (Γ_3), etc. It should be observed that at the surface where a fluid meets a solid

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δz and δz, which indicate respectively the displacements of the solid and the fluid, may have different values, but the components of these displacements which are normal to the surface must be equal.

From this equation, among other particular conditions of equilibrium, we may derive the following—

$$d\varsigma = g\,(\Gamma)\,dz, \tag{681}$$

[compare (614),] which expresses the law governing the distribution of a thin fluid film on the surface of a solid, when there are no passive resistances to its motion.

By applying equation (680) to the case of a vertical cylindrical tube containing two different fluids, we may easily obtain the well-known theorem that the product of the perimeter of the internal surface by the difference $\varsigma' - \varsigma''$ of the superficial tensions of the upper and lower fluids in contact with the tube is equal to the excess of weight of the matter in the tube above that which would be there, if the boundary between the fluids were in the horizontal plane at which their pressures would be equal. In this theorem, we may either include or exclude the weight of a film of fluid matter adhering to the tube. The proposition is usually applied to the column of fluid *in mass* between the horizontal plane for which $p' = p''$ and the actual boundary between the two fluids. The superficial tensions ς' and ς'' are then to be measured in the vicinity of this column. But we may also include the weight of a film adhering to the internal surface of the tube. For example, in the case of water in equilibrium with its own vapor in a tube, the weight of all the water-substance in the tube above the plane $p' = p''$, diminished by that of the water-vapor which would fill the same space, is equal to the perimeter multiplied by the difference in the values of ς at the top of the tube and at the plane $p' = p''$. If the height of the tube is infinite, the value of ς at the top vanishes, and the weight of the film of water adhering to the tube and of the mass of liquid water above the plane $p' = p''$ diminished by the weight of vapor which would fill the same space is equal in numerical value but of opposite sign to the product of the perimeter of the internal surface of the tube multiplied by ς'', the superficial tension of liquid water in contact with the tube at the pressure at which the water and its vapor would be in equilibrium at a plane surface. In this sense, the total weight of water which can be supported by the tube per unit of the perimeter of its surface is directly measured by the value of $-\varsigma$ for water in contact with the tube.

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Modification of the conditions of equilibrium by electromotive force.—Theory of a perfect electro-chemical apparatus.

We know by experience that in certain fluids (electrolytic conductors) there is a connection between the fluxes of the component substances and that of electricity. The quantitative relation between these fluxes may be expressed by an equation of the form

$$De = \frac{Dm_a}{\alpha_a} + \frac{Dm_b}{\alpha_b} + \text{etc.} - \frac{Dm_g}{\alpha_g} - \frac{Dm_h}{\alpha_h} - \text{etc.}, \qquad (682)$$

where De, Dm_a, etc. denote the infinitesimal quantities of electricity and of the components of the fluid which pass simultaneously through any same surface, which may be either at rest or in motion, and α_a, α_b, etc., α_g, α_h, etc. denote positive constants. We may evidently regard Dm_a, Dm_b, etc., Dm_g, Dm_h, etc., as independent of one another. For, if they were not so, one or more could be expressed in terms of the others, and we could reduce the equation to a shorter form in which all the terms of this kind would be independent.

Since the motion of the fluid as a whole will not involve any electrical current, the densities of the components specified by the suffixes must satisfy the relation

$$\frac{\gamma_a}{\alpha_a} + \frac{\gamma_b}{\alpha_b} + \text{etc.} = \frac{\gamma_g}{\alpha_g} + \frac{\gamma_h}{\alpha_h} + \text{etc.} \qquad (683)$$

These densities, therefore, are not independently variable, like the densities of the components which we have employed in other cases.

We may account for the relation (682) by supposing that electricity (positive or negative) is inseparably attached to the different kinds of molecules, so long as they remain in the interior of the fluid, in such a way that the quantities α_a, α_b, etc. of the substances specified are each charged with a unit of positive electricity, and the quantities α_g, α_h, etc. of the substances specified by these suffixes are each charged with a unit of negative electricity. The relation (683) is accounted for by the fact that the constants α_a, α_g, etc. are so small that the electrical charge of any sensible portion of the fluid varying sensibly from the law expressed in (683) would be enormously great, so that the formation of such a mass would be resisted by a very great force.

It will be observed that the choice of the substances which we regard as the components of the fluid is to some extent arbitrary, and that the same physical relations may be expressed by different equa-

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tions of the form (682), in which the fluxes are expressed with reference to different sets of components. If the components chosen are such as represent what we believe to be the actual molecular constitution of the fluid, those of which the fluxes appear in the equation of the form (682) are called the *ions*, and the constants of the equation are called their *electro-chemical equivalents*. For our present purpose, which has nothing to do with any theories of molecular constitution, we may choose such a set of components as may be convenient, and call those *ions*, of which the fluxes appear in the equation of the form (682), without farther limitation.

Now, since the fluxes of the independently variable components of an electrolytic fluid do not necessitate any electrical currents, all the conditions of equilibrium which relate to the movements of these components will be the same as if the fluid were incapable of the electrolytic process. Therefore all the conditions of equilibrium which we have found without reference to electrical considerations, will apply to an electrolytic fluid and its independently variable components. But we have still to seek the remaining conditions of equilibrium, which relate to the possibility of electrolytic conduction.

For simplicity, we shall suppose that the fluid is without internal surfaces of discontinuity (and therefore homogeneous except so far as it may be slightly affected by gravity), and that it meets metallic conductors (*electrodes*) in different parts of its surface, being otherwise bounded by non-conductors. The only electrical currents which it is necessary to consider are those which enter the electrolyte at one electrode and leave it at another.

If all the conditions of equilibrium are fulfilled in a given state of the system, except those which relate to changes involving a flux of electricity, and we imagine the state of the system to be varied by the passage from one electrode to another of the quantity of electricity δe accompanied by the quantity δm_a of the component specified, without any flux of the other components or any variation in the total entropy, the total variation of energy in the system will be represented by the expression

$$(V'' - V')\,\delta e + (\mu_a'' - \mu_a')\,\delta m_a + (\Upsilon' - \Upsilon'')\,\delta m_a,$$

in which V', V'' denote the electrical potentials in pieces of the same kind of metal connected with the two electrodes, Υ', Υ'', the gravitational potentials at the two electrodes, and μ_a', μ_a'', the intrinsic potentials for the substance specified. The first term represents the increment of the potential energy of electricity, the second the incre-

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ment of the intrinsic energy of the ponderable matter, and the third the increment of the energy due to gravitation.* But by (682)

$$\delta m_a = \alpha_a \, \delta e$$

It is therefore necessary for equilibrium that

$$V'' - V' + \alpha_a (\mu_a'' - \mu_a' - \Upsilon'' + \Upsilon') = 0. \qquad (684)$$

To extend this relation to all the electrodes we may write

$$V' + \alpha_a (\mu_a' - \Upsilon') = V'' + \alpha_a (\mu_a'' - \Upsilon'')$$
$$= V''' + \alpha_a (\mu_a''' - \Upsilon''') = \text{etc.} \qquad (685)$$

For each of the other cations (specified by $_b$ etc.) there will be a similar condition, and for each of the anions a condition of the form

$$V' - \alpha_g (\mu_g' - \Upsilon') = V'' - \alpha_g (\mu_g'' - \Upsilon'')$$
$$= V''' - \alpha_g (\mu_g''' - \Upsilon''') = \text{etc.} \qquad (686)$$

When the effect of gravity may be neglected, and there are but two electrodes, as in a galvanic or electrolytic cell, we have for any cation

$$V'' - V' = \alpha_a (\mu_a' - \mu_a''), \qquad (687)$$

and for any anion

$$V'' - V' = \alpha_g (\mu_g'' - \mu_g'), \qquad (688)$$

where $V'' - V'$ denotes the electromotive force of the combination. That is:—

When all the conditions of equilibrium are fulfilled in a galvanic or electrolytic cell, the electromotive force is equal to the difference in the values of the potential for any ion or apparent ion at the surfaces of the electrodes multiplied by the electro-chemical equivalent of that ion, the greater potential of an anion being at the same electrode as the greater electrical potential, and the reverse being true of a cation.

Let us apply this principle to different cases.

(I.) If the ion is an independently variable component of an electrode, or by itself constitutes an electrode, the potential for the ion (in any case of equilibrium which does not depend upon passive resistances to change) will have the same value within the electrode as on its surface, and will be determined by the composition of the electrode with its temperature and pressure. This might be illustrated by a cell with electrodes of mercury containing certain quantities of zinc in solution (or with one such electrode and the other of pure

* It is here supposed that the gravitational potential may be regarded as constant for each electrode. When this is not the case, the expression may be applied to small parts of the electrodes taken separately.

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zinc) and an electrolytic fluid containing a salt of zinc, but not capable of dissolving the mercury.* We may regard a cell in which hydrogen acts as an ion between electrodes of palladium charged with hydrogen as another illustration of the same principle, but the solidity of the electrodes and the consequent resistance to the diffusion of the hydrogen within them (a process which cannot be assisted by convective currents as in a liquid mass) present considerable obstacles to the experimental verification of the relation.

(II.) Sometimes the ion is soluble (as an independently variable component) in the electrolytic fluid. Of course its condition in the fluid when thus dissolved must be entirely different from its condition when acting on an ion, in which case its quantity is not independently variable, as we have already seen. Its diffusion in the fluid in this state of solution is not necessarily connected with any electrical current, and in other relations its properties may be entirely changed. In any discussion of the internal properties of the fluid (with respect to its fundamental equation, for example,) it would be necessary to treat it as a different substance. (See page 117.) But if the process by which the charge of electricity passes into the electrode, and the ion is dissolved in the electrolyte is *reversible*, we may evidently regard the potentials for the substance of the ion in (687) or (688) as relating to the substance thus dissolved in the electrolyte. In case of absolute equilibrium, the density of the substance thus dissolved would of course be uniform throughout the fluid, (since it can move independently of any electrical current,) so that by the strict application of our principle we only obtain the somewhat barren result, that if any of the ions are soluble in the fluid without their electrical charges, the electromotive force must vanish in any case of absolute equilibrium not dependent upon passive resistances. Nevertheless, cases in which the ion is thus dissolved in the electrolytic fluid only to a very small extent, and its passage from one electrode to the other by ordinary diffusion is extremely slow, may be regarded as approximating to the case in which it is incapable of diffusion. In such cases, we may regard the relations (687), (688) as approximately valid, although the condition of equilibrium

* If the electrolytic fluid dissolved the mercury as well as the zinc, equilibrium could only subsist when the electromotive force is zero, and the composition of the electrodes identical. For when the electrodes are formed of the two metals in different proportions, that which has the greater potential for zinc will have the less potential for mercury. [See equation (98).] This is inconsistent with equilibrium, according to the principle mentioned above, if both metals can act as cations.

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relating to the diffusion of the dissolved ion is not satisfied. This may be the case with hydrogen and oxygen as ions (or apparent ions) between electrodes of platinum in some of its forms.

(III.) The ion may appear in mass at the electrode. If it be a conductor of electricity, it may be regarded as forming an electrode, as soon as the deposit has become thick enough to have the properties of matter in mass. The case therefore will not be different from that first considered. When the ion is a non-conductor, a continuous thick deposit on the electrode would of course prevent the possibility of an electrical current. But the case in which the ion being a non-conductor is disengaged in masses contiguous to the electrode but not entirely covering it, is an important one. It may be illustrated by hydrogen appearing in bubbles at a cathode. In case of perfect equilibrium, independent of passive resistances, the potential of the ion in (687) or (688) may be determined in such a mass. Yet the circumstances are quite unfavorable for the establishment of perfect equilibrium, unless the ion is to some extent absorbed by the electrode or electrolytic fluid, or the electrode is fluid. For if the ion must pass *immediately* into the non-conducting mass, while the electricity passes into the electrode, it is evident that the only possible terminus of an electrolytic current is at the line where the electrode, the non-conducting mass, and the electrolytic fluid meet, so that the electrolytic process is necessarily greatly retarded, and an approximate ceasing of the current cannot be regarded as evidence that a state of approximate equilibrium has been reached. But even a slight degree of solubility of the ion in the electrolytic fluid or in the electrode may greatly diminish the resistance to the electrolytic process, and help toward producing that state of complete equilibrium which is supposed in the theorem we are discussing. And the mobility of the surface of a liquid electrode may act in the same way. When the ion is absorbed by the electrode, or by the electrolytic fluid, the case of course comes under the heads which we have already considered, yet the fact that the ion is set free in mass is important, since it is in such a mass that the determination of the value of the potential will generally be most easily made.

(IV.) When the ion is not absorbed either by the electrode or by the electrolytic fluid, and is not set free in mass, it may still be deposited on the surface of the electrode. Although this can take place only to a limited extent (without forming a body having the properties of matter in mass), yet the electro-chemical equivalents of all substances are so small that a very considerable flux of electricity

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may take place before the deposit will have the properties of matter in mass. Even when the ion appears in mass, or is absorbed by the electrode or electrolytic fluid, the non-homogeneous film between the electrolytic fluid and the electrode may contain an additional portion of it. Whether the ion is confined to the surface of the electrode or not, we may regard this as one of the cases in which we have to recognize a certain superficial density of substances at surfaces of discontinuity, the general theory of which we have already considered.

The deposit of the ion will affect the superficial tension of the electrode if it is liquid, or the closely related quantity which we have denoted by the same symbol σ (see pages 482–500) if the electrode is solid. The effect can of course be best observed in the case of a liquid electrode. But whether the electrodes are liquid or solid, if the external electromotive force $V' - V''$ applied to an electrolytic combination is varied, when it is too weak to produce a lasting current, and the electrodes are thereby brought into a new state of polarization, in which they make equilibrium with the altered value of the electromotive force, without change in the nature of the electrodes or of the electrolytic fluid, then by (508) or (675)

$$d\sigma' = -\Gamma_a' \, d\mu_a',$$
$$d\sigma'' = -\Gamma_a'' \, d\mu_a'';$$

and by (687),

$$d(V' - V'') = -\alpha_a (d\mu_a' - d\mu_a'').$$

Hence

$$d(V' - V'') = \frac{\alpha_a}{\Gamma_a'} d\sigma' - \frac{\alpha_a}{\Gamma_a''} d\sigma''. \tag{689}$$

If we suppose that the state of polarization of only one of the electrodes is affected (as will be the case when its surface is very small compared with that of the other), we have

$$d\sigma' = \frac{\Gamma_a'}{\alpha_a} d(V' - V''). \tag{690}$$

The superficial tension of one of the electrodes is then a function of the electromotive force.

This principle has been applied by M. Lippmann to the construction of the electrometer which bears his name.* In applying equations (689) and (690) to dilute sulphuric acid between electrodes of mercury, as in a Lippmann's electrometer, we may suppose that the

* See his memoir: "Relations entre les phénomènes électriques et capillaires." *Annales de Chimie et de Physique*, 5e série, t. v, p. 494.

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suffix refers to hydrogen. It will be most convenient to suppose the *dividing surface* to be so placed as to make the surface-density of mercury zero. (See page 397.) The matter which exists in excess or deficiency at the surface may then be expressed by the surface-densities of sulphuric acid, of water, and of hydrogen. The value of the last may be determined from equation (690). According to M. Lippmann's determinations, it is negative when the surface is in its natural state (i. e., the state to which it tends when no external electromotive force is applied), since σ' increases with $V'' - V'$. When $V'' - V'$ is equal to nine-tenths of the electromotive force of a Daniell's cell, the electrode to which V'' relates remaining in its natural state, the tension σ' of the surface of the other electrode has a maximum value, and there is no excess or deficiency of hydrogen at that surface. This is the condition toward which a surface tends when it is extended while no flux of electricity takes place. The flux of electricity per unit of new surface formed, which will maintain a surface in a constant condition while it is extended, is represented by $\frac{\Gamma_{\mathrm{h}}'}{\alpha_{\mathrm{h}}}$ in numerical value, and its direction, when Γ_{h}'' is negative, is from the mercury into the acid.

We have so far supposed, in the main, that there are no passive resistances to change, except such as vanish with the rapidity of the processes which they resist. The actual condition of things with respect to passive resistances appears to be nearly as follows. There does not appear to be any passive resistance to the electrolytic process by which an ion is transferred from one electrode to another, except such as vanishes with the rapidity of the process. For, in any case of equilibrium, the smallest variation of the externally applied electromotive force appears to be sufficient to cause a (temporary) electrolytic current. But the case is not the same with respect to the molecular changes by which the ion passes into new combinations or relations, as when it enters into the mass of the electrodes, or separates itself in mass, or is dissolved (no longer with the properties of an ion) in the electrolytic fluid. In virtue of the passive resistance to these processes, the external electromotive force may often vary within wide limits, without creating any current by which the ion is transferred from one of the masses considered to the other. In other words, the value of $V' - V''$ may often differ greatly from that obtained from (687) or (688) when we determine the values of the potentials for the ion as in cases I, II, and III. We may, however, regard these equations as entirely valid, when the potentials for the

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ions are determined at the surface of the electrodes with reference to the ion in the condition in which it is brought there or taken away by an electrolytic current, without any attendant irreversible processes. But in a complete discussion of the properties of the surface of an electrode it may be necessary to distinguish (both in respect to surface-densities and to potentials) between the substance of the ion in this condition and the same substance in other conditions into which it cannot pass (directly) without irreversible processes. No such distinction, however, is necessary when the substance of the ion can pass at the surface of the electrode by reversible processes from any one of the conditions in which it appears to any other.

The formulæ (687), (688) afford as many equations as there are ions. These, however, amount to only one independent equation additional to those which relate to the independently variable components of the electrolytic fluid. This appears from the consideration that a flux of any cation may be combined with a flux of any anion in the same direction so as to involve no electrical current, and that this may be regarded as the flux of an independently variable component of the electrolytic fluid.

General Properties of a Perfect Electro-chemical Apparatus.

When an electrical current passes through a galvanic or electrolytic cell, the state of the cell is altered. If no changes take place in the cell except during the passage of the current, and all changes which accompany the current can be reversed by reversing the current, the cell may be called a perfect electro-chemical apparatus. The electromotive force of the cell may be determined by the equations which have just been given. But some of the general relations to which such an apparatus is subject may be conveniently stated in a form in which the ions are not explicitly mentioned.

In the most general case, we may regard the cell as subject to external action of four different kinds. (1) The supply of electricity at one electrode and the withdrawal of the same quantity at the other. (2) The supply or withdrawal of a certain quantity of heat. (3) The action of gravity. (4) The motion of the surfaces enclosing the apparatus, as when its volume is increased by the liberation of gases.

The increase of the energy in the cell is necessarily equal to that which it receives from external sources. We may express this by the equation

$$d\varepsilon = (V' - V'')\, de + dQ + dW_G + dW_P, \qquad (691)$$

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in which $d\varepsilon$ denotes the increment of the intrinsic energy of the cell, de the quantity of electricity which passes through it, V' and V'' the electrical potentials in masses of the same kind of metal connected with the anode and cathode respectively, dQ the heat received from external bodies, dW_G the work done by gravity, and dW_P the work done by the pressures which act on the external surface of the apparatus.

The conditions under which we suppose the processes to take place are such that the increase of the entropy of the apparatus is equal to the entropy which it receives from external sources. The only external source of entropy is the heat which is communicated to the cell by the surrounding bodies. If we write $d\eta$ for the increment of entropy in the cell, and t for the temperature, we have

$$d\eta = \frac{dQ}{t}. \tag{692}$$

Eliminating dQ, we obtain

$$d\varepsilon = (V' - V'')\,de + t\,d\eta + dW_G + dW_P, \tag{693}$$

or

$$V'' - V' = -\frac{d\varepsilon}{de} + t\frac{d\eta}{de} + \frac{dW_G}{de} + \frac{dW_P}{de}. \tag{694}$$

It is worth while to notice that if we give up the condition of the reversibility of the processes, so that the cell is no longer supposed to be a perfect electro-chemical apparatus, the relation (691) will still subsist. But, if we still suppose, for simplicity, that all parts of the cell have the same temperature, which is *necessarily* the case with a perfect electro-chemical apparatus, we shall have, instead of (692),

$$d\eta \geqq \frac{dQ}{t}, \tag{695}$$

and instead of (693), (694)

$$(V'' - V')\,de \leqq -d\varepsilon + t\,d\eta + dW_G + dW_P. \tag{696}$$

The values of the several terms of the second member of (694), for a given cell, will vary with the external influences to which the cell is subjected. If the cell is enclosed (with the products of electrolysis) in a rigid envelop, the last term will vanish. The term relating to gravity is generally to be neglected. If no heat is supplied or withdrawn, the term containing $d\eta$ will vanish. But in the calculation of the electromotive force, which is the most important application of the equation, it is generally more convenient to suppose that the temperature remains constant.

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The quantities expressed by the terms containing dQ and $d\eta$ in (691), (693), (694), and (696) are frequently neglected in the consideration of cells of which the temperature is supposed to remain constant. In other words, it is frequently assumed that neither heat nor cold is produced by the passage of an electrical current through a perfect electro-chemical combination (except that heat which may be indefinitely diminished by increasing the time in which a given quantity of electricity passes), and that only heat can be produced in any cell, unless it be by processes of a secondary nature, which are not immediately or necessarily connected with the process of electrolysis.

It does not appear that this assumption is justified by any sufficient reason. In fact, it is easy to find a case in which the electromotive force is determined entirely by the term $t\frac{d\eta}{de}$ in (694), all the other terms in the second member of the equation vanishing. This is true of a Grove's gas battery charged with hydrogen and nitrogen. In this case, the hydrogen passes over to the nitrogen,—a process which does not alter the energy of the cell, when maintained at a constant temperature. The work done by external pressures is evidently nothing, and that done by gravity is (or may be) nothing. Yet an electrical current is produced. The work done (or which may be done) by the current outside of the cell is the equivalent of the work (or of a part of the work) which might be gained by allowing the gases to mix in other ways. This is equal, as has been shown by Lord Rayleigh,* to the work which may be gained by allowing each gas separately to expand at constant temperature from its initial volume to the volume occupied by the two gases together. The same work is equal, as appears from equations (278), (279) on page 217, (see also page 220,) to the increase of the entropy of the system multiplied by the temperature.

It is possible to vary the construction of the cell in such a way that nitrogen or other neutral gas will not be necessary. Let the cell consist of a U-shaped tube of sufficient height, and have pure hydrogen at each pole under very unequal pressures (as of one and two atmospheres respectively) which are maintained constant by properly weighted pistons, sliding in the arms of the tube. The difference of the pressures in the gas-masses at the two electrodes must of course be balanced by the difference in the height of the two columns of acidulated water. It will hardly be doubted that such an apparatus

* *Philosophical Magazine*, vol. xlix, p. 311.

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would have an electromotive force acting in the direction of a current which would carry the hydrogen from the denser to the rarer mass. Certainly the gas could not be carried in the opposite direction by an external electromotive force without the expenditure of as much (electromotive) work as is equal to the mechanical work necessary to pump the gas from the one arm of the tube to the other. And if by any modification of the metallic electrodes (which remain unchanged by the passage of electricity) we could reduce the passive resistances to zero, so that the hydrogen could be carried reversibly from one mass to the other without finite variation of the electromotive force, the only possible value of the electromotive force would be represented by the expression $t\frac{d\eta}{de}$, as a very close approximation. It will be observed that, although gravity plays an essential part in a cell of this kind by maintaining the difference of pressure in the masses of hydrogen, the electromotive force cannot possibly be ascribed to gravity, since the work done by gravity, when hydrogen passes from the denser to the rarer mass, is negative.

Again, it is entirely improbable that the electrical currents caused by differences in the concentration of solutions of salts, (as in a cell containing sulphate of zinc between zinc electrodes, or sulphate of copper between copper electrodes, the solution of the salt being of unequal strength at the two electrodes,) which have recently been investigated theoretically and experimentally by MM. Helmholtz and Moser,* are confined to cases in which the mixture of solutions of different degrees of concentration will produce heat. Yet in cases in which the mixture of more and less concentrated solutions is not attended with evolution or absorption of heat, the electromotive force must vanish in a cell of the kind considered, if it is determined simply by the diminution of energy in the cell. And when the mixture produces cold, the same rule would make any electromotive force impossible except in the direction which would tend to increase the difference of concentration. Such conclusions as would be quite irreconcilable with the theory of the phenomena given by Professor Helmholtz.

A more striking example of the necessity of taking account of the variations of entropy in the cell in *a priori* determinations of electromotive force is afforded by electrodes of zinc and mercury in a solution of sulphate of zinc. Since heat is absorbed when zinc is dissolved

* *Annalen der Physik und Chemie*, Neue Folge, Band iii, February, 1878.

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in mercury,* the energy of the cell is increased by a transfer of zinc to the mercury, when the temperature is maintained constant. Yet in this combination, the electromotive force acts in the direction of the current producing such a transfer.† The couple presents certain anomalies when a considerable quantity of zinc is united with the mercury. The electromotive force changes its direction, so that this case is usually cited as an illustration of the principle that the electromotive force is in the direction of the current which diminishes the energy of the cell, *i. e.*, which produces or allows those changes which are accompanied by evolution of heat when they take place directly. But whatever may be the cause of the electromotive force which has been observed acting in the direction from the amalgam through the electrolyte to the zinc (a force which according to the determinations of M. Gaugain is only one twenty-fifth part of that which acts in the reverse direction when pure mercury takes the place of the amalgam), these anomalies can hardly affect the general conclusions with which alone we are here concerned. If the electrodes of a cell are pure zinc and an amalgam containing zinc not in excess of the amount which the mercury will dissolve at the temperature of the experiment without losing its fluidity, and if the only change (other than thermal) accompanying a current is a transfer of zinc from one electrode to the other,—conditions which may not have been satisfied in all the experiments recorded, but which it is allowable to suppose in a theoretical discussion, and which certainly will not be regarded as inconsistent with the fact that heat is absorbed when zinc is dissolved in mercury,—it is impossible that the electromotive force should be in the direction of a current transferring zinc from the amalgam to the electrode of pure zinc. For, since the zinc eliminated from the amalgam by the electrolytic process might be re-dissolved directly, such a direction of the electromotive force would involve the possibility of obtaining an indefinite amount of electromotive work, and therefore of mechanical work, without other expenditure than that of heat at the constant temperature of the cell.

None of the cases which we have been considering involve combinations by definite proportions, and, except in the case of the cell with electrodes of mercury and zinc, the electromotive forces are quite small. It may perhaps be thought that with respect to those cells in which combinations take place by definite proportions the electromotive force may be calculated with substantial accuracy from

* J. Regnauld, *Comptes Rendus*, t. li, p. 778.

† Gaugain, *Comptes Rendus*, t. xlii, p. 430.

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the diminution of the energy, without regarding the variation of entropy. But the phenomena of chemical combination do not in general seem to indicate any possibility of obtaining from the combination of substances by any process whatever an amount of mechanical work which is equivalent to the heat produced by the direct union of the substances.

A kilogramme of hydrogen, for example, combining by combustion under the pressure of the atmosphere with eight kilogrammes of oxygen to form liquid water, yields an amount of heat which may be represented in round numbers by 34000 calories.* We may suppose that the gases are taken at the temperature of 0° C., and that the water is reduced to the same temperature. *But this heat cannot be obtained at any temperature desired.* A very high temperature has the effect of preventing to a greater or less extent, the combination of the elements. Thus, according to M. Sainte-Claire Deville,† the temperature obtained by the combustion of hydrogen and oxygen cannot much if at all exceed 2500° C., which implies that less than one-half of the hydrogen and oxygen present combine at that temperature. This relates to combustion under the pressure of the atmosphere. According to the determinations of Professor Bunsen‡ in regard to combustion in a confined space, only one-third of a mixture of hydrogen and oxygen will form a chemical compound at the temperature of 2850° C. and a pressure of ten atmospheres, and only a little more than one-half when the temperature is reduced by the addition of nitrogen to 2024° C., and the pressure to about three atmospheres *exclusive* of the part due to the nitrogen.

Now 10 calories at 2500° C. are to be regarded as reversibly convertible into one calorie at 4° C. together with the mechanical work representing the energy of 9 calories. If, therefore, all the 34000 calories obtainable from the union of hydrogen and oxygen under atmospheric pressure could be obtained at the temperature of 2500° C., and no higher, we should estimate the electromotive work performed in a perfect electro-chemical apparatus in which these elements are combined or separated at ordinary temperatures and under atmospheric pressure as representing nine-tenths of the 34000 calories, and the heat evolved or absorbed in the apparatus as representing one-tenth of the 34000 calories.§ This, of course, would give an electromotive

* See Rühlmann's *Handbuch der mechanischen Wärmetheorie*, Bd. ii, p. 290.

† *Comptes Rendus*, t. lvi, p. 199; and t. lxiv, 67.

‡ *Pogg. Ann.*, Bd. cxxxi (1867), p. 161.

§ These numbers are not subject to correction for the pressure of the atmosphere, since the 34000 calories relate to combustion under the same pressure.

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force exactly nine-tenths as great as is obtained on the supposition that all the 34000 calories are convertible into electromotive or mechanical work. But, according to all indications, the estimate 2500° C. (for the temperature at which we may regard all the heat of combustion as obtainable) is far too high,* and we must regard the theoretical value of the electromotive force necessary to electrolyze water as considerably less than nine-tenths of the value obtained on the supposition that it is necessary for the electromotive agent to supply all the energy necessary for the process.

The case is essentially the same with respect to the electrolysis of hydrochloric acid, which is probably a more typical example of the process than the electrolysis of water. The phenomenon of dissociation is equally marked, and occurs at a much lower temperature, more than half of the gas being dissociated at 1400° C.† And the heat which is obtained by the combination of hydrochloric acid gas with water, especially with water which already contains a considerable quantity of the acid, is probably only to be obtained at temperatures comparatively low. This indicates that the theoretical value of the electromotive force necessary to electrolyze this acid (*i. e.*, the electromotive force which would be necessary in a reversible electrochemical apparatus), must be very much less than that which could perform in electromotive work the equivalent of all the heat evolved in the combination of hydrogen, chlorine and water to form the liquid submitted to electrolysis. This presumption, based upon the phenomena exhibited in the direct combination of the substances, is corroborated by the experiments of M. Favre, who has observed an absorption of heat in the cell in which this acid was electrolyzed.‡ The

* Unless the received ideas concerning the behavior of gases at high temperatures are quite erroneous, it is possible to indicate the general character of a process (involving at most only such difficulties as are neglected in theoretical discussions) by which water may be converted into separate masses of hydrogen and oxygen without other expenditure than that of an amount of heat equal to the difference of energy of the matter in the two states and supplied at a temperature far below 2500° C. The essential parts of the process would be (1) vaporizing the water and heating it to a temperature at which a considerable part will be dissociated, (2) the partial separation of the hydrogen and oxygen by filtration, and (3) the cooling of both gaseous masses until the vapor they contain is condensed. A little calculation will show that in a continuous process all the heat obtained in the operation of cooling the products of filtration could be utilized in heating fresh water.

† Sainte-Claire Deville, *Comptes Rendus*, t. lxiv, p. 67.

‡ See *Mémoires des Savants Étrangers*, Sér. 2, t. xxv, No. 1, p. 142; or *Comptes Rendus*, t. lxxiii, p. 973. The figures obtained by M. Favre will be given hereafter, in connection with others of the same nature.

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electromotive work expended must therefore have been less than the increase of energy in the cell.

In both cases of composition in definite proportions which we have considered, the compound has more entropy than its elements, and the difference is by no means inconsiderable. This appears to be the rule rather than the exception with respect to compounds which have less energy than their elements. Yet it would be rash to assert that it is an invariable rule. And when one substance is substituted for another in a compound, we may expect great diversity in the relations of energy and entropy.

In some cases, there is a striking correspondence between the electromotive force of a cell and the rate of diminution of its energy per unit of electricity transmitted, the temperature remaining constant. A Daniell's cell is a notable example of this correspondence. It may perhaps be regarded as a very significant case, since of all cells in common use, it has the most constant electromotive force, and most nearly approaches the condition of reversibility. If we apply our previous notation [compare (691)] with the substitution of finite for infinitesimal differences to the determinations of M. Favre,* estimating energy in calories, we have for each equivalent (32.6 kilogrammes) of zinc dissolved

$$(V'' - V')\,\Delta e = 24327^{\text{cal.}}, \quad \Delta\varepsilon = -25394^{\text{cal.}}, \quad \Delta Q = -1067^{\text{cal.}}.$$

It will be observed that the electromotive work performed by the cell is about four per cent. less than the diminution of energy in the cell.† The value of ΔQ, which, when negative, represents the heat evolved in the cell when the external resistance of the circuit is very great, was determined by direct measurement, and does not appear to have been corrected for the resistance of the cell. This correction would diminish the value of $-\Delta Q$, and increase that of $(V'' - V')\,\Delta e$, which was obtained by subtracting $-\Delta Q$ from $-\Delta\varepsilon$.

It appears that under certain conditions neither heat nor cold is produced in a Grove's cell. For M. Favre has found that with different degrees of concentration of the nitric acid sometimes heat and sometimes cold is produced.‡ When neither is produced, of course

* See *Mém. Savants Étrang.*, loc. cit., p. 90; or *Comptes Rendus*, vol. lxix, p. 35, where the numbers are slightly different.

† A comparison of the experiments of different physicists has in some cases given a much closer correspondence. See Wiedemann's *Galvanismus*, etc., 2te Auflage, Bd. ii, §§ 1117, 1118.

‡ *Mém. Savants Étrang.*, loc. cit., p. 93; or *Comptes Rendus*, t. lxix, p. 37, and t. lxxiii, p. 893.

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the electromotive force of the cell is exactly equal to its diminution of energy per unit of electricity transmitted. But such a coincidence is far less significant than the fact that an absorption of heat has been observed. With acid containing about seven equivalents of water (HNO_6+7HO), M. Favre has found

$$(V''-V')\,\Delta e = 46781^{\text{cal.}},\quad \Delta\varepsilon = -41824^{\text{cal.}},\quad \Delta Q = 4957^{\text{cal.}};$$

and with acid containing about one equivalent of water (HNO_6+HO),

$$(V''-V')\,\Delta e = 49847^{\text{cal.}},\quad \Delta\varepsilon = -52714^{\text{cal.}},\quad \Delta Q = -2867^{\text{cal.}}.$$

In the first example, it will be observed that the quantity of heat absorbed in the cell is not small, and that the electromotive force is nearly one-eighth greater than can be accounted for by the diminution of energy in the cell.

This absorption of heat in the cell he has observed in other cases, in which the chemical processes are much more simple.

For electrodes of cadmium and platinum in hydrochloric acid his experiments give*

$$(V''-V')\,\Delta e = 9256^{\text{cal.}},\qquad \Delta\varepsilon = -8258^{\text{cal.}},$$
$$\Delta W_P = -290^{\text{cal.}},\qquad \Delta Q = 1288^{\text{cal.}}.$$

In this case the electromotive force is nearly one-sixth greater than can be accounted for by the diminution of energy in the cell with the work done against the pressure of the atmosphere.

For electrodes of zinc and platinum in the same acid one series of experiments gives†

$$(V''-V')\,\Delta e = 16950^{\text{cal.}},\qquad \Delta\varepsilon = -16189^{\text{cal.}},$$
$$\Delta W_P = -290^{\text{cal.}},\qquad \Delta Q = 1051^{\text{cal.}};$$

and a later series,‡

$$(V''-V')\,\Delta e = 16738^{\text{cal.}},\qquad \Delta\varepsilon = -17702^{\text{cal.}},$$
$$\Delta W_P = -290^{\text{cal.}},\qquad \Delta Q = -674^{\text{cal.}}.$$

In the electrolysis of hydrochloric acid in a cell with a porous partition, he has found§

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* *Comptes Rendus*, t. lxviii, p. 1305. The total heat obtained in the whole circuit (including the cell) when all the electromotive work is turned into heat, was ascertained by direct experiment. This quantity, 7968 calories, is evidently represented by $(V''-V')\,\Delta e - \Delta Q$, also by $-\Delta\varepsilon + \Delta W_P$. [See (691).] The value of $(V''-V')\,\Delta e$ is obtained by adding ΔQ, and that of $-\Delta\varepsilon$ by adding $-\Delta W_P$, which is easily estimated, being determined by the evolution of one kilogramme of hydrogen.

† *Ibid.*

‡ *Mém. Savants Étrang.*, loc. cit., p. 145.

§ *Ibid*, p. 142.

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$$(V' - V'')\,\Delta e = 34825^{\text{cal.}} \qquad \Delta Q = 2113^{\text{cal.}},$$

whence

$$\Delta\varepsilon - \Delta W_{\text{P}} = 36938.$$

We cannot assign a precise value to ΔW_{P}, since the quantity of chlorine which was evolved in the form of gas is not stated. But the value of $-\Delta W_{\text{P}}$ must lie between $290^{\text{cal.}}$ and $580^{\text{cal.}}$, probably nearer to the former.

The great difference in the results of the two series of experiments relating to electrodes of zinc and platinum in hydrochloric acid is most naturally explained by supposing some difference in the conditions of the experiment, as in the concentration of the acid, or in the extent to which the substitution of zinc for hydrogen took place.* That which it is important for us to observe in all these cases is that there are conditions under which heat is absorbed in a galvanic or electrolytic cell, so that the galvanic cell has a greater electromotive force than can be accounted for by the diminution of its energy, and the operation of electrolysis requires a less electromotive force than would be calculated from the increase of energy in the cell,—especially when the work done against the pressure of the atmosphere is taken into account.

It should be noticed that in all these experiments the quantity represented by ΔQ (which is the critical quantity with respect to the point at issue) was determined by direct measurement of the heat absorbed or evolved by the cell when placed alone in a calorimeter. The resistance of the circuit was made so great by a rheostat placed outside of the calorimeter that the resistance of the cell was regarded as insignificant in comparison, and no correction appears to have been made in any case for this resistance. With exception of the error due to this circumstance, which would in all cases diminish the heat absorbed in the cell (or increase the heat evolved), the probable error of ΔQ must be very small in comparison with that of $(V' - V'')\,\Delta e$, or with that of $\Delta\varepsilon$, which were in general determined by the compar-

* It should perhaps be stated that in his extended memoir published in 1877 in the *Mémoires des Savants Étrangers*, in which he has presumably collected those results of his experiments which he regards as most important and most accurate, M. Favre does not mention the absorption of heat in a cell of this kind, or in the similar cell in which cadmium takes the place of zinc. This may be taken to indicate a decided preference for the later experiments which showed an evolution of heat. Whatever the ground of this preference may have been, it can hardly destroy the significance of the absorption of heat, which was a matter of direct observation in repeated experiments. See *Comptes Rendus*, t. lxviii, p. 1305.

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ison of different calorimetrical measurements, involving very much greater quantities of heat.

In considering the numbers which have been cited, we should remember that when hydrogen is evolved as gas the process is in general very far from reversible. In a perfect electrochemical apparatus, the same changes in the cell would yield a much greater amount of electromotive work, or absorb a much less amount. In either case, the value of ΔQ would be much greater than in the imperfect apparatus, the difference being measured perhaps by thousands of calories.*

It often occurs in a galvanic or electrolytic cell that an ion which is set free at one of the electrodes appears in part as gas, and is in part absorbed by the electrolytic fluid, and in part absorbed by the electrode. In such cases, a slight variation in the circumstances, which would not sensibly affect the electromotive force, would cause all of the ion to be disposed of in one of the three ways mentioned, if the current were sufficiently weak. This would make a considerable

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* Except in the case of the Grove's cell, in which the reactions are quite complicated, the absorption of heat is most marked in the electrolysis of hydrochloric acid. The latter case is interesting, since the experiments confirm the presumption afforded by the behavior of the substances in other circumstances. (See page 514.) In addition to the circumstances mentioned above tending to diminish the observed absorption of heat, the following, which are peculiar to this case, should be noticed.

The electrolysis was performed in a cell with a porous partition, in order to prevent the chlorine and hydrogen dissolved in the liquid from coming in contact with each other. It had appeared in a previous series of experiments (*Mém. Savants Étrang.*, loc. cit., p. 131; or *Comptes Rendus*, t. lxvi, p. 1231,) that a very considerable amount of heat might be produced by the chemical union of the gases in solution. In a cell without partition, instead of an absorption, an evolution of heat took place, which sometimes exceeded 5000 calories. If, therefore, the partition did not perfectly perform its office, this could only cause a diminution in the value of ΔQ.

A large part at least of the chlorine appears to have been absorbed by the electrolytic fluid. It is probable that a slight difference in the circumstances of the experiment—a diminution of pressure, for example,—might have caused the greater part of the chlorine to be evolved as gas, without essentially affecting the electromotive force. The solution of chlorine in water presents some anomalies, and may be attended with complex reactions, but it appears to be always attended with a very considerable evolution of heat. (See Berthelot, *Comptes Rendus*, t. lxxvi, p. 1514.) If we regard the evolution of the chlorine in the form of gas as the normal process, we may suppose that the absorption of heat in the cell was greatly diminished by the retention of the chlorine in solution.

Under certain circumstances, oxygen is evolved in the electrolysis of dilute hydrochloric acid. It does not appear that this took place to any considerable extent in the experiments which we are considering. But so far as it may have occurred, we may regard it as a case of the electrolysis of water. The significance of the fact of the absorption of heat is not thereby affected.

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difference in the variation of energy in the cell, and the electromotive force cannot certainly be calculated from the variation of energy alone in all these cases. The correction due to the work performed against the pressure of the atmosphere when the ion is set free as gas will not help us in reconciling these differences. It will appear on consideration that this correction will in general increase the discordance in the values of the electromotive force. Nor does it distinctly appear which of these cases is to be regarded as normal and which are to be rejected as involving secondary processes.*

If in any case secondary processes are excluded, we should expect it to be when the ion is identical in substance with the electrode upon which it is deposited, or from which it passes into the electrolyte. But even in this case we do not escape the difficulty of the different forms in which the substance may appear. If the temperature of the experiment is at the melting point of a metal which forms the ion and the electrode, a slight variation of temperature will cause the ion to be deposited in the solid or in the liquid state, or, if the current is in the opposite direction, to be taken up from a solid or from a liquid body. Since this will make a considerable difference in the variation of energy, we obtain different values for the electromotive force above and below the melting point of the metal, unless we also take account of the variations of entropy. Experiment does not indicate the existence of any such difference,† and when we take account of variations of entropy, as in equation (694), it is apparent that there ought not to be any, the terms $\frac{d\varepsilon}{de}$ and $t\frac{d\eta}{de}$ being both

* It will be observed that in using the formulae (694) and (696) we do not have to make any distinction between *primary* and *secondary* processes. The only limitation to the generality of these formulae depends upon the *reversibility* of the processes, and this limitation does not apply to (696).

† M. Raoult has experimented with a galvanic element having an electrode of bismuth in contact with phosphoric acid containing phosphate of bismuth in solution. (See *Comptes Rendus*, t. lxviii, p. 643.) Since this metal absorbs in melting 12.64 calories per kilogramme or 885 calories per equivalent ($70^{kil.}$), while a Daniell's cell yields about 24000 calories of electromotive work per equivalent of metal, the solid or liquid state of the bismuth ought to make a difference of electromotive force represented by .037 of a Daniell's cell, if the electromotive force depended simply upon the energy of the cell. But in M. Raoult's experiments no sudden change of electromotive force was manifested at the moment when the bismuth changed its state of aggregation. In fact, a change of temperature in the electrode from about fifteen degrees above to about fifteen degrees below the temperature of fusion only occasioned a variation of electromotive force equal to .002 of a Daniell's cell.

Experiments upon lead and tin gave similar results.

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affected by the same difference, viz., the heat of fusion of an electrochemical equivalent of the metal. In fact, if such a difference existed, it would be easy to devise arrangements by which the heat yielded by a metal in passing from the liquid to the solid state could be transformed into electromotive work (and therefore into mechanical work) without other expenditure.

The foregoing examples will be sufficient, it is believed, to show the necessity of regarding other considerations in determining the electromotive force of a galvanic or electrolytic cell than the variation of its energy alone (when its temperature is supposed to remain constant), or corrected only for the work which may be done by external pressures or by gravity. But the relations expressed by (693), (694), and (696) may be put in a briefer form.

If we set, as on page 144,

$$\psi = \varepsilon - t\,\eta,$$

we have, for any constant temperature,

$$d\psi = d\varepsilon - t\,d\,\eta;$$

and for any perfect electrochemical apparatus, the temperature of which is maintained constant,

$$V'' - V' = -\frac{d\psi}{de} + \frac{d\,W_G}{de} + \frac{d\,W_P}{de}; \tag{697}$$

and for any cell whatever, when the temperature is maintained uniform and constant,

$$(V'' - V')\,de \leqq -d\psi + d\,W_G + d\,W_P. \tag{698}$$

In a cell of any ordinary dimensions, the work done by gravity, as well as the inequalities of pressure in different parts of the cell may be neglected. If the pressure as well as the temperature is maintained uniform and constant, and we set, as on page 147,

$$\zeta = \varepsilon - t\,\eta + p\,v,$$

where p denotes the pressure in the cell, and v its total volume (including the products of electrolysis), we have

$$d\zeta = d\varepsilon - t\,d\eta + p\,dv,$$

and for a perfect electro-chemical apparatus,

$$V'' - V' = -\frac{d\zeta}{de}, \tag{699}$$

or for any cell,

$$(V'' - V')\,de \leqq -d\zeta. \tag{700}$$

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ERRATA TO VOL. III.

Page 1, line 13, for Hagerman, read Hagenman.

Page 5, line 30, for *Vetumnus*, read *Vertumnus*.

Page 9, last line, for 1873, read 1874.

Page 11, line 31, for *virticillata*, read *verticillata*.

Page 13, line 34, for *cappilare*, read *capillare*.

Page 23, line 4, for *Smythella*, read *Samythella*.

Page 28, line 19, for **Caridon**, read **Caridion**.

Page 35, line 3, for **Scapellum**, read **Scalpellum**.

Page 58, line 14, for branches, read branchlets.

Page 60, line 12, for Plate X, read IX.

Page 167, formula (168), for m_1, read μ_1.

Page 167, formula (169), for $m_1, \ldots m_{n-1}$, read $\mu_1, \ldots \mu_{n-1}$.

Page 239, formula (333), for $\frac{C}{t}$ read $\frac{C}{a_t t}$.

Page 295, note †, 1st line, for Ueber de Gliedmassen, read Ueber das Skelett der Gliedmassen.

Page 304. note, for prominal, read proximal.

Page 356, last line but two, for crystalline solid, read solid of continuous crystalline structure.

Page 385, line 13, for M′, read M.

Pages 391, 394, 395, 400, in headings, after *Discontinuity*, add *between Fluid Masses*.

Page 403, line 16, after any other film, add of the same components.

Page 405, line 29, after this, add case.

Page 432, line 15 of foot-note, for H, read H_s.

In figure 40, plate XVI, there is a series of ovals around one-half of the real double points. There should be added to the curve, as represented, a like series of ovals around each of the remaining real double points.

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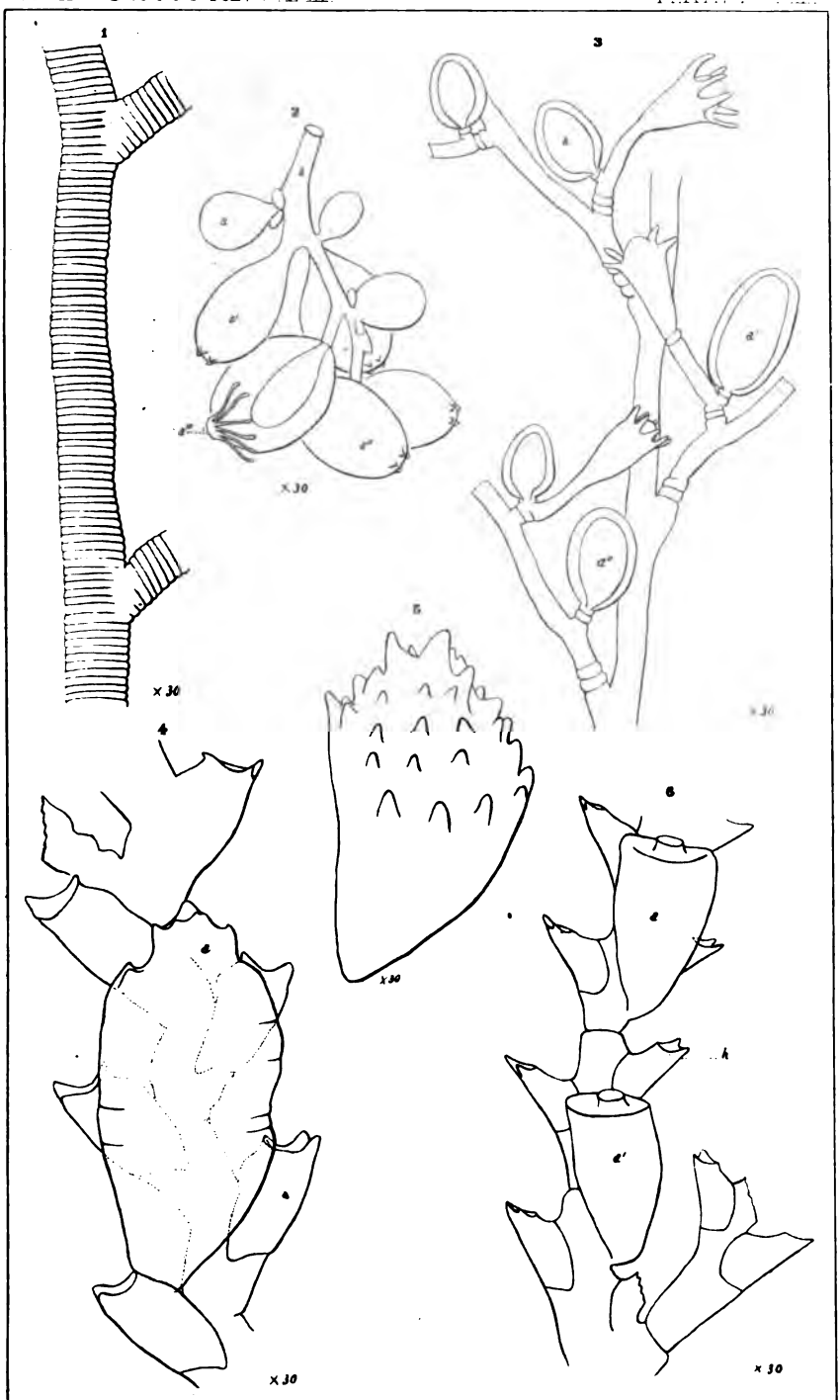

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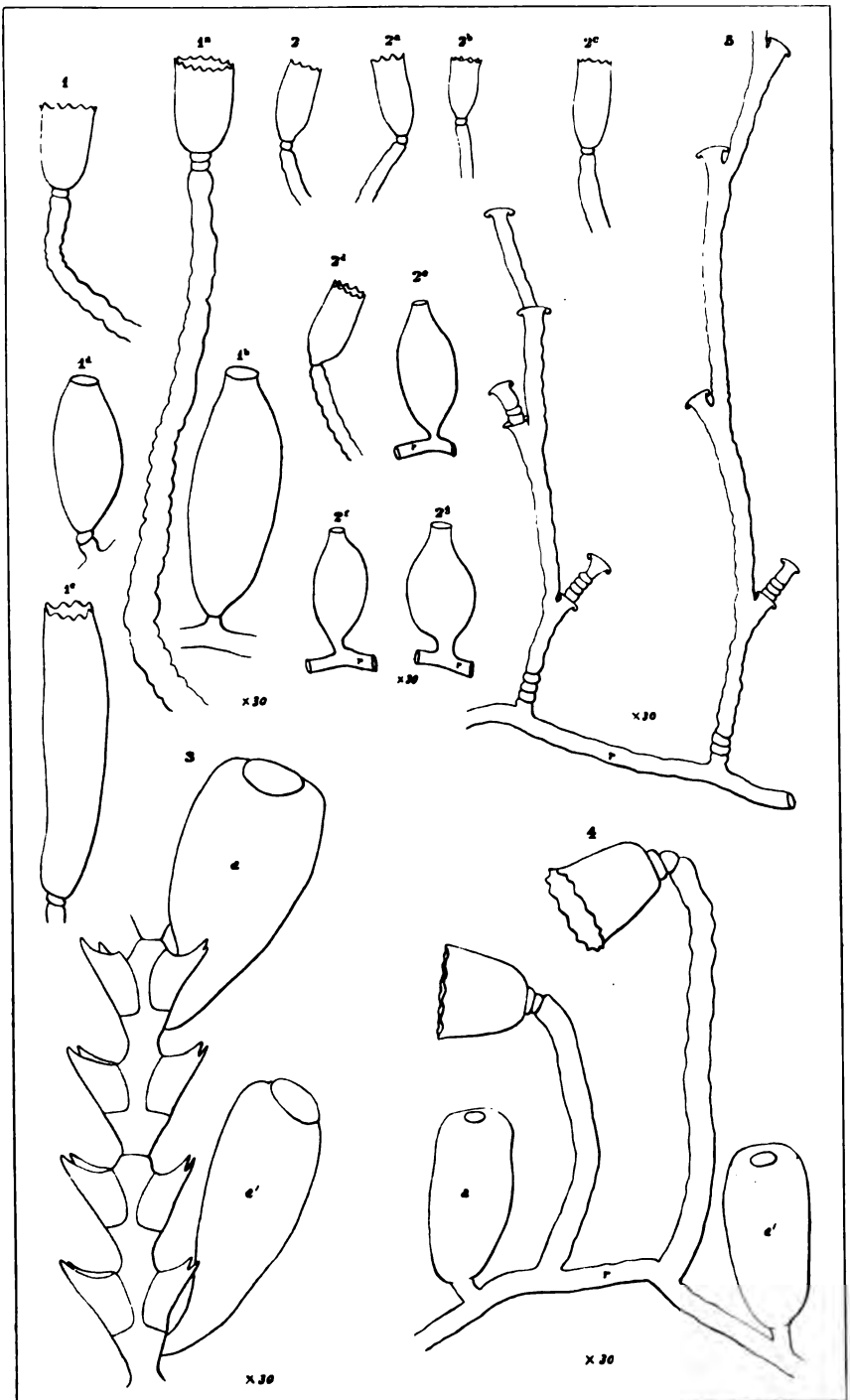

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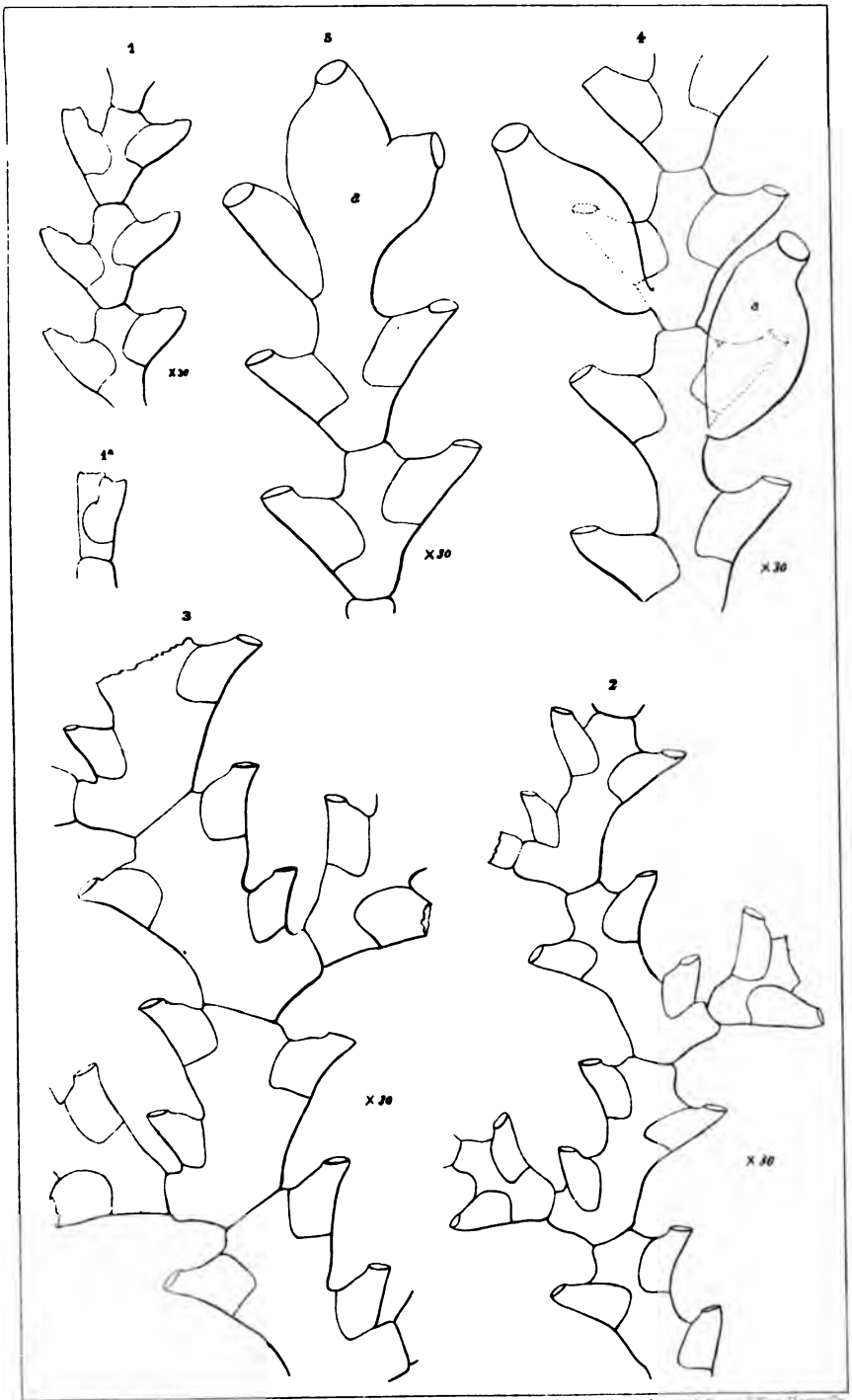

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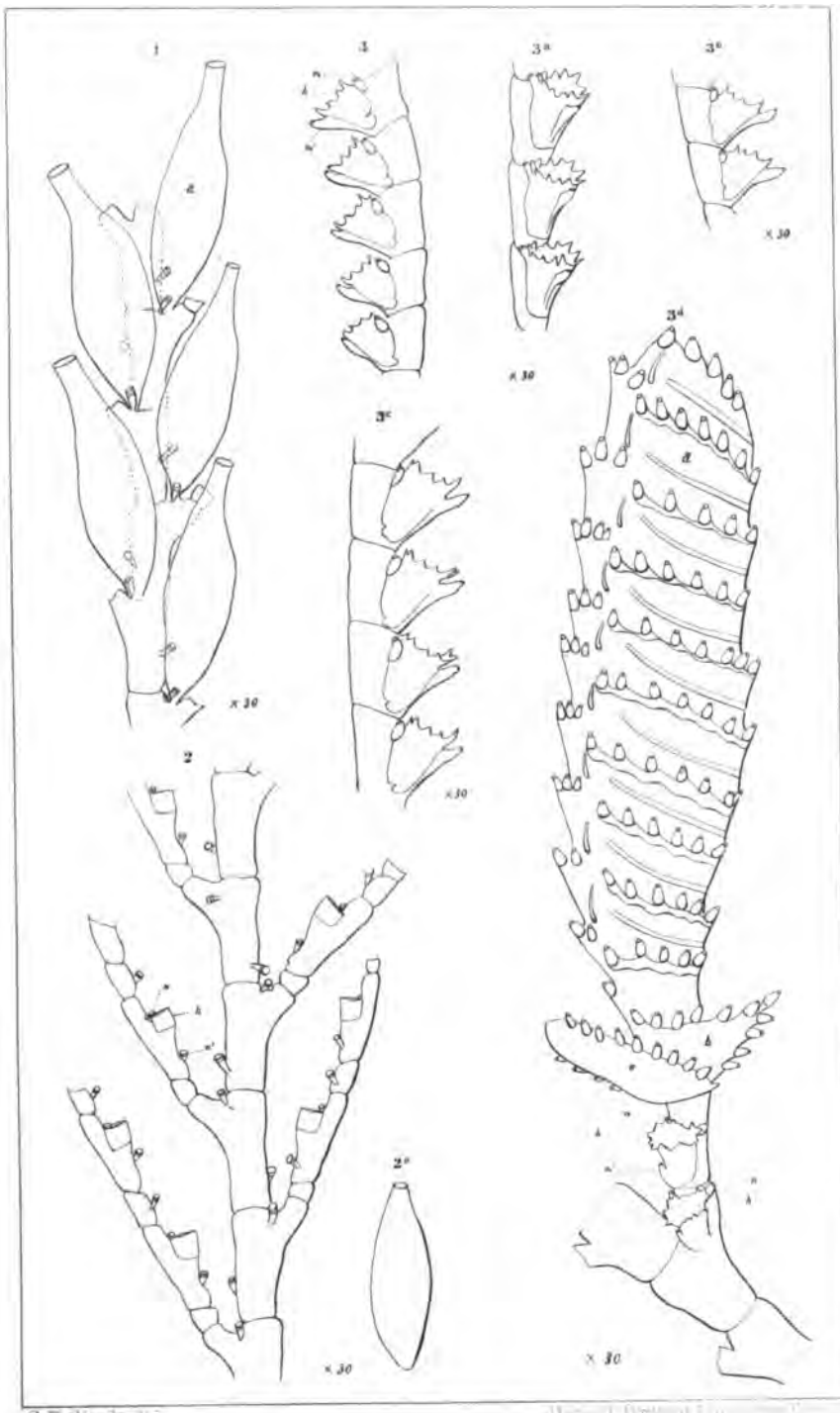

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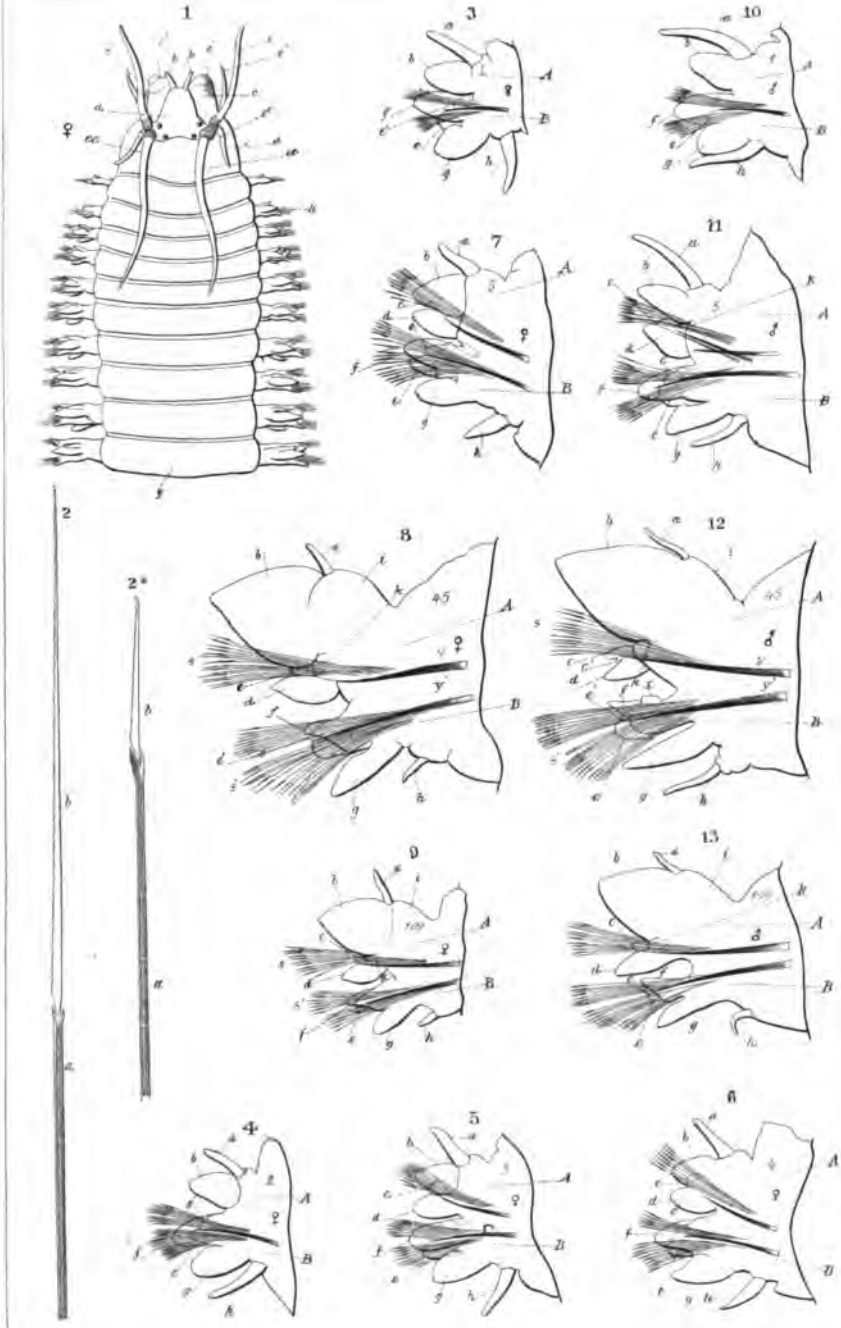

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Punderson & Crosland New Haven, Ct.

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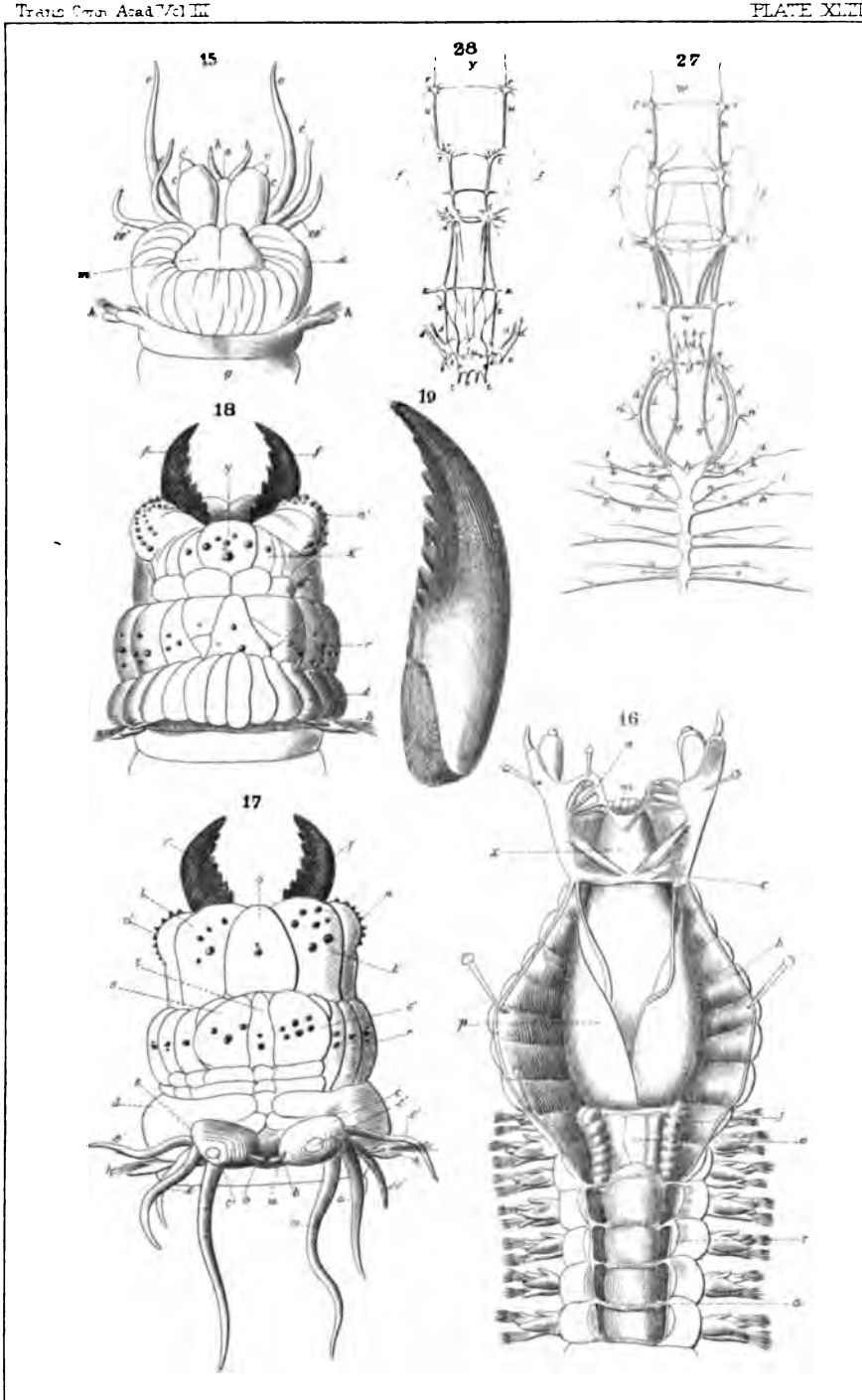

F M Tuttle del. Sundersen & Greene New Haven

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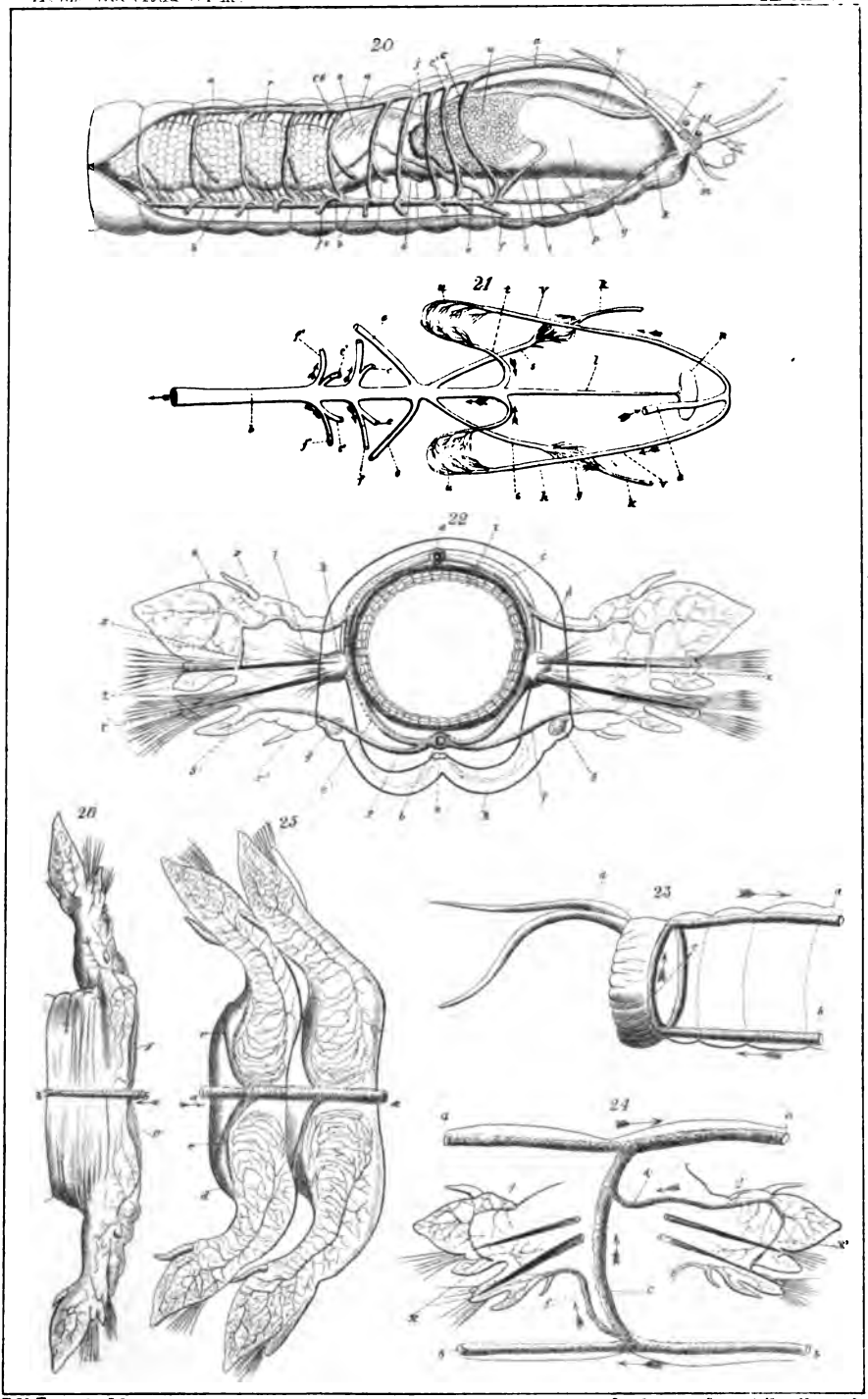

F. M. Turnbull del. Engraved on Steel New Haven Ct.

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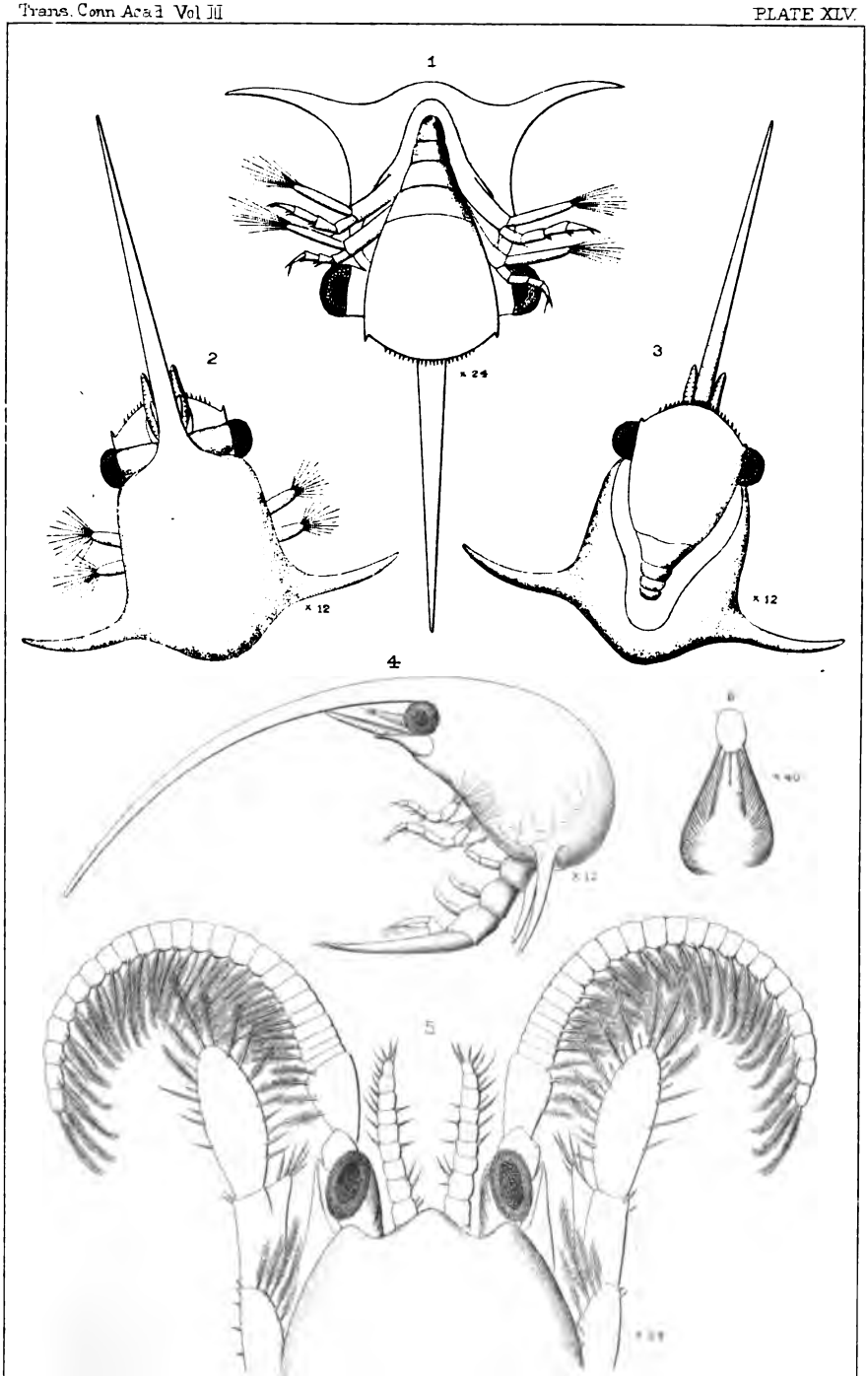

Charles Blake del. Pancoast & Spear, New Haven

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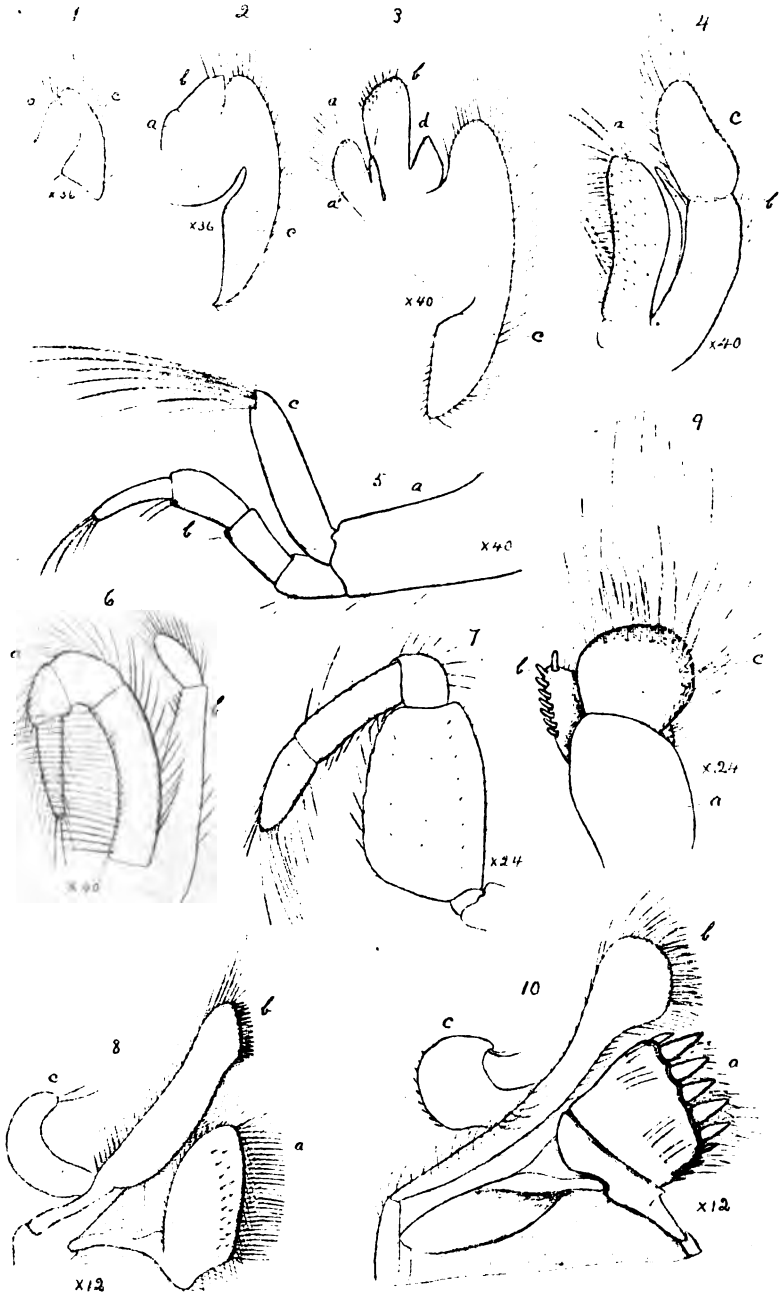

S. I. Smith del.

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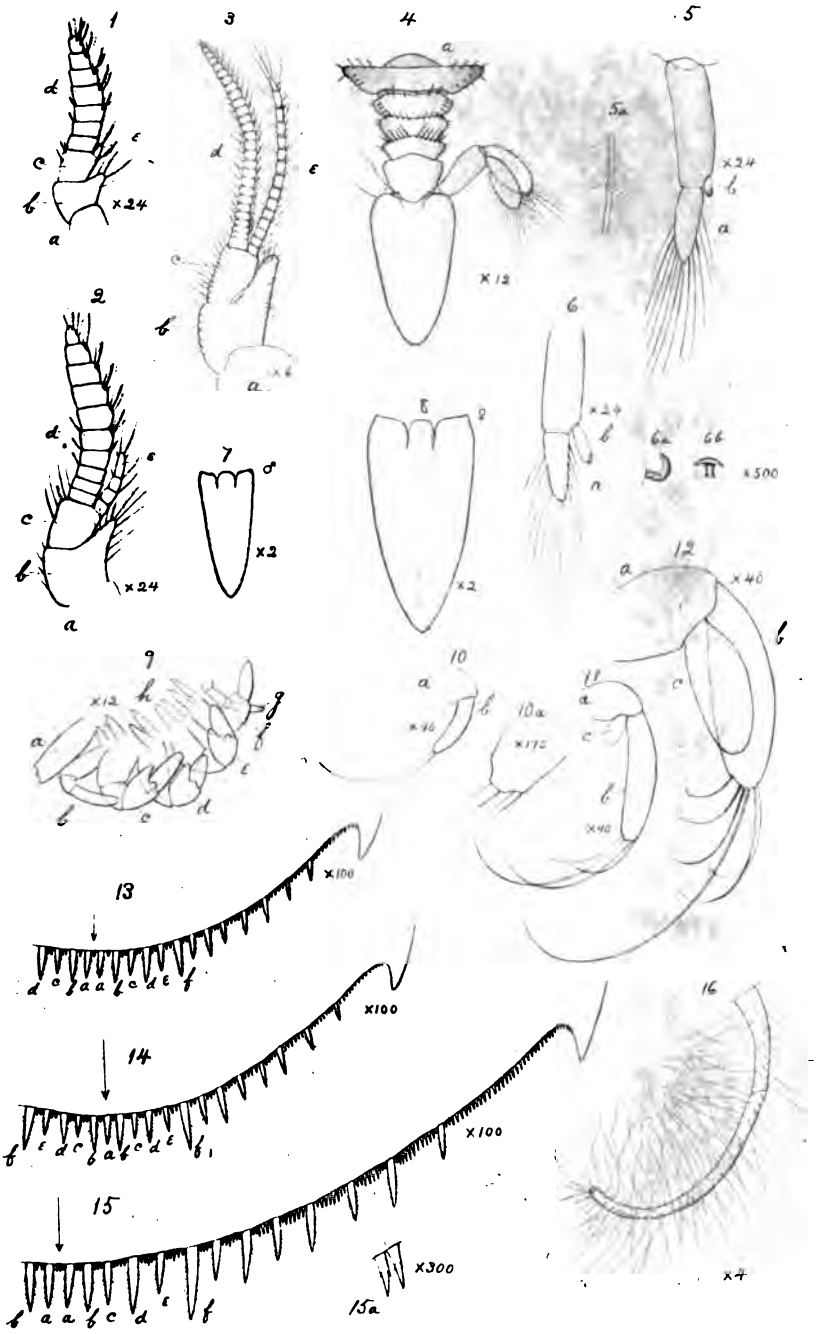

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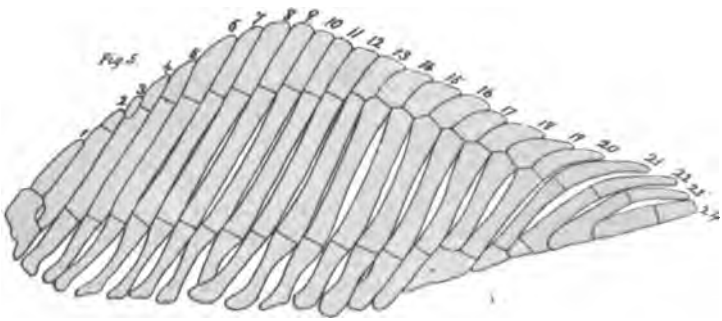

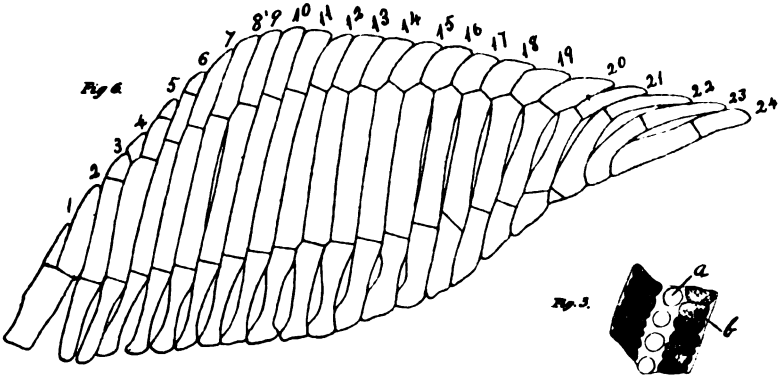

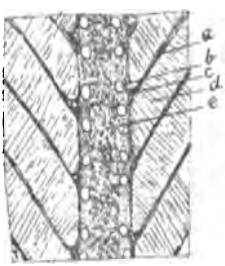

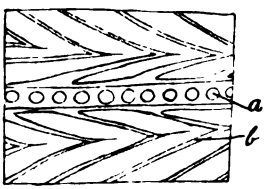

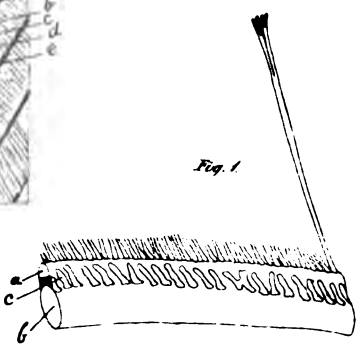

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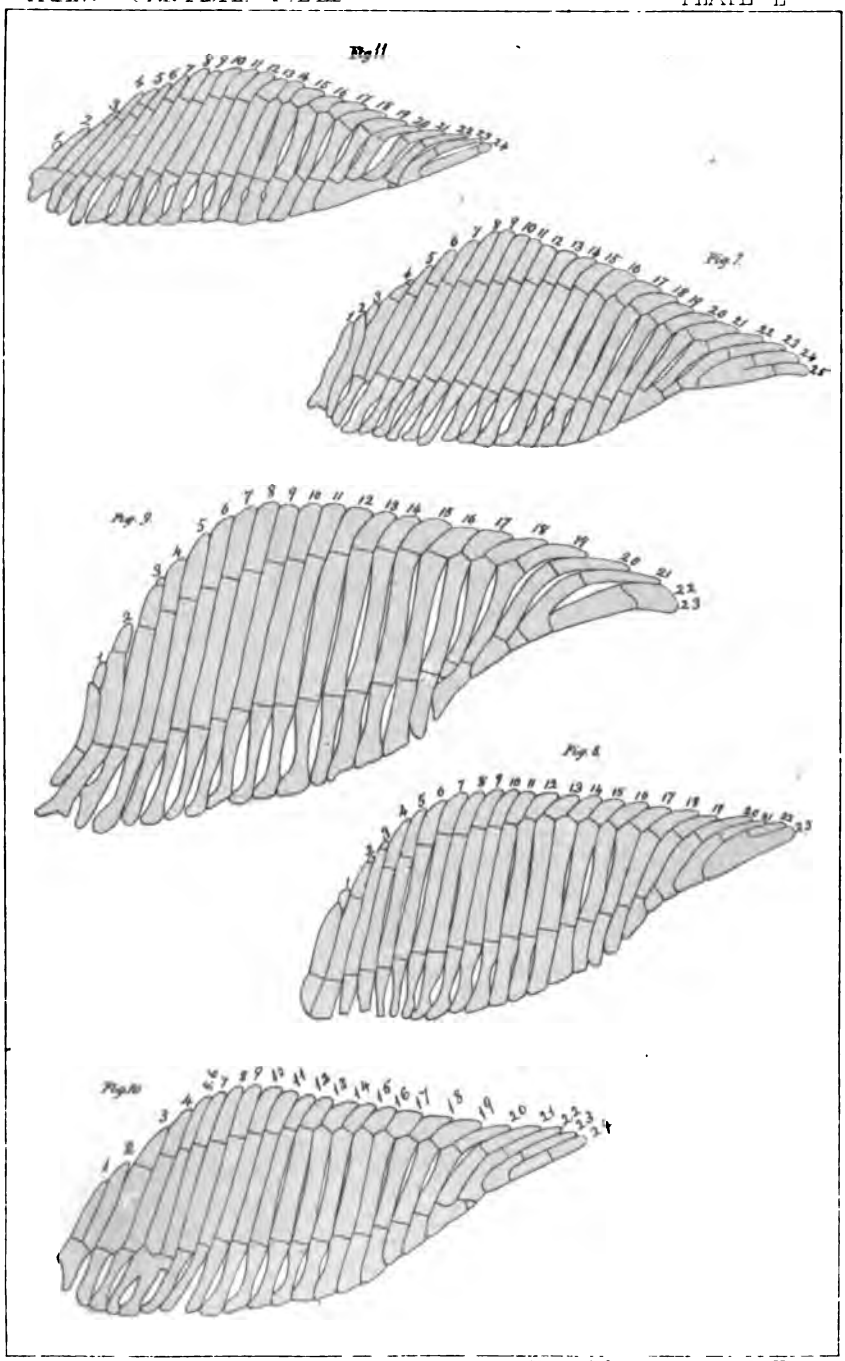

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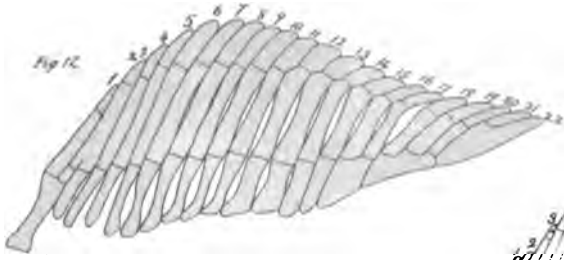

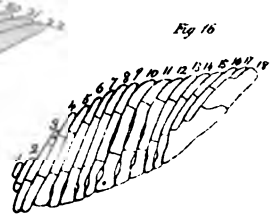

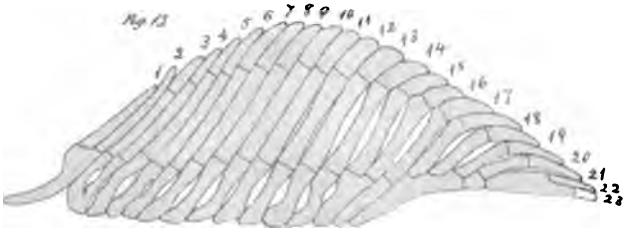

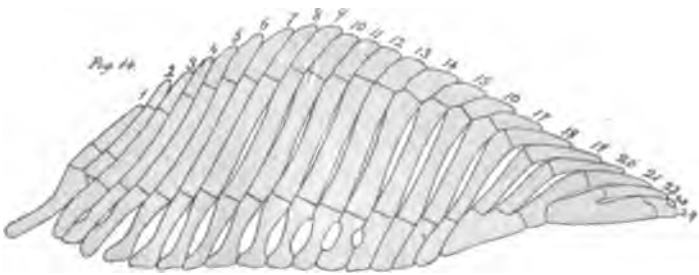

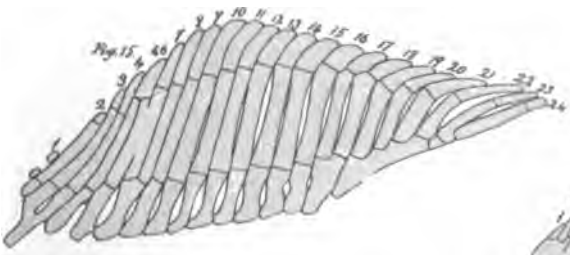

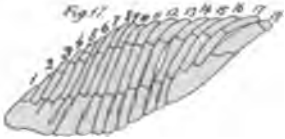

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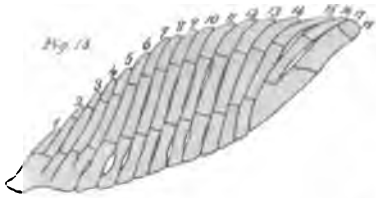

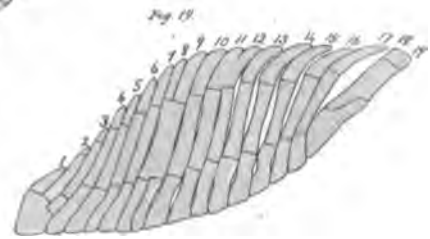

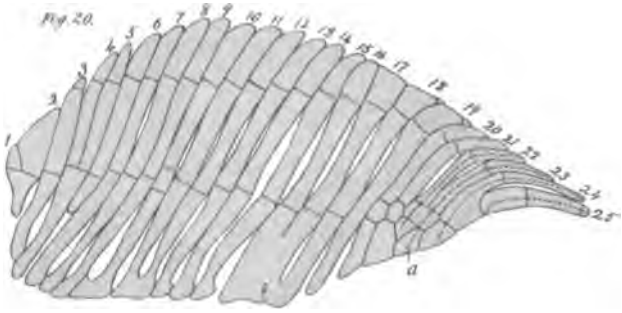

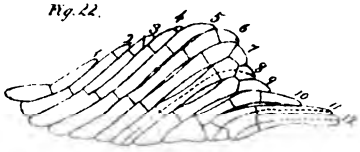

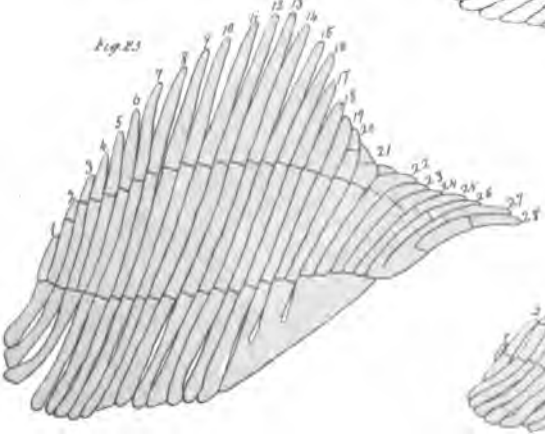

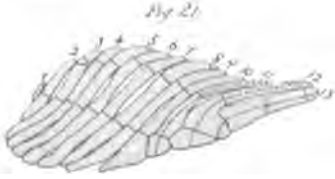

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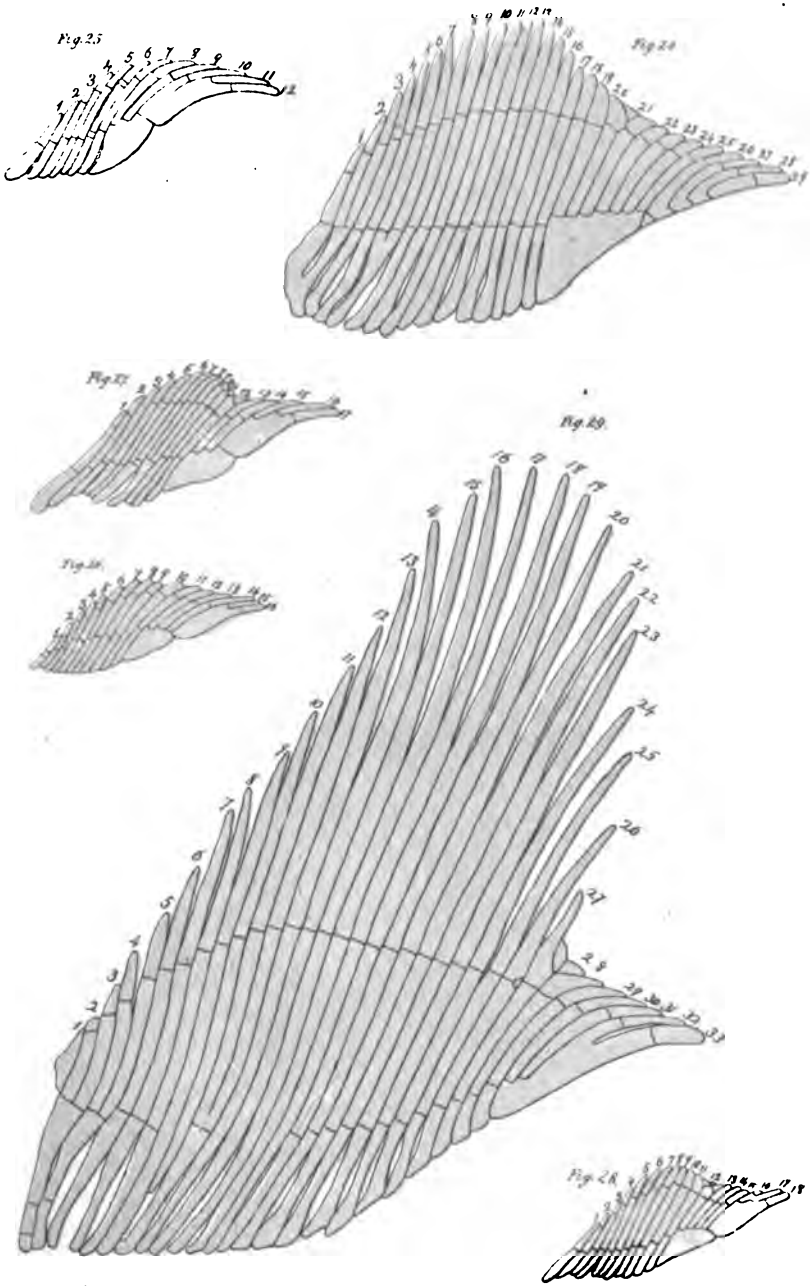

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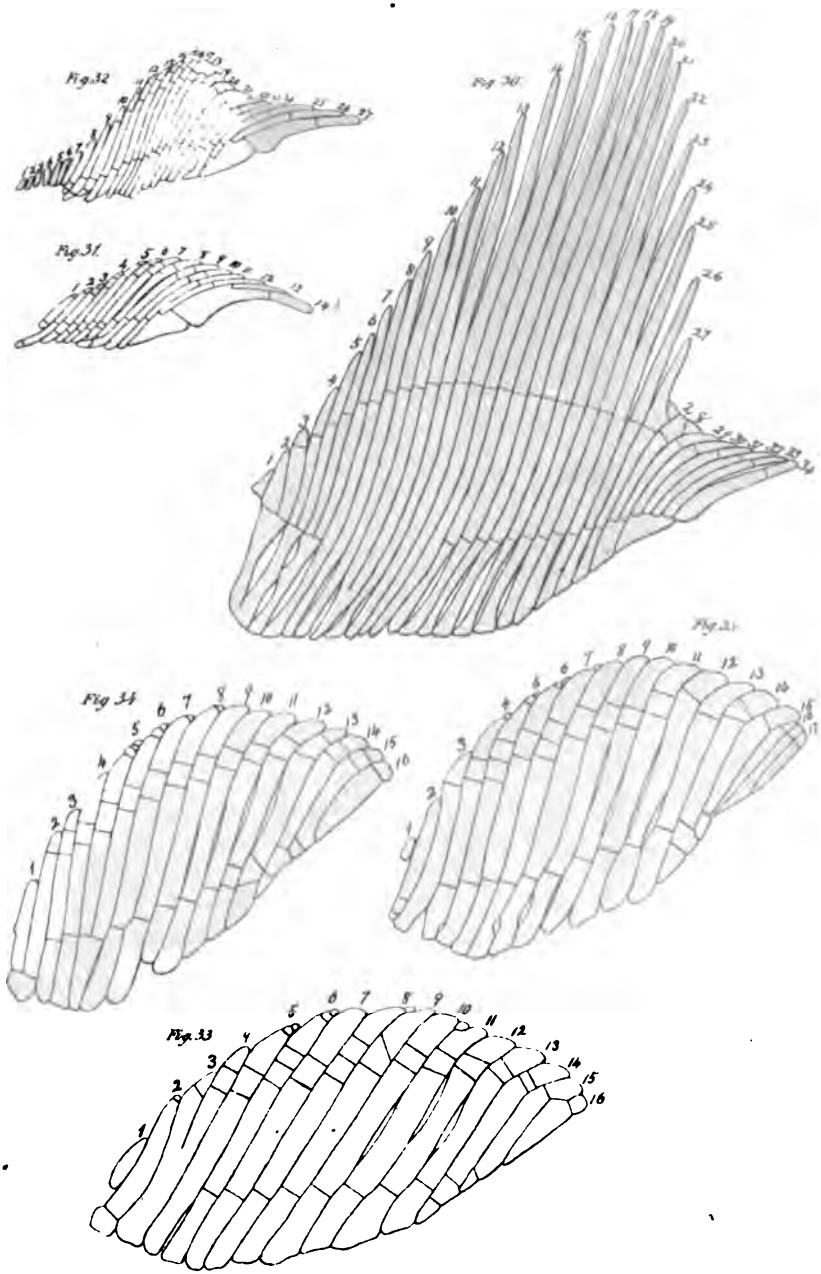

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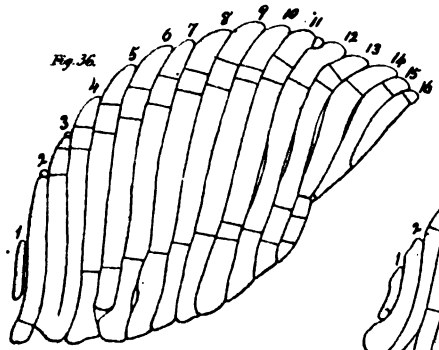

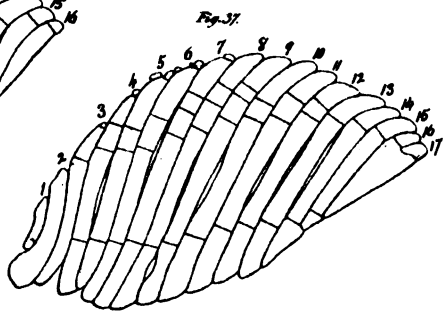

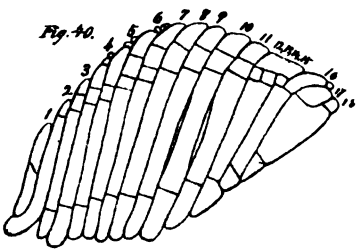

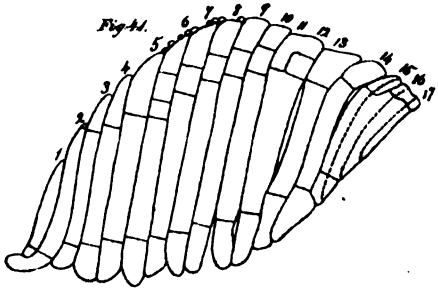

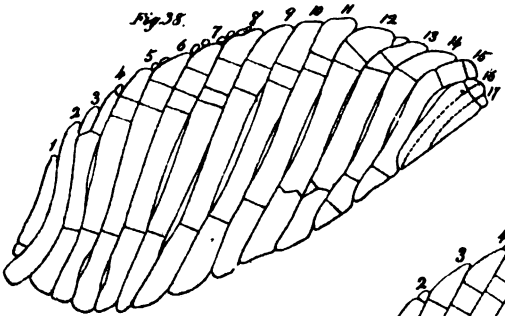

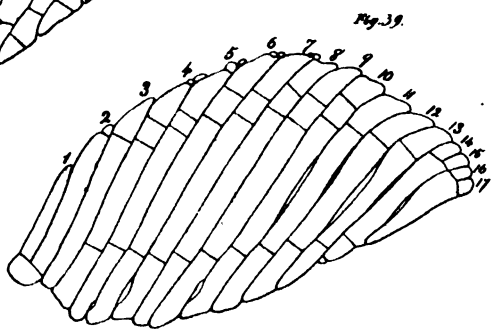

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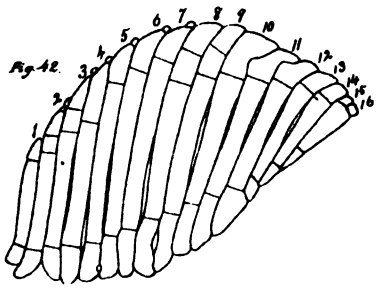

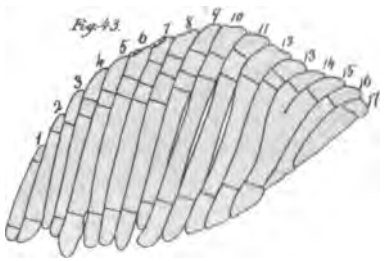

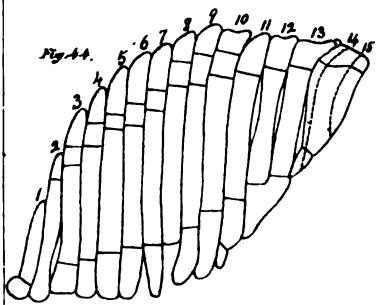

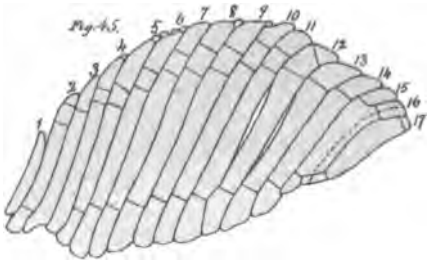

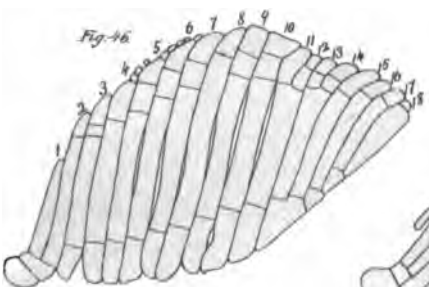

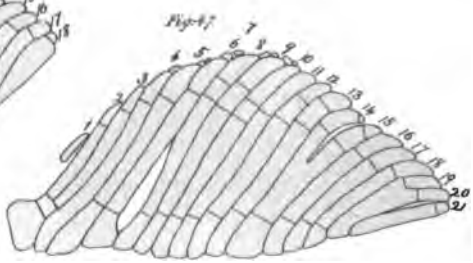

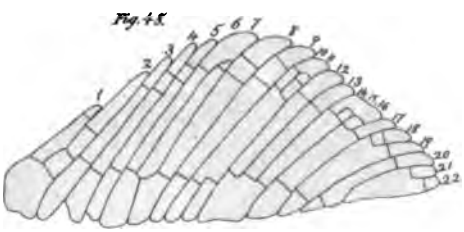

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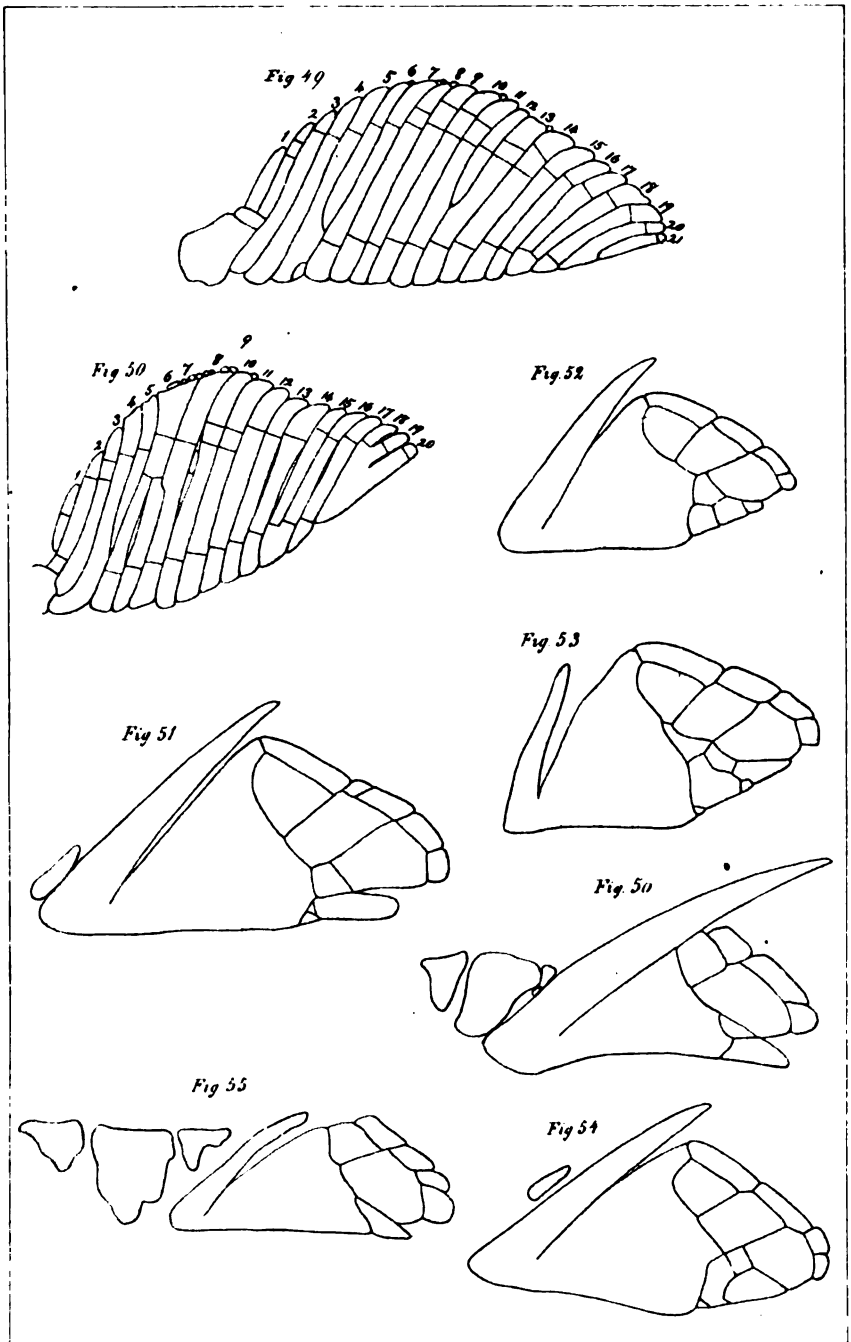

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Fig 61.

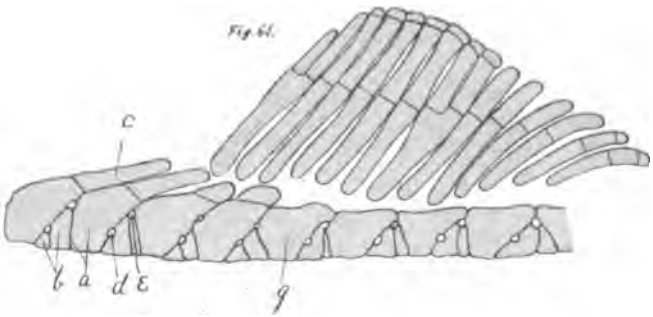


Fig 60

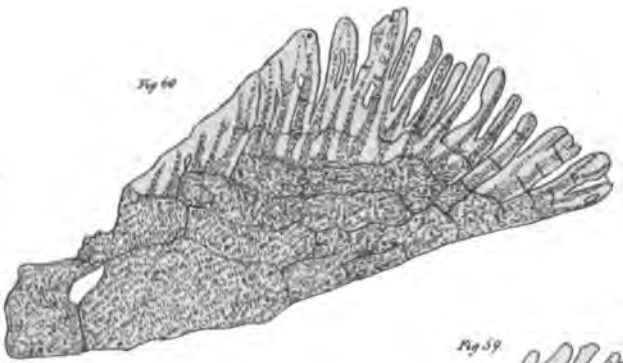


Fig 59

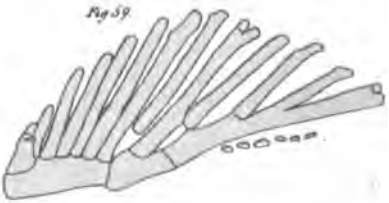
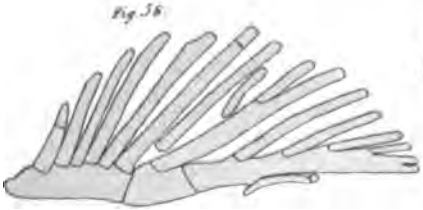


Fig 56



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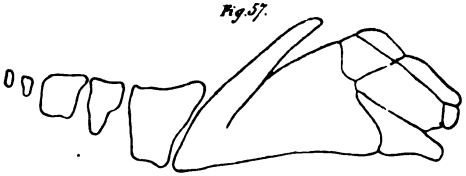

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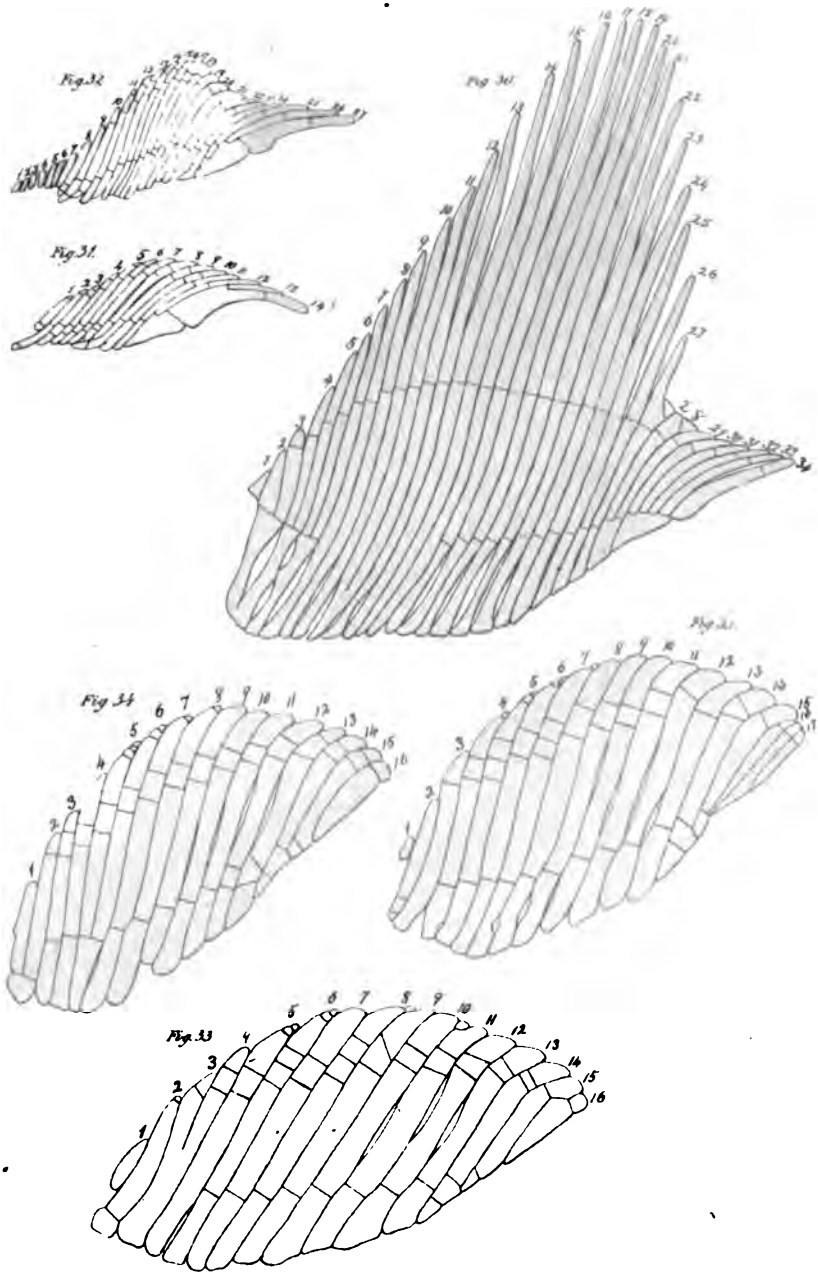

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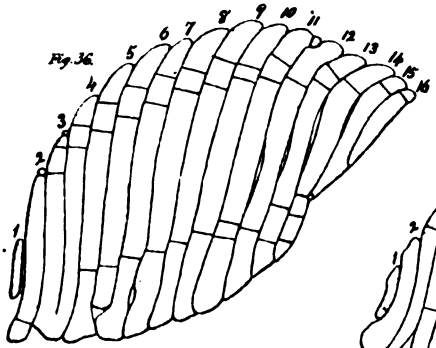

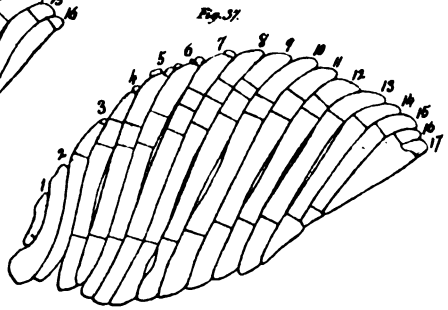

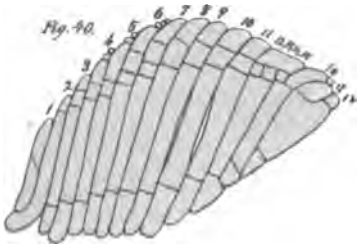

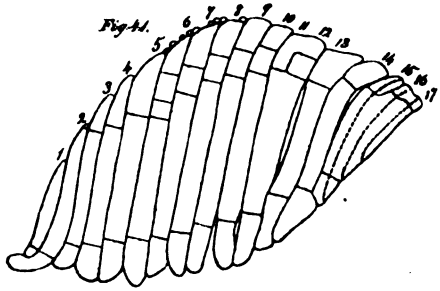

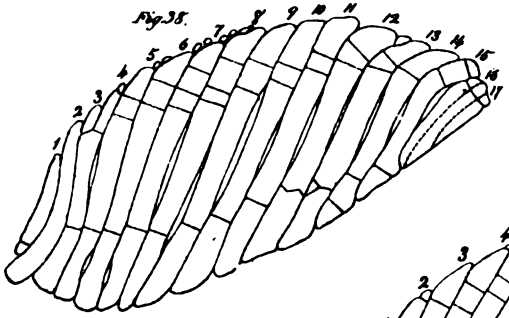

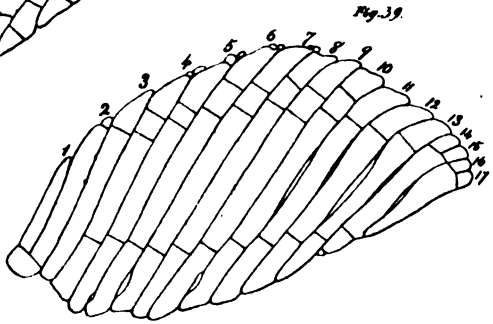

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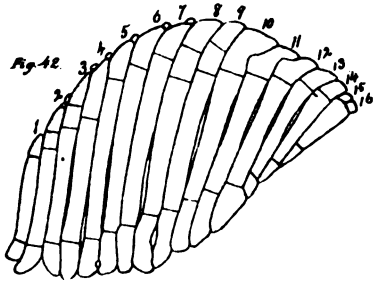

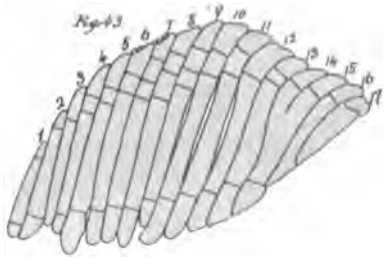

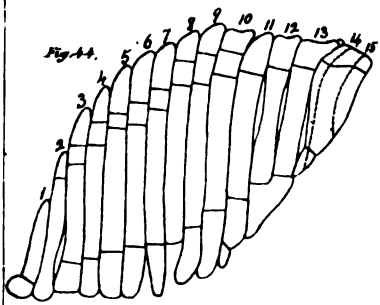

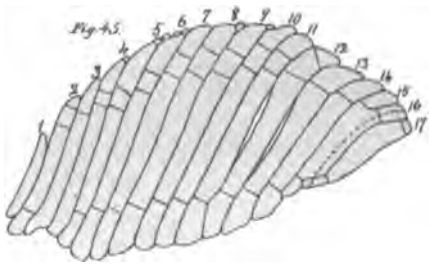

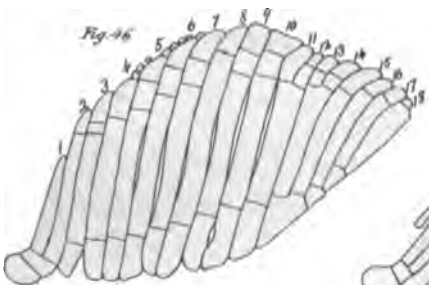

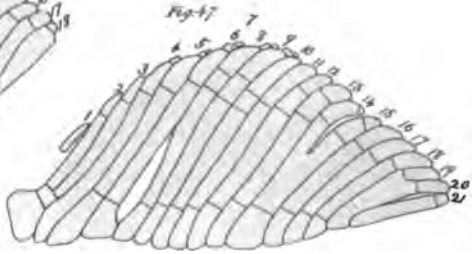

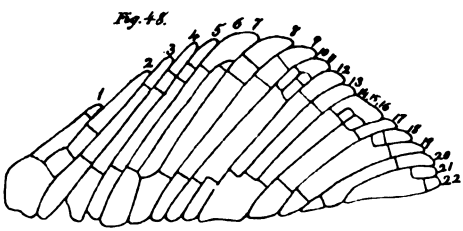

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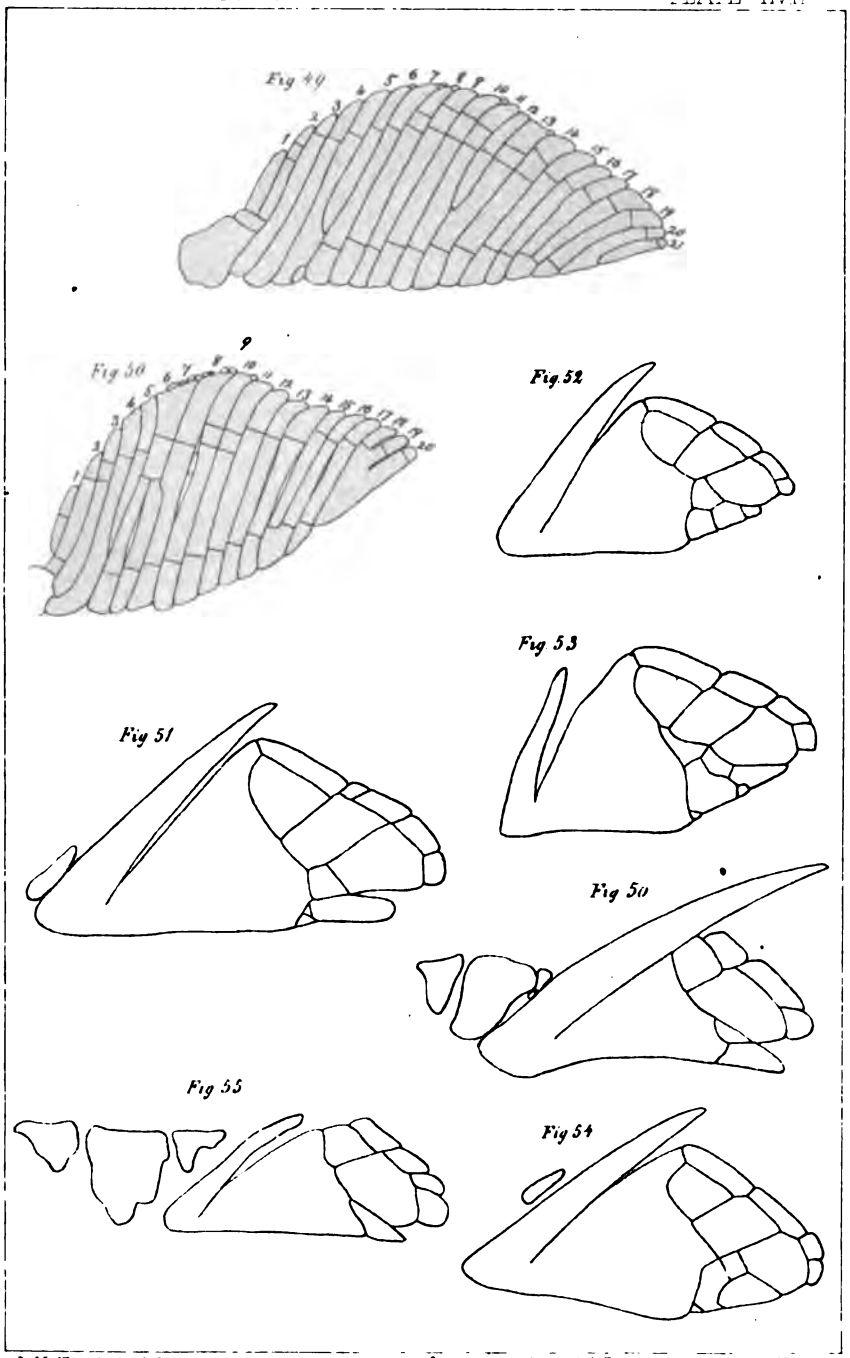

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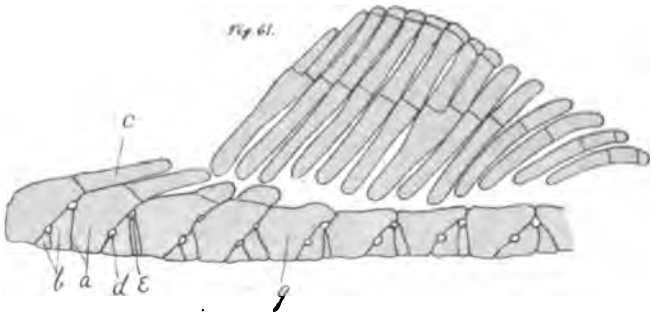

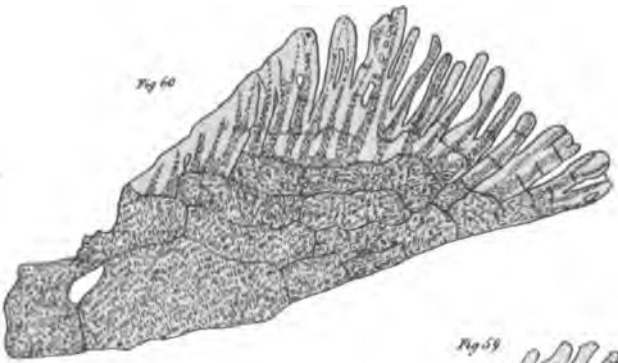

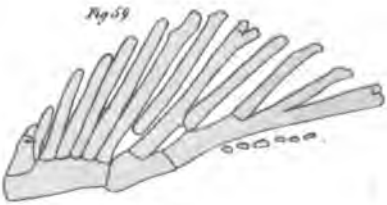

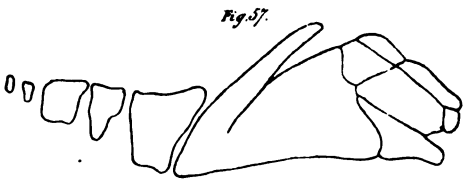

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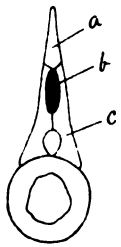

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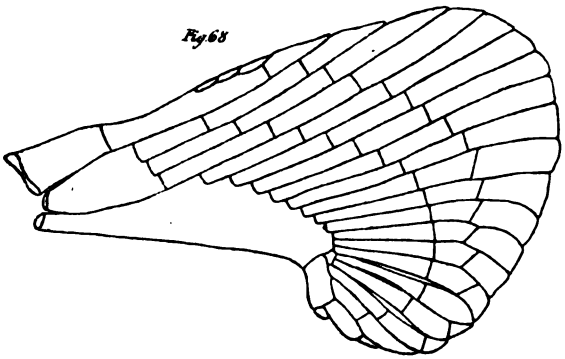

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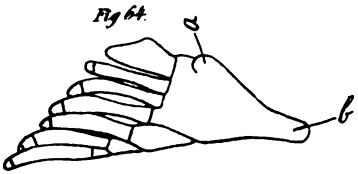

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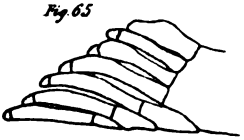

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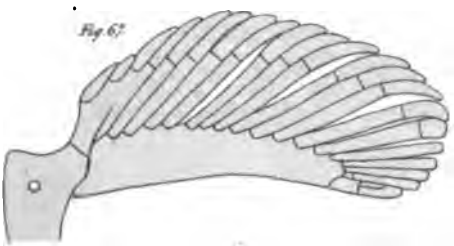

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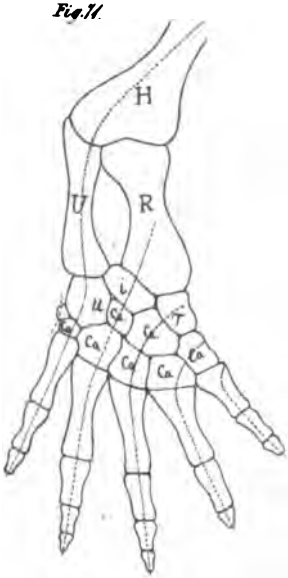

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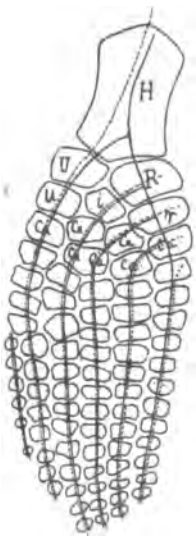

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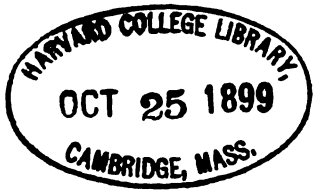

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I. A Review of the Birds of Connecticut, with Remarks on their Habits. By C. Hart Merriam.

Read June 20, 1877.

The little State of Connecticut, forming, as it does, the southwestern corner of New England, and lying between the forty-first and forty-second parallels (exactly 41° to 42° 3′ N.), and extending, in an east and west direction, from the meridian 71° 55′ to 73° 50′ west longitude, contains an area of but 4,674 square miles. Nevertheless it is highly probable, so favorably is it situated, that no equal area in the country can boast a greater number of species of birds than may be found within its limits. Indeed, nearly half the total number of species in the United States have already been detected inside its borders, and it is certain that future investigation will decidedly increase this number.

Zoölogically speaking, Connecticut belongs to that division of country known as the Alleghanian Fauna. Still, as Mr. Purdie has said,* its southern border has an evident tinge of the Carolinian, and I may add that this "tinge" runs up the valley of the Connecticut River, extending completely through the State, and even into Massachusetts.

Some years ago Prof. A. E. Verrill stated that whenever such birds as the Crossbills, the Spruce Partridge and the Canada Jay "breed abundantly in any region, it may safely be considered as belonging to the Canadian Fauna."† With equal truth it may be said that whenever such birds as the Worm-eating, Blue-winged Yellow, and Hooded Warblers, together with the Large-billed Water Thrush and Yellow breasted Chat, *breed regularly and in considerable numbers*, the region may safely be included in the Carolinian Fauna. That these species are so found in southern Connecticut—notably about the mouth of the Connecticut River—is now too well known to need further confirmation.

The dividing lines between the several faunæ are always more or less irregular, and never very sharply defined. The distribution

* Am. Nat., vol. vii, No. 11, p. 693, Nov., 1873.

† Proceed. Essex Inst., vol. iii, p. 138. 1863.

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of those species by which the different faunæ are distinguished, is unquestionably governed, in great measure (as shown by Professor Verrill),* by the temperature *during the breeding season*, of the regions in which they commonly breed. Hence the dividing lines between faunæ do not follow, when placed on the chart, such smooth, regular curves as serve to represent the isothermal lines (which show the average temperature for the entire year), but more nearly coincide with lines drawn to indicate the average temperature during the months of April, May, and June,—the period in which the great majority of singing birds breed. It has been stated by Professor Verrill that "a line drawn upon the map of Eastern North America representing the mean temperature of 50° F., during these three months, will coincide with the southern boundary of the Canadian Fauna, as previously determined from the examination of the birds breeding in that sub-division. Another line representing the temperature of 65° will represent the southern boundary of the Alleghanian Fauna,"* as distinguished from the Carolinian.

The fact that, during the thirty-four years that have elapsed since the publication of Linsley's "*Catalogue of the Birds of Connecticut*," no enumeration of the birds of this State has appeared, is sufficient excuse for the present attempt. Moreover, the Connecticut Academy resolved, some time since, to publish a series of papers on the fauna of this State, and Professor Verrill did me the honor to request that I should prepare, as the first of this series, "A Review of the Birds of Connecticut," which I have done as well as the limited time at my disposal would permit.

In the year 1861, Dr. William Wood, of East Windsor Hill, Conn., published, in the Hartford Times, a series of twenty-one admirable and most interesting articles on our "Birds of Prey," and it is much to be regretted that he did not, in like manner, take up the remaining groups. A few brief notices of the occurrence of some of the rarer species within our limits have, from time to time, appeared in the American Naturalist and Bulletin of the Nuttall Ornithological Club, but nothing like a systematic list of the birds of the State has been attempted.

My object has not been to give the largest possible number of species, but to represent faithfully our true knowledge of the Avifauna of Connecticut. An acknowledged fault with most local lists is that their authors include, not only those species which have actually been detected, but also those which, from their occurrence

* Am. Jour. Sci. and Arts, vol. xli, p. 249, (Second Series,) 1866.

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in neighboring districts, may, at some future time, be looked for with a degree of success directly proportionate to the ornitho-distributional knowledge of the writer. In the preparation of the following list I have scrupulously examined the evidence on which each species stands, and have rigorously excluded all those whose occurrence rests on insufficient proof.

In a note, under each genus or family, will be found those species which should, and probably do, occur, but which, owing to the small number of collectors in this State, have not as yet been detected. Information has been solicited from all, so far as I have been able to ascertain, who are familiar with our birds—whether active collectors or not—and I take this means of expressing my thanks and gratitude for the many favors received, believing, at the same time, that they should not be regarded as mere personal favors, but as real contributions to science, for in preparing this list I act but as the exponent of the present state of our knowledge concerning the birds of Connecticut. I am indebted to my friends, Mr. George Bird Grinnell, Mr. A. J. Dayan, Prof. Wm. D. Whitney, Dr. Wm. H. Hotchkiss, Mr. Thomas B. Osborne, Mr. E. B. Wilson, Dr. F. W. Hall, Walter R. Nichols, the Stadtmüller Brothers, Robert F. Morris, and Fred. Sumner Smith, for information relating to species found in the immediate vicinity of New Haven; to Capt. O. N. Brooks, of Faulkner's Island Light, for a list of the water birds of that portion of the Sound; to Messrs. W. W. Coe and John H. Sage, of Portland, Erwin I. Shores, of Suffield, and J. N. Clark, of Saybrook, Conn., for notes on the species observed in the Connecticut Valley; to Dr. William Wood, of East Windsor Hill, Conn., for valuable information on the time of occurrence and relative abundance of many of the rarer species—especially in regard to the Hawks and Owls; to Prof. G. Brown Goode, of Middletown, Conn., not only for throwing open for my inspection the cases of birds contained in the Museum of Wesleyan University, but also for laboriously searching the Museum Catalogue for records of the capture of the rarer species within the State; to Josiah G. Ely, Esq., for notes on the rarer species found about Lyme, New London County, Conn.; and to Drs. D. Crary and D. Crary, Jr., of Hartford, Conn., for information concerning the less common birds of that vicinity. To each of these gentlemen due credit is given under the species to which their notes pertain. I am also under obligations to Mr. Robert Ridgway, of the Smithsonian Institution, Dr. Thomas M. Brewer, of Boston, and Mr. William Brewster, of Cambridge, for the identification of birds and eggs, and for other information.

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My thanks are especially due to W. W. Coe, John H. Sage, Erwin I. Shores, Thomas B. Osborne, Capt. O. N. Brooks, and the Stadtmüller Brothers, for placing their manuscript notes at my disposal; also, to Professors A. E. Verrill, Sidney I. Smith, and Franklin B. Dexter, for the use of books not in the College Library; and to Dr. William H. Hotchkiss for aid in the preparation of the special lists appended to this paper, as well as for much kindly assistance rendered from time to time during the progress of the work.

Numerous references have been made to the writings of others, and I hold myself responsible for all dates and statements of any kind, where the contrary is not expressly stated. Neither have quotations been omitted, when they seemed desirable, either for the value of the facts they contain, or for their antiquity; and, as a rule, I have taken pains to refer to the older and less accessible works, rather than to those with which we are all familiar.

Although Ornithology, as a science, is, in this country, of comparatively recent date, still that the birds of New England early attracted the attention of our forefathers is evident from their writings. In the year 1616, Capt. John Smith said that there were, in New England, "Eagles, Gripes, diuers sorts of Haukes, Cranes, Geese, Brants, Cormorants, Ducks, Sheldrakes, Teale, Meawes, Guls, Turkies, Diuedoppers, and many other sorts, whose names I knowe not."* And a few years later (1622) one William Hilton writes that the land "*affoords beasts of diuers sorts, and great flocks of Turkies, Quailes Pigeons and Patriges: many great lakes abounding with fish, fowls, Beuers and Otters. The sea affoords us as great plenty of all excellent sorts of sea-fish, as the riuers and Iles doth varietie of wild fowle of most vseful sorts.*"† And there are numerous others of a like nature, some of which have been reproduced under the species to which they refer.

It is worthy of note that many birds once common along the coast, and some throughout the greater part of New England, are now either extremely rare or not to be met with at all within its limits, while a few species have unquestionably increased in numbers since the country has become settled. Among the former class may be mentioned the Great Auk (*Alca impennis*), the Crane (*Grus Canadensis*, and perhaps *G. Americana* also), the Swan (*Cygnus Americanus*, and perhaps *C. buccinator*), the Wild Turkey (*Meleagris gallopavo*), the Pinnated Grouse or Prairie Chicken (*Cupidonia cupido*),

* A Description of New England, by Capt. John Smith. p. 16. 1616.

† New England's Trials. p. 14. 1622.

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the Swallow-tailed Kite (*Nauclerus forficatus*), the Raven (*Corvus corax*) now common only in northern New England, the Mocking-bird (*Mimus polyglottus*), and many others. That these birds were common here two or three hundred years ago is clear from the numerous references to them in the writings of the time. And during this period many others have greatly decreased in numbers—such as the Pigeon (*Ectopistes migratoria*), Pileated Woodpecker (*Hylotomus pileatus*), black-birds, water fowl, and waders of all descriptions.* The disappearance of some of these birds is readily accounted for by the advance of civilization with its concomitant evils—destruction of forests, increased use of fire arms, etc.—but in other cases the cause is less easily explained.

Few people, living away from the coast, have any idea that hundreds of thousands of birds are killed each year, during migrations, by flying against lighthouse towers. Capt. O. N. Brooks, of Faulkner's Island Light, tells me that between two and three hundred dead birds were picked up at the foot of the tower on the morning of May 16th, 1877, and that at the same time thousands of living ones stopped at the Island, devouring all the tender plants and newly sown seeds in his garden. During the same night about three hundred birds killed themselves against the light at the entrance of New Haven Bay, and the daily papers stated that over seven hundred were picked up on the decks of one of the New York steamers. When we take into consideration the number of light-houses and steamships along our whole coast it becomes clear that the number of birds that perish annually by this means is simply immense. Nearly all our common, and many rare, migrants are found among the dead, but the Maryland Yellow-throats (*Geothlypis trichas*) greatly predominate, constituting full one third of the total number examined. It is lamentable that this wholesale slaughter of our song-birds seems to be unavoidable.

The nomenclature and arrangement of species adopted in the present paper is that given in Coues' "Key to North American Birds," with such corrections as have since appeared in his "Birds of the Northwest" and elsewhere.

* See also an interesting article by J. A. Allen in Bull. Nutt. Ornith. Club, vol. i, No. 3. Sept. 1876.

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A. *General List, giving times of occurrence and relative abundance of the species; together with notes on the habits and distribution of some of the less common forms.*

Family, TURDIDÆ.

1. Turdus migratorius Linné. Robin.

An abundant summer resident; breeds abundantly, and several times during the season. A few winter. The summer residents arrive during the middle and latter part of February (Feb. 18, 1876; Feb. 27, 1875), and remain in considerable numbers till near the close of November.

2. Turdus mustelinus Gmelin. Wood Thrush.

A common summer resident; breeds. Arrives about May 3d (common before the 10th), remaining till October. Mr. John H. Sage, of Portland, Conn., informs me that he and Mr. J. N. Clark, and also Mr. W. W. Coe, have found its eggs "covered with fine black specks." Mr. Fred. Sumner Smith, of this city, has recently shown me a large egg of this species which is marked with a few small, brownish-red spots. On sending the egg to Dr. Brewer, I received the following: "The spots I never noticed before, but any egg, always excepting a woodpecker's, is liable to be marked by minute effusions of colored lymph of the parent, in its exclusion." Mr. Smith tells me that the nest, which contained these speckled eggs, was placed on a railway bridge, and within *six inches* of the rail. A number of trains pass daily over this bridge.

3. Turdus Pallasi Cabanis. Hermit Thrush.

Common during its migrations, particularly so in the fall. Arrives from the South before the middle of April (April 7, 1877, Coe; April 6, 1875, Sage; April 13, 1874, Sage; April 14, 1876, Osborne; April 16, 1877, April 19, 1875), remaining till early in May; on their return are with us from early in October till the latter part of November (Nov. 14, 18, 20, 1875). The woods abound with them during the middle and latter part of October.

4. Turdus Swainsoni Cabanis. Olive-backed Thrush.

Common during its migrations, but never so abundant as the last. It is met with in spring from May 10th to 26th, and in autumn from

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Sept. 4th (1874, Hall) to Oct. 13th. In the fall it is not uncommonly found in flocks of a dozen or fifteen in low swampy woods where there is an abundance of undergrowth.

4a. Turdus Swainsoni, var. **Aliciæ** (Baird) Coues. Gray-cheeked Thrush.

Occurs, during migrations, with the foregoing, and is more common in the vicinity of New Haven—judging from the proportionate number of skins in private collections about here. Near Portland, Conn., on the contrary, the true *Swainsoni* type seems to predominate—as shown by specimens in the cabinets of W. W. Coe and John H. Sage.

5. Turdus fuscescens Stephens. Wilson's Thrush.

A summer resident; breeds. Arrives early in May (May 8). Common in the immediate vicinity of New Haven, especially during the spring migrations.

6. Mimus polyglottus (Linné) Boie. Mocking-bird.

A rare summer visitant; used to breed here and may occasionally* do so now. Mr. Geo. Bird Grinnell has taken it near Milford, Conn. Mr. Erwin I. Shores writes me, on the authority of Milton Lester, that it has been killed at Suffield, Conn. Mr. J. N. Clark thinks that it bred pretty regularly at Saybrook, Conn., many years ago; and I am informed by Mr. Osborne that he saw one above the Whitney Lakes, near New Haven, May 30th, 1877. Also taken late in the fall of 1874, by Mr. William Brewster, near Concord, Mass.† "Stratford and New Haven," Linsley.

There can be little doubt but that Mocking-birds were once common in Connecticut, Rhode Island, and Massachusetts, if not still farther north. In an appendix to the Mass. Agricultural Report for 1863, Samuels gives it as a "rare summer visitant," stating that it "occasionally breeds;" and in 1864 Allen says that they "have been known to breed in Springfield several times within five years, and in 1860 two pairs nested here. June 20th, 1860, I found a nest containing three freshly laid eggs. . . . . Locality, a sandy field growing up to pitch pines, in one of which the nest was placed, about three feet from the ground."‡

* A Catalogue of the Birds of Connecticut, arranged according to their natural families; by Rev. James H. Linsley, published in Am. Jour. of Sci. and Arts, vol. xliv, No. 2, p. 255. April, 1843.

† Rod and Gun, vol. v, No. 24, p. 370, Mar. 13, 1875.

‡ Catalogue of the Birds found at Springfield, Mass., by J. A. Allen. pp. 67–8. 1864.

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7. Mimus Carolinensis (Linné) Gray. Cat-bird.

An abundant summer resident, arriving early in May (May 4, 1877, Portland; May 5, 1874, Sage; May 6, 1876, C. H. M., and 6, 1877, E. B. Wilson), and remaining till November (Nov. 4, 1874 and 1875); breeds.

8. Harporhynchus rufus (Linné) Cabanis. Brown Thrush; "Thrasher."

A common summer resident; breeds. Arrives during the last of April or first of May (April 30, 1876, Osborne; May 1, 1877) departing early in October (Oct. 2).

Family, SAXICOLIDÆ.

9. Sialia sialis (Linné) Haldeman. Common Bluebird.

Resident; abundant during summer ; breeds in holes in trees and posts, etc. They arrive early in February (Feb. 8, 1876), remaining till about the middle of November (Nov. 13, 1876). Have found it singing as early as March 5th. Though generally evenly distributed and everywhere a common species, Mr. Shores "could not find it in 1872" about Suffield, Conn.* A few small flocks frequently winter in secluded spots.

Family, SYLVIIDÆ.

10. Regulus calendula (Linné) Licht. Ruby-crowned Kinglet.

Common during its migrations. Arrives about the middle of April (April 11, 1875, Sage); remaining till the middle of May (16, Osborne). Have taken it in the fall from Oct. 13th to Nov. 24th.

11. Regulus satrapa Lichtenstein. Golden-crested Kinglet.

A winter resident; have seen it at repeated intervals from Oct. 13th till spring (May). Commonly found, with the preceding, associated with small flocks of chickadees (*Parus atricapillus*), and frequently accompanied by a few nuthatches of both species (*Sitta Carolinensis* and *S. Canadensis*).

12. Polioptila cærulea (Linné) Sclater. Blue-gray Gnatcatcher.

A rare summer visitant. "Stratford," Linsley. "Two were shot at Wauregan (Windham County), Conn., by Mr. C. M. Carpenter,—a male in 1874 and a female in 1876. Three or four were seen by me

* MS. notes of Erwin I. Shores, Esq.

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at Providence, R. I., May 23d, 1875."* Mr. Jencks writes me that the Connecticut record is a mistake, as all the specimens were taken near Providence, R. I.

Mr. William Brewster thus speaks of their habits and appearance in the south: "When seen one hundred feet or more above the earth they remind one more of insects than of birds, so active, and so very frail and slender do they seem. In motions, they bear, perhaps, a greater resemblance to the Redstart (*Setophaga ruticilla*) than to any other bird, like him launching out frequently after insects and alighting with spread tail and drooping wings. They have withal an impertinent, quizzical air, savoring strongly of Cat-bird ways; the song is indeed quite that of the latter bird, but in miniature (if I may apply such an expression to sound), a quaint mocking little strain, continued half a minute or more at a time, and full of mewings and harsh chatters, with an occasional full round note, but altogether so feeble as scarcely to be audible at twenty yards' distance. The note used by both sexes is a harsh but rather faint lisp."†

Family, PARIDÆ.

13. Lophophanes bicolor (Linné) Bonaparte. Tufted Titmouse.

A rare visitor from the south. "New Haven," Linsley. Josiah G. Ely, Esq., writes me from Lyme, New London Co., Conn., that he shot a Tufted Titmouse, Feb. 27th, 1872, in that vicinity (the ground being covered to a considerable depth with snow), and saw another in Jan., 1874. They were found flitting about among dense hemlocks in a very wild portion of country. It has also been taken near Hartford, Conn., by Dr. D. Crary, but must be regarded as accidental, though a common resident in northern New Jersey (Elizabeth, 1871–72). Also breeds on Long Island "seeking retirement in the lonely part of the woods among the heavy timber."‡ It has been found as far north as New Hampshire.§

14. Parus atricapillus Linné. Black-capped Chickadee.

A common resident; breeds in holes in trees. Gregarious except during the breeding season.

* Mr. Jencks to H. A. Purdie, see Bulletin of the Nuttall Ornithological Club, vol. ii, pp. 20 and 21. Jan. 1877.

† Annals Lyceum Nat. Hist. N. Y., vol. xi, p. 132, June, 1875.

‡ Birds of Long Island, J. P. Giraud, Jr. p. 78. 1844.

§ Belknap's History of New Hampshire, vol. iii, p. 173. 1792.

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15. Parus Hudsonicus Forster. Hudsonian Chickadee.

A rare accidental visitor from the north. In the July (1876) number of the Bulletin Nuttall Ornith. Club, I called attention to the first instance of the capture of this northern chickadee in Connecticut: "On Nov. 13, 1875, Mr. Robert Morris, while shooting in a wooded ravine a few miles from town [New Haven], killed a female Hudsonian Titmouse (*Parus Hudsonicus*). The specimen is now in the collection of Mr. Thomas Osborne of this city." Its occurrence so far south must be regarded as purely accidental. Mr. Wm. Brewster took a specimen at Concord, Mass., Oct. 30, 1870.* This was supposed to be the most southern, and only Massachussetts, record, but as long ago as 1839, Peabody tells us that it "has been found by S. Eliot Greene, Esq., near his house in Brookline."†

Family, SITTIDÆ.

16. Sitta Carolinensis (Gmelin) Latham. White-bellied Nuthatch.

Resident; breeds. Not particularly abundant immediately about New Haven, especially in summer. Its nest, however, has been taken within city limits by the Stadtmüller Brothers.

17. Sitta Canadensis Linné. Red-bellied Nuthatch.

A tolerably common winter resident, arriving about the middle of October (Oct. 13, 1875, abundant), and remaining till after the middle of April (April 19, 1876, Osborne).

Family, CERTHIIDÆ.

18. Certhia familiaris Linné. Brown Creeper.

A rather common resident; breeds. Particularly abundant in winter, when it may frequently be seen running about on the elms in the heart of the city.

Family, TROGLODYTIDÆ.

19. Troglodytes aëdon Vieillot. House Wren.

A summer resident; breeds. Not abundant. Arrives late in April or early in May (Apr. 27, 1869, Hartford; 27, 1877, Portland; May

* Am. Nat., vol. vi, No. 5. p. 306. May, 1872.

† A Report on the Ornithology of Massachusetts, by Wm. B. O. Peabody. p. 402. 1839.

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2, 1863; 3, 1876; 4, 1861, 1873, all Portland, Conn., Jno. H. Sage), remaining till late in October (Oct. 26 and 30, 1875).

20. Anorthura troglodytes, var. **hyemalis** (Vieill.) Coues. Winter Wren.

A winter resident; rather common during migrations. Have seen it early in October (Oct. 2d), and secured a specimen on the 24th of November, 1875. Mr. Grinnell has taken it every month during winter.

21. Telmatodytes palustris (Wilson) Baird. Long-billed Marsh Wren.

A common summer resident; breeds abundantly in the brackish water marshes bordering the Quinnipiac river, near New Haven. Mr. A. J. Dayan and myself found dozens of completed nests, a few of which contained from 1 to 5 eggs each, on the 7th of June, 1876. Also breeds in suitable fresh water marshes throughout the State.

22. Cistothorus stellaris (Licht.) Cabanis. Short-billed Marsh Wren.

A rather rare summer resident; breeding in suitable localities along our southern border and in the Connecticut Valley. "New Haven," Linsley. Dr. Wood tells me that it is not common about Hartford, Conn.; and Mr. Erwin I. Shores writes me that it breeds in a marsh near Southwick (or Congannick) Ponds, in the town of Suffield, Hartford Co., Conn. (close to the Massachusetts border). Mr. J. G. Ely, of Lyme, Conn., writes me that the Short-billed Marsh Wren breeds abundantly in that vicinity.

NOTE.—The Great Carolina Wren, *Thryothorus Ludovicianus* (Latham) Bonap., has been known to breed in Massachusetts* and on Long Island, and doubtless occurs, as a rare summer resident from the South, in the Connecticut Valley, and along our southern border.

Family, ALAUDIDÆ.

23. Eremophila alpestris (Forst.) Boie. Horned Lark.

A winter resident, occurring along the shore, and also in barren, sandy, tracts, throughout the State. Mr. Dayan, Mr. Osborne, Mr. Grinnell, and myself, have seen it here in fall (Nov. 18, 1874, C. H. M.) and winter, and Mr. W. W. Coe has found it at Portland, Conn., as late as March 26, (1875).

* Bull. Nutt. Ornith. Club, vol. i, No. 3, p. 76. Sept. 1876.

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Family, MOTACILLIDÆ.

24. Anthus Ludovicianus (Gmelin) Licht. Titlark.

Common along the coast during migrations. Have taken it as late as Nov. 14. It passes north in the month of May (Dr. Wood). Dr. Brewer says that he has "found it, sometimes in large flocks, in open country near the coast, in Massachusetts, in midwinter."*

Family, SYLVICOLIDÆ.

25. Mniotilta varia (Linné) Vieillot. Black-and-white Creeper.

A summer resident; breeds. Particularly abundant during migrations. Arrives late in April or early in May (April 30, 1877, May 2, 1876), remaining till October (Oct. 2). Mr. J. H. Sage writes that, in company with Mr. J. N. Clark, he took a nest containing five eggs of this species, at Saybrook, Conn., May 30, 1877. I shot it at New Haven on June 23, and have seen several others during the breeding season. Mr. Sage also took another nest containing five eggs ("young well developed"), at Portland, Conn., June 12, 1875.

26. Parula Americana (Linné) Bonap. Blue Yellow-backed Warbler.

A summer resident; breeds. One of our commonest migrants. Arrives early in May (May 2d, Osborne). In the fall it is very abundant during the middle and latter part of September, and I have taken it as late as Oct. 13th. Mr. W. W. Coe showed me a number of their beautiful hanging nests, composed entirely of *Usnea*, which he had found about Portland, Conn. It has also been found breeding near New Haven (Maltby Park—Stadtmüller Brothers).

27. Helmitherus vermivorus (Gmelin) Bonap. Worm-eating Warbler.

A rare summer resident; breeds. As long ago as the year 1843, Dr. J. D. Whelpley observed this species at New Haven (Linsley), and it has since been taken in this vicinity by several collectors. In the "Bull. Nutt. Ornith. Club," (vol. ii, No. 1, Jan., 1877, p. 21), Mr. H. A. Purdie states that "Mr. Shores shot a male at Suffield (Hartford County), Conn., August 22, 1874. This is, I think, its most northerly record in the Atlantic States yet noted." Also, in the American Naturalist for Nov., 1873 (vol. vii, No. 2, p. 692), the same author remarks that Mr. J. N. Clark, of Saybrook, Conn., writes him that he has taken "in the nesting season, *Helmitherus vermivorus*,

* Bull. Nutt. Ornith. Club, vol. i, No. 4, p. 92. Nov. 1876.

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but as yet has failed to find the nest." Mr. Clark tells me he has seen as many as five individuals in a single day. Mr. Thomas Osborne, of New Haven, has a mounted specimen in his cabinet, procured May 17, 1875. Two or three others specimens were shot near here in May, 1875, and Mr. George Bird Grinnell tells me that he has known of the capture of several in this vicinity.

So little is really known concerning the habits of this rare warbler, that I take great pleasure in quoting the following interesting account of it from the observations of my friend, Mr. William Brewster, of Cambridge, "On the Birds of Ritchie County, West Virginia." Mr. Brewster writes that it is "most partial to the retired thickets in the woods, along water courses, and is seldom or never found in the high open groves. They keep much on the ground, and *walk* about rather slowly, searching for food among the dried leaves. In general appearance they are quite unique, and I rarely failed to identify one with an instant's glance, so very peculiar are all their attitudes and motions. The tail is habitually carried at an elevation considerably above the line of the back, which gives them quite a smart, jaunty, air, and if the dorsal aspect be exposed, in a clear light, the peculiar marking of the crown is quite conspicuous. Seen as they usually are, however, dimly flitting ahead through the gloom and shadow of the thickets, the impression received is that of a dark little bird which vanishes unaccountably before your very eyes, leaving you quite uncertain where to look for it next: indeed, I hardly know a more difficult bird to procure, for the slightest noise sends it darting off through the woods at once. Occasionally you will come upon one winding around the trunk of some tree, exactly in the manner of *Mniotilta varia*, running out along the branches with nimble motion, peering alternately under the bark on either side, and anon returning to the main stem, perhaps the next instant, to hop back to the ground again. On such occasions they rarely ascend to the height of more than eight or ten feet. The males are very quarrelsome, chasing one another through the woods with loud, sharp, chirpings, careering with almost inconceivable velocity up among the tops of the highest oaks, or darting among the thickets with interminable doublings until the pursuer, growing tired of the chase, alights on some low twig or mossy log, and, in token of his victory, utters a warble so feeble that you must be very near to catch it at all, a sound like that produced by striking two pebbles very quickly and gently together, or the song of *Spizella socialis* heard at a distance, and altogether a very indifferent performance."*

* Annals Lyceum Nat. Hist. N. Y., vol. xi, pp. 134–5. June, 1875.

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28. Helminthophaga pinus (Linné) Baird. **Blue-winged Yellow Warbler.**

A summer resident in southern Connecticut and in the Connecticut Valley; breeds. Not common at New Haven. Mr. L. C. Bragg took a female May 12, and I captured a male on the 24th, 1876. Mr. J. N. Clark finds it breeding regularly, and in considerable numbers, at Saybrook, Conn.* Mr. W. R. Nichols shot one at Branford, Conn., May 12th, 1877. On the 23d of June last (1877) while collecting with Mr. Wm. H. Taylor, in a piece of woods within a mile of the city of New Haven, I succeeded in shooting a fine adult male of this species, which Prof. Wm. D. Whitney was kind enough to mount for the collection of New England birds now on exhibition in the Peabody Museum of Yale College. The bird kept flitting about so actively in the dense undergrowth that it was with difficulty secured. It unquestionably had a nest in the immediate vicinity.

29. Helminthophaga chrysoptera (Linné) Cabanis. **Blue Golden-winged Warbler.**

A rather rare summer resident; breeds in the northern part of the State. Mr. Walter R. Nichols shot one near New Haven August, 24, 1876, and Mr. Osborne tells me that he saw one in May, 1875, killed by a friend of his. I have not yet met with it. The finest specimens of this beautiful warbler that I have ever seen are in the cabinets of Mr. Coe and Mr. Sage, of Portland, Conn. They were taken in that vicinity. Mr. Sage informs me that he has seen it from May 11th, (May 11 and 27, 1876) to the 30th, (May, 30, 1877, Saybrook). Mr. J. N. Clark, of Saybrook, Conn., tells me that he has seen but one specimen there. Mr. Erwin I. Shores writes me from Suffield, Conn.: "Not common; breeds. On July 7th, 1876, I obtained a female and three young just able to fly. Have taken it at two other times." Dr. Crary has taken it near Hartford.

30. Helminthophaga ruficapilla (Wilson) Baird. **Nashville Warbler.**

A summer resident; abundant spring and autumn migrant. Arrives early in May. Breeds sparingly throughout the State.

31. Helminthophaga peregrina (Wilson) Cabanis. **Tennessee Warbler.**

A rather rare migrant. Two specimens were taken near New Haven during the latter part of May, 1876 (May 24, Osborne; May 26, Bragg). Mr. Erwin I. Shores shot one near Suffield, Conn., June 8,

* Am. Nat., vol. vii, No. II, p. 692. Nov. 1873.

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1875, which is so late one might almost suspect it of breeding within our limits. Mr. J. N. Clark, of Saybrook, has a specimen in his cabinet in unusually high plumage; it is a rare bird there. I am told by Mr. Clark and others that its note is peculiarly loud and clear, serving to distinguish the species when some distance away.

NOTE.—The Orange-crowned Warbler, *Helminthophaga celata* (Say) Baird, has been taken both in Massachusetts* and Rhode Island,† and also in New York State (by Mr. E. P. Bicknell, and but a few miles from our western border)‡ and unquestionably occurs within our limits.

32. Dendrœca æstiva (Gmelin) Baird. Yellow Warbler.

A common summer resident; breeds. Arrives about the first of May (May 7, Osborne; May 4, 1874, and May 5, 1873, Portland, Conn., Sage).

33. Dendrœca virens (Gmelin) Baird. Black-throated Green Warbler.

A summer resident; very abundant during its migrations. Arrives about May 1st, remaining through the month (May 23); have taken them in the fall, from September 15th to October 18th. Mr. W. W. Coe tells me that they remain through the entire summer in dense swamps about Portland, Conn., and doubtless breed, though he has not yet taken the nest. I am also informed by Mr. J. N. Clark that a few breed at Saybrook, where he has found two nests after the young had hatched. I am surprised at this, for it is the last place in the State where they *ought, theoretically*, to breed.

34. Dendrœca cærulescens (Linné) Baird. Black-throated Blue Warbler.

A summer resident. Common spring migrant; Mr. Osborne and myself have also taken it here in autumn (October 7, 12, and 19). Arrives about May 10th; breeds sparingly in the northeastern part of the State (Eastford), where two nests were found in June (8 and 13, 1874,) by the Rev. C. M. Jones.§

35. Dendrœca cærulea (Wilson) Baird. Cærulean Warbler.

A very rare summer visitant from the Southern and Central States. This rare warbler was observed at Stratford, Conn., by Linsley, as

* Bull. Nutt. Ornithol. Club, vol. i, No. 4, p. 95. Nov. 1876.

† Ibid, vol. ii, No. 1, p. 21. Jan. 1877.

‡ In letter from E. P. Bicknell.

§ Bull. Nutt. Ornith. Club, vol. i, No. 1, p. 11. April, 1876.

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long ago as April, 1841. H. A. Purdie says, "a male was obtained by Mr. Shores at Suffield, June 12, 1875."* Mr. Shores writes me that he "obtained it purely by accident while trying to shoot a Warbling Vireo (*Vireo gilvus*)." So far as I am aware these two are the only instances of its capture within the State.

36. Dendrœca coronata (Linné) Gray. Yellow-rumped Warbler.

Very common during migrations. Have taken it as early as April 18th (1875). In the fall it may be seen from the last of September till the first of November, some remaining still later (Nov. 15), and Mr. Grinnell informs me that he has taken it every month during the entire winter.

37. Dendrœca Blackburniæ (Gmelin) Baird. Blackburnian Warbler.

Common during the migrations, being particularly abundant in spring. A few sometimes breed. Arrives about May 9th, remaining till the last of the month (May 27). Also occurs from the middle of September till early in October (Oct. 3). In the spring great numbers of them frequent the elms in the heart of the town. Mr. Josiah G. Ely writes me, from Lyme, Conn., that he "shot a female Blackburnian Warbler in July, 1873, with its belly minus of feathers. It evidently had a nest somewhere in that vicinity."

38. Dendrœca striata (Forster) Baird. Black-poll Warbler.

One of our most abundant migrants. Arrives from the south about May 13th, remaining till the last of the month (May 30; June 2, 1872, Portland, Conn., Sage). In the autumn they are quite numerous by Sept. 10th, and their number does not decrease much till after the middle of October—some staying till the last of the month (Oct. 26).

39. Dendrœca castanea (Wilson) Baird. Bay-breasted Warbler.

Sometimes quite abundant during the spring migration (1876, for example); at other times extremely rare, if occurring at all. Arrives about the middle of May, remaining till the last of the month. Mr. Osborne has seen it in fall (Sept. 30 and Oct. 17, 1876). Mr. Shores gives it as "rare," never having taken "but two specimens," at Suffield, Conn.†

* Bull. Nutt. Ornith. Club, vol. ii, No. 1, p. 21. January, 1877.

† MS. notes of Erwin I. Shores.

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40. Dendrœca Pennsylvanica (Linné) Baird. Chestnut-sided Warbler.

A summer resident; breeds. Very abundant during migrations. Arrives early in May (7), the greater number passing north before the first of June.

41. Dendrœca maculosa (Gmelin) Baird. Black-and-yellow Warbler.

A common spring and autumn migrant. Arrives about May 12th, remaining till June 1st. Have taken it in the fall, from the middle of September to October 1st, and it probably remains still later.*

42. Dendrœca tigrina (Gmelin) Baird. Cape May Warbler.

A rather rare migrant, though a few are taken every season. Have known of its occurrence from May 10th, (Osborne) to the 25th. Mr. Shores writes me that it is generally rare about Suffield, Conn., but that it was not uncommon there in the spring of 1876.

43. Dendrœca discolor (Vieillot) Baird. Prairie Warbler.

A summer resident; not common about New Haven. Breeds sparingly. Mr. Dayan has taken it on May 13th and Sept. 15th. Mr. W. W. Coe tells me that it breeds near Portland, Conn., but is rather rare. I shot one near New Haven, May 14th, 1877. I am informed by Mr. E. I. Shores, of Suffield, that it is not rare about Enfield, Conn. Mr. J. N. Clark tells me that it breeds plentifully about Saybrook, Conn.

44. Dendrœca dominica (Linné) Baird. Yellow-throated Warbler.

A rare accidental visitor from the south Dr. Daniel Crary, of Hartford, Conn., writes me that during fifteen years of bird collecting in that vicinity (in the Connecticut Valley) he has secured several specimens of this rare species. Dr. E. L. R. Thompson also assures me that he has seen it about New Haven. Coues, in his "List of the Birds of New England," states, and very properly too, that "there is no good evidence," that this species "ever straggles as far north as New England,"† but, for some unaccountable reason, in his "Birds of the Northwest" (p. 66) disregards his former correct statement and says that the bird occurs "north to Connecticut (Linsley)." A glance at Linsley's catalogue shows that he did not know of its occurrence in this State.

* For an interesting account of the habits of this bird, see an article by William Brewster, in Bull. Nutt. Ornith. Club, vol. ii, No. 1, pp. 1–7. Jan., 1877.

† Proceed. Essex. Inst., vol. v, p. 270.

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45. Dendrœca palmarum (Gmelin) Baird. Red poll Warbler.

A common spring and autumn migrant. Arrives, in spring, about the middle of April (13, 1877; 18, 1875), usually remaining but a few days; in fall I have taken it from Oct. 18th to Nov. 4th. But few were seen this spring (1877), and they straggled along in the most unaccountable manner. I shot one as late as May 5th! Along with *D. pinus* and *D. coronata* it may be seen, in spring, long before the hosts of other warblers make their appearance. It is also one of the last to depart in the fall, at which time it is found along fences, and among low bushes, by the road-side, and in open fields. In early spring, before the snow has quite all disappeared, large numbers of them may sometimes be seen, in company with the English Sparrows, running about on the plots of bare ground, and the roots of the elm trees, in the *City Green*, in the heart of New Haven. In fact they seem to pay but little attention to the weather, as may be seen from the circumstance that they were really abundant on the 18th of April, 1875, at a time when the ground was covered with snow nearly a foot deep, with only here and there a bare spot; while none were seen this season (1877) till April 13th, notwithstanding the fact that the ground had been bare since the latter part of March, and the weather unusually mild. Apparently in great haste to reach the breeding grounds, their stay with us, in spring, is generally brief—a few days and all are gone. During the autumnal migrations, however, the case is quite different. Having safely accomplished a long and perilous journey to the far north, and having successfully struggled with the elements and the hungry animate world in rearing and providing for a family of children, its anxiety is over, and now it lingers, for two or three weeks, in its favorite haunts by the road-side, before taking its final departure for Cuba or the Bahamas; or, perchance, it may not pass beyond our own limits, but, finding some suitable spot in the Gulf States, concludes to go no farther, and winters there.

The abundance of this species during its migrations varies considerably in different years. Thus in the spring of 1875 they were very numerous, while this season (1877) less than half a dozen were seen. This remark applies with equal force to *D. pinus* and several other species.

46. Dendrœca pinus (Wilson) Baird. Pine-creeping Warbler.

A summer resident; tolerably common during migrations. It breeds in pines, and is more often heard than seen. Arrives before the

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middle of April* (April 14, and probably earlier) and departs in September (Sept. 29, 1875, Sage). They are so partial to the tree whose name they bear that, in speaking of their habits in the "Pine-barrens" of South Carolina and Georgia, I remarked that "it was very appropriately named the *pine*-creeping warbler, as I never, except on one occasion, saw it alight, even for an instant, on anything but a pine tree; here it would sit by the hour and warble out its sweet song."† I might also state that at such times it is generally so concealed among the branches that one may not unfrequently spend half an hour in vainly endeavoring to "get a shot"—the bird, meanwhile, uttering, at intervals, its characteristic note, which is not unlike a certain strain in the pleasant and perplexing song of the common field-sparrow (*Spizella pusilla*). On referring to my MS. notes on the birds of Central Massachusetts, I find the following: "Seldom found except in pine woods, where a few were found breeding on June 25th, 1874." Here, however, though evidently preferring similar tracts, they may be met with, especially during the migrations, in a variety of situations. In the spring of 1875, for example, they were frequently seen in the City Green hopping about on the ground in company with *D. palmarum* and the English Sparrows, and running up and down on the trunks of the large elms searching for insects. Mr. C. J. Maynard, in his beautiful, but lamentably incomplete, work on the "Birds of Florida," says: "While walking in the piny woods of Florida one will suddenly observe that the trees over his head are filled with birds, where but a moment before not a living thing was to be seen; and his ears will be saluted by a variety of sounds. Beside the loud, harsh notes of the woodpeckers and nuthatches, and the mellow whistle of the bluebirds, the slowly given trill of the pine warblers will occasionally be heard. There are hundreds of these little birds in every passing flock, yet but few of them ever sing. They are extremely active, now searching for insects among the swaying foliage of the pines high overhead, then clinging to the brown trunks to peer into the crevices of the bark, or alighting on the ground. . . . . Of all the thousands of this species which spend the colder season in Florida but few remain to breed, and by the middle of March the greater portion leave for the North. They arrive in New England in early April, and by the first of May

* Dr. J. A. Allen stated, in the "New England Farmer" for 1861 (p. 540): "The present year [1861] I observed them the 4th of April, at which time a recent snow covered the ground to the depth of several inches."

† Am. Nat., vol. viii, No. 1, p. 7. Jan. 1874.

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begin to construct their nests, which are commonly placed in a fork of the topmost limb of a pine tree."*

47. Siurus auricapillus (Linné) Swainson. Golden-crowned Thrush; "Oven Bird."

A common summer resident, arriving early in May (May 3), and remaining till the latter part of September. Breeds abundantly. Its characteristic loud, but not particularly pleasing, note frequently annoys one while listening for the softer tones of some of the rarer warblers.

48. Siurus nævius (Boddaert) Coues.† (*Noveboracensis* of Authors.) Water-Wagtail; Water Thrush.

Rather common during migrations. Possibly a few occasionally remain and breed. Arrives early in May. Have taken it in spring from May 5th to 27th; and in fall from Sept. 28th to Oct. 18th. Mr. Gentry, in his late work on the "Birds of Eastern Pennsylvania," says that it "cannot be considered either abundant or rare." The same remark applies here in spring, when they frequent their favorite haunts near some small stream or swampy place in woodland. In the fall, however, they sometimes congregate in rather large flocks and may then be seen on hill-sides away from water.

49. Siurus motacilla (Vieillot) Coues. (*Ludovicianus* of Authors.) Large-billed Water Thrush.

Not rare in southern Connecticut, where it breeds regularly, and probably in considerable numbers. Arrives earlier than either of the other species. I shot a fine male on the 27th of April, 1877. That they also breed very early is shown by the fact that a female, containing an egg ready for the nest, was shot by my friend, Mr. Dayan, on the 17th of May, 1876; and Mr. J. N. Clark, of Saybrook, Conn., tells me that the young are generally hatched *before* May 25th. The fact of its breeding regularly in southern New England has now become so well established that it would be superfluous to cite the numerous published records of its capture.

In habits it seems to agree with *S. nævius*—at least in its fondness for shaded streams. Its note is peculiarly loud and clear, and does

* The Birds of Florida. By. C. J. Maynard. Part II, p. 49. Salem, 1873.

† For the nomenclature of this genus, see Coues, in Bulletin of the Nuttall Ornith. Club, vol. ii, No. 2. April, 1877.

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not resemble that of any other bird I have ever heard. Dr. F. W. Hall shot a pair in Maltby Park (near New Haven) June 10th, 1874; they had evidently bred there. W. R. Nichols and Thos. B. Osborne saw several specimens on the 2d of May, one of which they secured; and on the 15th I saw still another. I am informed by Mr. J. N. Clark, of Saybrook, that a favorite site for their nests is among the upturned roots of a fallen tree.

In writing of the habits of this species in West Virginia, Mr. Brewster says: "While the northern Water Thrush [*S. nævius*] was confined to the muddy banks of the creek—and I will remark *en passant* that I never saw one other than in a muddy locality—this species seemed to delight in the pebbly streams of the hills; just such streams as in the New England mountains would be called good trout brooks, overshadowed by mighty forest trees, frequently choked up by fallen logs, and abounding in beautiful cascades, still, deep pools, and wild rocky ravines. In the deepest, darkest retreats you were most likely to find him, and here, on several occasions, I was fortunate enough to hear his song. It is somewhat like that of *S. Noveboracensis* [*S. nævius*], being quite as loud, almost as rapid, and commencing in nearly the same way, but lacking the beautiful crescendo termination, and, altogether, a less fine performance. Represented by words, it would be nearly as follows: *pseur, pseur, per sèe ser.* This is usually uttered several times in succession from some shelving rock, or fallen log overhanging the stream; the bird in the intervals between each repetition tilting its body incessantly, and looking nervously about as if he didn't half like your appearance and had a good mind to be off, and this expression, in the majority of cases, soon finds vent in action, for he is a very shy little fellow the moment he begins to suspect that he is wanted to grace your cabinet. Come upon him suddenly, however, as he is running nimbly along the margin of some quiet pool or rippling eddy, and at times he will seem to pay little regard to your presence, and you may have a fine chance to observe his motions and sandpiper-like ways, as he wades knee deep into the water, or splashes through it in hot pursuit of some aquatic insects."*

50. Oporornis agilis (Wilson) Baird. Connecticut Warbler.

A rare spring and autumn migrant. The only specimen that I have ever seen alive was a female, which I shot near New Haven, Oct. 2d,

* Annals Lyceum Nat. Hist. N. Y., vol. xi, pp. 136–37. June, 1875.

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1875. Nearly four years ago Mr. H. A. Purdie called attention to the fact that it had been obtained, in September, at Saybrook, Conn., by Mr. J. N. Clark;* and Mr. Clark has recently informed me that he has taken several specimens there in fall, and one "full-plumaged male in spring." This last is the finest specimen I have ever seen, and, at first sight, I mistook it for the Mourning Warbler (*Geothlypis Philadelphia*), the male of which it closely resembles, both in the deep ash of the throat and breast (which is almost black where it joins the yellow below), and in the shade and limited extent of the yellow of the belly. The ring around the eye, however, is well marked, and pure white.

As its name indicates, this species was first discovered in Connecticut, and by the distinguished Ornithologist, Alexander Wilson. He met with it but once in this State, but afterwards found it in Pennsylvania, and says, "It was found, in every case, among low thickets, but seemed more than commonly active, not remaining for a moment in the same position."† Its habits are such as to render it unlikely to be observed unless present in considerable numbers.

51. Oporornis formosus (Wilson) Baird. Kentucky Warbler.

A rare summer visitor from the South. Mr. Erwin I. Shores, who has favored me with so many valuable notes concerning our rarer birds, writes that he "obtained a male of this species at Suffield, Conn., Aug. 16th, 1876,"‡ thus adding another bird, not only to the Avi-fauna of Connecticut, but also to New England. I am aware that Dr. Coues, in his "Birds of the Northwest" (p. 73), states that the species occurs "north to the Connecticut Valley," but on what authority I am unable to surmise. Perhaps the learned Doctor's knowledge of the distribution of birds, and of that something in their hearts which ofttimes causes those inexplicable peregrinations, together with his marvelous power of intuition, told him that it did occur in the Connecticut Valley, and had long been waiting to be discovered by Mr. Shores. Indeed, nearly ten years ago, Dr. Coues prophesied that "the occurrence of this species as a rare or casual summer visitor in southern New England is to be confidently anticipated,"§ and it is an old saying that probabilities become facts if only given time enough.

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* Am. Nat., vol. vii, No. 2, p. 693, Nov. 1873.

† American Ornithology. Vol. ii, p. 160. Edinburgh. 1831.

‡ MS. notes of Erwin I. Shores.

§ Proceed. Essex Inst., vol. v, p. 269. 1868.

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Its nest was found at Sing Sing, Westcheter Co., N. Y., in June, 1875, by Mr. A. K. Fisher,* and Mr. E. P. Bicknell writes me that he secured a male at Riverdale, in the same county, May 30, 1876. Giraud found it on Long Island many years ago.† Wilson says, "This species is seldom seen among the high branches, but loves to frequent low bushes and cane swamps, and is an active, sprightly bird. Its notes are loud, and in threes, resembling *tweedle, tweedle, tweedle.* . . . . . It appeared to me to be a restless, fighting species, almost always engaged in pursuing some of its fellows; though this might have been occasioned by its numbers, and the particular season of spring, when love and jealousy rage with violence in the breasts of the feathered tenants of the grove; who experience all the ardency of those passions no less than their lord and sovereign man."‡

Since writing the above I learn from Mr. J. G. Ely of Lyme, Conn., that he has "shot one Kentucky Warbler."

52. Geothlypis trichas (Linné) Cabanis. Maryland Yellow-throat.

A common summer resident, breeding plentifully. Arrives early in May (May 5), remaining till late in the fall (Nov. 1st, 1876).

53. Geothlypis Philadelphia (Wilson) Baird. Mourning Warbler.

A rather rare migrant, but probably more abundant than commonly supposed—not coming till after most of the warblers are gone. Have only seen it in spring. Mr. Bragg shot a male on May 24th, 1876, in low bushes. On the following day Mr. Dayan secured two males; and I saw several on the morning of the 27th. Mr. Thomas Osborne has seen it as early as May 15th (May 15 and 17, 1876). I shot a beautiful male near Savin Rock, Conn., May 25th, 1877. Mr. Grinnell tells me that he killed a pair (♂ and ♀) late in May, 1875, near Milford, Conn. Mr. J. N. Clark, also, has killed it, late in May, at Saybrook, Conn. At Easthampton, Mass., in the spring of 1874, I shot two specimens: "The first, May 27th, in a brush-heap; and the other, May 28th, on a large birch tree." Large numbers of them breed regularly, in suitable localities, in Lewis and Herkimer Counties, in northern New York. It also breeds abundantly about Umbagog Lake, Oxford Co., Maine (Brewster), and in Minnesota (T. Martin Trippe), even as far west as "the Red River, between Dakota and Minnesota" (Coues).

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* Am. Nat., vol. ix, No. 10, p. 573. Oct., 1875.

† Birds of Long Island. By J. P. Giraud. Jr., p. 50. 1844.

‡ Wilson's American Ornithology. Vol. ii, p. 151–2. 1831.

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Regarding its song, Samuels, in his "Birds of New England," says, "Its note was a simple chirp, with a warbling termination like the syllables *chirpchree*, *chirpchree*, uttered in a soft, pensive tone" (p. 207). I am perfectly familiar with the song of the bird in question, but never heard it utter a note that could, by any legitimate stretch of the imagination, be construed into Samuels' "*chirpchree.*" Its common song consists of a simple, clear, warbling whistle, resembling the syllables *'trúē*, *'trúē*, *'trúē*, *'trú*, *'toó*, the voice rising on the first three syllables and falling on the last two. Sometimes, when otherwise occupied, the first, or first two, syllables are omitted. All through the breeding season, and till late in July, they have a very characteristic habit of perching, at frequent intervals during the day, on some branch, generally a dead one, and commonly ten or fifteen feet from the ground, and singing for half an hour at a time.

54. Icteria virens (Linné) Baird. **Yellow-breasted Chat.**

A common summer resident, breeding in dense undergrowth. Arrives before the middle of May (May 13, 1876, Osborne; 14, 1877). Extends up the Connecticut Valley to Massachusetts ("not rare" at Suffield).* Given by Emmons, in 1833, as an "occasional visitant" in Massachusetts.† Their loud, ringing, notes surpass those of the Catbird and almost equal the richly varied song of the Brown Thrush (*Harporhynchus rufus*). Its habits so closely resemble those of its western congener (var. *longicauda*) that I take the liberty to quote the following brief extract, relating to some of the peculiarities of that bird, from my report on the Mammals and Birds collected by the U. S. Geol. Survey of the Territories in the year 1872: "They are shy, suspicious creatures, and, although when disturbed they flit about in a scolding, angry manner, generally manage to keep out of sight. You hear them in the bushes imitating the mewing of a cat, the shrill notes of the Jay, sometimes singing like a Catbird, and again they sing sweetly in their own peculiar manner. They have a strange habit of elevating themselves in the air to the height of thirty or forty feet, then, poising for a moment, descend again to the bushes. During the descent they jerk themselves about in the air, at the same time uttering clear, ejaculated notes, which can be heard for quite a distance, and are not altogether unpleasant to the ear."‡

* MS. notes of Erwin I. Shores, Esq.

† See Hitchcock's Report, p. 547. 1833.

‡ See Hayden's Report for 1872, p. 675.

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Wm. Brewster, Esq., speaking of its habits in West Virginia, remarks that they arrived there about May 1st, "and for a few days were silent, but soon became very noisy, especially when their retreats were invaded. Their notes are so varied as almost to defy description. What I took to be the *song* of the male was a series of about eight very loud bell-like whistles, commencing quickly, and becoming slower and more emphatic toward the end, then, after an interval of a few seconds, would follow a scolding chatter, to be immediately succeeded by a single very clear note, then the series of whistles again, but all these notes were varied to an almost infinite extent. All this time the bird would be dodging through the bushes ahead, keeping always in the thickest places, and perhaps, after a moment of silence, would suddenly strike up directly behind you. In this way I have frequently pursued one for fifteen or twenty minutes without so much as getting a glimpse at him. Several times, however, when I came upon him suddenly, he would put on a very innocent and injured air and vociferate his notes directly *at* me, as if to dispel any possible suspicion, on my part, that he had been running, or, to speak more literally, flying away."*

55. Myiodioctes mitratus (Gmelin) Audubon. Hooded Warbler.

A summer resident, breeding in the Connecticut valley and along our southern border. Although breeding abundantly at Saybrook, Conn., according to Mr. J. N. Clark (recorded by H. A. Purdie),† I have, as yet, seen but a single specimen from this vicinity. An adult female was shot at Hamden (near New Haven) June 2, 1874, by Dr. F. W. Hall. It was evidently breeding. Still it was found here in June, by Dr. Whelpley, as long ago as 1842.‡ Mr. Clark writes me from Saybrook: "The Hooded Warbler is very abundant here in dense woods, breeding everywhere in suitable places, always in a low Laurel (Kalmia) bush." Mr. E. I. Shores has taken it at Suffield, Conn., near the Massachusetts border, but in the Connecticut Valley.

My friends, Messrs. John H. Sage of Portland, Conn., and Eugene P. Bicknell of Riverdale, Westchester Co., N. Y., have recently called my attention to an interesting state of plumage, in the female of this species, which was alluded to by Wilson§ and Nuttall,‖ but

* Ann. Lyc. Nat. Hist. N. Y., vol. xi, pp. 137–8. June, 1875.

† Am. Nat., vol. vii, No. 11, p. 692. Nov., 1873.

‡ Linsley's Catalogue of the Birds of Connecticut, p. 257. 1843.

§ American Ornithology, vol. ii, p. 136. 1831.

‖ Manual of Ornithology. Vol. i, p. 374. 1832.

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has since been denied by both Coues* and Baird,† who positively, assert that the female has no black on the head. Concerning it, Mr. Sage favors me with the following note: "When collecting at Saybrook, Conn., with Mr. J. N. Clark, May 30th, 1877, he shot a female *M. mitratus* in a plumage not mentioned in our Ornithological works, and thinking a description would be interesting, I send the following: Bill black. Feet flesh-color. Head and fore part of breast black (but not so pure as in the adult male), with slight indications of black on the throat. A broad patch on the forehead, extending down on the cheeks, with the under parts bright yellow. Ear-coverts tinged with olive. Upper parts olive-green. Wings unmarked. Greater part of inner webs of outer three tail feathers white. The ova of this specimen were large. We took the nest with four eggs."

I have lately seen, in the cabinet of Mr. Sage, the bird from which the above description was taken, and have now before me another female (killed by Dr. F. W. Hall, near New Haven, June 2, 1874) which agrees well with the description of Mr. Sage's specimen, but has the crown of the head, or "hood," deep black—as rich as in the male. The lores also, in this specimen, are black, and the auriculars lack the olive tinge, being bright yellow. Since the birds from which Prof. Baird and Mr. Ridgway, and Dr. Coues, took their descriptions were dried skins, and not "in the flesh," it is not so surprising that the mature females were mistaken for young males. From the limited amount of material I have been able to examine, and from the notes given me by Mr. Sage and Mr. Bicknell, I am inclined to believe that the female bird, like the male, is several years—at least three—in attaining its full plumage; and that the two sexes, when fully adult, can only be distinguished by the fact that, in the female, the throat, though strongly tinged with black, is never *pure black* as in the male.

56. Myiodioctes pusillus (Wilson) Bonaparte. Green Black-capped Fly-catching Warbler.

A tolerably common spring migrant, arriving about the middle of May. Have not taken it later than the 25th. This species is by no means so common here as it seems to have been formerly.

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‡ Coues' Key to N. Am. Birds. p. 109. 1872.

§ Baird, Brewer and Ridgway, History N. Am. Birds, vol. i, p. 314. 1874.

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57. Myiodioctes Canadensis (Linné) Audubon. Canadian Fly-catching Warbler.

Common during the migrations. Arrives early in May. Have taken it from the 6th to the 29th. Mr. W. W. Coe informs me that it doubtless breeds about Portland, Conn., as he has seen it there throughout the summer.

58. Setophaga ruticilla (Linné) Swainson. Redstart.

A common summer resident; breeds. Arrives early in May. Have taken it from May 8th to September 22d. Mr. Stadtmüller has a very beautiful nest of this species which "was placed in the fork of an oak tree about fifteen feet from the ground. It was built of grass and bits of paper, and covered with cotton all over outside, and lined with a few horse hairs."* It contained four eggs, June 10, 1874.

Family, TANAGRIDÆ.

59. Pyranga rubra (Linné) Vieillot. Scarlet Tanager.

A summer resident; not uncommon. Most frequent during the spring migrations; breeds. Arrives about May 10th, after which date they are common till the first of June. Took a female Sept. 11th, 1875.

60. Pyranga æstiva (Linné) Vieillot. Summer Redbird.

A rare summer visitant from the South. "Stratford and New Haven, Linsley," and in a foot-note he further states that "The Summer Redbird is more rare than the Scarlet Tanager, though I have taken both here [Stratford] during the season of cherry blossoms."† In the Bulletin of the Nuttall Ornithological Club for Jan., 1877 (vol. ii, No. 1, p. 21), Mr. Purdie says, "Mr. Jencks informs me that a male was shot, a few years since, on Ten-mile River, six or eight miles northeast of Providence," R. I. I have never met with it except in Florida, where it has a very pleasant song. Dr. Thompson, of this city, informs me that he shot one near New Haven, in summer, about five years ago; and I am told by Prof. Wm. D. Whitney that a friend of his, who was perfectly familar with the species, saw two individuals, at different seasons, in Hillhouse Avenue, New Haven.

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* MS. notes of the Stadtmüller Brothers.

† Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 261. April, 1843.

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Mr. Erwin I. Shores writes me that he killed one at Suffield, Conn. (in the Connecticut Valley) July 21st, 1876. Does it not sometimes breed here?

Family, HIRUNDINIDÆ.

61. Hirundo horreorum (Barton.) Barn Swallow.

A common summer resident, breeding abundantly under the roofs of barns. Arrives during the latter part of April (April 21, 1877), remaining till September. Under the head of this species, Peter Kalm wrote, from Raccoon, New Jersey, in 1750: "I observed them for the first time on the 10th of April [new style); the next day in the morning, I saw great numbers of them sitting on posts and planks, and they were as wet as if they had been just come out of the sea," and the translator of his "Travels," John Reinhold Forster, F.A.S., puts in a foot-note, saying: "It has been a subject of contest among naturalists, to determine the winter-retreat of *Swallows*. . . . . The question must therefore be decided by facts; nor are they wanting here: *Dr. Wallerius*, the celebrated *Swedish* Chemist, wrote in 1748, *September* the 6th, O. S., to the late Mr. *Klein*, Secretary of the City of *Dantzick:* 'That he has seen more than once *Swallows* assembling on a reed, till they were all immersed and went to the bottom; this being preceded by a dirge of a quarter of an hour's length. He attests likewise, that he had seen a *Swallow* caught during winter out of a lake with a net, drawn, as is common in northern countries, under the ice: this bird was brought into a warm room, revived, fluttered about, and soon after died. Mr. *Klein* applied to many *Fermiers generaux* of the King of *Prussia's* domains, who had great lakes in their districts. . . . . All the people questioned made affidavits upon oath before the magistrates."

"*First*, The mother of the Countess *Lehndorf* said, that she had seen a bundle of Swallows brought from the *Frith-Haff* (a lake communicating with the *Baltic* at *Pillau*) which when brought into a moderately warm room, revived and fluttered about. *Secondly*, Count *Schlieben* gave an instrument on stamped paper, importing, that by fishing on the lake belonging to his estate of *Gerdauen* in winter, he saw several *Swallows* caught in the net, one of which he took up with his hand, brought it into a warm room, where it lay about an hour, when it began to stir, and half an hour after it flew about in the room. *Thirdly*, Fermier general (*Amtman*) *Witkowski* made affidavit, that in the year 1740, three *Swallows* were brought

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up with the net in the great pond at *Didlacken*; in the year 1741, he got two *Swallows* from another part of the pond, and took them home, (they all being caught in his presence); after an hour's space they revived all in a warm room, fluttered about, and died three hours after. 4*thly*, *Amtman Bönke* says, that having had the estate *Kleskow* in farm, he had seen nine *Swallows* brought up in the net from under the ice, all which he took into a warm room, where he distinctly observed how they gradually revived; but a few hours after they all died. Another time his people got likewise some *Swallows* in a net, but he ordered them again to be thrown into the water. 5*thly*, *Andrew Rutta*, a master fisherman, at *Oletsko*, made affidavit, 1747, that 22 years ago, two *Swallows* were taken up, by him, in a net, under the ice, and being brought into a warm room, they flew about. 6*thly*, *Jacob Kosiulo*, a master fisherman, at *Stradauen*, made affidavit, that in 1736, he brought up in winter, in a net, from under the ice of the lake at *Raski*, a seemingly dead *Swallow*, which revived in half an hour's time, in a warm room, and he saw, a quarter of an hour after, the bird grow weaker, and soon after dying. 7*thly*, I can reckon myself among the eye-witnesses of this *paradoxon* of natural history. In the year 1735, being a little boy, I saw several *Swallows* brought in winter by fishermen, from the river *Vistula*, to my father's house, where two of them were brought into a warm room, revived, and flew about. I saw them several times settling on the warm stove (which the *Northern* nations have in their rooms) and I recollect well that the same forenoon they died, and I had them, when dead, in my hand. . . . . In *January* [1754] the lake of *Lybshau*, belonging to these estates, being covered with ice, I ordered the fishermen to fish therein, and in my presence several *Swallows* were taken; which the fishermen threw in again; but one I took up myself, brought it home, which was five miles from thence, and it revived, but died about an hour after its reviving. These are facts, attested by people of the highest quality. . . . . It is therefore highly probable, or rather incontestibly true, that *Swallows* retire in the *Northern* countries during winter, into the water, and stay there in a torpid state, till the return of warmth revives them again in spring."*

Not many years ago I brought upon myself the everlasting odium of an old lady, in the northern part of New York State, by presuming to question her statement that she had seen, "with

* Peter Kalm's Travels into North America, vol. ii, pp. 140–44. 1771.

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her own eyes," a number of Swallows that had been exhumed, in a torpid state, from the muddy bottom of a pond in the neighborhood of her dwelling. She further stated that, on taking them into the kitchen, "they soon came to life and flew about the room," but whether this premature "thawing out" was followed, like the cases above recorded by Forster, by an equally speedy death, I do not remember.

62. Tachycineta bicolor (Vieillot) Cabanis. White-bellied Swallow.

A common summer resident, breeding in holes in trees (generally standing in or about ponds). The earliest to arrive and last to depart. Mr. Grinnell has observed it as early as April 7th, and I have found it quite common by the 13th.

In looking over Mr. W. W. Coe's note book I find the following interesting item: "While collecting in the vicinity of Portland, Conn., June 14th, 1873, I found, in the same tree, three Bluebird's eggs and four White-bellied Swallow's eggs, and thereby hangs a tale: in cutting away the hole to take out the Bluebird's eggs, I noticed a white feather sticking up through the nest. Never having seen feathers in a Bluebird's nest, and knowing that the White-bellied Swallows build theirs almost entirely of them, I thought the Bluebirds might have driven off the Swallows and appropriated the nest to their own use. On removing the Bluebird's nest I found this to be the case, for, underneath it was the nest of the White-bellied Swallow complete and containing four fresh eggs—the eggs of both are now in my cabinet."*

63. Petrochelidon lunifrons (Say) Sclater. Cliff Swallow; Eave Swallow.

A common summer resident, breeding abundantly under the eaves of barns. Arrives about April 20th (April 15, 1877, Osborne; 21, 1877, A. J. Dayan), remaining till September. Professor Verrill, some years since, showed conclusively that the Cliff Swallow had not, as formerly supposed, immigrated from the West, but was actually known in New England as long ago as the year 1800—long before its discovery in the West. He also states that a large colony of them was known to breed at Windsor, Conn., in 1830.†

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* MS. notes of W. W. Coe.

† Proceed. Bost. Soc. Nat. Hist., vol. ix, p. 276. July 1, 1863.

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64. Cotyle riparia (Linné) Boie. Bank Swallow.

A common summer resident, breeding in colonies in holes in banks, generally near some pond or river. Arrives early in May (April 21, 1877, A. J. Dayan). Williams, writing of the Swallows of Vermont, in 1794, relates the following, which I transcribe for what it is worth: "The usual times of the appearance and disappearance of these birds, serve to mark the temperature of the climate, with as much precision, as any of the phenomena of nature. But they do not seem to be properly birds of passage. At *Danby* in this State, the inhabitants report, that some of them were taken out of a pond in that town, some years ago. A man was employed in the winter, to procure the roots of the pond lily, for medicinal purposes. Among the mud and roots which he threw out, several swallows were found enclosed in the mud; alive, but in a torpid state. . . . . It has been doubted by some able naturalists, whether it is possible for the Swallow to live in such a situation. I saw an instance which puts the possibility of the fact beyond all room for doubt. About the year 1760, two men were digging in the salt marsh, at Cambridge, in Massachusetts: on the bank of the Charles River, about two feet below the surface of the ground, they dug up a Swallow, wholly surrounded and covered with mud. The Swallow was in a torpid state, but being held in their hands, it revived in about half an hour. The place where this Swallow was dug up was every day covered with the salt water, which at every high tide, was four or five feet deep. The time when this Swallow was found was the latter part of the month of February."*

65. Stelgidopteryx serripennis (Audubon) Baird. Rough-winged Swallow.

A rare summer visitant. "A female of this species was shot at Suffield, Conn., by Mr. Shores, June 6th, 1874."† My friend, Mr. Eugene P. Bicknell, informs me that he has found it in numbers at Riverdale, N. Y., within a few miles of the Connecticut line, and that it breeds there—sometimes placing its nest "under a bridge."

66. Progne purpurea (Linné) Boie. Purple Martin.

A summer resident, breeding in "Martin Boxes" in villages. Arrives during the middle or latter part of April (April 17, 20, 25,

* The Natural and Civil History of Vermont. By Samuel Williams. pp. 115–16. Printed at Walpole, N. H. 1794.

† Purdie in "Nuttall Ornith. Bulletin, vol. ii, No. 1, p. 21. Jan. 1877.

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1877). A large colony of them is located over the First National Bank, in the center of New Haven. They are abundant at Guilford, Conn., and a single pair breeds on Faulkner's Island, occupying a house erected for them by Capt. Brooks.

Family, AMPELIDÆ.

67. Ampelis garrulus Linné. Bohemian Waxwing.

A rare, almost accidental, winter visitant. On the 11th of February, 1875, Prof. S. I. Smith saw a large Ampelis which he thinks was, without doubt, *A. garrulus*. It was in the old wooded cemetery in New Haven. Prof. Smith is familiar with the bird in question, having seen it in Maine. A number of cases of its occurrence in Massachusetts have been recorded, but the only instance of its capture in this State is that given by J. A. Allen. "It has been taken in Connecticut [near Hartford] by Dr. Wood,"* and the Doctor tells me that he shot them by accident, when firing into a flock of Cedar Birds. "Several were shot on Long Island in 1830 and 1832."†

68. Ampelis cedrorum (Linné) Sclater. Cedar Bird; Cherry Bird.

A resident. Generally common. Breeds in orchards and low thickets. Gregarious. Almost the only bird that feeds, to any great extent, on canker worms.

Family, VIREONIDÆ.

69. Vireo olivaceus (Linné) Vieillot. Red-eyed Vireo.

An abundant summer resident, breeding chiefly in high woodland. Arrives during the first week in May (May 6), remaining as late as the first of October (Oct. 2). Superfluously noisy.

70. Vireo gilvus (Vieillot) Bonaparte. Warbling Vireo.

A common summer resident, breeding in orchards. Arrives before the middle of May (May 7, 1876, Osborne; 12, 13, 14, 1877.) Coues thus briefly alludes to the habits of this pleasing little songster: "Not born to 'waste its sweetness on the desert air,' the Warbling Vireo forsakes the depths of the woodland for the park and orchard and

* Notes on some of the Rarer Birds of Massachusetts, p. 25. 1869.

† Giraud's Birds of Long Island, p. 165. 1844.

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shady street, where it glides through the foliage of the tallest trees, the unseen messenger of rest and peace to the busy, dusty haunts of men."

71. Vireo flavifrons Vieillot. Yellow-throated Vireo.

A tolerably common summer resident; particularly abundant in spring and fall. Arrives early in May (May 6), remaining till after the middle of September (Sept. 15, 17, 1875).

72. Vireo solitarius (Wilson) Vieillot. Blue-headed, or Solitary Vireo.

Not uncommon during the migrations. A few breed. Arrives during the first week in May (May 2, 1876, Osborne; 6, 1875, C. H. M.; May 5, 1877, Nichols). Have taken it as late as Oct. 13th. A nest of this species, found June 18th, 1875, near New Haven (Beaver Swamp), by the Stadtmüllers, "was out on the limb of a Chestnut tree about 30 feet from the ground, and was constructed of the bark of different kinds of trees, lined with grass. Moss and caterpillars' silk were plastered outside. It contained four eggs, which were two-thirds hatched."*

73. Vireo Noveboracensis (Gmelin) Bonaparte. White-eyed Vireo.

A summer resident, but not particularly common except along southern Connecticut, and in the Connecticut Valley, where it is common all the way up to the Massachusetts line (Suffield, E. I. Shores). Arrives before May 10th (May 7, Osborne).

NOTE.—*Vireo Philadelphicus* Cassin, undoubtedly occurs within our limits, as a rare, or accidental, visitant, but as yet no record of its capture has appeared. On Sept. 7th, 1875, Mr. William Brewster "shot a female of this beautiful little species in Cambridge, Mass."†

Family, LANIIDÆ.

74. Collurio borealis (Vieillot) Baird. Butcher Bird; Shrike.

A somewhat irregular winter resident. Arrives in October or November (Oct. 29, 1876, Sage;) remaining till the latter part of March (March 24, 1876, Osborne). Very abundant some years, and equally scarce in others. Mr. Maynard says, "when the Lesser Red-

* MS. notes of the Stadtmüller Brothers.

† Bull. Nutt. Ornith. Club, vol. i, No. 1, p. 19. April, 1876.

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polls or the Pine Finches are common, the Shrikes follow them and prey upon them." During the winters of 1874–5 and 1875–6, when the Red-polls (*Ægiothus linaria*) and the Pine Finches (*Chrysomitris pinus*) were here, Butcher Birds were notably scarce; while during the past winter (1876–7) Shrikes have been uncommonly plenty, but neither of the other birds were seen at all. It is but fair to state, however, that during the first two winters mentioned neither the Linnets nor Finches were here in large enough numbers to have afforded the Butcher Birds a reliable source of diet. Last winter the English Sparrows supplied all deficiencies and it was no uncommon thing to see a Butcher Bird flying across the street with one in its talons. The "poor sparrows," unused to danger of any sort, were utterly helpless, and at one time it seemed as if we were actually going to be rid of the little pests, but warm weather came to their relief, driving the intruders back to their boreal homes, and now the sparrows are apparently as numerous as ever they were. Mr. Nichols shot one Jan. 13th, 1876, whose stomach contained grasshoppers! Where could it have found them at that season?

75. Collurio Ludovicianus (Linné) Baird. Loggerhead Shrike.

A rare visitor from the South and West. Mr. W. W. Coe has a fine specimen of this species in his cabinet, which he shot near Portland, Conn., in Nov., 1876. Franklin Benner, Esq., writes me that he "saw a Shrike of some species at Mt. Carmel [near New Haven] May 24th, 1873," which, if he was not mistaken in the bird, could hardly have been the northern Butcher Bird, and might have been this species.

Family, FRINGILLIDÆ.

76. Pinicola enucleator (Linné) Cabanis. Pine Grosbeak.

An irregular winter visitant. They were here through the months of Feb. and Dec., 1875, and during Jan. and Feb., 1876, but none were seen last winter. Messrs. Coe and Sage, of Portland, Conn., tell me that it is found there regularly *every winter*, though the red males are not always seen, and that they have observed it as late as March 12th (1873), and as early as Nov. 24th (1875, and 25, 1874).

77. Carpodacus purpureus (Gmelin) Gray. Purple Finch.

Resident, breeding abundantly. Gregarious. Mr. Grinnell informs me that he has taken it during every month of the year. Dr.

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Wood, of East Windsor Hill, tells me that they were almost unknown here twenty years ago, and have gradually become common since. I am likewise informed by Mr. Clark, of Saybrook, that the bird has only recently become a common species in that vicinity.

78. Loxia leucoptera Gmelin. White-winged Crossbill.

An irregular winter visitant. They were not uncommon during the months of Jan., Feb. and March, 1875.

79. Loxia curvirostra, var. **Americana** (Wilson) Coues. Red Crossbill.

An irregular winter visitant, like the last, and more frequently seen. Linsley gave it in his list, stating that it had "been repeatedly seen in Trumbull, in this [Fairfield] county, by a Mr. Beers." Mr. W. W. Coe has taken a number of this, as well as the preceding species, about Portland, Conn. A few were seen about New Haven early in Dec., 1876. It has been known to breed near New York City.*

80. Ægiothus linaria (Linné) Cabanis. Red-poll Linnet.

Also an irregular winter visitant. They were quite common here during the month of March, 1875 (and Mr. Sage saw a flock at Portland, Conn., March 31), but none were seen in the winters of 1875–6 and 1876–7. Mr. W. W. Coe took it at Portland, Conn., March 22d, 1873, and saw there large flocks of them (including many highly plumaged males) in March, 1875.†

81. Chrysomitris pinus (Wilson) Bonaparte. Pine Finch; Pine Linnet.

An irregular fall and winter visitant. Quite common from Oct., 1874, till March, 1875. Also seen in Oct., 1875. Linsley says: "I took one specimen from a large flock, which was here in my yard [Stratford, Conn.] as late as Nov. 7th, 1840." In March and April, 1873, I found them extremely abundant near the boundary line between Georgia and South Carolina, notwithstanding the statement, in Baird, Brewer, and Ridgway's great work, that: "according to Dr. Coues, this species occasionally strays as far to the South as the Carolinas, but it is not common there," (p. 481). This case simply affords another example of their irregular migrations.

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* Am. Nat., vol. x, No. 4, p. 237. April, 1876.

† MS. notes of W. W. Coe, Esq.

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82. Chrysomitris tristis (Linné) Bonaparte. Common Yellowbird.

An abundant resident, breeding late. Common everywhere, and is gregarious.

83. Plectrophanes nivalis (Linné) Meyer. Snow Bunting.

A common winter visitor, generally seen during snow-storms. Linsley, in his "Catalogue of the Birds of Connecticut," so frequently cited, says, "Large flocks of the Snow Bunting were repeatedly seen here in the winters of 1840, 1841 and 1842. Previous to that period, *I had not seen one here in eighteen years.*" (p. 261, foot-note).

84. Plectrophanes Lapponicus (Linné) Selley. Lapland Longspur.

Occurs as a winter visitant, along with *P. nivalis*, though no record of its capture has yet been published. My friend, Mr. Geo. B. Grinnell informs me that he has seen half a dozen specimens, killed near the Connecticut River (Portland, Conn.), by Mr. W. W. Coe. Mr. Walter P. Nichols has taken it near New Haven, and Mr. Erwin I. Shores "obtained a male out of a flock of *P. nivalis*, in Nov., 1874" at Suffield, Conn. Mr. Coe has taken it as late as March 16th, (1875).

85. Passerculus princeps Maynard. Maynard's Sparrow.

Probably occurs quite regularly along the coast in Nov. and Dec. (and perhaps also in March) in such sparing numbers as to escape notice. The only specimen of this beautiful bird yet taken in this State, so far as I am aware, is the one shot by myself while collecting invertebrates, at low water, with my friend, Mr. S. F. Clark. "On Nov. 4th, 1875, while collecting along the beach at 'South End,' a few miles below New Haven, I was fortunate enough to secure a fine specimen of the Ipswich Sparrow (*Passerculus princeps* Maynard). The specimen was a female, and in excellent condition. Its mate was seen, but escaped capture."* The day was cold and chilly, with occasional flurries of snow.

86. Passerculus Savanna (Wilson) Bonaparte. Savanna Sparrow.

An abundant summer resident, breeding in open fields and pastures. Arrives early in April, remaining, in large numbers, till after the middle of November.

* Bull. Nutt. Ornithol. Club, vol. i, No. 2, p. 52. July, 1876.

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87. Pooecetes gramineus (Gmelin) Baird. Bay-winged Bunting; Grass Finch.

A common summer resident, breeding abundantly, like the last, in open fields. Arrives early in April (April 6, 1875 and same date 1877, Sage); remaining till about the middle of November.

88. Coturniculus passerinus (Wilson) Bonaparte. Yellow-winged Sparrow.

A common summer resident in some parts of Connecticut (notably in the Connecticut Valley and along our southern border); not found at all in others. Arrives early in May. Frequents dry, sandy, treeless wastes. Linsley found it breeding here and at Stratford. "In the vicinity of Hartford, Conn., this bird appears also to be a not uncommon summer resident."* Mr. Purdie states that "at Saybrook, Conn., its notes were to be heard in every field."† I have taken it near Savin Rock (May 29, 1877), and Mr. Grinnell tells me it is not uncommon about Milford, Conn. It also breeds about Portland, Conn. (Coe and Sage). I am informed by Messrs. Grinnell and Clark that it builds a beautiful covered nest, not unlike that of the Oven Bird (*Siurus auricapillus*). Mr. Erwin I. Shores writes me that he has taken it at Enfield, Conn., in the northern part of the State, but in the Connecticut Valley, in July (July 6, 1874).

89. Coturniculus Henslowi (Audubon) Bonaparte. Henslow's Sparrow.

A rare summer resident. I am very glad to be able to include this species on the strength of a specimen just received from my friend Dr. F. W. Hall. He shot it at Killingworth, Middlesex Co., Conn., July 18th, 1873, and it is in the worn breeding plumage. I had searched in vain for it in the many collections of the birds of this State to which I have had access, and had really despaired of finding it at all. Many specimens have been taken in Massachusetts and it doubtless occurs regularly in the Connecticut Valley, if not in other parts of the State. Giraud says that on Long Island "it is not so rare as is generally supposed. In general, it frequents the low, wet meadows, and passes most of its time on the ground among the tall grass, and is exceedingly difficult to flush, even when pursued with dogs; it will not fly until nearly within their reach, when it starts from the ground, moves on only a few yards, and again drops among the grass."‡

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* Baird, Brewer and Ridgway, vol. i, p. 554.

† Bull. Nutt. Ornith. Club, vol. ii, No. 1, p. 17. January, 1877.

‡ Giraud's Birds of Long Island, p. 104. 1844.

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90. Ammodromus maritimus (Wilson) Swainson. Seaside Finch.

A common summer resident, breeding abundantly in the salt and brackish water marshes near the coast. Remains into September.

91. Ammodromus caudacutus (Gmelin) Swainson. Sharp-tailed Finch.

A common summer resident, like the last, and breeds in the same situations. Arrives about the middle of April (Apr. 14, Osborne). Their eggs are laid during the last of May and first of June. Mr. Dayan found a nest containing four fresh eggs on May 31, 1876, at Savin Rock, and on the 7th of June both species were breeding abundantly. We found great numbers of both along the Quinnipiac River, and so far inland that the water was almost, if not quite, fresh. In some particular spots the *maritimus* outnumbers the *caudacutus*, but on the whole the latter greatly predominates.

92. Melospiza palustris (Wilson) Baird. Swamp Sparrow.

A common summer resident, breeding plentifully. Arrives early in April, remaining till late in November (Nov. 20).

93. Melospiza melodia (Wilson) Baird. Song Sparrow.

An abundant resident; particularly numerous from February to December, but not uncommon all through December and January. In the winter season they frequent old fences alongside which a copious growth of brushwood has sprung up; also found in back-yards in the city. Mr. W. W. Coe, of Portland, Conn., found a Song Sparrow's nest in a hole in a tree ten feet from the ground (early June, 1873).*

94. Melospiza Lincolni (Audubon) Baird. Lincoln's Finch.

A rather rare summer resident. Mr. J. N. Clark, of Saybrook, Conn., showed me a mounted specimen of this species which he had shot in a brush heap in his garden in the spring of 1875. He thinks he saw another a few days before, but it was very shy and he did not kill it. Mr. Erwin I. Shores, of Suffield, Conn., writes me that it is "*not* rare" in that vicinity, where he took one specimen in 1874, and three more this spring (1877). Mr. Shores says that on May 23, and again on June 2d, he "saw one with small twigs in its bill," hence, although he did not actually find the nest, there can be no reasonable

* MS. notes of W. W. Coe.

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doubt of its breeding. He further states: "There is a small piece of woodland in this place where surely they cannot be considered rare. Have seen several that I've not been able to shoot. They are very shy. You just barely get a glimpse of one and have just time to get an idea of what it is, when down he goes into the thick shrubbery, and no amount of patient waiting will tempt him to come in sight again. Provoked, you determine to kill every one that comes in sight, and after the slaughter of half a dozen innocent Song or Swamp Sparrows, you conclude that that won't do. *Then*, perhaps, almost the first bird you leave will be Lincoln's Sparrow. I think they are much more common than generally supposed, but are so shy, and inhabit such bushy pastures, that they are hard to find."* J. G. Ely, of Lyme, Conn., "took three this spring" (1877).

95. Junco hyemalis (Linné) Sclater. Slate-colored Snowbird.

A common winter resident, arriving, from the north before the middle of October (Oct. 8, 1876, Osborne), and remaining till the first week in May (last seen May 2, 1877, Osborne; May 3, 1876).

96. Spizella monticola (Gmelin) Baird. Tree Sparrow.

A common winter resident, but more numerous in spring and fall. Arrives from the north about the last of October (Oct. 28, 1875), and remains till near the middle of April (Apr. 12, Osborne).

97. Spizella socialis (Wilson) Bonaparte. Chipping Sparrow.

An abundant summer resident, arriving as early as the latter part of February (Feb. 23, 1877, G. B. Grinnell), and remaining, in numbers, till the last of November. Sometimes places its frail nest on the ground, as well as on trees and bushes (Coe). A few sometimes spend the winter with the English Sparrows about town. They often awake in the night, sing once, and go to sleep again. Scarcely a night passes (in June and early July) but that I hear one sing several times—generally about midnight.

98. Spizella pusilla (Wilson) Bonaparte. Field Sparrow.

An abundant summer resident, generally breeding in open fields. Arrives early in April (Apr. 6, 1877, Portland, Conn., Sage), remaining till December (have taken it all through November and on Dec.

* MS. notes of Erwin I. Shores.

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2, 1875). They are untiring songsters, particularly in early spring, and may be found everywhere except in dense woods. The song of the Field Sparrow is loud, clear, and pleasing, and he may consider himself fortunate, who has mastered its numerous variations. Minot says of it, "No sounds are more refreshing, on a warm afternoon of early summer, than those which they produce." It nests both on the ground and in low bushes.

99. Zonotrichia albicollis (Gmelin) Bonaparte. White-throated Sparrow.

Abundant during the migrations, sometimes remaining all winter. Arrives from the north about Sept. 22d, remaining through November. Returns in April and departs about May 20, at which date it was last seen, both in 1876 and 1877. During the winter of 1874–5 they were very numerous all about the city, as well as in the surrounding country. Mr. Erwin I. Shores, of Suffield, Conn., writes me that on May 15, 1877, they were extremely abundant in the woods in his vicinity—outnumbering all the other species together.

100. Zonotrichia leucophrys (Forst.) Sw. White-crowned Sparrow.

An irregular migrant; sometimes quite common. (Nov. 28 and Dec. 5, 1874; Oct. 25, 1877, common.) Mr. Dayan shot a beautiful specimen May 10, 1876. Mr. Osborne saw it on May 15, 1877. Mr. W. W. Coe shot one at Portland, Conn., March 20, 1875.

101. Passer domesticus Brisson. English Sparrow; House Sparrow.

Introduced. An abundant resident in all the larger, and most of the smaller, towns throughout the State. The opening sentence, under the head of this species, in H. E. Dresser's magnificent work on "The Birds of Europe," shows that its habits have remained unchanged, in one respect at least, notwithstanding the great distance it has been carried, and the varied conditions to which it has been subjected: "Throughout Europe the House Sparrow is very generally distributed almost wherever there are human habitations (except in the extreme north); for it follows the footsteps of man almost like a domestic animal, and where he fixes his habitation there the Sparrow also takes up its abode." Their pugnacity seems to vary in different parts of the country and at different times of the year. In New Haven their attitude toward other species is not generally offensive, and they are commonly seen on the friendliest terms with the Chipping Sparrows, throughout the entire season. In early spring I have

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observed them feeding quietly with the Red-poll Warblers (*Dendrœca palmarum*) on the City Green, and in fall it is no uncommon thing to find them associated with *Passerella iliaca*, *Zonotrichia albicollis*, *Melospiza melodia*, and *Spizella socialis* and *pusilla*, without showing the slightest sign of animosity. In New York and other cities, however, they are sometimes quite belligerent, attacking and driving off the birds that once occupied the same ground. This is especially noticeable in spring, when they appropriate the houses put up for Bluebirds, Wrens, and other species. That they do some good in destroying injurious insects is certain. "In open places where there are a few trees in the towns, such as the gardens in the squares or in the parks, it is eminently useful in ridding the foliage of the insects which would otherwise destroy the leaves and tender shoots; . . . . . and destroys myriads of the small smooth caterpillers and larvæ which feed on the buds of the trees, and is one of the best guardians of the orchard."* A few days ago my attention was attracted by the peculiar actions of one of these Sparrows. It was hopping along on the uppermost rod of a fence, and whenever it came to a post (into which the fence rods were inserted) it would stand on tip-toe and peep up under the ornamental top-board of the post, as if looking for something. On coming nearer I discovered that the bird was searching for moths, of which it secured one or more at each post, and, after proceeding in this manner for several rods, it flew away, evidently having had enough for one meal. In passing the fence since, I have, on several occasions, seen a Sparrow, apparently and probably the same individual, going through with precisely the same procedure, and with the same result. Do not cases like the above go far to prove that many birds are highly intelligent and possess a good degree of memory? The Sparrow in question discovered a moth, doubtless by accident, concealed beneath the overlaping edge of the top-board of a fence-post. Was he content with simply gobbling this one up? By no means; if there is one there may be more, and sure enough he soon spies another, and, going to the next post, still another, and so on till he is satisfied. Next day, when searching for breakfast, does he forget yesterday's experience? Not at all; he returns to the fence-posts and readily secures another meal. The chance finding of the first moth has led to the knowledge that similar insects take refuge, by day, in certain places; and in the

* A History of the Birds of Europe, including all the species inhabiting the Western Palæarctic Region. By H. E. Dresser. Part XLVIII, March, 1876.

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establishment of a regular habit of searching for them. In spring, when the trees are in flower, and the swollen buds give place to tender green leaves, the House Sparrows join the early Warblers in running about among the branches in pursuit of one another as well as of the small beetles that come there to feed upon the fresh foliage. But so far as the accomplishment of the object for which they were imported, viz., the extermination of the canker worm (*Anisopteryx vernata* and *A. pometaria*) in our larger towns and cities, they have most signally failed. That they occasionally devour the full-grown moth can not be doubted; neither do I feel justified in denying that they sometimes eat the worms themselves, though I have never seen them touch one—not even when hopping about in the midst of hundreds of canker worms. They seem to prefer the small beetles and seeds abounding in the horse droppings about the streets. It is true that there was a great diminution in the ranks of the canker worms soon after the introduction of the House Sparrows, but I am informed that this was due to a parasite (*Platygaster*) which preyed upon and destroyed great numbers of the worms by depositing its eggs in the eggs of the canker worm moth, and not, as generally supposed, to the Sparrows. It may, perhaps, be considered as somewhat of a digression in a mere local list like this, to devote so much space to the biography of a species, but I believe that notes recording actual observations on the habits of any bird should always prove acceptable, especially when there is difference of opinion, resulting in controversy, regarding the merits of a species, for in this way the possibility that individuals, or colonies, may differ in habits in different localities, or at different seasons, is suggested, and warns us not to be too dogmatic in our statements, or too hasty in drawing conclusions.

For abundant and unquestionable evidence of the pugnacity and disagreeable qualities of this bird, in some sections, see the writings of Coues, Gentry, and others. Velmont de Bomare, writing in 1791, says: "In Brandebourg, in order to diminish the ravages committed by Sparrows, a price is set on their heads, and the peasants are compelled by law to bring in a certain number yearly; in each village there are Sparrow hunters who sell the birds to the peasants to enable them to pay their tribute. . . . It follows the farmer while sowing, harvesting, threshing, or feeding his poultry; it enters the Dovecot, and with its bill pierces the throats of young pigeons, to obtain the grain in their craw."*

* Quoted by Charles Pickering in Proceed. Bost. Soc. Nat. Hist., vol. xi, p. 158, April 17, 1867.

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The English House Sparrow was first introduced into New England, so far as I am aware, in the fall of 1858, when "Six birds were set at liberty in a large garden" in the city of Portland, Maine. "They were introduced into Boston by the city government in 1868,"* and have since spread over the greater part of New England.

102. Passerella iliaca (Merrem) Swainson. Fox-colored Sparrow.

Common during the migrations, arriving from the North during the middle or latter part of October (Oct. 10, 1876, Osborne; 23, 1877), and remaining about a month (Nov. 20). Have taken it, in spring, from March 6th to April 19th. Gregarious. The largest and surely one of the handsomest of our Sparrows. Speaking of its song, Dr. Brewer says: "His voice is loud, clear, and melodious; his notes full, rich, and varied; and his song is unequalled by any of this family that I have ever heard."

103. Euspiza Americana (Gmelin) Bonaparte. Black-throated Bunting.

Dr. Linsley, in his "*Catalogue*," takes particular pains to state that this species was "very common" at New Haven (p. 261), but I have never seen it here and can find no other record of its capture in this State. It must be regarded as very rare in New England, although several have been taken, breeding, in Massachusetts. Its former abundance and present scarcity in this section affords another excellent example of the irregular migrations of birds. Giraud gave it as breeding commonly on Long Island,† but says the eggs are "white, speckled with black," whereas they are blue, unmarked.

104. Goniaphea Ludoviciana (Linné) Bowditch. Rose-breasted Grosbeak.

A common summer resident, breeding in dense undergrowth. Arrives before the middle of May (May 11, 1874, Portland, Conn., Sage), and I shot a female as late as Nov. 25th, (1874), in the city. I am informed by Prof. G. Brown Goode, of Middletown, Conn., that he knew an individual of this species to live eighteen years in confinement. Mr. J. Hammond Trumbull tells me that it was almost unknown about Hartford, Conn., thirty-five years ago.

NOTE.—*G. cærulea* probably occurs as an accidental visitant from the South. It has been taken on the Island of Grand Menan,‡ and

* Birds of North America, Baird, Brewer & Ridgway, vol. i, p. 526, 1874.

† Birds of Long Island, p. 100. 1844.

‡ Herrick's Partial Catalogue of the Birds of Grand Menan, p. 8. 1873.

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Mr. Boardman states that it was "common in the spring of 1861" in the vicinity of Calais, Maine!*

105. Cyanospiza cyanea (Linné) Baird. Indigo Bird.

A common summer resident, breeding, like the last, in thick patches of bushes. Arrives before the middle of May (May 14, 1874, J. H. Sage; 14, 1876, Osborne; 14, 1877, C. H. M.; also May 9, 1875, Sage), remaining into September.

106. Cardinalis Virginianus (Brisson) Bonaparte. Cardinal Grosbeak.

A rare and accidental summer visitor from the South. Mr. Thomas Bostwick of this city found one lying dead during the summer of 1874, and Mr. Geo. Bird Grinnell, who examined the specimen, tells me that its bill and feet, as well as the plumage, proved it to be a wild bird. Numerous specimens have, from time to time, been taken or seen along the Connecticut Valley, and it is highly improbable that they were all escaped cage birds. My friend, Mr. E. P. Bicknell, has taken a number of individuals about Riverdale, Westchester Co., N. Y., and it is certain that most, if not all, of them were wild.

Regarding the age to which our small birds attain, it is worthy of mention that my grandmother kept a Cardinal Bird caged for twenty-one years—it was noisy to the last. Its colors began to fade several years before its death, till finally it looked very like a worn female.

It winters as far north as southern New Jersey and Pennsylvania (Turnbull).†

107. Pipilo erythrophthalmus (Linné) Vieillot. Chewink; Towhee Bunting.

A common summer resident, breeding on the ground, in undergrowth, and in the woods. Arrives early in May (May 1, 1876, Osborne; 4, 1877), remaining till November (Oct. 28, Nov. 8.) Mr. J. H. Sage has a fine male in his cabinet which was shot near Portland, Conn., *Jan.* 22*d*, 1876! A characteristic nest, found May 24th, 1874, "under a Virginia Juniper," by the Stadtmüller Brothers, "was composed externally of cedar bark, lined with grass and horse hair."‡

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* Proceed. Bost. Soc. Nat. Hist., vol. ix, p. 127. 1862.

† The Birds of Eastern Pennsylvania and New Jersey. By Wm. P. Turnbull, LL.D. p. 24. 1869.

‡ MS. notes of the Stadtmüller Brothers.

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Family, ICTERIDÆ.

108. Dolichonyx oryzivorus (Linné) Swainson. Bobolink; Reed-bird, Rice-bird.

A common summer resident, breeding in meadows. Arrives before the middle of May (May 7, 1876, Osborne; same date at Portland, Sage; May 12, 1877).

109. Molothrus pecoris (Gmelin) Swainson. Cow-bird.

An abundant summer resident, arriving in March or early in April and remaining till November (Nov. 4, 1874). Sometimes winters. Mr. Geo. Bird Grinnell writes me that he took it twice in mid-winter (Jan. 15, 1874, and Jan. 16, 1875).

110. Agelæus phœniceus (Linné) Vieillot. Red-winged Blackbird.

A common summer resident, breeding abundantly in swampy places. Sometimes winters. Arrives during the latter part of February or early in March (March 4), remaining till late in the fall. Though generally choosing clumps of bushes for their characteristic nests, they sometimes place them on the ground. On the 6th of June, 1876, Mr. Dayan and myself, while collecting on a part of the Quinnipiac marshes where there were no bushes, found several nests containing fresh eggs. The nests were extremely shallow—very unlike those commonly found in bushes—and were placed on the bare ground, in the grass, with no attempt at concealment. I am informed by Mr. Grinnell that he has taken it in January (Jan. 15, 1874, and Jan. 16, 1875). This is the bird concerning which Josselyn wrote (in 1675) that there were, in New England, "*Starlings* black as *Ravens* with scarlet pinions."* Mr. W. W. Coe once wounded one of these birds (probably striking it in the head) which "started into the air and kept going up—up—up—sailing in larger circles—still ascending till lost to sight."†

111. Sturnella magna (Linné) Baird. Meadow-lark.

Resident, but particularly abundant from early spring till late in the fall. A few small flocks winter along the coast. The Rev. Mr. Peabody observes that this bird "has few enemies excepting Hawks, snakes, and young sportsmen."

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* An Account of two Voyages to New England, p. 100. 1675.

† MS. notes of W. W. Coe.

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112. Icterus spurius (Linné) Bonaparte. Orchard Oriole.

A common summer resident, breeding chiefly in orchards. Arrives during the first week in May (May 7, 1876, Osborne).

113. Icterus Baltimore (Linné) Daudin. Baltimore Oriole.

A common summer resident, breeding plentifully in the city as well as country throughout the State. Arrives about May 10th. Have taken it on the 8th, and Mr. Osborne saw one as early as the 6th (1876), while this year (1877) it did not come, in any numbers, till the 13th, when the whole country was literally "alive with them." Mr. J. H. Sage saw it at Hartford, May 6th, 1868 and 7th, 1872; also at Portland, Conn., May 7th, 1876, and May 10th, 1874 and 1875.

114. Scolecophagus ferrugineus (Gmelin) Swainson. Rusty Grackle.

Abundant during the migrations, sometimes wintering. Arrives before the middle of February (Grinnell) remaining through March into April (April 19, Osborne). In the fall it returns before the middle of September (Sept. 11, 1875 several flocks seen), remaining into November. Mr. Grinnell informs me that he took it, at Milford, Conn., Jan. 16th and 29th, and Feb. 6th, 1875.

115. Quiscalus purpureus (Linné) Licht. Crow Blackbird.

A common summer resident, breeding in evergreen trees in the city, as well as outside. Arrives about March 1st, though a few are generally seen in February (Feb. 13, 1876). Departs in November.

Such was the abundance, in early colonial times, of some of our commoner, and at present harmless, birds, that "premiums were paid by the local governments for the destruction of many of these species, and not without cause."* The town of Lynn, on March 8th, 1697, voted "that every householder in the town, should, sometime before the fifteenth day of May next, kill or cause to be killed, twelve blackbirds, and bring the heads of them, at or before the time aforesaid, to Ebenezer Stocker's, or Samuel Collins's, or Thomas Burrage's, or John Gowing's, who are appointed and chose by the town to receive and take account of the same, and take care this order be duly prosecuted; and if any householder as aforesaid shall refuse or neglect to kill and bring in the heads of twelve blackbirds, as aforesaid, every

* J. A. Allen in Bull. Nutt. Ornith. Club, vol. i, No. 3, p. 54. Sept., 1876.

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such person shall pay three pence for every blackbird that is wanting, as aforesaid, for the use of the town."*

Peter Kalm, in his "Travels into North America," thus speaks of their depredations: "A species of birds, called by the Swedes, maize-thieves, do the greatest mischief in this country. They have given them that name, because they eat maize, both publicly and secretly, just after it is sown and covered with ground, and when it is ripe. The English call them blackbirds. There are two species of them, both described and drawn by Catesby.† Though they are very different in species, yet there is so great a friendship between them, that they frequently accompany each other in mixed flocks. However, in Pennsylvania, the first sort are more obvious, and often fly together without any of the red-winged stares. . . . . Their chief and most agreeable food is maize. They come in great swarms in spring, soon after the maize is put under ground. They scratch up the grains of maize and eat them. As soon as the leaf comes out, they take hold of it with their bills, and pluck it up, together with the corn or grain; and thus they give a great deal of trouble to the country people, even so early in spring. To lessen their greediness of maize, some people dip the grains of that plant in a decoct of the root of the *veratrum album*, or white hellebore, (of which I shall speak in the sequel), and plant them afterwards. When the maize-thief eats a grain or two, which are so prepared, his head is disordered, and he falls down: this frightens his companions, and they dare not venture to the place again. But they repay themselves amply towards autumn, when the maize grows ripe; for at that time they are continually feeding. They assemble by thousands in the maize-fields, and live at discretion. They are very bold; for when they are disturbed, they only go and settle in another part of the field. In that manner, they always go from one end of the field to the other, and do not leave it till they are quite satisfied. They fly in incredible swarms in autumn; and it can hardly be conceived whence such immense numbers of them should come. When they rise in the air they darken the sky, and make it look quite black. They are then in such great numbers, and so close together, that it is suprising how they find room to move their wings. I have known a person shoot a great number of them on one side of a maize-field, which was far from frightening the rest; for they only just took flight, and dropped

* History of Lynn, by Alonzo Lewis and James R. Newhall, p. 298.

† See Catesby's Nat. Hist. of Carolina, vol. i, tabs. 12 and 13.

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at about the distance of a musket-shot in another part of the field, and always changed their place when their enemy approached. They tired the sportsman, before he could drive them off the maize, though he killed a great many of them at every shot. They likewise eat the seeds of the *aquatic tare-grass* (*Zizania aquatica*) commonly late in autumn, after the maize is got in. I am told, they likewise eat buckwheat, and oats. Some people say, that they even eat wheat, barley, and rye, when pressed by hunger; yet, from the best information I could obtain, they have not been found to do any damage to these species of corn. In spring, they sit in numbers on the trees, near the farms; and their note is pretty agreeable. As they are so destructive to maize, the odium of the inhabitants against them is carried so far, that the laws of *Pennsylvania* and *New Jersey* have settled a premium of three pence a dozen for dead maize-thieves. In *New England*, the people are still greater enemies to them; for Dr. *Franklin* told me, in the spring of the year 1750, that, by means of the premiums which have been settled for killing them in *New England*, they have been so extirpated, that they are very rarely seen, and in a few places only. But as, in the summer of the year 1749, an immense quantity of worms appeared on the meadows, which devoured the grass, and did great damage, the people have abated their enmity against the maize-thieves; for they thought they had observed that those birds lived chiefly on these worms before the maize is ripe, and consequently extirpated them, or at least prevented their spreading too much. They seem therefore to be entitled, as it were, to a reward for their trouble."†

NOTE.—The Boat-tailed Grackle (*Quiscalus major*, Vieillot) has been accredited to New England by Linsley and others, but an unusually large Crow Blackbird was probably mistaken for it.*

Family, CORVIDÆ.

116. Corvus Americanus Audubon. Common Crow.

An abundant resident; generally lays five eggs and sometimes six (Coe). On the 25th of January, 1875, I saw a flock of several hundred Crows near New Haven. "It is related of a certain ancient philosopher, walking along the sea-shore to gather shells, that one of

* Peter Kalm's Travels into North America, vol. ii, pp. 73–78. 1771.

† See Coues' Birds of the Northwest, p. 204. 1874.

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these unlucky birds, mistaking his bald head for a stone, dropped a shell-fish upon it, and thus killed at once a philosopher and an oyster."*

117. Corvus ossifragus Wilson. Fish Crow.

This species must be regarded as a rare summer visitor so far north as this State, although it has been seen in Massachusetts by Mr. William Brewster, who writes: "On the morning of March 16th, 1875, I saw a bird of this species flying swiftly over our place in Cambridge. It was pursued by at least twenty-five or thirty of our common species (*Corvus Americanus*), and at each renewal of their attacks gave utterance to its peculiar and unmistakable notes."† Linsley gave it as occurring at "Stratford," Conn.‡

118. Cyanurus cristatus (Linné) Swainson. Blue Jay.

An abundant resident; frequently seen about the city.

Family, TYRANNIDÆ.

119. Tyrannus Carolinensis (Linné) Temminck. King-bird; Bee-bird.

A common summer resident, arriving early in May (Mr. Dayan informs me that he saw one as early as April 13, 1877). Departs about the middle of September (Sept. 15) or later. Mr. Sage has twice seen it at Portland, Conn., as early as May 7th (1864 and 1876) and once on the first (1877), but it generally comes on the 10th.

The habit, so characteristic of this species, of attacking, and driving away from the vicinity of its nest, Hawks, Crows, and other large birds, early attracted attention, for we find, in Josselyn's Voyages to New England (published in 1675, p. 96), the following account of it: "There is a small Ash-color Bird that is shaped like a *Hawke* with talons and beak that falleth upon *Crowes*, mounting up into the air after them, and will beat them till they make them cry."§ The description of the bird would fit a Shrike even better than the one in question were it not that it "falleth upon Crows" which pastime this bird is not supposed to indulge in; still I have seen a Northern

* Nuttall's Manual of Ornithology, vol. i, p. 211. 1832.

† Bull. Nutt. Ornith. Club, vol. i, No. 1, p. 19. April, 1876.

‡ Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 260. April, 1843.

§ Reprinted in Collections of the Mass. Historical Society, vol. iii, 3d series, p. 275. 1833.

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Butcher Bird chase a Barred Owl for the space of half an hour, closely following him to and fro through the woods, till I put an end to his misery by shooting both.

120. Milvulus forficatus (Gmelin) Sw. Swallow-tailed Flycatcher.

An extremely rare accidental visitor. The only record of its capture in this State is that recently published by Mr. H. A. Purdie: "Mr. Jencks informs me that a specimen of this species was shot by Mr. Carpenter, at Wauregan, Conn., about April 27, 1876. The bird first attracted Mr. Carpenter's attention by its opening and closing the tail while flying about a small sheet of water in quest of insects. The only other Eastern United States capture of this species, is a male taken at Trenton, New Jersey, a few years ago, as recorded by Dr. C. C. Abbott."* Dr. Abbott's specimen was shot on the 15th of April, 1872, and "when captured, was busily engaged in picking semi-dormant insects from the bark of the trees; creeping about very much as is the custom of *Certhia familiaris*, and all the while opening and shutting the long scissor-like tail."† Its proper habitat is the lower part of the Mississippi Valley and Texas, thence southward into South America.

121. Myiarchus crinitus (Linné) Cabanis. Great-crested Flycatcher.

A common summer resident, generally placing its well-known snake-skin-lined nest in the hollow limb of some old apple tree, or rotten fence-post. Arrives early in May (May 8, 1873, Hartford, Sage), and Mr. W. W. Coe has taken its nest (four eggs) as late as June 13th, (1873). The history of this bird affords us a remarkably good example of the change in habitat of a species during a comparatively brief period of years.

Mr. T. Martin Trippe, in one of his interesting articles on "The Irregular Migrations of Birds,"‡ thus details his experience with the bird in question: "In a series of several years close observation at Orange, New Jersey, I searched for the Great-crested Flycatcher (*Myiarchus crinitus*), year after year, but all in vain; and what made the fact very singular was, that twelve or fifteen miles off, I had seen the bird sufficiently often to convince me that if not common, it was by no means rare. Yet for some inexplicable reason it did not

* Bulletin Nuttall Ornithological Club, vol. ii, No. 1, p. 21, Jan., 1877.

† American Naturalist, vol. vi, No. 6, p. 367, June, 1872.

‡ Am. Nat., vol. vii, No. 7, p. 390–91, July, 1873.

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inhabit the country immediately about Orange, for, although in the woods nearly every week for years, I never saw it until, after I had almost despaired of ever finding it, I did succeed in shooting a single specimen. This was in the fall; the next spring I saw a pair . . . . ., and, after an absence of two years, returning to Orange, I strolled through the woods, my old hunting grounds, and, to my surprise, almost the first bird I saw was the Great-crested Flycatcher. Subsequently I scarcely ever took a walk through the woods without seeing or hearing it." A precisely parallel case occurred in the vicinity of my home in Lewis County, northern New York. There, prior to the year 1870, they were unknown, at least so far as I can ascertain, and it is safe to say that they were extremely rare. In 1870 my cousin, Mr. C. L. Bagg, shot one specimen, the first we had ever seen. During the next season I shot a pair. I was away from home in 1872, but Mr. Bagg informs me that he took several Great-crested Flycatchers that year, and that they were quite common. In August, 1873, I shot eight in about an hour's time, and since then they have been one of our commonest species, breeding abundantly in the tall maple and birch forests, where their characteristic, but rather harsh cry, may be heard at any hour of the day throughout the entire season.

Regarding its former scarcity in Connecticut, Linsley said "a specimen of the Great-crested Fly-catcher was shot by me in the spring of 1838, in my front yard, the only living individual of this bird I have ever seen in this State,"* and Nuttall observed that it was "nearly unknown in New England."† That it is now really a common bird in *southern* Connecticut, at least, is certain. However, we must take into consideration the fact that these gentlemen (Nuttall and Linsley) were probably not familiar with its characteristic—I may even say diagnostic—note, and my experience with the bird has been that it is rarely seen, unless, guided by its note, it is persistently followed up, and even then one is often at his wits end to get a shot, so well does the bird keep concealed amongst the foliage. And surely it is not particularly conducive to tranquillity of mind to stand, up to one's knees in water, amidst myriads of mosquitoes, in a hot day in summer, gazing intently up into a tall tree, where, directly overhead, the cry of the Great-crested Flycatcher is constantly heard, and yet the most careful search fails to reveal the exact where-

* Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 259, April, 1873.

† Manual of the Ornithology of the United States and Canada. By Thomas Nuttall. Vol. i, Land Birds, p. 271, 1832.

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abouts of its author; and, when one's neck is nearly broken, to hear the cry again, this time issuing from another and still deeper portion of the swamp.

In support of this view I now propose to bring forward pretty conclusive proof: Mr. W. W. Coe, who early became familiar with the note of this species, tells me that it has been a common bird in the vicinity of Portland, Conn., for at least fifteen years—ever since he commenced collecting birds. Prof. William D. Whitney, of New Haven, on inquiry, informs me that he has known the bird for at least thirty years, and that he has always regarded it as common. His observations extend over portions of Massachusetts (about Northampton) as well as Connecticut. Mr. J. N. Clark, of Saybrook, Conn., says that it was not rare, about the mouth of the Connecticut, twenty-five years ago; and Dr. Wm. Wood, of East Windsor Hill, Conn., tells me that it has bred regularly, and has not been uncommon, in that vicinity, for the last twenty-five or thirty years.

In the face or these facts what is to be done? For my own part, I am willing to admit that in northern New York the bird may not have been rare prior to the year 1870 (although I feel pretty sure that this was not the case), and that it escaped notice because I was then unfamiliar with its note. In New Jersey, however, a similar supposition will not hold, for Mr. Trippe *was* familiar with its note and habits, from observations in a neighboring district, and asserts positively that it suddenly became abundant in a locality where before it was nearly, if not quite, unknown. Hence it is only fair to conclude that, while some of the supposed cases of change of habit may be explained on the ground that the observers were not sufficiently familiar with the bird, yet there are others concerning which the proof is ample, and the *cause* of the change only remains to be accounted for. Whether this be due to changed conditions in the physical features of the country (such as the cutting away of timber, drying up of streams and swamps, etc., for example), or to an increase in some species of insects on which the bird feeds (caused perhaps by the abundance of some particular food-plant—due, may be, to altered climatic conditions), or to an actual increase in the number of birds themselves, I will not take upon myself to decide, so imperfect is the present state of our knowledge on these points.

122. Sayornis fuscus (Gmelin) Baird. Pewee Flycatcher; Phœbe-bird.

A common summer resident. Comes very early: Dr. F. W. Hall tells me he saw it on Feb. 25th. 1876, and Mr. Osborne saw one March

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11th of the same year. I first noticed it on the 24th of March this season (1877), and Mr. Sage found it at Portland, Conn., on the 30th. In the fall it remains till, or after, the middle of October (Oct. 7, 1874; Oct. 13, 18, 1875). Breeds abundantly under the gable-ends, or on exposed beams, of houses and outbuildings, in old sheds, and under bridges. The primitive habit of building on the side of some rocky cliff is still adhered to in some parts of the State, and the most beautiful nest I have ever seen was found by Prof. Daniel C. Eaton and myself, while botanizing near Mt. Carmel, on the 23d of May, 1875. It was placed in a small wedge-shaped niche in the face of the rock, and its exterior was composed entirely of delicate green mosses. It contained five pure white, unspotted, fresh eggs. Another beautiful nest, similarly placed, was found on the "Hanging Hills of Meriden," by my friend, Mr. William H. Patton, on the 12th of May, 1877. It contained four fresh eggs, one of which is distinctly sprinkled, chiefly at the larger end, with small brownish-red spots. It also breeds on the faces of Pine and West Rocks near New Haven.

123. Contopus borealis (Swainson) Baird. Olive-sided Flycatcher.

Rare; probably a few sometimes breed in the more northern and hilly portions of the State, as they are known to do in Massachusetts. Not previously recorded from Connecticut, except by Linsley, who gave it, with a query, from Stratford. On the 18th of October, 1875, attracted by its characteristic note, "which is a short whistle resembling the syllables *O-whêŏ*, *O-whêŏ*, *O-whêŏ*," uttered several times in succession, "with the accent on the *whê*, and the voice falling on the last *ŏ*,"* I caught a momentary glimpse of one, perched on the top of a tall tree; but the bird was very shy and I did not succeed in getting a shot. Mr. Erwin I. Shores writes me that he took a male at Suffield, Conn., August 5, 1874. This is strong evidence of their breeding in the hills about that portion of the State, for migrants would hardly appear in Connecticut during the *first* of August. On turning to my manuscript notes on the birds of Easthampton, Mass., I find the following: "Breeds. Not so rare as it should be. Four specimens procured: one shot Sept. 10, 1873, and another May 23, 1874, in a small grove of pines northwest of town; two shot May 28, 1874, on Mt. Nonotuck [part of Mt. Tom], where they appear to be quite common. Their alimentary tracts contained coleopterous insects,

* Am. Nat., vol. vii, No. 12, p. 750, Dec., 1873.

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wasps, and humble-bees." In Prof. Whitney's private collection, and in the beautiful cabinet of mounted birds presented by him to the Peabody Museum of Yale College, are two or three specimens of this bird, which he killed in the vicinity of Northampton, Mass., some years ago —one as late as June 4th, (1846). Mr. Ruthven Deane writes: "During the past three or four years I have observed each year several specimens of this beautiful Flycatcher in the vicinity of Cambridge, Mass., and, although I consider this a rare bird with us, I am inclined to think a few breed within the limits of the State every year."* The species was described by Nuttall, from specimens taken in the vicinity of Cambridge, "in the woods of Sweet Auburn," in June, 1830 and 1831, and it has since been observed in eastern Massachusetts by Audubon (1832), Welch (1858), Brewster, Maynard, Cabot, Minot, and others. Nuttall says that his friend, W. Cooper, Esq., "received this bird likewise the preceding summer from the vicinity of Cape May, and Egg-harbor, in New Jersey,"† but Turnbull gives it as "very rare" in eastern Pennsylvania and New Jersey, and states that it only occurs as a migrant, being "generally seen early in May on its way north, . . . returns in September."‡ The occurrence in New Jersey, in "summer" (the month is, unfortunately, not stated) would indicate its breeding there, which hardly seems credible from what is now known concerning its distribution, though it is true that the "Pine-barrens" would afford it many congenial spots for nesting. Concerning their habits in the Adirondack region, in northern New York, where they breed plentifully, I have already observed that "they all seemed to have the same habit of choosing a large hemlock tree with a few dead branches on top, and were sure to light on the uppermost twig."§ Messrs. Maynard and Brewster gave it as "quite common and breeding at Umbagog," Maine, where "it has the habit of perching on dead stubs on the edge of clearings."‖ Professor Verrill gave it as a "summer visitant, not very common,"¶ at Norway, Maine (about forty miles south of

* Am. Nat., vol. viii, No. 5, p. 308, May, 1874.

† Manual of Ornithology, p. 284, 1832.

‡ The Birds of Eastern Pennsylvania and New Jersey, by William P. Turnbull, 1869.

§ Am. Nat.. vol. vii, p. 750, Dec., 1873.

‖ A Catalogue of the Birds of Coos Co., N. H., and Oxford Co., Me., by C. J. Maynard, with notes by William Brewster. From Proceed. Boston Soc. Nat. Hist., vol. xiv, p. 21, Oct. 18, 1871.

¶ Catalogue of the Birds found at Norway, Oxford Co., Me., by A. E. Verrill. From Proceed. Essex Institute, vol. iii, p. 144, May, 1863.

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Umbagog), and Prof. Charles E. Hamlin also observed it, in Kennebec County, in the same State.* Dr. W. H. Gregg found that it was "not a very common species" near Elmira, Chemung Co., southern New York, "where he met with only two specimens during several years of bird collecting."† In central Ohio it is, according to the high authority of Dr. J. M. Wheaton, a "rare migrant."‡ The record of its occurrence in the east, south of New Jersey, is not well authenticated. My friend, Mr. E. P. Bicknell, informs me that he has taken several specimens "and saw six or seven individuals be tween August 21st and Sept. 26th, 1876, at Riverdale, Westchester Co., N. Y.

Now, on the other hand, on going northward, we find it breeding at Randolph, Vermont (Chas. S. Paine),§ and Mr. Osborne has seen it about Mt. Washington. "Audubon found it in Maine, on the Magdeline Islands, and on the coast of Labrador,"‖ as mentioned by Dr. Brewer, who further states that "Mr. Boardman reports the Olive-sided Flycatcher as having of late years been very abundant during the summer in the dead woods about the lakes west of Calais [eastern Maine] where formerly they were quite uncommon," and that he is informed by Mr. Hoy "that this species used to be quite common near Racine [Wisconsin], frequenting the edges of thick woods, where they nested."

From the above references it will be seen that the Olive-sided Flycatcher belongs, in the east, to the Canadian fauna, while it occasionally extends down into the Alleghanian, and, if Cooper's record can be relied on, stragglers have been known to breed in the Carolinian. Going westward, however, the case is quite different, and we find *Contopus borealis* breeding in numbers from the "Cumberland House, on the Saskatchewan, in latitude 54°,"¶ where it was obtained by Sir John Richardson, and described by Swainson in 1831 (this description having priority over Nuttall's, which was not published till 1833), to Camp Bowie, Arizona, latitude 32°, "within one hundred miles of Mexico," where both "young and old were secured in

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* Report Sect. Maine Board Agriculture, p. 170, 1865.

† Catalogue of the Birds of Chemung Co., N. Y., by W. H. Gregg, M.D. From Proceed. Elmira Academy of Sciences, 1870.

‡ The Food of Birds as related to Agriculture, by J. M. Wheaton, M.D. From Ohio Agricultural Report, p. 8, 1874.

§ Appendix to Zadock Thompson's History of Vermont, p. 21. 1853.

‖ History N. Am. Birds, Baird, Brewer and Ridgway, vol. ii, p. 354.

¶ Coues, Birds of the Northwest, p. 244. 1874.

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August," by my friend, Mr. H. W. Henshaw, who, in his recent excellent work on the birds of that region, observes that "Its favorite perching places are the tops of the high pine stubs. From these stations it makes frequent sallies after passing insects, and seems rarely to miss its prey. When thus engaged, the clicking noise of its bill may be heard quite a distance. About the first of June, in southern Colorado, they had all mated, and each pair maintained a most jealous watch over the neighborhood chosen as its summer residence, never allowing the intrusion of the larger birds to pass unnoticed. The loud call notes of the male are, at this season, almost incessantly repeated."* Mr. Henshaw also states that "specimens were taken near Camp Apache [Arizona] in July, which doubtless were breeding," and that he "found it almost as numerous in eastern Arizona, quite far to the south, as in Colorado."

Numerous specimens have been taken in Colorado by Allen,† Henshaw, Aiken, Trippe, and others. Regarding its occurrence at Idaho Springs, Colorado, T. Martin Trippe writes Dr. Coues that "It is quite uncommon, only three or four pairs having been observed throughout the summer, and these at widely different points, each pair apparently monopolizing a wide range. It keeps in the tops of the trees, and is an active flycatcher; its noise is loud and distinct; and its nest is placed in the top of a pine, and jealously guarded from all intrusion with as much fierceness and energy as the Kingbird's."‡ It is given by Snow as "rare in western Kansas."§ Notices of its occurrence in Utah have been published by Allen "several seen among the cottonwoods along Weber River",‖ Henshaw,¶ Ridgway "breeding in Parley's Park, Wahsatch Mts.",** Nelson (who "obtained one specimen from the top of a dead pine," among the mountains, thirty miles south of Fort Bridger),†† and myself;‡‡

* Report upon the Ornithological Collections made in portions of Nevada, Utah, California, Colorado, New Mexico and Arizonia, during the years 1871, 1872, 1873 and 1874. By H. W. Henshaw. Forming Chapter III of the Zoölogical Volume published under the direction of Lieut. Geo. M. Wheeler, in charge of the Geographical and Geological Explorations and Surveys west of the 100th meridian. p. 350. Washington. 1875.

† Bulletin Museum Comp. Zoöl. Cambridge, vol. iii, No. 6, p. 158. 1872.

‡ In Coues' Birds of the Northwest, p. 245. 1874.

§ Catalogue of the Birds of Kansas. By F. H. Snow. 3d ed., p. 8. 1875.

‖ Bull. Mus. Comp. Zoöl. Cambridge, vol. iii, No. 6, pp. 169, 179. 1872.

¶ Report upon Ornithological Specimens, pp. 22, 46. Wash. 1874.

** Bull. Essex Inst., vol. vii, No. 2, p. 33. Feb., 1875.

†† Proceed. Boston Soc. Nat. Hist., vol. xvii, p. 344. Jan., 1875.

‡‡ Sixth Annual Report U. S. Geol. Survey of the Territories for the year 1872. p. 691.

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and it was taken in Nevada by Bischoff (recorded by Henshaw), and Ridgway,* who writes me as follows: "Although I found it at very few localities, I am certain of its occurrence wherever suitable localities exist. . . . . It there inhabits the coniferous woods at an altitude of from about 8,000 feet up to timber line, and I noticed that it preferred localities where the trees had been deadened by fire. The only places, in Nevada, where I met with this bird, were the pine woods of the East Humboldt and Ruby Mountains." Concerning its occurrence in California, Dr. Cooper says: "It seems to be resident in most parts of the State where it is found, but not occurring south of Monterey, where Dr. Gambel found young in July. I found them rather common in the Coast Range toward Santa Cruz, where they had nests in May, but I could not examine any of these, their location being generally on a high inaccessible branch. I also found this bird at Lake Tahoe in September. It is rather silent, keeping mostly on tops of the trees, and catching passing insects."† Mr. Ridgway also found it breeding in California, "on the eastern slope of the Sierra Nevada."‡ In Oregon and Washington Territory it is, according to Dr. Cooper, "a much more abundant bird near the Columbia River and throughout the northern Rocky Mountains. There they migrate, remaining at the Lower Columbia from May to October."† Dr. Brewer writes: "In Washington Territory this bird appears to be somewhat more common than in other portions of the United States. Dr. Suckly obtained a specimen at Fort Steilacoom, July 10th, 1856. It was not very abundant about Puget Sound, and showed a preference for shady thickets and dense foliage [contrary to its usual habit], where it was not easily shot." "A single specimen," continues Dr. Brewer, "was taken, Aug. 29th, 1840, at Nenortalik, Greenland [by Reinhardt], and sent to Copenhagen."§ Hence the distribution of this beautiful and interesting Flycatcher affords an excellent illustration of the manner in which some species that are confined, during the breeding season, to the Canadian and Alleghanian faunæ in the Eastern Province, are found, during the same period, in the far West, extending from the Saskatchewan, in British America, almost, if not quite, to Mexico.

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* Bull. Essex. Inst., vol. vii, No. 1, pp. 21, 38. Jan., 1875.

† Ornithology of California, vol. i, p. 324. 1870.

‡ Bull. Essex. Inst, vol. vi, No. 10, p. 174. Oct., 1874.

§ Baird, Brewer and Ridgway, Birds N. Am., vol. ii, p. 356. 1874.

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124. Contopus virens (Linné) Cabanis. Wood Pewee.

A common summer resident, generally placing its beautiful lichen-covered nest astride the horizontal branch of some tall birch, or maple, in deep forests, but sometimes choosing an orchard or garden, in close proximity to occupied buildings. Arrives before the middle of May (May 10, 1876, Osborne), remaining through September. It is one of the few birds that may be found alike, during the breeding season, in the heart of the city, near the farm house, and in the darkest swamps and most secluded forests, far from the habitations of man. Its mournful note may sometimes be heard at all hours of the night.

125. Empidonax Acadicus (Gmelin) Baird. Acadian Flycatcher; Small Green-crested Flycatcher.

A rare summer visitant from the Middle States; may breed in the Connecticut Valley.

It affords me great pleasure to be able, through the kindness of Mr. Erwin I. Shores, to replace this species among the birds of New England. As is well known, it was formerly included in all New England lists, but, as shown by Dr. Coues, Mr. H. W. Henshaw, and others, the records were founded on erroneous identifications—the bird having been mistaken for *E. minimus* or *Traillii*, generally the former. In a letter to Dr. Coues, Dr. Brewer says "I do not think the bird occurs in New England, even in the Connecticut Valley,"* and Mr. William Brewster, of Cambridge, the highest authority on the Birds of New England, has expressed similar views. Hence it was with a peculiar sense of gratification that I received, a few days since, an unmistakable example of this species, from Mr. Shores, who states that he shot it "in Suffield, Conn., June 24th, 1874, in a piece of woods known as Beech Swamp." For the benefit of those who might not feel disposed to accept my identification in so important a matter, and to avoid all possibility of mistake, I at once sent the bird to my friend Mr. Robert Ridgway, of the Smithsonian Institution, and he pronounces it to be a "typical *E. Acadicus*."

126. Empidonax Traillii (Audubon) Baird. Traill's Flycatcher.

A common summer resident, arriving early in May (May 6, 1877). Frequents swampy lowlands, where it breeds.

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* Birds of the Northwest, p. 251. 1874.

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127. Empidonax minimus Baird. Least Flycatcher.

An abundant summer resident, arriving early in May (May 5, 1877), and remaining till late in September. Breeds abundantly in gardens throughout the city.

128. Empidonax flaviventris Baird. Yellow-bellied Flycatcher.

Not uncommon during migrations; arrives about the middle of May (May 20, 1876), remaining till the last of the month, and Mr. Grinnell tells me he has seen it in early June. Have taken it in fall, about the middle of September (Sept. 17, 1875). Not known to breed.

Family, CAPRIMULGIDÆ.

129. Antrostomus vociferus (Wilson) Bonaparte. Whippoorwill.

A common summer resident. Arrives early in May (May 2, 1877, Osborne). Nocturnal, spending the day on the ground beneath some tree, or among thick bushes. When disturbed, its flight is short and irregular, and it may easily be shot. The Stadtmüller Brothers have an unusually pale set of eggs taken on Mill Rock (near New Haven) May 30th, 1875. "They were not placed by the side of a rock or log, as Samuels says."*

130. Chordeiles Virginianus (Brisson) Bonaparte. Night-hawk; Bull-bat.

A common summer resident, arriving early in May (May 11, 1875, Sage). Mr. W. W. Coe has taken its eggs as late as June 23d (1872). By no means strictly nocturnal like the last, but may be seen, especially in autumn, flying about high in the air and constantly uttering its peculiar cry, at any hour of the day, though generally preferring the twilight. Commonly roosts on old logs or horizontal limbs (sometimes quite high up) on which they lie longitudinally.

131. Chætura pelagica (Linné) Baird. Chimney Swift.

An abundant summer resident, arriving in April or May (April 14, 1876, Osborne; April 30, 1877, May 5, 1874, Portland, Sage; May 8, 1874), and remaining till late in September (Sept. 27, Osborne); breeds abundantly in old chimneys. Before the days of

* MS. notes of the Stadtmüller Brothers.

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brick chimneys, and to some extent since, these birds used to congregate in multitudes about certain large hollow trees, where they bred regularly, and, according to some, spent the winter also. Williams, in his "Natural and Civil History of Vermont," published in 1794, relates the following concerning three "Swallow trees" which came under his observation. "The species called the house or Chimney Swallow, has been found during the winter, in hollow trees. At *Middlebury* in this State, there was a large hollow elm, called by the people in the vicinity the Swallow tree. From a man who, for several years, lived within twenty rods of it, I procured this information: He always thought the Swallows tarried in the tree through the winter, and avoided cutting it down, on that account. About the first of May, the Swallows came out of it, in large numbers, about the middle of the day; and soon returned. As the weather grew warmer, they came out in the morning with a loud noise, or roar, and were soon dispersed; about half an hour before sun down, they returned in millions, circulating two or three times round the tree, and then descending like a stream, into a hole about sixty feet from the ground. It was customary for persons in the vicinity, to visit this tree, to observe the motions of these birds. And when any persons disturbed their operations, by striking violently against the tree, with their axes, the Swallows would rush out in millions, and with a great noise. In November, 1791, the top of this tree was blown down, twenty feet below where the Swallows entered. There has been no appearance of the Swallows since. Upon cutting down the remainder, an immense quantity of excrements, quills, and feathers, were found; but no appearance or relicks of any nests. Another of these swallow trees was at *Bridport.* The man who lived the nearest to it, gave this account: The Swallows were first observed to come out of the tree, in the spring; about the time that the leaves first began to appear on the trees. From that season, they come out in the morning, about half an hour after sunrise. They rushed out like a stream, as big as the hole in the tree would admit, and ascended in a perpendicular line, until they were about the height of the adjacent trees; then assumed a circular motion, performing their revolutions two or three times, but always in a larger circle, and then dispersed in every direction. A little before sundown, they returned in immense numbers, forming several circular motions, and then descended like a stream into the hole, from whence they came out in the morning. About the middle of September, they were seen entering the tree, for the last time." "Neither of these accounts are attended

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with the highest degree of evidence, which the subject may admit of: but I am led to believe from them, that the House Swallow, in this part of America, generally resides during the winter, in the hollow of trees."*

The tree above described, from Middlebury, Vermont, finally blew down, and, more than half a century (in 1852) after Williams wrote the account above given of it, its *remains* were visited by his successor, in the historical line, Zadock Thompson, and were found scarcely less interesting than the tree itself when inhabited by thousands of Swifts. Thompson gave this account of its condition and contents in 1852: "The tree had rotted away, leaving little besides the cylindrical mass, which had filled its hollow. The length of this mass was about seven feet, and its diameter fifteen inches. Of the materials which composed it, about one-half consisted of the feathers of the Chimney Swallow, being, for the most part, wing and tail feathers. The other half was made up of exuvia of insects, mostly fragments and eggs of the large wood-ant, and a brown substance probably derived from the decayed wood of the interior of the tree. This discovery at Middlebury, though interesting, would not have been regarded as very remarkable, if the materials which filled the hollow of the tree, had been promiscuously and disorderly mingled together. Such a jumbled mass would be what we should expect to find in a hollow tree which had been, for centuries, perhaps, the roosting place of myriads of Swallows. But this is not the case. In their general arrangement, the larger feathers have nearly all their quills pointing outward, while their plumes, or ends on which their webs are arranged, point inward. . . . . But this is not the most remarkable circumstance connected with the subject. In various parts of the mass, are found, in some cases, all the primary feathers of the wing; in others, all the feathers of the tail, lying together in contact, and in precisely the same order and position, in which they are found in the living Swallow. In a lump of the materials, measuring not more than seven inches by five, and less than three inches thick, five wings and two tails were plainly seen, with their feathers arranged as above mentioned, and in one of the wings, all the secondary quills were also arranged in their true position with regard to the primaries. Now, we cannot conceive it possible that these feathers could be shed by living birds, and be thus deposited. We may

* The Natural and Civil History of Vermont. By Samuel Williams, pp. 116–18. 1794.

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suppose that the birds died there, and that their flesh had been removed by decay, or by insects, without deranging the feathers. But in that case what has become of the skeletons? I do not learn that a bone, beak, or a claw, has been found in any part of the whole mass. What then has become of these? They could hardly have been removed by violent means, without disturbing the feathers. But, if done quietly, what did it? What insect would devour the bones, and beak, and claws, and not meddle with the quills? Or would the formic, or any other acid, which might be generated within the mass, dissolve the former without affecting the latter?"*

Family, TROCHILIDÆ.

132. Trochilus colubris Linné. Ruby-throated Hummingbird.

A common summer resident. Arrives early in May (May 5, 1877), remaining till the middle of October (Oct. 5, 1876; Oct. 16, 1874). They usually breed early in June, but Dr. F. W. Hall informs me that a friend of his found a nest completed, and containing two eggs, as early as May 27th (1877).

Thomas Morton, in 1632, expressed his astonishment at the dimensions and habits of this bird, in the following words: "There is a curious bird to see to, called a hunning bird, no bigger than a great Beetle; that out of question lives upon the Bee, which he eateth and catcheth amongst Flowers. For it is his custome to frequent these places. Flowers hee cannot feed upon by reason of his sharp bill, which is like the poynt of a Spanish needle, but shorte. His fethers have a glasse like silke, and as hee stirres, they show to be of a chaingable coloure: and has bin, and is admired for shape, coloure, and size."† And two years later, William Wood, in his "New England's Prospect" informs us that "The Humbird is one of the wonders of the Countrey, being no bigger than a Hornet, yet hath all the demensions of a Bird, as bill, and wings, with quills, spider-like legges, small clawes: For colour she is as glorious as the Raine-bow, as she flies, she makes a little humming noise like a Humble-bee: wherefore shee is called the Humbird."‡

"The Humberd for some Queene's rich Cage more fit,
Than in the vacant Wildernesse to sit."‡

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* Zadock Thompson, *Appendix* to the History of Vermont, p. 26, 1853.

† Force's Historical Tracts, vol. ii, Tract 5, p. 50.

‡ New England's Prospect, by William Wood. p. 31, 1634.

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Family, ALCEDINIDÆ.

133. Ceryle alcyon (Linné) Boie. Belted Kingfisher.

A common summer resident about water courses and along the coast. Arrives during the latter part of March (Mar. 29, 1877), remaining into November (Nov. 13, 1875). A few sometimes winter. One was seen near Hartford, Feb. 13, 1874, by Mr. K. C. Humphrey.* Mr. Fred. S. Smith, of this city, informs me that he once found the eggs of this species placed on a bed of fish bones, about six feet from the entrance to the hole. This was near Lake Whitney, Conn. In this latitude their eggs are generally deposited during the latter part of May. Josselyn, in enumerating the birds of New England, in 1675, says there are "*Kings fishers*, which breed in the spring in holes in the Sea-bank, being unapt to propagate in Summer, by reason of the driness of their bodies, which becomes more moist when their pores are closed by cold."†

Family, CUCULIDÆ.

134. Coccyzus erythrophthalmus (Wilson) Bonaparte. Black-billed Cuckoo.

A common summer resident, arriving about the middle of May (May 15, 1876), and remaining till the latter part of September (Sept. 28, 1875).

135. Coccyzus Americanus (Linné) Bonaparte. Yellow-billed Cuckoo.

A common summer resident. Arrives early in May (May 7, 1876), remaining into October (Oct. 11, 1873, shot, Hall). Both Mr. Coe and myself have found, on the same day, and in the same Cuckoo's nest, fresh eggs and young birds.

Family, PICIDÆ.

136. Hylotomus pileatus (Linné) Baird. Pileated Woodpecker.

A rare winter visitant; it was once common throughout the State, but is now almost exterminated and driven into the less civilized districts. Linsley gave it from Stratford and New Haven. Mr. W. W. Coe tells me that a specimen was killed near Portland, Conn., in

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* MS. notes of John H. Sage, Esq.

† Voyages to New England, p. 101, 1675.

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November, 1876; and "Mr. Wm. King took a specimen, several years ago, at Suffield, Conn."* No longer ago than 1839, Peabody gave it as resident in Massachusetts, where it was "not uncommon in the woodlands."†

137. Picus villosus Linné. **Hairy Woodpecker.**

Resident, but not common. Have taken four specimens and seen several others in the immediate vicinity of New Haven. Found chiefly in winter. Mr. Coe tells me that it is quite common about Portland, Conn.

138. Picus pubescens Linné. **Downy Woodpecker.**

A common resident; found everywhere except in open fields devoid of stumps and fences.

139. Picoides arcticus (Swainson) Gray. **Black-backed Three-toed Woodpecker.**

A rare winter visitor from the North. In the Museum at Middletown is a specimen, from the Shurtleff Collection, taken at Simsbury, Conn., in 1860. Dr. Wood has also taken it at East Windsor Hill, Conn.

140. Sphyrapicus varius (Linné) Baird. **Yellow-bellied Woodpecker.**

Rare about New Haven. Have seen but four individuals (Sept. 28, and Oct. 2, 1875, and March 30, and May 3, 1876). Mr. J. N. Clark finds it to be "abundant in fall" at Saybrook.‡ Linsley gave it from New London, Conn. It is rare about Portland, Conn., as I am informed by Messrs. Coe and Sage. Mr. Thos. Osborne tells me that they were quite common about New Haven last fall (1876) and that he secured four specimens. Mr. Grinnell says that it is not uncommon about Milford, Conn., in fall; and Mr. J. N. Clark, of Saybrook, writes me that he has observed it to be "very common both in spring and fall," but that he "never saw it after April till autumn."

Though most Woodpeckers are residents where they are found at

* MS. notes of Erwin I. Shores.

† Peabody's Report on the Ornithology of Mass., p. 334. 1839.

‡ Am. Nat., vol. vii, No. 11, p. 693. Nov., 1873.

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all, in any numbers, this is truly, though not strictly, a migrant.* Dr. Wm. O. Ayres writes me: "At New Canaan, Conn., I used to find them each autumn, though never very numerous; I never saw them in the spring.

141. Centurus Carolinus (Linné) Swainson. Red-bellied Woodpecker.

A rare accidental visitor from the South. Linsley saw one "ascending an apple tree, on the 16th of October, 1842."† Dr. Crary says he has killed it near Hartford, Conn.; and Mr. Erwin I. Shores writes me that he "took a female, July 30th, 1874," at Suffield, Conn. Giraud gave it as breeding regularly on Long Island, though not very common.‡

142. Melanerpes erythrocephalus (Linné) Swainson. Red-headed Woodpecker.

A rare summer resident; breeds at Saybrook, sometimes remaining all winter.§ Linsley gave it from Stratford, and Dr. J. D. Whelpley informed him that it was "rare at New Haven"† thirty-five years ago, and, so far as I am aware, its numbers have not increased since. In fact, as Dr. Coues has remarked: "Comparisons of the older with more recent local authorities indicate that the species is now much less numerous than formerly."‖ Dr. F. W. Hall took one at West Haven in December, 1872, and has seen several other specimens in this vicinity. W. W. Coe and J. H. Sage have each taken it about Portland, Conn., but find it rarely. Dr. Wood, of East Windsor Hill, tells me that they were really abundant, in that vicinity, thirty-five years ago, being more numerous than the Flicker (*Colaptes auratus*). Four years later a large flock was seen, and since then they have been one of our rare birds. A few, however, still breed pretty regularly in the vicinity of East Windsor, Conn. Mr. J. N. Clark informs me that, during the latter part of September, 1872, they suddenly became abundant in a grove of oaks near Saybrook, Conn. The adult birds were mostly killed off by boys from the neighborhood, while the young-of-the-year, lacking the brilliant and

* For a valuable contribution to the biography of this beautiful Woodpecker, see an article by William Brewster, in Bull. Nutt. Ornith. Club, vol. i, No. 3, pp. 63–70. Sept., 1876.

† Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 263. April, 1843.

‡ Birds of Long Island, p. 179. 1844.

§ Am. Nat., vol. vii, No. 11, p. 693. Nov., 1873.

‖ A List of the Birds of New England. [From Proc. Essex Inst., vol. v, p. 15.] 1868.

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much prized red heads, were spared and remained through the winter —some even breeding there the next summer (1873). Mr. J. H. Sage killed one at Hartford, Dec. 31st, of the same winter (1872). Their food consisted chiefly of acorns. In this connection I will state that in northern New York (Lewis Co.), where they are usually one of our commonest Woodpeckers, they subsist almost exclusively on beech-nuts, of which they evidently are extremely fond, eating them, apparently with equal relish, whether green or fully matured. It is truly a beautiful sight to watch these magnificent birds, together with their equally abundant cousins, the Yellow-bellied Woodpeckers (*Sphyrapicus varius*), creeping about, after the manner of the Warblers, among the small branches and twigs, which bend low with their weight, while picking and husking the tender nuts—the bright crimson of the head, neck and breast, the glossy blue-black back, and creamy-white belly, together with the scarcely less striking colors of their yellow-bellied companions, contrast handsomely with the deep green foliage.

143. Colaptes auratus (Linné) Swainson. Golden-winged Woodpecker; Yellow-hammer; High-holder; Flicker; Yaffle.

Resident; common from early April till November. A few generally winter. It breeds in holes in old stubs and trees. The migrants arrive during the latter part of March (March 28), remaining till the middle or latter part of November (Nov. 14 and 28). My friend, Mr. A. J. Dayan, saw one, near New Haven, on the 26th of January, 1877. Mr. Geo. Bird Grinnell tells me that he has taken it, near Milford, Conn., in December (Dec. 25, 1876), January (Jan. 1, 1877), and February (Feb. 27, 1875). Mr. W. W. Coe writes me that they also spend the winter in the vicinity of Portland, Conn.

Kalm states, in his *Travels*, that "this species is destructive to maize-fields and orchards, for it pecks through the ears of maize, and eats apples. . . . . Some years ago there was a premium of two pence per head, paid from the public funds, in order to extirpate this pernicious bird."*

Family, STRIGIDÆ.

144. Strix flammea, var. **Americana** (Audubon) Schl. Barn Owl.

A rare accidental visitor from the South. Taken by Linsley at Stratford, Conn.† Prof. J. A. Allen states that Dr. Wood "has a

* Peter Kalm's Travels into North America, vol. ii, pp. 86–7. 1771.

† Catal. Birds Conn., p. 253. 1843.

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specimen in his cabinet that was shot 'at Sachem's Head, Connecticut, Oct. 28th, 1868.'"* Massachusetts also lays claim to two specimens: the first "was killed near Springfield, Mass., about the last of May," 1868,† and the other "was shot near Lynn, in the same State, in 1863."‡

Since writing the above I have seen, in the cabinet of Capt. O. N. Brooks, at Faulkner's Island, Conn., a mounted specimen of this species which was killed at Madison, Conn., some years ago; and Dr. Wm. O. Ayres, now of Easthampton, Long Island, writes me that he found one at Hartford, Conn., about the year 1841. Hence at least four have been killed in this State.

145. Bubo Virginianus (Gmelin) Bonaparte. Great Horned Owl.

Resident; not rare. Breeds in suitable localities throughout the State. It feeds on the larger game (hares, grouse, and the like), not forgetting the poultry yard, and seems particularly fond of turkeys, of which it seldom touches more than the head, if there are a plenty about. Indeed I have known one to kill and decapitate three turkeys and several hens in a single night, leaving the bodies uninjured and fit for the table. Hence, if not so prone to select valuable fowls the loss would not be great. Their tenacity to life is something remarkable. A number of years ago I kept one in confinement for six or eight months, during which time I was twice attacked by him and bear the marks of his talons to this day. On both of these occasions I kicked and pounded him with sticks till he was, as I supposed, dead, but always on returning to skin the bird I found him sitting up on his perch, blinking, snapping his bill, and making faces at me as usual. Dr. Wood writes that a gentleman, while viewing one of this species in his collection, said: "I suppose that is the kind of bird that once scared me almost to death. While riding on horseback through a tract of large woods in New York State, one night, with a white beaver on my head, something, without the least noise or warning, struck my head with such force that it took my hat, and I supposed for a time the top of my head off. I thought the devil was after me, and the way that old horse went for the next three miles would have astonished John Gilpin. It seemed to me my heart beat

* Notes on some of the Rarer Birds of Massachusetts, p. 17. 1869. I have lately seen this specimen.

† Proceed. Essex Institute, vol. v, p. 312.

‡ Coues' Birds of the Northwest, p. 300. 1874.

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louder than an ordinary church bell, and I had to swallow fast to keep it from coming into my throat and choking me."*

My friend, Mr. W. W. Coe, of Portland, Conn., has had such remarkable success in finding the nests, and securing the eggs, of this species, that I was induced to trouble him for an account of some of his exploits in that direction, and he has been kind enough to favor me with the following: "In 1868, a farmer's boy told me that a pair of Great Horned Owls and their two young had taken up their abode in a piece of woods near his place, and that a pair had lived there for several years. He had often seen them when at work in the woods in winter, and observed them one morning on the lower limbs of a large hemlock. One sat on a branch directly above the other and had a rabbit in its claws, upon which they both seemed to be feeding. He watched them for sometime and then frightened them off. The bird with the rabbit carried it with him a short distance, but finally dropped it. The boy offered to shoot one of the Owls for me, but I told him not to do it, for I wanted to get the eggs, if possible, the next year.

"On the first of May, 1869, we went to look for the nest and found it in a large chestnut tree. It contained two young Owls almost large enough to fly. I brought them both home, stuffed one, and kept the other alive sometime, when it got killed while I was away. During the latter part of March, 1870, we went again and found the Owls occupying the same nest. One old bird was on, and I thought myself sure of the eggs this time, but on ascending to the nest found two young Owls covered with down. They were, I should think, about two weeks old, so we left them for seed and determined to be in time next season, but sickness and extra work forced me to neglect it. In the following year (1872) we started out on the 2d of March and visited the old nest, but for some reason it had been abandoned. We then visited another piece of woods near by and soon found a vary large nest in a chestnut tree, but it showed no sign of life. While deliberating whether it was best to climb it or not, I gave the tree several hard raps with a club, when off went the old bird with a *hoo, hoo, hoo.* My companion climbed the tree and found two eggs. I thought they might lay again, since it was so early, and accordingly, on the 12th of April, visited the same nest, and, on getting near, a bird jumped up from it which I at first took to be the Owl, but as it started off I saw it was a Red-tailed Hawk, which we found had

* Hartford Times, chap. xvii, July 20, 1861.

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taken possession and layed two eggs. We then went to the old Owl's nest, where we had found young in 1869 and 1870, and found the old bird on. This nest now contained two eggs in which the young were well advanced—so the old birds must have laid again soon after the first nest was robbed.

"About the first of March, 1873, we again visited the same place but could not find the birds, though I noticed one of their feathers, and, since snow had fallen a day or two before, I know they could not be far off. Finally, on the 13th of March, I found them occupying a nest about two miles from the old place. It was the old nest of a Red-tailed Hawk from which I took three eggs April 29th, 1872. The nest now contained one young Owl, apparently about five or six days old, and one rotten egg. I think the egg had been frozen, for it was badly cracked. I am, of course, unable to say whether or no all the eggs above mentioned were deposited by the same pair of Owls, since three or four old birds have been killed in this vicinity. However, I am quite sure that they all belonged to the same family. I also took their nests in 1874 (Feb. 28), 1875 (took young in May), 1876 (Feb. 22), and 1877 (took eggs twice, Feb. 19 and Mar. 23). During the whole time I have known of them they have not built a new nest, but have either occupied the same one for at least three years, or taken some vacant Hawk's nest.

"To sum up: my experience with Great Horned Owls has been that they lay in old nests of Red-tailed Hawks, in hollow trees, and occasionally in ledges of rocks. I never knew them to build a nest, or to lay more than two eggs, and I have known of many nests not mentioned above."

146. Scops asio (Linné) Bonaparte. Mottled Owl; Screech Owl.

A common resident throughout the State. It lays five eggs, in a hollow tree, about the last of April. This is, I think, the most nocturnal of our Owls. At least it can generally be approached in broad day-light more easily than the other species, and seems to be dazzled by the light. It sometimes catches fish through a hole in the ice, like the Snowy Owl.*

147. Otus vulgaris, var. **Wilsonianus** (Lesson) Allen. Long-eared Owl.

A common resident. Its large nest is commonly placed on some thick tree—generally a pine—but sometimes in low bushes. It is

* Bull. Nutt. Ornithol. Club, vol. ii, No. 3, p. 80. July, 1877.

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said to take possession of the deserted nests of Crows and Hawks. "Mr. J. S. Brandigee, of Berlin, Conn., found a nest, early in April, in a hemlock tree, situated in a thick dark evergreen woods. The nest was flat, made of coarse sticks, and contained four fresh eggs."* Have taken it, in company with the following species, in clumps of low bushes near the coast.

148. Brachyotus palustris (Bechst.) Gould. Short-eared Owl.

Resident about the salt marshes near the coast; also found throughout the State. Nests on the ground. Not uncommon. In the year 1856, on the island of Grand Menan, "A nest of this bird was found by Mr. Cabot in the midst of a dry, peaty bog. It was built on the ground, in a very slovenly manner, of small sticks and a few feathers, and presented hardly any excavation."†

149. Syrnium cinereum (Gmelin) Audubon. Great Gray Owl.

A rare straggler from the north. Linsley captured a specimen at Stratford, Conn., Jan. 6, 1843.‡

During the winter of 1852 a large Great Gray Owl was killed near Boston, Mass., after creating some little excitement among the inhabitants: "A light snow fell on Sunday evening, March 21st, and the next morning mysterious footprints were discovered in the vicinity of Nahant street and Long Beach. They were of a shape that excited much curiosity, and no one was able to determine what sort of a creature had made them. But on Monday evening Mr. John Barry shot a very large gray owl, on the marsh, near the foot of Pleasant street, and it was concluded that the wonderful tracks were made by him. He measured more than five feet from tip to tip of the wings."§

150. Syrnium nebulosum (Forster) Boie. Barred Owl.

A common resident throughout the State. Mr. J. N. Clark, of Saybrook, Conn., tells me that he once found it breeding, in a hollow tree, within twenty-five rods of the nest of a pair of Great Horned Owls.

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* Baird, Brewer & Ridgway, vol. iii, p. 22, 1874.

† A List of Birds observed at Grand Menan and at Yarmouth, N. S., June 16 to July 8, 1856. Proceed. Boston Soc. Nat. Hist., vol. vi, p. 115. March, 1857.

‡ Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 253. April, 1843.

§ History of Lynn, Mass., pp. 432–3, 1865.

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Dr. Wm. Wood, of East Windsor Hill, Conn., relates the following amusing anecdote, which serves well to illustrate "the nature of the beast" under certain conditions, and at the same time affords us an excellent example of conjugal affection and innate wisdom in our own species: "A gentleman who fancied himself a taxidermist because, forsooth, he had seen a single specimen stuffed, determined to allow that talent no longer to be buried, but bring it to light by practically demonstrating to the world in general, and mankind in particular, the way by which the feathered species can forever look 'as natural as life and a little more so.' Having obtained a specimen of the Barred Owl that was only slightly winged, he let it loose in the cellar till a leisure day. The bird, with only the tip of the wing injured, and having free use of its limbs and feet, could visit any part of her prison, and finding herself sole mistress, was not long in appropriating anything and everything that would promote her own comfort and ease. The potato and apple bins, as well as the soap and pork barrels were indiscriminately and alike used for places of rest and repose. The day arrived when this nuisance could no longer be tolerated. Approaching, with hand extended, to seize the Owl, my friend received the benefit of her claws in and around his fingers. Grasping the bird's leg to disengage it from its gripe, the other claw was instantly brought in requisition and seized the other hand. Both hands being now securely held by the bird, no wonder that our hero called out lustily for help when he saw the Owl expanding herself almost indefinitely, and snapping her bill in anticipation of the dainty repast which, according to appearance, she was about to swallow! The terrified scream alarmed the whole household, and the rush down the cellar stairs reminds one very forcibly of the second and fourth illustrations in Bachelor Butterfly, on page 14th, where he jumped overboard, and, in order to save the life of the Naturalist, was followed by his omnipresent Dorothy and the whole crew. His better half becoming alarmed for the safety of her beloved, seized the enraged bird by the head and pulled with all the force that conjugal fidelity or nervous excitement could arouse. The more she pulled the deeper were the talons of the Owl inserted, as was evidenced by the musical entertainment given by our friend, which comprised a scale of at least two octaves, with trills and variations that would have astonished an Italian prima donna! What more could be done? The adage that 'necessity is the mother of invention' proved true in this case, for the imminent danger, to say nothing of the excruciating pain, roused our hero to the importance of trying what

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virtue there was in his incisors. Being blest with a good set he with much difficulty succeeded in biting off all the claws of the Owl, and relieving himself from his perilous position, probably a wiser man for his first lesson in Ornithology."*

Surely Thomas Morton, Esq., could not have been familiar with the love calls of this species, or he would never have written: "There are Owles of divers kindes: but I did never heare any of them whop as ours doe."†

151. Nyctea Scandiaca (Linné) Newton. Snowy Owl.

This magnificent bird seems to be a pretty regular winter visitant along our coast. Linsley had secured five specimens from Stratford, Conn., and Dr. Whelpley had observed it at New Haven, prior to the year 1843.‡ Two specimens came to my notice during the winter of 1875–6 (the first on Nov. 10), and no less than a dozen specimens were killed in the immediate vicinity of New Haven in October (Oct. 17, Osborne), and November last (1876). They unquestionably belonged to the immense flock of these Owls that passed through eastern Massachusetts about the same time. Over *two hundred* specimens were shot about Boston in October and November. For a detailed account of this wonderful and really perplexing migration, consult an article by Ruthven Deane, Esq., in the Bulletin of the Nuttall Ornithological Club, No. 1, vol. ii, p. 9, for January, 1877. Dr. Wm. Wood, of East Windsor Hill, Conn., writes that they were unusually abundant here during the winter of 1858–9, when fifteen or twenty were shot in Hartford County. He further observes that "it hunts either by day, or in the twilight, occasionally pursuing game on the wing and securing it after the manner of the true falcons, but generally devouring it on the spot like the Marsh Hawk. When annoyed and teased by Crows it will now and then seek shelter in a hollow tree. Some years since I was pursuing one of these birds, in our meadows, that was followed and tormented by a great number of Crows. It finally alighted on an apple tree about a hundred rods distant, and in a few moments the whole tree was black with his tormenters, and still they came from all directions, attracted by the noise and confusion of those diving at the Owl. Soon they began to

* Hartford Times, chap. xx, August 17, 1861.

† New English Canaan, p. 49. (Reprinted in Force's Historical Tracts, Tract 5, vol. ii.)

‡ Am. Jour. Sci. and Arts, No. 2, vol. xliv, p. 253, 1843.

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leave, and before my arrival all was quiet. This was a mystery to me, for I had not seen the Owl fly, and if he had, the crows would certainly have given the alarm and followed him, yet no Owl was in sight. Having passed the tree some fifteen or twenty rods I heard a noise, and turning about saw the Owl coming out of a hollow apple-tree stump close by the tree. This was rather provoking, for I had been, just a moment before, near enough to the stump to have touched it with my gun."*

About the middle of March, 1863, "four large Arctic Owls were shot during one week, at Nahant, and on the beaches" about Boston.†

152. Surnia ulula, var. **Hudsonia** (Gmelin) Coues. Hawk Owl.

A rare winter visitor from the North. To Dr. F. W. Hall belongs the honor of adding this species to the birds of our State. In Nov. 1869, he saw the bird in an elm tree in an open field. Not having a gun with him he returned for it and secured a fine specimen of the Hawk Owl. It was mounted by the Rev. C. M. Jones, and is now in the cabinet of Dr. Hall. In the vicinity of Hudson's Bay, during the winter season, the white Ptarmigan (*Lagopus albus*) constitutes its chief article of diet; and it is said to follow the hunter, pouncing upon his game before he has time to reach it. As early as 1833 the Hawk Owl was recorded, by Ebenezer Emmons, M.D., as a rare "autumn" visitant in Massachusetts.‡ It has been shot as far South as Haddington, near Philadelphia.§

153. Nyctale Tengmalmi, var. **Richardsoni** (Bonaparte) Ridgway. Richardson Owl.

An extremely rare accidental winter visitor from the North. Dr. Wm. Wood has a fine specimen of this boreal Owl in his cabinet. It was captured some twenty years ago, in mid-winter, near East Windsor, Conn. The doctor gives the following account of the manner in which he came in possession of this specimen, in one of his interesting articles published in the "Hartford Times," Aug. 24th, 1861: "Visiting a patient some three miles from my office, I was

* Hartford Times, chap. xvi, July 6, 1861.

† History of Lynn, Mass., by Alonzo Lewis and James R. Newhall, p. 473, 1865.

‡ Report on the Geology, Mineralogy, Botany, and Zoology of Massachusetts. By Edward Hitchcock, Amherst. p. 546. 1833.

§ Birds of East Pennsylvania and New Jersey. By William P. Turnbull, M.D., p. 41. 1869.

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informed that word had been left for me to call at a house near by. On driving over and meeting the occupant at the gate, I inquired who was sick? After giving me an evasive answer I was invited into the house. Being well acquainted with the gentleman, and knowing that he had quite a taste for natural history, I began to suspect that it was not professional advice that was wanting. Leaving the room for a few minutes, he returned with an Owl. This bird, he informed me, sat upon the verandah early the previous morning, and approaching cautiously, he captured it without its making the least resistance or effort to escape, and put it into a cage. The next morning, from some unknown cause, the bird was found dead, and 'thinking it might be something a little different from any in my collection,' he saved it for me. No one but a naturalist can imagine the satisfaction it afforded me when I beheld Richardson's Owl; and I assure you that I would not only have gone three miles, but ten times three to obtain it." Several specimens have been taken in Massachusetts.*

154. Nyctale Acadica (Gmelin) Bonaparte. Acadian Owl; Saw-whet Owl.

Resident, though rather rare, or at least not often seen, for, owing to its diminutive size and nocturnal habits, it might be quite common and yet escape notice. Linsley never saw "but two individuals," one of which "was found lying upon his back in a barn-yard in a cold morning in March, 1841, though still living." A specimen was taken at East Haven, Conn., Nov. 26th, 1874, by Dr. F. W. Hall.

Mr. Coe and Mr. Sage have each fine examples of this species in their cabinets, and the former gentleman showed me a set of five eggs found in a hole in a tree near Portland, Conn. Dr. Wood, of East Windsor Hill, has several specimens taken in that vicinity, and I have lately seen quite a number, killed about New Haven, and throughout the State. It feeds on small birds, mice, grasshoppers, beetles, etc. "For rearing its young the Saw-whet takes possession of the old nest of a crow, or some other large bird, or of a hollow cavity of an old tree."†

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* J. A. Allen's Notes on some of the Rarer Birds of Mass., pp. 47–8. 1869.

† Zadock Thompson's History of Vermont, pp. 66–7. 1842.

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Family, FALCONIDÆ.

155. Circus cyaneus, var. **Hudsonius** (Linné) Schl. Marsh Hawk; Harrier; "Mouse Hawk."

A tolerably common summer resident, especially about salt marshes, where it breeds, placing its nest on the ground. Arrives in March or April (April 9, 1876, Osborne), remaining into November (Nov. 6).

I take the following interesting account of the audacity sometimes displayed by this bird, from one of Dr. Wood's articles in the Hartford Times: "If once introduced to your young poultry a faithful supervision will be exercised over them to see that none *are left* to suffer from want of attention or hunger. I know of an instance where the old bird returned daily, and about the same hour, until all were taken. Coming one day when the owner was there, the Hawk evidently intended to convince him that he was sole proprietor of that brood. The chickens having fled into the coop, he made himself quite at home by alighting on the shed near by and waiting for them to appear. The farmer caught up a stick and threw it at him, which so enraged the bird that it flew about his head, diving at him and squealing at a most furious rate, when he returned again to the shed, scolding vehemently. The farmer remarked that 'he was the sauciest little thing he ever saw.' In capturing mice partly concealed they sometimes 'wake up the wrong passenger' and get captured. An instance of this occurred in East Hartford. A Mouse Hawk was observed sailing over some tall grass, evidently intent on game. Poising himself for a moment by the slow, easy flappings of his wings, he suddenly dove, and soon appeared, ascending slowly and with difficulty. The very singular movements of the Hawk attracted the notice of several persons who were talking together, and they watched every motion with interest. He gradually rose higher and higher, till his strength began to fail, when he began to descend obliquely for a little way, then headlong to the ground. On arriving at the place they found that the Hawk had seized a weasel, that had killed its assailant by eating into the sides of the bird, where he was still at work!"

"Some years since, a student in the seminary came running up to me in a great hurry, almost out of breath, puffing like a locomotive, and exclaiming, 'Doc — doc—doctor,—can—you—shoot—flying?' I replied that sometimes I did such things. "Well, I wish you would go out East and shoot a Hawk that has chased me for half a

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mile, diving so close to my head that I expected every moment he would have hold of me.' Taking my gun, we walked towards the swamp, when he informed me that 'he came out there to practice elocution, so as not to disturb any one, and before he had finished the vowel sounds the Hawk came at him.' On arriving near the place I requested him to commence his vocal exercises again, so as to afford me an opportunity for practicing on the wing. He commenced, and, if that was a specimen of elocution, no wonder the Hawk drove him from the premises, for of all the earthly or unearthly sounds ever uttered, those exceeded anything I ever heard. If blindfold, one would have supposed that there was a concert of Great Horned Owls of all ages, echoing through the forest 'waugh ho! oo, ó, ah,

'Making night hideous,'

almost tempting one to think, if not to utter, the penult of the word named. The bird without doubt recognized the vowel sounds, for she soon came around, but either not liking the looks of me or my gun, kept at a respectful distance, flying about, uttering the peculiar cry of the Marsh Hawk. Neither the attempts of Audubon or Nuttall to interpret their song has elucidated it according to my ear. It appears to me to sound like the male falsetto descending quickly from high F to D, key of two sharps, with the syllables hey, ha, ha, in compound time, accenting the first note very forcibly. The next day we went again to the swamp, but taking the precaution to conceal myself in the bushes before he commenced his elocutionary exercises, the bird dove at him as before, when he sung out lustily, 'shoot! shoot! shoot!' Discovering me the hawk changed her tactics, much to the relief of my friend, and, poising herself for a moment, dove with so much force at my head as to make a distinct rustling sound. After allowing her to repeat this several times to see if she would really take hold of my cranium, my old gun went off, carrying death in its track, much to the joy of my companion, who now, for the first time, ventured to take a full breath since coming into the swamp."*

156. Nauclerus forficatus (Linné) Ridgway. Swallow-tailed Kite.

A rare straggler from the South, at present. Mr. Josiah G. Ely, of Lyme, New London County, Conn., writes me that while hunting on the 2d of July (1877), one of these magnificent birds flew over his

* Hartford Times, chap. xv. June 29, 1861.

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head, and that "there is no doubt as to its identity." Mr. W. W. Coe, of Portland, Conn., tells me that he saw a Swallow-tailed Kite in that vicinity, during the summer of 1861. He was crossing an open meadow when the Kite suddenly dove, close to where he was standing, and arose with a snake in its talons. This it devoured while in the air, much to the surprise of Mr. Coe, who was not then familiar with its habits. A specimen has also been observed in Massachusetts quite recently.* It has been shot on Long Island (Giraud). The cause of its restricted northern range, in the eastern province, at the present time, can scarcely depend on the temperature (as generally supposed), for it was once not only common in New England, but actually wintered here. Williams, writing in 1794, says that they were then found in Vermont, and "seem to be fitted by nature, to endure all the diversity of our climate, and are to be seen in the coldest weather of our winters."† The power of wing exhibited by this splendid species is truly wonderful. Its swift flight and abrupt turnings can only be compared to those of the Swallow. I have often seen them, in Florida, dart down and pick a wasp's nest from the under side of a palmetto leaf, devouring the grubs it contained while on the wing.‡ Dr. Coues writes thus of it: "The Swallow-tailed Kite is a marked feature of the scene in the Southern States, alike where the sunbeams are redolent of the orange and magnolia, and where the air reeks with the pestilent miasm of the moss-shrouded swamps that sleep in perpetual gloom. But, imbued with a spirit of adventure, possessed of unequaled powers of flight, it often wanders far from its southern home; it has more than once crossed the ocean, and become a trophy of no ordinary interest to the ardent collector in Europe." It extends northward in the Mississippi Valley, "where it regularly occurs above the mouth of the Missouri." "Marked among its kind by no ordinary beauty of form and brilliancy of color, the Kite courses through the air with a grace and buoyancy it would be vain to rival. By a stroke of the thin-bladed wings and a lashing of the cleft tail, its flight is swayed to this or that side in a moment, or instantly arrested. Now it swoops with incredible swiftness, seizes without a pause, and bears its struggling captive aloft, feeding from its talons as it flies; now it mounts in airy circles till it is a speck in the blue ether and disappears."§

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* Rarer Birds of Mass., J. A. Allen, p. 46, 1869.

† The Natural and Civil History of Vermont, by Samuel Williams, p. 112, July 16, 1794.

‡ Am. Nat., vol. viii, No. 2. p. 88, Feb., 1874.

§ Coues' Birds of the Northwest, p. 332, 1874.

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157. Accipiter fuscus (Gmelin) Bonaparte. Sharp-shinned Hawk.

A common summer resident, breeding on trees and on high rocky cliffs throughout the State. Mr. W. W. Coe writes me, that, of all our Hawks, they are the latest to breed, laying their eggs, generally five in number, in June. He has found them "in an old grey squirrel's nest fixed up for the occasion." Arrives in March, remaining into November. F. W. Putnam, in 1856, gave it as "resident," but "not abundant," in Essex County, Mass. I have no authentic record of its occurrence here in winter. Dr. Wood writes: "Its flight is quick, irregular, and so rapid, that, if your gun is not cocked it will pass out of range before you can get aim. It pounces upon its prey with such velocity that no time is allowed for escape. While riding one day, one passed within a few feet of me and dashed into a cluster of alders, interwoven with grapevines and briars, and seized a half-grown quail, passing out with it, without scarcely checking its speed. Although the hedge was within twenty feet of me, it was so thick that I could not discern the bird until it appeared on the opposite side in the talons of the Hawk. When hungry and in search of game, it knows no fear, often diving within a few feet of you and seizing a chicken. Sometimes you will see one flying along very swift and low, wheeling right and left, taking a 'bird's-eye view' of every hedge and bush, until it starts some little bird, whose fate is surely sealed."* Nuttall writes: "Descending furiously and blindly upon its quarry, a young Hawk of this species broke through the glass of the green-house, at the Cambridge Botanic Garden; and fearlessly passing through a second glass partition, he was only brought up by the third, and caught, though little stunned by the effort."†

158. Accipiter Cooperi (Bonaparte) Gray. Cooper's Hawk; Chicken Hawk.

A common summer resident, generally placing its nest in some tall pine or other high tree. "Their four eggs are usually deposited about the middle of May. They frequently build a new nest every year but often take possession of the old nest of a Red-tailed or Red-shouldered Hawk."‡ Arrives in March, remaining into November. Dr. Wood writes that he knew of six of their nests in the vicinity of East Windsor Hill, Conn., in a single season, and that he obtained

* Hartford Times, chap. x, May 25th, 1861.

† Nuttall's Manual of Ornithology, vol. i, p. 88. 1832.

‡ MS. notes of W. W. Coe.

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eggs from five of them. He says, "It is bold and fearless, often diving within a few rods of the farmer and seizing his chickens. If once introduced to the young poultry, you may rest assured of a daily call till all are gone, unless you are fortunate enough to secure the intruder. I once saw one of these Hawks seize a chicken on a very steep side-hill, close beside the old hen. In an instant the enraged mother flew upon the thief, and both came tumbling down the hill, clinched together; running up, I was just about to grasp the Hawk when they parted. It sometimes attacks full grown poultry with success. A gentleman once informed me that, 'while standing by his wood-pile, close by the house, one of these birds dove upon a full grown rooster, within six or eight rods of him. The fowl ran some two or three rods and dropped dead. The Hawk soon returned to devour his game, as it was too heavy for him to carry away, but his audacity cost him his life.'"*

In the old colonial days, when every man spelled as best suited his own fancy, the different kinds of Hawks did not pass unnoticed, for in 1632 Morton wrote: "There are Hawkes in New England of 5. sorts, and these of all other fether fowles I must not omitt, to speake of, nor neede I to make any Apology for my selfe, concerning any trespass, that I am like to make upon my judgment, concerning the nature of them, having bin bred in so genious a way, that I had the common use of them in England: and at my first arrivall in those parts practiced to take a Lannaret, which I reclaimed, trained, and made flying in a fortnight, the same being a passinger at Michuelmas. I found that these are most excellent Mettell, rank winged, well conditioned, and not tickleish footed, and having whoods, bels, luers, and all things fitting, was desirous to make experiment of that kinde of Hawke, before any other. And I am perswaded: that nature hath ordained them to be of a farre better kinde, then any that have bin used in England."† It seems to me that there is little doubt but that the above remarks refer to Cooper's Hawk.

159. Astur atricapillus (Wilson) Jardine. Goshawk.

Rather rare, and somewhat irregular, winter visitant. Said to breed, occasionally, as far south as Massachusetts (Maynard and Minot).

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* Hartford Times, chap. ix, May 18th, 1861.

† New English Canaan. By Thomas Morton, p. 49. 1632. [Reprinted in Force's Hist. Tracts, vol. ii, T. 5.]

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Professor Wm. D. Whitney has a finely mounted specimen, in his cabinet, which was shot, some years ago, at Hamden Plains, near New Haven, Conn. W. W. Coe, and Jno. H. Sage, of Portland, Conn., have each several fine specimens taken in the State—one near Hartford, Nov. 9th, 1867. Mr. Geo. Bird Grinnell secured one, late in the fall (Nov. or Dec.) of 1876, at Milford, Conn. For several days previous to its capture it had been in the habit of lunching on a neighbor's chickens. Mr. Erwin I. Shores writes me that, in the vicinity of Suffield, Conn., he has seen it "four or more times during the last two winters." I am informed by Dr. Wm. Wood, of East Windsor Hill, Conn., that they are really common in that vicinity about once in ten years (he once had seven specimens, in the flesh, on hand at one time) but are seldom seen between times. The doctor relates the following anecdote as illustrating well the boldness and daring often displayed by this species: An old man, over eighty years of age, was sitting quietly in the kitchen with his maiden daughter. The door was open and their quiet was suddenly broken by a hen who rushed frantically into the room, followed closely by a large Goshawk. There, right on the kitchen floor, and in the presence of the two, the bold Hawk seized the hen. The feeble old man came to the rescue, and, with stick in hand, finally succeeded in beating off the intruder, who now made for the door. But it was too late—the daughter had closed the door and actually caught the furious bird in her hands and put him to death!

Zadock Thompson says: "Its disposition is very savage, and it is withal so much of a cannibal as sometimes to devour its own young!"* Dr. Wm. Wood, of East Windsor Hill, writes that a specimen which he once kept alive in a small room "refused food until the thirteenth day, when it devoured an entire hen. . . . . It died the next night, a victim to its voraciousness."† "The poet Chaucer in alluding to it says,—

> 'Riding on hawking by the river,
> With gray Goshawk in hand.'

Falconry and hawking, as defined by our lexicographers, are synonymous, but formerly birds of sport were divided into two classes, those of falconry, and those of hawking. This bird came under the latter class." "The Goshawk does not usually soar high, like the longer winged Hawks, nor dart upon its prey by a direct descent, as do the

* History of Vermont, p. 62. 1842.

† Hartford Times, chap. ix, May 18th, 1861.

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true Falcons, but by a side glance. It is restless, seldom alighting but for a moment, except to devour its quarry, and then it stands almost erect. Its flight is so rapid that it can easily overtake the swift Pigeon on the wing. . . . . When looking for prey it skims along near the surface of the ground with great velocity, and catches its game so quickly and easily as scarcely to be seen by the looker-on." "The Goshawk is the most daring and venturesome of any of our diurnal birds of prey. A farmer who resides a few miles from my office, wishing to perpetuate the old New England custom of having a chicken-pie for Thanksgiving dinner, caught some fowls, took them to a log, severed the neck of one, and threw it down beside him. In an instant a Goshawk seized the struggling fowl, and, flying off some ten rods, alighted and commenced devouring its prey. The boldness of the attack so astonished the farmer that he looked on with blank amazement. Recovering from his surprise, he hastened into the house and brought out his gun, which secured him both the Hawk and the fowl."*

Pennant, in 1785, tells us that Goshawks "are used by the Emperor of *China* in his sporting progresses, attended by his grand falconer, and a thousand of the subordinate. . . . . The Emperor often carries a Hawk on his hand, to let fly at any game which presents itself; which are usually Pheasants, Partridges, Quails, or Cranes. *Marco Polo* saw this diversion about the year 1269."†

160. Falco communis Gmelin. Peregrine Falcon; Duck Hawk.

Resident, but rare; breeds on Talcott Mountain, near Hartford, Conn. Linsley tells us that, previous to the year 1842, a specimen of this celebrated and powerful bird was wounded by a gun-shot in Stratford, [Conn.], and after he was taken soon recovered, and was kept on poultry until he became too expensive to the owner,"‡ who then released him. Mr. Geo. Bird Grinnell informs me that he saw, on the 23d of February, 1876, a Duck Hawk (or Great-footed Hawk, as it is often called), flying about the Sound, near Milford, Conn.

On the 29th of June, 1877, while enjoying the hospitality of Capt. O. N. Brooks, at Faulkner's Island, Conn., in company with my friend, Mr. Jno. H. Sage, the Terns breeding on Goose Island (one mile

* Dr. Wood, in Am. Nat., vol. x, No. 3, pp. 132–4. March, 1876.

† Arctic Zoology. By Thomas Pennant, vol. ii, p. 204. 1785.

‡ Linsley's Catal. Birds Conn., p. 250, 1843.

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distant) were observed to be in an unusual state of commotion. The Captain remarked that they had been agitated, the day previous, by a Duck Hawk which might still be prowling in the vicinity, and he kindly offered to take us over there. We landed, and, on rounding the island, the Falcon suddenly started from her retreat among the rocks and weeds, which, as we afterwards discovered, was strewn with feathers. But the captain's gun was in readiness, and, through his kindness, the bird now graces my collection. During her brief visit she had made sad havoc among the Terns, and her crop was greatly distended with their remains, which had been swallowed in incredibly large pieces—whole legs, and the long bones of the wings were found entire and unbroken! Indeed she was perfectly gorged, and contained the remains of at least two adult Terns, besides a mass of newly hatched young! The only other great-footed Hawk that Capt. Brooks has ever taken, he killed, at this same place, twelve years ago. Query: did this bird come from Talcott Mountain, or did it breed far beyond our limits? It is said that they are able to fly many hours without resting, and at the enormous rate of an hundred miles per hour!* They sometimes live nearly 200 years!

In the Hartford Times of June 29th, 1861, Dr. Wm. Wood, of East Windsor Hill, Conn., published the first account of the first Duck Hawk's nest found in New England. It was discovered by Dr. Moses, on Talcott Mountain, near Hartford, Conn., May 25th, 1861, and contained four young almost large enough to fly. The finding of this nest was, at the time, of particular interest, since, as Dr. Wood remarked, "it settled beyond dispute three points: 1st, that they breed on cliffs; 2d, that they breed in Connecticut; and 3d, that they nest very early" (sometime in March).†

They have since been known to breed regularly on Mount Tom, Mass., where their eggs were first taken, April 19, 1864, by Mr. C. W. Bennett,‡ of Holyoke. Their nest was again robbed, by Mr. Bennett, in 1866 and in 1869. Mr. J. A. Allen, in his "Notes on some of the Rarer Birds of Massachusetts," writes: "During a visit to this Mountain [Mt. Tom], in company with Mr. Bennett (Apr. 28, 1869), we had the great pleasure of discovering their second eyrie, from which, with considerable difficulty, three freshly laid eggs were obtained. Not discouraged by this second misfortune, they nested

* See an interesting Article in Am. Nat., vol. v, No. 2, p. 82, April, 1871.

† Hartford Times, June 29, 1861.

‡ See Proceed. Essex Inst., vol. iv, p. 153.

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again, this time depositing their eggs in the old eyrie from which all except the last set of eggs have been obtained. Again they were unfortunate, Mr. Bennett removing their second set of eggs, three in number, May 23d, at which time incubation had just commenced. The birds remained about the mountain all the summer, and from the anxiety they manifested in August it appears not improbable that they had laid a third time, and at this late period had unfledged young."* Mr. Harold Herrick states that it is common on the Island of Grand Menan, N. B., where it "breeds on the cliffs, but in such inaccessible situations that its nest is rarely taken. There is a place between 'Fish Head' and the 'Old Bishop' known as the 'Seven Day's Work,' where the cliff is divided into seven strata as sharply defined as lines of masonry. On an indentation in the face of this cliff, about one hundred feet from the top, and one hundred and fifty feet from the bottom, a pair of these Falcons have had their eyrie for a succession of years, secure alike from the assaults of the most zealous naturalist, and the small boy of bird's-egging proclivities."†

Mr. W. W. Coe, of Portland, Conn., tells me that while duck shooting a few years since, as the birds rose at the report of his gun, a Duck Hawk dove, struck a Teale, on the wing, and carried it off! Dr. Wood writes: "In the vicinity of their breeding places they are a terror to the poultry as well as a dread to the farmer, for there they usually hunt in pairs, one following directly after the other, and if the first one misses the game, the other is sure to pick it up; there is no escaping the two. This is the universal testimony of all the farmers living in the vicinity of the cliffs where they breed. One of my collectors went over one hundred miles to get a nest of their eggs, from only hearing a farmer in the vicinity of the cliff describe their manner of hunting; knowing from this circumstance alone that it must be the Duck Hawk."‡

161. Falco columbarius Linné. Pigeon Hawk.

Resident, but rare in summer, and not often seen in winter. It is not uncommon here in spring and fall. Dr. Wm. Wood tells me that he has found it about East Windsor Hill, Conn., in May, June, and July, but failed to discover the nest. He is not, of course, perfectly

* Notes on some of the Rarer Birds of Mass., p. 10–11, 1869.

† Herrick's Partial Catalogue of the Birds of Grand Menan, p. 10, 1873.

‡ Am. Nat., vol. v, No. 2, p. 82, April, 1871.

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certain that they breed there at all, though their occurrence in early summer certainly looks like it. I have seen it, in the vicinity of New Haven, in April, and again in August and September, and now have notes of its presence in this State in every month of the year. In one of his interesting articles on the "Game Falcons of New England," Dr. Wood writes: "In May, 1860, a gentleman who resides some five miles distant, informed me that a small Hawk came almost every day and carried off a chicken for him—that it never missed, for it went so like lightning that there was no escaping its grasp. He said that it always came in the same direction from a tract of woods near his house ..... Accompanied by my friend, we carefully searched the woods without finding anything except the nest of the Red-shouldered Hawk. The next day the same little Hawk returned and was shot, and is now in my collection, a beautiful representative of the Pigeon Hawk. I have no doubt that it had a nest about there, as it was the season for nesting, and it always came from, and went to, the same piece of woods, and in the same direction. If it had not young it must have been carrying food to its mate while incubating. If a mere straggler it would come and go without any definite place of resort. Our inability to find the nest was not strange, as there were some sixty or eighty acres of heavy-timbered oaks and pines in the tract."* Mr. Geo. A. Boardman states that it "breeds in hollow trees,"† which would, of course, render its nest still more difficult to find. I am strongly inclined to believe that a few pairs do occasionally breed in Connecticut, for, though its nest has not yet been actually discovered, the fact that the birds are sometimes found here throughout the entire year is strong evidence that they breed, and mere negative evidence, in such cases, amounts to little or nothing.

162. Falco sparverius Linné. Sparrow Hawk.

A rather rare resident; only a few pairs breed within the State, and it is seldom seen in winter. Mr. W. W. Coe tells me that it breeds near Portland, Conn., and I am informed by Dr. Wood that it sometimes breeds in the vicinity of East Windsor Hill.

Dr. Wood thus describes a novel site for a nest of this Hawk which was found at Granby, Conn.: "A farmer made a dove house inside of his barn with holes through the sides of the building com-

* Am. Nat., vol. vii, No. 6, pp. 342–3, June, 1873.

† Proceed. Bost. Soc. Nat. Hist., vol. ix, p. 122, Sept., 1862.

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municating with it. A pair of doves that had nested there were attacked and killed by a pair of Sparrow Hawks who took possession of their nest, laid four eggs, and commenced to sit. During incubation they found the farmer's chickens very convenient for food —too much so for their own good. I saw both birds after they were killed; also their four eggs, two of which are now in my cabinet."* In Elizabeth, New Jersey, several years ago, I saw a pair of Sparrow Hawks fly up under the eaves of an old barn, and drag a couple of Swallows out of their nests! Mr. Sage has seen it in January (Jan. 6, 1877), near Portland, Conn., and Dr. Wood writes me that he has occasionally taken it in winter.

"Sparhawkes there are also," wrote Morton in 1632, "the fairest, and best, shaped birds that I have ever beheld of that kinde, those that are litle, no use is made of any of them, neither are they regarded, I onely tried conclusions with a Lannaret at first comming; and when I found, what was in that bird, I turned him going: but for so much as I have observed of those birds, they may be a fitt present for a prince; and for goodnesse too be preferred before the Barbary, or any other used in Christendome, and especially the Lannars and Lannarets."† The above may have referred to the Sharp-shinned Hawk.

163. Buteo borealis (Gmelin) Vieillot. **Red-tailed Hawk; Hen Hawk.**

A common resident, but more numerous in early spring and during the fall migrations than at other times of the year. The migrants arrive early in March, and in fall, may be seen, in considerable numbers, in September, October, and November. On the 25th of September, 1875, I saw, near New Haven, a flock of twenty-six Red-tailed Hawks, soaring high, and sailing slowly southward. The day was clear and cool, and there was little wind.

Mr. W. W. Coe, of Portland, Conn., has been remarkably fortunate in finding the nests of this Hawk, having taken, during the past five years, nearly forty of its eggs! The best time to go for them seems to be about the middle of April, but he has found nests, containing eggs, at all dates from March 30th (1875) to May 23d (1873), though the young generally hatch during the latter part of April or early in May. The Stadtmüller brothers found a nest, near New Haven, April 13, 1877, containing three eggs which "were just beginning to

* Am. Nat., vol. viii, No. 5, p. 268, May, 1874.

† New English Canaan, p. 50. [Reprinted from Force's Hist. Tracts, vol. ii, T. 5.]

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hatch."* The Red-tailed Hawk generally lays two, frequently three, and sometimes four, eggs. In writing of them, from Portland, Conn., Mr. Coe says: "Their nests are easily identified, being the largest nests built in this locality, and the earliest. They lay in the same nest year after year if not disturbed, and often if they are. I once found a nest in a young chestnut, not over twenty feet from the ground, but they commonly build high up in large trees."†

The Rev. Mr. Peabody, in his Report on the Birds of Massachusetts (in 1839), remarked that these Hawks, "like the Crow, seem to have an intuitive perception of the use and reach of the gun, for if they see a person armed, they give a scream of disgust, and sail away far beyond his reach."‡

They are often called Hen Hawks from their frequent visits to the poultry yard. I have also found, in their alimentary canals, the remains of mice, snakes, and frogs.

164. Buteo lineatus (Gmelin) Jardine. Red-shouldered Hawk.

A common resident, like the last, but more frequently seen in winter. Also breeds in high trees. Mr. Coe writes me that it generally lays a little later than the Red-tail, makes a smaller nest, and is more likely to build a new one every year. He has taken their eggs from April 16th (only one egg, just deposited) till May 19th. They generally lay three or four eggs, and sometimes five. Dr. Wood once found six! I once took from the stomach of one of these Hawks a snake (*Eutainia saurita*) measuring twenty-two inches in length!

A young bird, which Dr. Wood raised, became very much attached to him. He says: "by giving a whistle he would answer with his *ka-hee*, *ka-hee*, *ka-hee*, and fly from the tree and alight on my shoulder, expecting his accustomed morsel. I did not confine him at all, but allowed him to fly where he pleased, knowing that he would not go far off. The domestic fowls became so well acquainted with him that they showed no signs of fear when he alighted on a tree or post near them. One morning he did not come at my call, nor at noon. In the afternoon I went in search of him and whistled till my lips would not pucker any longer, yet no bird answered to the music. The next morning he was brought to me as a rare specimen with the following

* MS. notes of the Stadtmüller Brothers.

† MS. notes of W. W. Coe.

‡ A Report on the Ornithology of Massachusetts, by William B. O. Peabody, p. 268, 1839.

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sequel: 'He came and alighted on my fence within a few feet of me when I was feeding my fowls. They flew in every direction. Carefully stepping up I caught the pretty creature and thought I would save him for the doctor. Putting him into a box, I gave him some *corn; but he did not seem hungry, and would not eat a kernel.*' "* Again, under the name of "Winter Hawk (*Buteo hyemalis*)," the doctor continues: "Sitting patiently upon a tree near some spring or marshy ground, it will watch by the hour for a frog to make its appearance, when it is immediately seized and drowned. There is a side-hill, some few miles from my office, from which springs constantly run in the coldest weather, forming quite a wet, marshy place, offering great inducements to the Winter Hawk. Here you may see one or more of these birds every winter, perched upon a tree near by watching for its favorite food. I received two specimens shot from that tree in one day."†

165. Buteo Pennsylvanicus (Wilson) Bonaparte. Broad-winged Hawk.

A rather rare resident, seldom seen in winter. It breeds sparingly about New Haven, and Mr. W. W. Coe has taken quite a number of their nests, together with several of the finest specimens of the bird that I have ever seen, in the vicinity of Portland, Conn. He informs me that they generally lay later than the Red-shouldered Hawks, and, like them, often build a new nest every year.

Mr. F. W. Putnam (in 1856) gave it as a rare winter visitant so far north as Essex Co., Mass.‡

166. Archibuteo lagopus, var. **Sancti-Johannis** (Gmelin) Ridgway. Rough-legged Hawk; Black Hawk.

A winter visitor; not common. I saw one near New Haven, Nov. 20th, 1875. Mr. Geo. Bird Grinnell tells me that he has seen it, in spring, near North Haven, Conn. It is sometimes quite abundant on the low meadows bordering the Connecticut River, where, in the vicinity of East Windsor Hill, Conn., Dr. William Wood has secured a large number of specimens. The splendid series thus obtained, enabled him, many years ago, to prove the identity of the two forms, *lagopus* and *Sancti-Johannis*, then considered, by our

* Hartford Times, chap. xii, June 8th, 1861.

† Hartford Times, chap. xiii, June 15th, 1861.

‡ Proceed. Essex Inst., vol. i, p. 203. 1856.

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best Ornithologists, to be specifically distinct. As long ago as 1861 Dr. Wood published the following: "The difference in size is no more than frequently occurs in birds of the same species. The shape and general form, the small claws, the same habits in every respect, their arrival at the same time, associating and sailing together, the plumage of the one running into the other as it changes, so that it is difficult, if not impossible, to tell where the dividing line comes, some being jet-black, others not quite as dark, others slightly mixed, some more so, certainly make a strong case in favor of their identity."* Three years later the doctor wrote J. A. Allen that he had then taken and examined about forty specimens, and could now state positively that: "The Rough-legged Falcon and Black Hawk *are the same.*"† Both Baird and Coues, in their late works on our birds, state that the examination of a large number of specimens leaves little doubt as to the identity of the two forms under consideration, each using such language as to indicate an original discovery dependant on his own investigations, and neither alludes to the published records of Dr. Wood, who, long before, arrived at the same conclusion, and from a larger amount of material, and of better quality, than is to be found in any other collection in the world.

167. Pandion haliaëtus (Linné) Cuvier. Fish Hawk; Osprey.

A summer resident, breeding abundantly along the coast, both on the main land and on islands in the Sound. It is particularly abundant during the spring migrations. It arrives late in March (March 28, 1877), remaining through October (Oct. 23). The migrants pass northward during the latter part of April, and return again in September. Mr. Fred. Sumner Smith, of this city, tells me that a friend of his found a Fish Hawk's nest in Heron Swamp (near New Haven) as late as July 4th, (1870). It was a small one, being little larger than a bushel basket, was placed in a clump of thick bushes, and contained three eggs. Mr. W. W. Coe informs me that they do not breed so far up the Connecticut River as Middletown and Portland, but are common at its mouth (about Saybrook), and that he has taken their nests, along the Sound, all the way from Saybrook to New London, Conn. "Immense numbers of them breed regularly at Plumb Island, Conn., where I saw, last spring, at least five hundred nests, and over a thousand birds. There is only one small piece of

* Hartford Times, chap. xiv, June 22d, 1861.

† Allen's notes on some of the Rarer Birds of Mass., p. 14. 1869.

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timber on the island, and every tree contains a Fish Hawk's nest, or from eight to ten Night Herons' nests. There is quite a colony of Night Herons there. There not being trees enough for the Hawks to nest in, many of them build on the ground and some lay their eggs in the sand. They occupy the same nest for years, adding a little to it each season, till some of them, that were originally placed flat on the ground, had become so large that I could not look into them. Many were seven feet high and measured six or eight feet across the top! On the 4th of June I found both young birds and fresh eggs in some of the nests. The Crow Blackbirds had built their nests in among the large sticks on the sides of the Fish Hawk's nests, there being often four or five of the former placed about the sides of one of the latter. Besides the Fish Hawks, Night Herons, and Crow Blackbirds, many other birds breed upon this island, among which might be mentioned the Upland and Killdeer Plover, and large numbers of Terns."*

The Rev. Wm. B. O. Peabody, in 1839, thus wrote of their habits: "The Fish Hawk is on excellent terms with the fishermen, though they are of the same trade. Its coming announces the arrival of the shoals of fish that crowd our rivers in the spring. Perhaps its exemption from persecution may be owing also to its well known gentleness of disposition. Unlike other birds of prey, the Fish Hawks are social and friendly to each other. They come to us in flocks of eight or ten, who build near each other, and rear their young in perfect harmony, and this spirit of hospitality and kindness is extended to other birds that seem to have no claim upon them. The Crow Blackbirds are permitted to shelter in the interstices of their nests, which are huge constructions, made of a cartload of heavy materials firmly matted together."†

168. Aquila chrysaëtus Linné. Golden Eagle.

A rare winter visitant. Dr. Wood informs me that it is sometimes seen about Hartford, Conn. Mr. J. N. Clark, of Saybrook, Conn., writes me that he sees one or two there every year, and that one remained in that vicinity a week, about the middle of May last (1877). One was secured at Deep River, Conn., Nov. 13th, 1875, by Mr. Harry Flint. Zadock Thompson stated that they sometimes live in Vermont, and says that "the nest is placed upon the inaccessible shelf of some

* MS. notes of W. W. Coe.

† Peabody's Report on the Ornithology of Mass., p. 265. 1839.

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rugged precipice, and consists of a few sticks and weeds barely sufficient to keep the eggs from rolling down the rocks. . . . . These eagles feed upon young fawns, hares, raccoons, wild turkeys, partridges, and other quadrupeds and birds, but will feed on putrid flesh only when severely pressed by hunger."*

169. Haliaëtus leucocephalus (Linné) Savigny. **Bald Eagle.**

A resident; not uncommon during the migrations. Saw one flying over the city, Nov. 20th, 1875. Also observed five individuals during Feb. and March, 1876. They unquestionably breed about four miles above the mouth of the Housatonic River, Conn., as I am told by my friend, Mr. Geo. Bird Grinnell. One was shot near the mouth of the East Haven River in Nov., 1876, and is now in the collection of Mr. Thomas Osborne of this city. Used to breed in suitable localities throughout the State, and there were formerly two eyries within a few miles of New Haven, one at Mt. Carmel, and the other on Saltonstall Ridge.†

Linsley kept an immature specimen (called by him "Washington's Eagle") alive for some time, concerning which he writes: "I kept him awhile confined, but soon found it unnecessary, because if he left my premises he would return to the stand at night. I have known him to eat fourteen birds (mostly King-birds), and then he was satisfied for a week. He appeared to prefer this mode of living, and paid no attention to a daily supply. He, however, in the course of the summer, became so mischievous among the young ducks of my neighbors, that I was compelled to kill him. A single anecdote of his conduct may not be uninteresting: While he had possession of my front yard, occupying the centre as his stand (the walks making a semicircle to the door), he would remain perfectly quiet if *gentlemen* or *ladies* entered; but if a person with tattered garments, or such persons as were not accustomed to come in at the front door, entered the yard, it was actually dangerous for them, and they could only escape the tremendous grasp of his talons by running with their full strength and shutting the gate after them. Facts of this kind often occurred, and I was occasionally compelled to release from his grasp such individuals as he had taken captive. With one claw in the sward and grass, he would hold quietly any man with the other."

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* History of Vermont, by Zadock Thompson, p. 59, 1842.

† Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 251, April, 1843.

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In 1634, William Wood wrote: "The Eagles of the Countrey be of two sorts, one like the Eagles that be in *England*, the other is something bigger, with a great white head and white tayle: these bee commonly called Gripes; these prey upon Duckes and Geese, and such Fish as are cast upon the Sea-shore. And although an Eagle be counted King of that feathered regiment, yet is there a certaine blacke Hawke that beates him; so that he is constrayned to soare so high, till heate expell his adversary."*

Family, CATHARTIDÆ.

170. Cathartes aura (Linné) Illiger. Turkey Buzzard.

A rare visitor from the South, at present, although once "not uncommon"† according to Linsley, who further states: "I have known it in Connecticut from a child, having at that period counted twenty in a flock in Northford in the month of August." "At the South, where they abound, it is seldom one attacks domestic poultry; but many years since I saw in Northford, in this State, a splendid male Turkey Buzzard pounce down upon a chicken about three-quarters grown, and within about three rods of where I was standing with two other persons. As he turned his eye upon us, still standing upon the chicken, he appeared so much alarmed as to be unable to rise; we all ran upon him, and when within a few feet of him he rose, just clearing our heads, and dropping the chicken at our feet, he hurried off."‡ This fact is particularly interesting, since they are commonly believed to feed exclusively on carrion. Nuttall heard that they were "accused, at times, of attacking young pigs and lambs, beginning their assault by picking out the eyes." But that he did not believe it is evident, for he goes on to say: "Mr. Waterton, however, while at Demerara, watched them for hours together amidst reptiles of all descriptions, but they never made any attack upon them. He even killed lizards and frogs and put them in their way, but they did not appear to notice them till they had attained the putrid scent. So that a more harmless animal, living at all upon flesh, is not in existence, than the Turkey Vulture."§ Nevertheless, since our own mod-

* New England's Prospect, p. 30, 1634.

† Regarding its former abundance, Mr. J. N. Clark writes me that an old hunter told him "that they used to be very common" about the mouth of the Connecticut, where "he had shot a good many, but not recently."

‡ Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 250, April, 1843.

§ Nuttall's Manual of Ornithology, p. 45, 1832.

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ern classification of the Animal Kingdom is not sufficiently plastic to admit of grouping together chickens, pigs, and lambs, under the head of "Reptiles," along with "lizards and frogs," and the like, and since the word of so careful and conscientious an observer as Mr. Linsley is unimpeachable, we are forced to admit that these "harmless animals" do occasionally visit the farm-yard with "malice aforethought," and that a young fowl, safely lodged in the otherwise empty stomach, may not prove an altogether distasteful article, exerting, perchance, as soothing an effect over the sluggish intellect of one of these indolent scavengers as the most delicious morsel of putrescent carrion. Indeed, Audubon says of it: "they often watch the young kid, the lamb, and the pig, issuing from the mother's womb, and attack it with direful success." "Any flesh that they can at once tear with their very powerful bill in pieces, is swallowed, no matter how fresh; . . . . but it frequently happens that these birds are forced to wait until the hide of the prey gives way to the bill."* Mr. Grinnell tells me that one was shot at the mouth of the Housatonic River, Conn., in June, 1875, by C. Merwin, of Milford Point. Turkey Buzzards have been observed at Saybrook, Conn., by Mr. J. N. Clark) recorded by Purdie;† two were taken in Massachusetts,‡ and one even strayed as far to the north as Calais, Maine, where it was captured by Mr. G. A. Boardman (recorded by Prof. A. E. Verrill).§ Dr. Wood tells me that one was seen, feeding on carrion, near East Windsor, Conn., only three years ago (1874). The Rev. J. Howard Hand writes me as follows, concerning the occurrence of Turkey Buzzards in Connecticut: "I took one specimen at Cromwell, Conn., Sept. 23d, 1874; also one at Westbrook, Conn., Oct. 16th, 1875, and again eight specimens on Oct. 18th (two days afterwards). They are not common." Dr. Wm. O. Ayres writes me that he took one at New Haven in 1853.

Along our eastern coast it does not breed farther north than Southern New Jersey; but in the West its range is much more extensive, its northern limit being "about 53° in the region of the Saskatchewan, where it arrives in June,"‖ and was obtained by Sir John Richardson. Dr. Coues saw it at Fort Randall, Dakota, lat. 43° 11′, and I have

* Appendix to Wilson's American Ornithology, vol. iv, pp. 254 and 258, 1831.

† Am. Nat., vol. vii, No. 11, p. 693, Nov., 1873.

‡ Samuel's Descriptive Catalogue of the Birds of Massachusetts, p. 3, 1864. [From Agr. Mass., App., p. xviii, 1863.]

§ Proc. Bost. Soc. Nat Hist., vol. ix, p. 122, Sept., 1862.

‖ Coues' Birds of the Northwest, p. 380, 1874.

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myself observed it in Idaho, Wyoming, and Utah. It was once common throughout New England, as attested by numerous old writers. Josselyn must have been blessed with a keen appetite and an admirable digestion, for he says: "The turkie-buzzard, a kind of kite, but as big as a turkie; brown of color, and very good meat."*

NOTE.—The Black Vulture, *Cathartes atratus* (Ray) Lesson, may sometimes occur as a rare straggler from the South, and the Rev. J. Howard Hand writes me that he thinks he has killed three specimens of it at Westbrook, Conn. (Aug. 10, Sept. 12 and 21, 1874), but they may have been young Turkey Buzzards. Unfortunately the specimens were not preserved. Several individuals have been recorded from Massachusetts,† and it has even straggled as far north as Maine (Calais, G. A. Boardman)‡ and Nova Scotia.

Family, COLUMBIDÆ.

171. Ectopistes migratoria (Linné) Swainson. Wild Pigeon.

Sometimes quite abundant during the migrations. A few breed (late in May). Arrives about the first of April (Apr. 2, 1875, Sage). Mr. Coe tells me that numbers of them bred about Portland, Conn., in 1875, and that a few generally nest there.

Concerning the enormous flocks of Wild Pigeons which passed to and fro over the country in former years (and which, on a smaller scale, are still to be met with in some parts of the West), Gov. Thomas Dudley wrote, as early as 1631: "Vpon the 8 of March, from after it was faire day light, untill about 8 of the clock in the forenoone, there flew over all the tounes in our plantacons soe many flocks of doues, each flock conteyning many thousands and some soe many that they obscured the lighte, that it passeth credit, if but the truth should bee written, and the thing was the more strange, because I scarce remember to have seene tenne doues since I came into the country. They were all turtles as appeared by diverse of them wee killed flying, somewhat bigger than those of Europe, and they flew from the north east to the south west; but what it portends I know not."§ And in the following year (1632), Thomas Morton, of Clif-

* New England's Rarities Discovered. By John Josselyn, p. 11, 1672.

† Coues' List of the Birds of New England, p. 6, 1868; J. A. Allen's Rarer Birds of Mass., p. 47, 1869; etc.

‡ Am. Nat., vol. iii, p. 498, Nov., 1869.

§ Reprinted in Force's Historical Tracts, Tract 4, p. 17–18.

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ford's Inn, speaking of his impressions of the country, when first he landed in New England, said: "The more I looked the more I liked it. And when I had more seriously considered of the bewty of the place, with all her faire indowments, I did not thinke that in all the knowne world it could be paralel'd." "Contained within the volume of the Land, Fowles in abundance, Fish in multitude, and discovered besides; Millions of Turtledoves one the greene boughes: which sate pecking, of the full ripe pleasant grapes, that were supported by the lusty trees, whose fruitfull loade did cause the armes to bend, which here and there dispersed (you might see) Lillies and of the Daphnean-tree, which made the Lande to mee seeme paradice, for in mine eie, t'was Natures Master-peece."*

In looking over a curious old pamphlet, printed in 1630, and entitled "New-England's Plantation. Or, a Short and trve description of the Commodities and discommodities of that countrey. Written by a reuerend Diuine now there resident.", I find the following notice of the Wild Pigeon: "In the Winter time I haue seene Flockes of Pidgeons, and haue eaten of them: they doe flye from Tree to Tree as other Birds doe, which our Pidgeons will not doe in *England:* they are of all colours as ours are, but their wings and tayles are farr longer, and therefore it is likely they fly swifter to escape the terrible Hawkes in this Countrey."† Samuel Williams, in his Natural and Civil History of Vermont (published in 1794), writes as follows: "In the Wild Pigeon, the multiplying power of Nature acts with great force and vigour. The male and female always pair: They sit alternately upon the eggs, and generally hatch but two at a time; but this is repeated several times in a season.—The accounts which are given of the number of pigeons in the uncultivated parts of the country, will appear almost incredible to those who have never seen their nests. The surveyor, *Richard Hazen*, who ran the line which divides Massachusetts from Vermont, in 1741, gave this account of the appearances, which he met with to the westward of Connecticut river. 'For three miles together the pigeon's nests were so thick, that five hundred might have been told on the beech trees at one time; and could they have been counted on the hemlocks, as well, I doubt not but five thousand at one turn round.' The remarks of the first settlers of Vermont, fully confirm this account. . . . . The settlement of the country has since set bounds

* Reprinted in Force's Historical Tracts, Tract 5, p. 42.

† Reprinted in Peter Force's Historical Tracts, vol. i, Tract 12, p. 11.

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to this luxuriancy of animal life; diminished the number of these birds, and drove them further to the northward."*

Two centuries after Morton's description was written, Nuttall remarks: "To talk of hundreds of millions of individuals of the same species habitually associated in feeding, roosting, and breeding, without any regard to climate or season as an operating cause in these gregarious movements, would at first appear to be wholly incredible. ..... The approach of the mighty feathered army with a loud rushing roar, and a stirring breeze, attended by a sudden darkness, might be mistaken for a fearful tornado about to overwhelm the face of nature. For several hours together the vast host, extending some miles in breadth, still continues to pass in flocks without diminution. The whole air is filled with them; their muting resembles a shower of sleet, and they shut out the light as if it were an eclipse. At the approach of the Hawk, their sublime and beautiful aerial evolutions are disturbed like the ruffling squall extending over the placid ocean; as a thundering torrent they rush together in a concentrating mass, and heaving in undulating and glittering sweeps towards the earth, at length again proceed in lofty meanders like the rushing of a mighty animated river." "Alighting, they industriously search through the withered leaves for their favorite mast [chiefly beech nuts and acorns]; those behind are continually rising and passing forward in front in such quick succession, that the whole flock, still circling over the ground, seems yet on the wing. As the sun begins to decline they depart in a body for the *general roost*, which is often hundreds of miles distant, and is generally chosen in the tallest and thickest forests almost divested of underwood. Nothing can exceed the waste and desolation of these nocturnal resorts; the vegetation becomes buried by their excrements to the depth of several inches. The tall trees, for thousands of acres, are completely killed, and the ground strewed with many branches torn down by the clustering weight of the birds which have rested upon them. The whole region for several years presents a continued scene of devastation, as if swept by the resistless blast of a whirlwind."† Wilson tells us that their breeding places are still more extensive than the roosts, mentioning one in Kentucky "which stretched through the woods in nearly a north and south direction; was several miles in breadth, and was said to be upwards of forty miles in extent!" "On some single trees

* The Natural and Civil History of Vermont, p. 114, 1794.

† Nuttall's Manual of Ornithology, vol. i, pp. 631–2, 1832.

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upwards of one hundred nests were found, each containing *one* young only,"* though undoubtedly two eggs are always laid—the one hatching first and crowding the other out of the nest, which is, at best, but a frail cradle, formed merely "of a few slender dead twigs, negligently put together, and with so little art that the concavity appears scarcely sufficient for the transient reception of the young,"† which, like the eggs, may readily be seen from below, through the delicate net-work of twigs. Wilson says it was dangerous to walk under these flying and fluttering millions, from the frequent fall of large branches, broken down by the weight of the multitudes above, and which, in their descent, often destroyed numbers of the birds themselves; while the clothing of those engaged in traversing the woods were completely covered with the excrements of the Pigeons." "The ground was strewed with broken limbs of trees, eggs, and young squab Pigeons, which had been precipitated from above, and on which herds of hogs were fattening. Hawks, Buzzards, and Eagles, were sailing about in great numbers, seizing the squabs from their nests at pleasure."‡ Audubon's description of a night passed at one of their roosting places deserves introduction here: Reaching it early in the afternoon, before the pigeons had come in, "many trees two feet in diameter" were observed "broken off at no great distance from the ground; and the branches of many of the largest and tallest had given way, as if the forest had been swept by a tornado." "Everything proved," continued Audubon, "that the number of birds resorting to this part of the forest must be immense beyond conception. As the period of their arrival approached, their foes [man] anxiously prepared to receive them. Some were furnished with iron pots containing sulphur, others with torches of pine-knots, many with poles, and the rest with guns. The sun was lost to our view, yet not a pigeon had arrived. Everything was ready, and all eyes were gazing on the clear sky, which appeared in glimpses amidst the tall trees. Suddenly there burst forth a general cry of 'here they come!' The noise which they made, though yet distant, reminded me of a hard gale at sea, passing through the rigging of a close-reefed vessel. As the birds arrived and passed over me, I felt a current of air that surprised me. Thousands were soon knocked down by the pole men. The birds continued to pour in. The fires were lighted, and a magnificent, as well as wonderful, and almost terrifying, sight presented itself. The Pigeons, arriving by thousands,

* American Ornithology, by Alexander Wilson, vol. ii, pp. 295–6. Edinburgh, 1831.
† Wilson, ibid., pp. 295–6. ‡ Nuttall, ibid., p. 633.

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alighted everywhere, one above another, until solid masses were formed on the branches all around. Here and there the perches gave way under the weight with a crash, and, falling to the ground, destroyed hundreds of the birds beneath, forcing down the dense groups with which every stick was loaded. It was a scene of uproar and confusion. I found it quite useless to speak, or even to shout to those persons who were nearest to me. Even the reports of the guns were seldom heard, and I was made aware of the firing only by seeing the shooters reloading. . . . . The Pigeons were constantly coming, and it was past midnight before I perceived a decrease in the number of those that arrived. . . . . Towards the approach of day the noise in some measure subsided; long before objects were distinguishable the Pigeons began to move off in a direction quite different from that in which they had arrived the evening before; and at sunrise all that were able to fly had disappeared. The howling of the wolves now reached our ears, and the foxes, lynxes, cougars, bears, racoons, opossums, and pole-cats were seen sneaking off, whilst Eagles and Hawks of different species, accompanied by a crowd of Vultures, came to supplant them, and enjoy their share of the spoil." Two farmers, "distant more than a hundred miles; had driven upwards of three hundred pigs to be fattened on the Pigeons which were to be slaughtered."*

"Audubon attempts to reckon the number of Pigeons in one of these flocks, and the daily quantity of food consumed by it. He takes, as an example, a column of one mile in breadth, and supposes it passing over us, without interruption, for three hours, at the rate of one mile per minute. This will give us a parallelogram of 180 miles by 1, averaging 180 square miles; and allowing two Pigeons to the square yard, we have one billion one hundred and fifteen millions one hundred and thirty-six thousand Pigeons in one flock: and as every Pigeon consumes fully half a pint per day, the quantity required to feed such a flock must be eight millions seven hundred and twelve thousand bushels per day!"†

"Indeed, for a time," Nuttall correctly remarks, "in many places nothing scarcely is seen, talked of, or eaten, but Pigeons!"

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* The Birds of America, by John James Audubon, vol. v, pp. 29–30.

† Wilson, ibid, Appendix, vol. iv, p. 323.

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172. Zenædura Carolinensis (Linné) Bonaparte. Carolina Dove; "Turtle Dove."

A rather common summer resident, sometimes remaining through the winter (Jan. 15, 1874; 16, 1875, Grinnell). Arrives early in May (May 5, 1875, shot, Sage). On May 24th, 1876, I found a nest containing two fresh eggs, on a maple sapling, fifteen feet above the ground. In the south and west they generally, though by no means exclusively, breed on the ground.* In central Massachusetts I have taken it as late as the middle of November (1873). It is particularly abundant throughout the far west, and near the Pacific coast has been seen as far north as "lat. 49° in summer, while a few winter in California"† about San Francisco, latitude 38°. Mr. Stadtmüller found a nest of this species, about twelve feet from the ground, in a pine grove, near New Haven, June 20th, 1874. "It was close to the trunk of the tree, and consisted of a few sticks placed loosely on top of a common squirrel's nest, and contained one egg and one young dove. I took the egg and four weeks later went to get the nest, but found another egg in it."‡ Surely the squirrel's nest must have been deserted, or it would hardly have constituted a safe base for bird's eggs.

NOTE.—The Wild Turkey, *Meleagris gallopavo*, var. *Americana* (Bartram) Coues, long since exterminated from this State, was once common here. Wild Turkies were plenty in 1780, and occasionally seen as late as 1790.§ Regarding their former abundance in New England, one Thomas Morton, of Clifford's Inn, Gent., wrote (printed by Charles Green, in 1632): "Turkies there are, which divers times in great flocks have sallied by our doores; and then a gunne (being commonly in a redinesse) salutes them with such a courtesie, as makes them take a turne in the Cooke roome. They daunce by the doore so well. Of these there hath bin killed, that have weighed forty-eight pound a peece. They are by mainy degrees sweeter than the tame Turkies of England, feede them how you can. I had a salvage who hath taken out his boy in a morning, and they have brought home

* Vide: Coues' Birds of the Northwest, p. 389, 1874; Allen, Bull. Mus. Comp. Zool., vol. iii, No. 6, p. 170, 1872; Cooper, Ornithology of California, p. 513, 1870; Merriam, Zool. Report in 6th Annual Report U. S. Geol. Survey Terr., p. 710, 1872; Henshaw, Report upon Ornithological Specimens, p. 68, 1874.

† Cooper, Ornithology of California, p. 513, 1870.

‡ MS. notes of the Stadtmüller Brothers.

§ A Statistical Account of the County of Middlesex, in Connecticut, by David D. Field, p. 19. 1819.

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their loades about noone. I have asked them what number they found in the woods, who have answered neent metawna, which is a thousand that day; the plenty of them is such in those parts. They are easily killed at rooste, because the one being killed, the other sit fast neverthelesse, and this is no bad commodity."* Linsley says: "The last Wild Turkey that I have known in Connecticut, was taken by a relative of mine, about thirty years since [about 1813], on Totoket Mountain, in Northford. It was overtaken in a deep snow, and thereby outrun. It weighed, when dressed, twenty-one pounds."† In 1842, Zadock Thompson wrote that a few "continue still to visit and breed upon the mountains in the southern part of the State" (Vermont).‡ As late as 1833 it was "frequently met with on Mt. Holyoke," but had "become scarce and nearly extinct" in other parts of the State.§ Professor Wm. D. Whitney once mounted a fine specimen of the Wild Turkey killed on Mt. Tom, Mass., Nov. 1st, 1847. It may now be seen in the beautiful case of birds given by Prof. Whitney to the Peabody Museum of Yale College, and is of particular value as being, in all probability, the last of its race seen in that State. In the month of October, "the *Turkey Moon* of the aborigines," they used to wander far and wide in quest of food, frequently assembling in vast numbers in districts where there was an abundance. Audubon tells us that "When they come upon a river, they betake themselves to the highest eminences, and there often remain a whole day, or sometimes two, as if for the purpose of consultation. During this time the males are heard gobling, calling, and making much ado, and are seen strutting about, as if to raise their courage to a pitch befitting the emergency. Even the females and young assume something of the same pompous demeanor, spread out their tails, and run round each other, *purring* loudly, and performing extravagant leaps. At length, when the weather appears settled, and all round is quiet, the whole party mount to the tops of the highest trees, whence, at a signal, consisting of a single *cluck*, given by a leader, the flock takes flight for the opposite shore."‖

Josselyn says "their eggs are very wholesome and restore decayed nature exceedingly."¶

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* Reprinted in Force's Historical Tracts, Tract 5, p. 48.

† Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 264. April, 1843.

‡ History of Vermont, Natural, Civil, and Statistical. By Z. Thompson, p. 101. 1842.

§ See Hitchcock's Report, p. 549. 1833.

‖ Audubon's Birds of America, vol. v, p. 43.

¶ Two Voyages to New England. p. 99. 1675.

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Family, TETRAONIDÆ.

173. Bonasa umbellus (Linné) Stephens. Ruffed Grouse.

A common resident. Breeds on both East and West Rock, near New Haven, as well as throughout the State. In May, 1877, Prof. Verrill found a nest, containing twelve eggs, within ten feet of a traveled road, near the city. In the vicinity of Easthampton, Mass., they were particularly abundant, and on one occasion I frightened one off from an apple tree directly behind the "Town Hall." Large numbers of them are caught in snares every fall, and the market is well supplied with native birds. Of it, in 1632, Morton wrote: "Partridges, there are much, like our Partridges of England, they are of the same plumes, but bigger in body. They have not the signe of the horseshoe-shoe on the breast as the Partridges of England; nor are they coloured about the heads as those are; they sit on the trees. For I have seene 40. in one tree at a time; yet at night they fall on the ground, and sit until morning so together; and are dainty flesh."*

174. Ortyx Virginianus (Linné) Bonaparte. Quail; Bob White.

A common resident, breeding in thick brushwood at South End and many other places near New Haven.

This species also attracted Morton's attention, for he says: "There are quailes also, but bigger then the quailes in England. They take trees also: for I have numbered 60. upon a tree at a time. The cocks doe call at the time of the yeare, but with a different note from the cock quailes of England."*

NOTE.—The Prairie Chicken, or Pinnated Grouse, *Cupidonia cupido* (Linné) Baird, was formerly a resident of New England, but, like the Wild Turkey, was exterminated many years ago—at least so far as the main land is concerned, for it is said that a few still exist on some of the islands south of Cape Cod (Naushon for example, and perhaps Martha's Vineyard). However, it is pretty certain that many years have elapsed since the last "wild chicken" was seen in Connecticut, for even Linsley, in 1842, gave it as a bird of the past. Nuttall, ten years earlier (in 1832), said that they were still met with "on the brushy plains of Long Island, *and in similar shrubby barrens*

* Force's Historical Tracts, vol. ii, Tract 5, p. 48.

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in Westford, Connecticut."* That our ancestors were fond of "fowling," and that it sometimes cost them their lives, may be seen from the following: In October, 1636, one "Joseph Tilly, master of a bark, came to anchor nearly opposite Calve's Island, and taking one man with him, went on shore for the purpose of fowling. As soon as he had discharged his piece, a large number of Pequots, rising from their concealment, took him and killed his companion; and then gratified their malice by putting him to torture. They first cut off his hands and then his feet; after which he lived three days. But as nothing which they inflicted upon him excited a groan, they pronounced him a stout man."† And this occurred in the town of Saybrook, Conn., at a time when many of our forefathers perished at the hands of the Indians, before bringing them to submission. One Thomas Morton, writing in 1632, speaks of the presence of this bird in New England in the following language: "There are a kinde of fowles which are commonly called Pheisants, but whether they be pheysants or no, I will not take upon mee, to determine. They are in form like our pheisant-henne of England. Both the male and the female are alike; but they are rough footed: and have stareing feathers about the head and neck, the body is as bigg as the pheysant-henne of England; and are excellent white flesh, and delicate white meate, yet we seldom bestowe a shoot at them."‡ The "white flesh" must have been a mistake unless he referred to the Ruffed Grouse which is immediately spoken of under the name of "Partridge." Nuttall says of its habits: "The season for pairing is early in the spring, in March or April. At this time the behavior of the male becomes remarkable. Early in the morning he comes forth from his bushy roost, and struts about with a curving neck, raising his ruff, expanding his tail like a fan, and seeming to mimic the ostentation of the Turkey. He now seeks out or meets his rival, and several pairs at a time, as soon as they become visible through the dusky dawn, are seen preparing for combat."§

* Manual of Ornithology, vol. i, p. 662. 1832.

† A Statistical Account of the County of Middlesex, in Connecticut. By David D. Field, p. 36. 1819.

‡ Force's Historical Tracts, vol. ii, Tract 5, p. 48.

§ Nuttall's Manual of Ornithology, vol. i, pp. 663-64. 1832.

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Family, CHARADRIIDÆ.

175. Squatarola helvetica (Linné) Brehm. Black-bellied Plover.

Occurs during the migrations. Taken at Stratford by Linsley. Mr. W. W. Coe has a splendid specimen of this bird, taken in fall, on the Sound near the mouth of the Connecticut. Dr. F. W. Hall shot several specimens about the middle of October (Oct. 18, 1873), and tells me that they were remarkably tame.

176. Charadrius fulvus, var. **Virginicus** (Bork.) Coues. Golden Plover.

Common during migrations. Capt. Brooks informs me that they are "plenty at Guilford, Conn., in spring and early fall," and that "sometimes a few stop in the fall and stay a few days" at Faulkner's Island. Linsley found it at Stratford. Mr. Coe has taken it as far inland as Portland, Conn., on the river.

177. Ægialitis vocifera (Linné) Bonaparte. Killdeer Plover.

A summer resident, but not very common. It generally arrives late in March (Feb. 24, 1875, plenty by last of March; April 5, 1872; Portland, Conn., W. W. Coe). Mr. Coe informs me that it breeds on Plumb Island, and in the vicinity of Portland, Conn., where it used to be very common.

178. Ægialitis Wilsonia (Ord) Cassin. Wilson's Plover.

Not common. Linsley took it at Stratford. It has also been taken on Long Island (Giraud), but seldom strays so far North.

179. Ægialitis semipalmata (Bonap.) Cabanis. Semipalmated Plover.

A common migrant. Arrives about, or before, the middle of May (May 17, 1876, Osborne). During the latter part of May, and first of June, they may be seen, in small flocks, running along the beach, in search of food, as the tide goes out. Mr. W. W. Coe writes me that he has taken it at Portland, Conn., twenty-five miles from the Sound.

180. Ægialitis meloda (Wilson) Bonaparte. Piping Plover; Ringneck.

A summer resident. Linsley found it breeding at Stratford. Mr. J. N. Clark also finds it breeding at Saybrook, Conn., and Mr. W. W. Coe has taken it at Portland.

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Family, HÆMATOPODIDÆ.

181. Hæmatopus palliatus Temminck. Oyster-catcher.

A rare migrant. Linsley says: "The Oyster-catcher is now rare here, but fifteen years since they were not very uncommon in autumn."*

182. Strepsilas interpres (Linné) Illiger. Turnstone.

A common migrant. Linsley gave it from Stratford, and Capt. Brooks writes me that it is "quite common in spring and fall" about Faulkner's Island, Conn. Mr. Sage, of Portland, has a beautiful male, which he killed at Westbrook, Conn., May 23d, 1877. In fall it returns during the latter part of August (Aug. 31, 1874, F. W. Hall).

Family, RECURVIROSTRIDÆ.

183. Recurvirostra Americana Gmelin. Avocet.

A rather rare migrant. Josiah G. Ely, Esq., writes me that he has seen but one specimen of the Avocet taken on our coast. "It was caught, in 1871, between Saybrook and East Lyme, in an old seine strung out on the beach to dry," and was kept alive for some time by a storekeeper.

Family, PHALAROPODIDÆ.

184. Steganopus Wilsoni (Sabine) Coues. Wilson's Phalarope.

Of rare and almost accidental occurrence in New England, though common throughout the West. Linsley says of it: "Wilson's Phalarope I have in my cabinet; it was killed in Bridgeport [Conn.] and sent to me by a friend, and is probably one of the rarest birds in New England. It is not only beautiful, but the great quantity of plumage on a bird so small and delicate, together with his unique bill, seems to render it one of the most peculiar of this class of animals."†

185. Phalaropus fulicarius (Linné) Bonaparte. Red Phalarope.

A rare visitor from the North. Mr. W. W. Coe has a specimen in his cabinet, killed at Portland, Conn., in September.

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* Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 265. April, 1843. † Op. cit., p. 268.

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Family, SCOLOPACIDÆ.

186. Philohela minor (Gmelin) Gray. Woodcock.

A resident; common from early spring till November. A few commonly, if not regularly, winter in low swamps. They arrive early in March ("Mar. 3, 1877, Middletown, Conn., killed by flying against a telegraph wire"*), and breed very early. On the 3d of April, 1877, my young friend, Walter R. Nichols, found, near Branford, Conn., a nest containing four fresh eggs. They may breed twice, for Mr. Nichols found a second nest, in the same locality, and containing the same number of eggs, as late as July 20th, 1877. The eggs were partially incubated, and the old bird was shot as she left the nest. Mr. W. W. Coe writes that he found one, near Portland, Conn., April 12, 1872, also containing four eggs: "The nest was on a bog, in the middle of a brook which ran through a swamp. It was not more than six inches above the water. The grass was short, and there were no bushes near, so that it was very much exposed, but still hard to find, for, although we had hunted the ground over carefully, the old bird did not fly off until my man stepped on the bog. I had my old dog Dincks with me, and his nose is first class, and yet he passed within a foot of her several times without scenting her, which satisfies me that a bird sitting on her eggs gives out no scent, for this is not the first time I have tried it. The nest was simply a shallow hole scraped in the top of the bog; there was a little coarse grass, a few leaves, and one or two of the Woodcock's feathers in it."† Mr. John H. Sage tells me that, while collecting with Mr. W. W. Coe near Portland, Conn., May 30th, 1874, they flushed a Woodcock with young, one of which she carried off in her claws! and Mr. Coe writes me, "in regard to the Woodcock carrying off its young: Mr. Sage and I were not four feet apart when the old bird got up between us, rose about three feet, and then dove down again and picked up a young bird with her feet, and, with her tail spread and held forward under the young, carried it off about eight rods, and came back for the others, but my boy frightened her away."

Thomas Morton, in 1632, thus alluded to the resemblance between our bird and the European Woodcock (*Scolopax rusticola*): "Simpes, there are like our Simpes in all respects, with very little difference. I have shot at them onely, to see what difference I could finde be-

* MS. notes of John H. Sage. † MS. notes of W. W. Coe.

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tweene them and those of my native country, and more I did not regard them."*

187. Gallinago Wilsoni (Temminck) Bonaparte. Wilson's Snipe.

A resident; common during the migrations; sometimes breeds. In October and November (some remain into December) large numbers are shot on our salt marshes. Mr. W. W. Coe took it Feb. 4th, 1872, near Portland, Conn., and thinks they sometimes winter there in low swampy places. Arrives in March (Mar. 18, 1874, Sage). Mr. W. W. Coe and Mr. J. H. Sage inform me that they took a nest containing three fresh eggs of this species at Portland, Conn., May 13th, 1874. The eggs were "fully identified, as the parent bird was found on the nest."† It was not previously known to breed as far south as Connecticut. The nearest approach to it is "a set of eggs in the Smithsonian labeled Oneida Co., N. Y."‡ Mr. Coe tells me that there were a number of Snipe in the field at the time, and he thinks there were other nests which they did not find.

188. Macrorhamphus griseus (Gmelin) Leach. Red-breasted Snipe.

Not rare during the migrations. "Stratford," Linsley. Mr. J. H. Sage of Portland has a specimen which he shot at Saybrook, Conn., Aug. 21st, 1874.

189. Ereunetes pusillus (Linné) Cassin. Semipalmated Sandpiper.

A summer resident; common along the shore during the migrations. Mr. W. W. Coe has seen it in June. On the 20th of July, 1877, Mr. Walter R. Nichols found, at Branford, Conn., four eggs of a small Sandpiper. They were placed on a few straws in a slight excavation in a corn field, about half a mile from the shore. Supposing them to be the eggs of this species I sent one to Dr. Brewer, who writes: "In the absence of my cabinet, for comparison, I cannot be certain, but I have little or no doubt that it is the egg of *Ereunetes pusillus*." It is unnecessary to state that this is the first authentic record of its breeding in southern New England. In fall, Dr. F. W. Hall has taken it as early as Aug. 25th (1874).

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* Reprinted in Force's Historical Tracts, vol. ii, Tract 5, p. 47.

† MS. notes of John H. Sage, Esq.

‡ Coues' Birds of the Northwest, p. 476.

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190. Tringa minutilla Vieillot. Least Sandpiper.

A common migrant. Maritime. Found along the shore in May and early June, and again in August and September.

191. Tringa maculata Vieillot. Pectoral Sandpiper; Jack Snipe.

Common during migrations. Mr. Coe tells me that it is common in fall as far up the Connecticut as Middletown. Arrives from the north early in August (Aug. 6, 1873, Hall).

192. Tringa fuscicollis Vieillot. Bonaparte's Sandpiper; White-rumped Sandpiper.

Not rare during migrations, though Linsley took only two specimens at Stratford, Conn. Dr. Hall has taken it late in August (Aug. 31, 1874).

193. Tringa maritima Brunnich. Purple Sandpiper.

Not uncommon during the migrations. Many winter on the islands along the coast. Not found by Linsley. Captain Brooks writes me from Faulkner's Island that "Purple Sandpipers come here in early fall and stay till spring," and that they are common and get to be quite tame.

194. Tringa alpina, var. **Americana** Cassin. Dunlin; "Ox-Bird."

A common migrant; a few may winter.

195. Tringa subarquata (Guld) Temminck. Curlew Sandpiper.

A rare visitor along our coast. Mr. Josiah G. Ely writes me that one was shot near Saybrook, Conn., some time ago, and I am informed by Dr. D. Crary, of Hartford, that a specimen of this species was killed, Oct. 3d, 1859, at Keeny's Cove, on the Connecticut River, in East Hartford (Hockanum), Conn. Also, Dr. E. L. R. Thompson, of this city, tells me that he shot three Curlew Sandpipers on the Quinnipiac River (near New Haven) in June, 1874. Dr. Wm. O. Ayres, now of Easthampton, Long Island, writes me that he "killed it once at Miller's Place, L. I., in 1839,"* and it has also been taken in Massachusetts.†

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* See also Giraud's Birds of Long Island, 1844.

† Catalogue of the Birds of New England. By T. M. Brewer, p. 13. 1875.

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196. Tringa canutus Linné. Red-breasted Sandpiper; Knot.

Common during migrations. Taken at Saybrook, by Mr. Sage, Aug. 21st, 1874; and Mr. Coe tells me that it is found about the Connecticut River, near Middletown, in summer.

NOTE.—The Stilt Sandpiper, *Micropalama himantopus* (Bonap.) Baird; and Baird's Sandpiper, *Tringa Bairdii* Coues, doubtless occur along the coast during migrations.

197. Calidris arenaria (Linné) Illiger. Sanderling.

Occurs during migrations, and is extremely abundant in fall. Linsley took it at Stratford. Mr. Grinnell informs me that they arrive during the latter part of September, remaining late into October. Thomas Morton wrote of them in 1632: "Sanderlings are dainty bird, more full bodied than a Snipe, and I was much delighted to feede on them, because they were fatt, and easie to come by, because I went but a stepp or to for them: and I have killed betweene foure and five dozen at a shoot which would lead me home. Their foode is at ebbing water on the sands, of small seeds, that grows on weeds there, and are very good pastime in August."*

198. Limosa fedoa (Linné) Ord. Great Marbled Godwit.

A rare migrant. Linsley found it at Stratford, Conn., in August, 1842, "in large flocks, but very shy."† Nearly an hundred years ago, Thomas Pennant, in his Arctic Zoölogy, stated that "it inhabits Hudson's Bay and Connecticut."‡

199. Limosa Hudsonica (Latham) Swainson. Hudsonian Godwit.

A rare migrant. Taken at Stratford by Linsley. Coues suggests that the *Limosa Edwardsii?* of Linsley (p. 267) is "perhaps an albino"§ of this species, but Pennant thought it was the Avocet (*Recurvirostra Americana*).‖ The good old preacher (Linsley), in speaking of these birds, could not take his Lord's name in vain on so slight a provocation—hence he called them "*Good*wits."

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* New English Canaan, p. 47, 1632. Reprinted in Force's Historical Tracts, vol. ii, Tract 5.

† Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 267, 1843.

‡ Arctic Zoölogy, vol. ii, p. 465, 1785.

§ List of the Birds of New England, p. 48, 1868.

‖ Arctic Zoölogy, vol. ii, p. 502, 1785.

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200. Totanus semipalmatus (Gmelin) Temminck. Willet; Tattler.

A summer resident; not common. Linsley found it breeding at Stratford, Conn. Mr. Grinnell has taken it, near Milford, late in the summer, and Mr. W. W. Coe took a nest, containing three eggs, at Madison, Conn., June 5th, 1873.*

201. Totanus melanoleucus (Gmelin) Vieillot. Greater Yellow-legs.

Common during migrations. Arrives in May (May 14, 1874, Sage), remaining till June (June 1, 1877, Sage). Found both coastwise and in the interior. I first became acquainted with it at Yellowstone Lake, where I killed four at one shot, Aug. 23d, 1872.

202. Totanus flavipes (Gmelin) Vieillot. Lesser Yellow-legs.

A common migrant. Killed one May 7th, 1877, on a fresh water pond near Meriden, Conn. Returns about the middle of August (Aug. 17, 1874, Hall).

NOTE.—**Totanus chloropus** Nilsson. Green-shanlis.

Linsley states that a specimen of this rare straggler was taken at Stratford, Conn., in the autumn of 1842.† This is, so far as I am aware, the only recorded instance of its capture north of Florida (Audubon shot three on Land Key, Fla., May 28, 1832),‡ where its occurrence seems to be purely accidental, its proper home being in the "Old World;" and since Dr. Coues states that it was given by Linsley "very possibly through an erroneous identification,"§ I do not feel justified in including it among the species ascertained to occur within our limits.

203. Totanus solitarius (Wilson) Audubon. Solitary Tattler.

Common during the migrations. Arrives early in May (May 2, 1877), frequenting muddy ponds and sluggish streams, in small flocks of about half a dozen. Solitary individuals may be seen as late as early June, (Coe; June 10, Grinnell).

204. Tringoides macularius (Linné) Gray. Spotted Sandpiper.

A common summer resident, arriving during the latter part of April or first of May (May 1, 1874, Sage). I once found its nest

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* MS. notes of W. W. Coe, Esq.

† Am. Jour. Sci. and Arts, vol. xliv, No. 2. p. 266, 1843.

‡ Audubon's Birds of North America, vol. v, p. 321.

Proceed. Essex Inst., vol. v, p. 296, 1868.

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within eight feet of a railroad track where trains passed every hour of the day! Remains into October (Oct. 6, 1874). Found wherever there is a pond or small stream.

205. Actiturus Bartramius (Wilson) Bonap. Bartramian Sandpiper; "Upland Plover."

A common summer resident. Breeds in open fields away from water. I am informed by Mr. Coe that large numbers of them breed on Plum Island, off New London, Conn. Arrives about May 1st (May 2, 1874, Portland, Conn., Sage).

206. Tryngites rufescens (Vieillot) Cabanis. Buff-breasted Sandpiper.

Occurs during the migrations, but is not common. Josiah G. Ely, Esq., writes me that two were killed near Saybrook, a few years ago, and Dr. Daniel Crary had one in his collection which "was shot near Hartford some years ago."

207. Numenius longirostris Wilson. Long-billed Curlew.

Not particularly rare during migrations, but excessively shy. Linsley took it at Stratford, Conn. Regarding its occurrence near Faulkner's Island, Conn., Capt. Brooks writes: "Not plenty; occasionally one stops here in the fall." It was seen at Milford, Conn., by Mr. Grinnell, during the summer of 1873. Mr. J. N. Clark, of Saybrook, Conn., tells me that it sometimes occurs there in the fall, but is rare. Dr. Crary tells me that it has been taken near Hartford, Conn.

208. Numenius Hudsonicus Latham. Hudsonian Curlew.

A rare migrant. Taken at Stratford, Conn., by Linsley. I am informed by Dr. Crary, of Hartford, Conn., that it has been killed in that vicinity. Under date of "July 20, 1877," the Rev. J. Howard Hand writes me, from Southampton, Long Island: "Have just taken three specimens of the Hudsonian Curlew this morning."

209. Numenius borealis (Forster) Latham. Eskimo Curlew.

Not common. Occurs during migration. Taken at Stratford, Conn., by Linsley. Mr. J. N. Clark has a fine mounted specimen in his cabinet, killed at Saybrook, Conn., Oct. 13, 1874.

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Family, TANTALIDAE.

210. Ibis falcinellus, var. **Ordii** (Bonaparte) Coues. Glossy Ibis.

A rare accidental visitor from the South. At Stratford, Conn., Linsley obtained five specimens of this species.* Stragglers have also been taken in Massachusetts. There is a specimen of this species in the Museum of Wesleyan University, at Middletown, Conn., taken in that vicinity, by Dr. Barrat about the year 1855.

211. Ibis alba (Linné) Vieillot. White Ibis.

One only recorded from New England. It was seen by Mr. Geo. Bird Grinnell within ten miles of New Haven: "Late in the afternoon of May 23, [1875] I observed near Milford, Conn., a specimen of *Ibis alba.* I recognized the bird as it flew over me, and following it to a small pond where it went down, discovered it perched upon a tree over the water. I carefully examined it with a good glass, at a distance of about one hundred and fifty yards, and by this means was enabled to note every detail of form and color. It was in full plumage, the white being pure, and the naked skin about the head, bright red. After watching it for a few moments I tried to approach it, but before I came within gunshot it flew, uttering a hoarse cackle as it went off."† Two specimens have been killed on Long Island.‡

Family, ARDEIDÆ.

212. Ardea herodias Linné. Great Blue Heron.

A summer resident. Common during the migrations. Arrives before the middle of April (Apr. 4, 1873, Coe; 12, Sage), and I have seen it as late as Nov. 26th (1875), along the coast. On April 17th, 1877, Mr. A. J. Dayan and I saw about a dozen of these splendid birds on the Whitney Lakes, within a couple of miles of New Haven, but they were very shy and remained only a few days. Mr. Grinnell once saw a flock of twenty-eight flying over the Sound. Mr. W. P. Nichols saw one near New Haven, June 2d, 1877.

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* Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 266, 1843.

† Am. Nat., vol. ix, No. 8, p. 470, 1875.

‡ Giraud's Bird's of Long Island, p. 275, 1844.

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213. Ardea egretta (Gmelin) Gray. Great White Egret.

A rare visitor from the South. Several specimens have been taken in Massachusetts, and I have myself seen it at the "Ox Bow" on the Connecticut. A specimen was shot near Middletown, Conn., some years ago, and is now in the Museum of Wesleyan University. Mr. Grinnell has seen it on the marshes near Milford, Conn., in September. Dr. Wood tells me that, several years ago, one spent a week on a marsh near East Windsor Hill, Conn. Mr. Fred. Sumner Smith tells me that he saw a pair of these birds at Lake Saltonstall (near New Haven), Conn., during the latter part of July, 1876, and two weeks later, at the same place, saw no less than seven individuals feeding together. They were exceedingly shy and he could not approach within gunshot.

214. Ardea candidissima (Jacquin) Gmelin. Little White Egret.

A rare accidental visitor from the South. Seen at Stratford, Conn., by Linsley. Also taken in Massachusetts, and one straggler even reached Nova Scotia ("Jones"). Dr. Crary says that he has taken it near Hartford, Conn.

215. Ardea cærulea Linné. Little Blue Heron.

A very rare accidental visitor from the South. Linsley took it at Stratford, Conn. Has been taken in Massachusetts. Mr. Dayan saw a small Heron on Lake Whitney, early in April, 1877, which he supposes to have been this species. In the Cabinet of Mr. Coe, of Portland, Conn., is a beautiful specimen of this species which he shot in that vicinity early in July, 1875. It was a young bird and is pure white all over, excepting the tips of the primaries, which show a little slate-blue color. There were two of them together, but the other escaped. Mr. Erwin I. Shores, of Suffield, Conn., writes me that one was shot there about the middle of May by Mr. Chas. Newton. Mr. Shores did not see the specimen, but says: "Dr. Newton described it to me as 'a small Heron blue all over,' and I have no doubt but that it was this species."

216. Ardea virescens Linné. Green Heron.

A common summer resident. Breeds in several places near New Haven—notably in "Pine Swamp." Arrives late in April or early in May (May 3), remaining into October. Capt. Brooks informs me

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that they sometimes stop at Faulkner's Island in spring. Breeds late in May and in early June.

217. Nyctiardea grisea, var. **nævia** (Boddert) Allen. Night Heron.

A common summer resident. Breeds in Pine and Heron Swamps, near New Haven, and in several other places about the State, and on islands off the coast. Mr. A. J. Dayan started a small flock on Lake Whitney, April 24th, 1877, and they alighted on a tree. On his near approach all took flight but one, which he brought down, and an examination proved that it had a well marked cataract in the eye facing the direction from which he approached. This is an interesting fact in Ornithological pathology. Remains into October. Mr. W. W. Coe, of Portland, Conn., on the 17th of April, 1872, visited a "*Heronry*" of this species, at Rocky Hill, Conn. He writes: "Saw hundreds of nests, each containing from two to five eggs. Eight and even ten nests were frequently found on one tree, and the same nest often contained fresh eggs, eggs half hatched, and young birds. The trees were white from the excrements of the birds, and looked as if they had all been whitewashed; nothing could grow under them."*

NOTE.—The Yellow-crowned Night Heron (*Nyctiardea violacea*) has been taken in Massachusetts, by Mr. Vickery (Oct., 1862),† and doubtless occurs as a rare accidental visitor.

218. Botaurus minor (Gmelin) Boie. Bittern; Stake-driver.

A common summer resident. Arrives in April (Apr. 26, 1875, Sage), remaining till November. Linsley relates the following amusing anecdotes concerning this species: "I obtained a fine specimen of the American Bittern two years since, which had previously given great alarm to many of our inhabitants by its peculiarly doleful and mournful sounds at evening. One man who was laboring near the swamp, it is said, ran a mile in the greatest consternation, alleging that 'the d—l was after him.' It is also stated by several of our most respectable inhabitants, that forty-seven years since, [1796] one hundred men united in a company on the Sabbath to traverse this swamp, and succeeded in killing one of these same birds, and that their sounds have not been heard in town since, until the former instance occurred which secured a specimen to me."‡

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* MS. notes of W. W. Coe, Esq.

† Allen, Rarer Birds of Massachusetts, p. 39, 1869.

‡ Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 265, 1843.

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219. Ardetta exilis (Gmelin) Gray. Least Bittern.

The Least Bittern seems to be, at present, a pretty regular summer resident, though formerly regarded as an accidental visitor. Linsley gave it from Northford, Conn., without comment. It has certainly bred here for several years past, and on June 27th, 1876, Mr. Nichols found its nest at Branford, Conn., containing one fresh egg. Have seen it in September. They were particularly abundant throughout the State during the season of 1875. Mr. W. W. Coe, who has seven beautiful specimens in his cabinet, showed me five eggs which he took from a nest at Portland, Conn., June 14th, 1873, and says that they breed regularly in that vicinity. Mr. Geo. Bird Grinnell also tells me that he takes two or three every year (generally in August or September). They follow up the Connecticut Valley to Massachusetts (Suffield, Conn., July, E. I. Shores).

NOTE.—The Sand-hill Crane, *Grus Canadensis* (Linné) Temm., though not occurring in New England at the present time, even as a rare straggler, was once common here. Thomas Morton, writing of the birds of New England, in 1632, says, of "Cranes, there are greate store, that even more came there at S. Davids day, and not before: that day they never would misse. These sometimes eate our corne, and doe pay for their presumption well enough; and serveth there in powther, with turnips to supply the place of powthered beefe, and is a goodly bird in a dishe, and no discommodity."* The fact that they ate corn, and were themselves, in turn, eaten by the inhabitants, clearly shows, as Prof. J. A. Allen has said, "that the Crane, and not a Heron, is the bird to which reference is made."† Moreover, Samuel Williams, more than an hundred and fifty years later (in 1794), says that the Sand-hill Crane ("*Ardea Canadensis*") was among the commonest of the "Water Fowl" found in Vermont at that time.‡ Belknap also gives it, in 1792, as one of the birds of New Hampshire.§ And even so recently as 1842, Zadock Thompson wrote that the Whooping Crane, *Grus Americana* (Linné) Temminck, was "occasionally seen during its migrations,"‖ in Vermont.

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* New English Canaan. Printed by Charles Greene, 1632. Reprinted in Force's Historical Tracts, vol. ii, Tract 5, pp. 47–8.

† Bull. Nutt. Ornith. Club, vol. i, No. 3, p. 58. Sept., 1876.

‡ The Natural and Civil History of Vermont, p. 119. 1794.

§ The History of New Hampshire, vol. iii. By Jeremy Belknap, p. 169. 1792.

‖ History of Vermont, p. 103. 1842.

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The present distribution of the Sand-hill Crane is interesting from its peculiarity. Common throughout the West, they extend northward even into Alaska, breeding about the Yukon (Dall),* but are not found east of the Mississippi Valley, except in Florida, where I have seen them, both on the Ocklawaha River,† and flying over the St. Johns. Both Mr. Grinnell‡ and myself§ found them to be abundant in the Yellowstone National Park, though so wary that it was difficult to obtain a shot at them. They were particularly numerous in the Lower Geyser Basin, in August, and used to make such an unnecessary amount of noise, mornings, that sleep, after daylight, was well nigh out of the question. Showing a decided preference for the grassy meadows (called "Parks," in the West) surrounded by heavy forests, and marking, perhaps, the course of some mountain stream, which is content to check its headlong speed while passing through a neighboring valley, the Sand-hill Crane, ever on the alert, keeps well away from the trees, and at the approach of any suspicious object, at once takes flight, uttering its warning cry for the benefit of those of its kind who happen to be near. The flats which they frequent are often studded with wooded knolls, and the best way to hunt them is to betake one's self, in company with a good rifle, to one of these "Islands" before the fog rises in the morning, and remain concealed and perfectly quiet. When the fog lifts, the stately forms of the Cranes may be seen scattered over the meadows, always peering about in search of danger; but at least one is pretty sure to be within rifle range. And at the report the alarm is given and the others lose no time in beating a hasty retreat. Their flight is heavy, and seemingly laborious. To their edible qualities I can testify with a good grace. The flesh is really excellent, deep red in color, and not unlike that of the Beaver. Few birds are more difficult to skin.

Thomas Pennant, in his Arctic Zoölogy (1785) says that "they arrive in May about Severn River, Hudson's Bay. Frequent lakes and ponds. Feed on fish and insects. Hatch two young; and retire southward in autumn.‖ I must observe, that they formerly made a halt in the Hurons country, at the season in which the Indians set their maize; and again on their return from the North, when the harvest was ready, in order to feed on the grain. The Indians, at

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* Alaska and its Resources. By William H. Dall. Appendix G, p. 583. 1870.

† Am. Nat., vol. viii, No. 2, p. 89. Feb., 1874.

‡ Ludlow's Report for 1875, p. 87. § Hayden's Report for 1872, p. 702.

‖ Ph. Trans., lxii. 409.

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those times, were used to shoot them with arrows headed with stone; for Theodat,* my authority, made his remarks in that country in the beginning of the last century."†

Family, RALLIDÆ.

220. Rallus longirostris Boddert. Clapper Rail.

Not common. Taken at Stratford by Linsley, who remarks that it "breeds abundantly" there.‡ Frequents *salt* marshes. Recently recorded from Massachusetts by Mr. Purdie. "The bird was captured by its flying on board a vessel in the [Boston] harbor, May 4, 1875."§

Several well authenticated instances of its occurrence in Connecticut have recently come to my notice.

221. Rallus elegans Audubon. King Rail.

Rather rare. Found breeding at Stratford, by Linsley.‡ Frequents *fresh-water* marshes. Mr. W. W. Coe has taken it at Portland, Conn. Mr. J. N. Clark of Saybrook, Conn., has a fine specimen in his cabinet taken there in *mid-winter* (Jan. 14, 1876).

222. Rallus Virginianus Linné. Virginia Rail.

A common summer resident, breeding plentifully in both salt and fresh-water marshes. They are quite abundant in the brackish-water marshes bordering the Quinnipiac River, and here my friend, Mr. Dayan, found a nest containing seven fresh eggs and secured the old bird, on June 7th, 1876. Concerning the nocturnal proclivities, and shrill, startling cry of the Rail, Dr. Coues thus graphically writes: "At nightfall some Mallard and Teal settled into the rushes, gabbling curious vespers as they went to rest. A few Marsh Wrens had appeared on the edge of the reeds, queerly balancing themselves on the thread-like leaves, sea-sawing to their own quaint music. Then they were hushed, and as darkness settled down, the dull, heavy croaking of the frogs played bass to the shrill falsetto of the insects. Suddenly they too were hushed in turn, frightened, may be, into silence; and from the heart of the bullrushes, '*crik-crik-rik-k-k-k*,'

* As quoted by De Buffon. † Arctic Zoölogy, vol. ii, p. 443. 1785.

‡ Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 267, 1843.

§ Bull. Nutt. Ornith. Club, vol. ii, No. 1, p. 22, January, 1877.

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lustily shouted some wide-awake Rail, to be answered by another and another, till the reeds resounded. Then all was silent again till the most courageous frog renewed his pipes. The Rail are, partially at least, nocturnal. During such moonlight nights as these they are on the alert, patrolling the marshes through the countless covered ways among the reeds, stopping to cry 'all's well' as they pass on, or to answer the challenge of a distant watchman. That they feed by night as well as by day, cannot be doubted. Their habit of skulking and hiding in the almost inaccessible places they frequent renders them difficult of observation, and they are usually considered rarer than they really are."*

223. Porzana Carolina (Linné) Cabanis. Carolina Rail; Sora.

An abundant summer resident. Large numbers are killed each year for the market. Have seen them as late as October (1874). Found both in fresh and salt-water marshes where there is an abundant growth of "Bullrushes," "Cat tails" and the like. Linsley says of it: "The Carolina Rail was so abundant here [Stratford, Conn.] last Autumn, in the marshes of the Housatonic, that something like hundreds were killed in a few hours, and that too for several days together. They were esteemed a great delicacy."† Regarding the difficulty of seeing them in their favorite haunts, Mr. Maynard writes: "I have been in a swamp where there were literally thousands of them, yet was unable to start more than two or three!"‡ I have had many similar experiences. One will suffice: Well do I remember finding a Rail's nest in a marshy swail near the mouth of a small cañon at the foot of the Wahsatch Range, just back of Ogden, Utah. It was early in June, and the nest, which was large and bulky, being composed of coarse marsh grass, was hidden in a clump of flags, whose arching blades met overhead, so concealing the enclosed treasures that they could only be seen by stooping over and peering through a small opening in the side, left for the passage of the parent bird. As yet but two eggs had been deposited, and the bird stole so silently and quickly through the reeds that I hardly felt sure it was not a snake till careful search revealed the nest. The next thing to be done was to secure the old bird, and with this end in view the place was visited at least once each day

* Coues, Birds of the Northwest, pp. 537–8, 1874.

† Am. Jour. Sci. and Arts, vol. xliv, No. 2. p. 267, April, 1843.

‡ Maynard's Naturalists' Guide, pp. 145–46, 1873.

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till the middle of June, but, notwithstanding the fact that I exercised the greatest caution in approach, I never so much as caught a momentary glimpse of her form, though once or twice a shadow seemed to flit hurriedly by and disappear in plain sight. What made it still more remarkable was that the number of eggs kept increasing day by day, and I always found them warm, showing that the bird had been gone but an instant. Once, while feeling of the eggs, I was so startled by her harsh crackling cry, uttered suddenly at my very feet, that I came near breaking them all, but still saw nothing of her. The time had come when we must move camp, so on the 15th of June I made a final effort to secure the old bird. The nest now contained twelve eggs, and I fancied I could hear the faint peeping of a young bird in his attempt to extricate himself from the shell. Stepping back a few paces, I waited, gun in hand, for the space of two long hours, standing first on one leg, then on the other, like a bashful country boy, till my patience was nearly exhausted and I was on the point of leaving, when something darted quickly toward the nest—it was enough; the mangled remains sufficed to determine the species. Meanwhile the egg had fairly hatched, and its noisy contents had already gained no little use of its tiny twigs. How the first hatched youngsters amuse themselves during the ten days, or two weeks, whilst the other eggs are coming to maturity, will doubtless be fully elucidated by he who attempts to explain how it is that a bird can give origin, in the coarse of a couple of weeks, to a dozen of eggs, each nearly as large and heavy as her own body. Certain it is that the processes of digestion, and assimilation of nutriment, must go on in them much more rapidly than in ourselves.

Wilson remarked that, "Of all our land or water fowl, perhaps none afford the sportsman more agreeable amusement, or a more delicious repast, than the little bird now before us. This amusement is indeed temporary, lasting only two or three hours in the day for four or five weeks in each year." The mode of procedure is thus described: "The sportsman furnishes himself with a light batteau, and a stout experienced boatman, with a pole of twelve or fifteen feet long, thickened at the lower end to prevent it from sinking too deep into the mud. About two hours or so before high water they enter the reeds, and each takes his post, the sportsman standing in the bow ready for action, the boatman on the stern seat pushing her steadily through the reeds. The Rail generally spring singly, as the boat advances, and at a short distance ahead, are instantly shot down, while the boatman, keeping his eye on the spot where the bird fell,

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directs the boat forward and picks it up while the gunner is loading. . . . . In this manner the boat moves steadily through and over the reeds, the birds flushing and falling, the gunner loading and firing, while the boatman is pushing and picking up. . . . . In these excursions it is not uncommon for an active and expert marksman to kill ten or twelve dozen in a tide!"* Mr. Grinnell informs me that the same method of hunting is practiced on the marshes bordering the Housatonic River, Conn.

224. Porzana Noveboracensis (Gmelin) Cassin. Yellow Rail.

Not common. Taken at Stratford, Conn., by Linsley. Though one of the rarer birds, it breeds about Middletown, Conn., as I am informed by Mr. Coe, who took it there in 1874 and 1875. Mr. Thos. Osborne has a specimen killed near New Haven. Mr. Grinnell favors me with the following note concerning its occurrence on the marshes near Milford, Conn.:

DEAR MERRIAM—The specimens of *Porzana Noveboracensis* about which you enquire were taken for the most part during the month of October, 1876, although I procured one individual as late as Nov. 10th. The securing of the first two or three was quite accidental.

I was working a young setter on Snipe (*Gallinago Wilsoni*) on a piece of wet meadow near Milford, Conn., and several times during the early part of the day was annoyed by the pertinacious way in which the dog would trail up some bird which neither he nor I could start. At length during one of these performances I saw the puppy grasp at something in the bogs before him, and immediately a small Rail rose and fluttered a few yards. Noticing its small size, and the fact that it had some white on its wings, and seeing from its flight that it was a Rail, I shot the bird before it had gone far, and when it was brought by the dog I was delighted to see that it was *P. Noveboracensis*, a species which I had never before seen alive. During the day several more individuals were secured. The next opportunity that I had of looking for these birds was, I think, Oct. 14th. That day my brother and I secured eight in an hour or two. They were ridiculously tame and would run along before the dog, creeping into the holes in the bogs and hiding there while we tried in vain to start them. I killed one with my dog whip, caught one alive in my hand, and the dog brought me another, uninjured, which he had

* Wilson's American Ornithology. vol. iii, p. 115, 1831.

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caught in his mouth. From what I saw of their habits, I am convinced that the only successful way of collecting these birds is to look for them with a dog. Without one they could never be forced from the ground. Yours sincerely,

GEO. BIRD GRINNELL.

225. Porzana Jamaicensis (Gmelin) Cassin. Black Rail.

An extremely rare summer resident. But three specimens of this rare bird have as yet been obtained in New England. The second Connecticut record is that given by Mr. H. A. Purdie, who writes: "Of this species Mr. Clark, of Saybrook, Conn., writes me that a neighbor of his, while mowing at that place, July 10th, 1876, swung his scythe over a nest of ten eggs on which the bird was sitting, unfortunately cutting off the bird's head and breaking all but four of the eggs."* I have recently seen the eggs in question, in Mr. Clark's collection. They agree precisely with Coues' description of the eggs of this bird, "being creamy-white, sprinkled all over with fine dots of rich, bright reddish-brown," and are totally unlike those of any other species of Rail. The bird was not preserved, but there seems to be no reasonable doubt of its identity. Mr. Purdie further states that he has "lately seen a skin of this species belonging to Mr. Browne, of Framingham. The bird was picked up dead, in August, 1869, by a relative of his, on Clark's Island, Plymouth Harbor [Mass.], and was forwarded to him as something entirely new to our shores. This instance adds a new bird to the Fauna of Massachusetts."* The only other recorded instance of its capture in New England is that given by Dr. Thos. M. Brewer: "Hazenville, Conn., Batty."

226. Gallinula galeata (Licht.) Bonaparte. Florida Gallinule.

A rather common summer resident, as I am told by Mr. Grinnell, who has taken a number of specimens about Milford, Conn. Mr. W. W. Coe has also taken it near Portland, Conn.

227. Porphyrio Martinica (Linné) Temminck. Purple Gallinule.

A rare accidental visitor from the South. A specimen of this species was killed near Middletown, Conn., about the year 1855, and is now in the Museum of Wesleyan University. It has been taken in Massachusetts, as recorded by G. P. Whitman: "A fine specimen

* Bull. Nutt. Ornith. Club, No. vol. ii, 1, p. 22, January, 1877.

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of the Purple Gallinule was shot at 'Henry's Pond,' 'South-end,' Rockport, Mass., on April 12th, [1875] by Mr. Robert Wendell."*

228. Fulica Americana Gmelin. Common Coot; Mud Hen.

Common during the migrations, particularly in fall. May breed. Linsley took five specimens at Stratford, remarking that it was "by no means common" there. Last October (1876), my friend, Mr. A. J. Dayan, shot several on Lake Whitney, and ascertained that no less than fifty specimens were killed there during that month!

Family, ANATIDÆ.

229. Cygnus Americanus Sharpless. Whistling Swan.

A rare, almost accidental, visitor, occurring only in winter. Linsley mentions the occurrence of four specimens, at Stratford, Conn., two of which were killed. My friend, Dr. Wm. H. Hotchkiss, of this city, tells me that he was informed, by William Beers, Esq., that two Swans were seen in Branford Harbor, during a severe gale, about the middle of March, 1876. I take it for granted that they were of this species, since there is no positive record of the occurrence of the Trumpeter Swan (*Cygnus buccinator*) within our limits. Mr. Grinnell also informs me that several Swans were seen near Milford, Conn., about the same time. Swans were once common in New England, during the migrations, as seen from Morton's remarks concerning them (in 1632): "And first of the Swanne, because she is the biggest of all the fowles of that Country. There are of them in Merrimack River, and in other parts of the Country, greate Store at the seasons of the yeare. The flesh is not much desired of the inhabitants, but the skinnes may be accompted a commodity, fitt for divers uses, both for fethers, and quiles."†

NOTE.—The Trumpeter Swan (*Cygnus buccinator* Richardson), may, and very probably does, sometimes occur within our limits. Dr. Wood, of East Windsor Hill, informs me that a hunter in his vicinity, who was perfectly familiar with this bird in the West, where they were common, once told him that he had heard the unmistakable note of the Trumpeter Swan, but did not see the bird. A short

* Am. Nat., vol. ix, No. 10, p. 573, Oct., 1875.

† Force's Historical Tracts, vol. ii, Tract 5, p. 46.

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time afterwards, however, on again meeting the Doctor, the hunter said that, a few days before, he heard the Swans coming and rushed into the house for his gun, but before he returned with it they had passed over, between the house and barn, that he had a good sight at them and was confident that they were Trumpeters. Proof is wanting to show that some of the birds mentioned under the last species were not really *Cygnus buccinator.*

230. Anser hyperboreus Pallas. Snow Goose.

A rare winter visitor. Linsley records seven specimens from Stratford, Conn. Mr. W. W. Coe, of Portland, has a magnificent specimen of this species in his cabinet. It was killed on the coast near Saybrook, Conn., in the fall of 1875.

NOTE.—The Barnacle Goose (*Branta leucopsis* Boie), a rare accidental visitor to our coast, from Europe, is given by Linsley from Stonington, Conn., but on insufficient evidence. Stragglers have been taken from South Carolina to Hudson's Bay, and a fine specimen of this Goose was killed on Long Island, N. Y., in October, 1876 (recorded by Mr. Lawrence),* hence it may occur as a rare accidental straggler.

231. Branta bernicla (Linné) Scop. Brant Goose; Black Brant.

A tolerably common spring and autumn migrant, sometimes remaining through the winter. Linsley said: "The Brant is common here [Stratford, Conn.] in winter" (p. 269). Captain Brooks writes me that they are "not common" at Faulkner's Island, where he has "only taken one." Mr. Grinnell informs me that two specimens of this species were killed off Stratford Light, Conn., last spring (1877). Mr. Osborne also saw three individuals near the mouth of the East Haven River, Conn., April 14th, 1876. Dr. Wood, of East Windsor Hill, Conn., has a fine specimen in his cabinet. It was shot on the Connecticut River, *above* Hartford, in the spring of 1876.

This species is first recorded from New England by Thomas Morton, who, in 1632, wrote: "There are Geese of three sorts, vize, brant Geese, which are pide, and white Geese which are bigger, and gray Geese, which are as big and bigger then the tame Geese of England, with black legges, black bills, heads and necks black, the flesh farre more excellent, then the Geese of England, wild or tame,

* Bull. Nutt. Ornith. Club, vol. ii, No. 1, p. 18, Jan., 1877.

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yet the purity of the air is such, that the biggest is accompted but an indifferent meale for a couple of men. There is of them great abundance. I have had often 1000. before the mouth of my gunne. I never saw any in England for my part so fatt, as I have killed there in those parts, the fethers of them makes a bedd, softer than any down bed that I have lyen on: and is there a very good commodity, the fethers of the Geese that I have killed in a short time have paid for all the powther and shott, I have spent in a yeare, and I have fed my doggs with as fatt Geese there, as I have ever fed upon myself in England."*

232. Branta Canadensis (Linné) Gray. Canada Goose; Wild Goose.

A winter resident, common during migrations; they arrive in November (Nov. 24, 1872, Sage), some remaining through April and sometimes even into May (May 22, 1864, Sage; May 10, 1877, C. H. M.). Linsley writes that "Hundreds of the common 'Wild Geese' winter at the mouth of the Housatonic, and so near my own dwelling that I often with my telescope present a distinct view of their *eyes* to my friends who call. Birds are said to be near enough to shoot when their eyes are visible to the sportsman. Many are killed here merely for sale by gunners, who frequently send them to New York."† Capt. Brooks says that they occasionally stop about the islands off Guilford, Conn., and that he killed two last November. Mr. Grinnell tells me that generally a few still winter about the mouth of the Housatonic River, Conn., and that over two hundred remained there last winter (1876–7). They begin to go north (some passing nearly due east) during the latter part of March (from Mar. 24th on), and all through April large flocks may be seen and heard overhead. Many of these contain upwards of one hundred and fifty birds, and I should say that they average about seventy-five. They were particularly numerous last spring (1877), and scarcely a day passed during April but one or more flocks were seen.

233 *a*. Branta Canadensis, var. **Hutchinsii** (Rich.) Coues. Hutchins' Goose; Southern Goose.

Not common. "Stratford," Conn. (Linsley). Mr. Geo. Bird Grinnell tells me that the hunters about Milford, Conn., all make a distinction between the common or Canada Goose and the Southern

* Force's Historical Tracts, Tract 5, (vol. ii,) p. 46.

† Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 269, 1843.

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Goose. That the latter does not arrive till after the Canada Geese have all come, and that they do not stay long, but pass southward—hence their name. They are also noticeably smaller than the Canada Goose.

233. Anas boschas Linné. **Mallard.**

A rare migrant. Have notes of its occurrence here September 30th, October, and November 13th, 1875, and Mr. Grinnell saw it in October and November, 1876. Linsley gives it from Stratford. Capt. Brooks writes me from Faulkner's Island, Conn., that they are not abundant: "occasionally see a few with Black Ducks in the fall."

234. Anas obscura Gmelin. **Black Duck.**

A resident, but most abundant during the migrations. Capt. Brooks writes me that they "come in September and stay through the winter. Leave in May and June." Linsley said that they occasionally bred about Stratford, Conn. That their edible qualities were early appreciated is seen from the writings of Thomas Morton, who said (in 1632): "Ducks, there are of three kindes, pide Ducks, gray Ducks, and black Ducks, in greate abundance: the most about my habitation were black Ducks: and it was a noted custome at my howse, to have every mans Duck upon a trencher, and then you will thinke a man was not hardly used, they are bigger boddied, then the tame Ducks of England: very fatt and dainty flesh. The common doggs fees were the gibletts, unlesse they were boyled now and than for to make broath."*

235. Dafila acuta (Linné) Bonaparte. **Pintail; Sprigtail.**

A rather rare winter resident. Have seen but few specimens. Linsley found it at Stratford, Conn. Mr. Osborne saw one in March (23), 1877. Mr. J. N. Clark, of Saybrook, Conn., tells me that he does not consider it particularly rare; in fact that he thinks it is rather common in spring and fall in that vicinity—about the mouth of the Connecticut. Dr. Wood, of East Windsor Hill, has two specimens killed on the Connecticut River above Hartford, but they are extremely rare there.

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* New English Canaan, p. 47. Reprinted in Peter Force's Historical Tracts, Tract 5.

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236. Chaulelasmus streperus (Linné) Gray. Gadwall; Gray Duck.

It occurs during the migrations; not common. Captain Brooks tells me that they are "occasionally seen" about Faulkner's Island, Conn., "but are not plenty." Linsley writes: "Flocks of the Gray Duck were here as early as August last season [1842], and were among the best of ducks for the table."*

237. Mareca Americana (Gmelin) Stephens. Widgeon; Baldpate.

Not particularly rare during the migrations; may winter. Taken at Stratford, Conn., by Linsley, and all along the coast by others, too numerous to mention. Dr. Wood has seen three specimens, one of which he shot, near East Windsor, Conn., but they are rare in this State so far inland. Dr. Coues states that "the Widgeon breeds in abundance in Northern Dakota and Montana along the banks of the streams and pools."† Thomas Morton found them in New England in 1632, for he writes: "Widggens there are, and abundance of other water foule, some such as I have seene, and such as I have not seene else where, before I came into those parts, which are little regarded."‡

238. Querquedula Carolinensis (Gmelin) Stephens. Green-winged Teal.

A common migrant. Have seen it in March. Taken by Linsley at Stratford, Conn. Concerning this and the following species Morton wrote, in 1632: "Teales, there are of two sorts greene winged, and blew winged: but a dainty bird, I have bin much delighted with a rost of these for a second course, I had plenty in the rivers and ponds about my howse."

239. Querquedula discors (Linné) Stephen. Blue-winged Teal.

A rather common migrant. Mr. Dayan secured a beautiful male of this species on Lake Whitney, September 25th, 1875. Arrives in August (Grinnell), remaining through October (Oct. 20, 1874, Sage).

240. Spatula clypeata (Linné) Boie. Shoveller; "Spoonbill-duck."

This splendid species is a rare migrant along our shores. Linsley "obtained two fine males" at Stratford, Conn. Mr. Grinnell informs

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* Am. Jour. of Sci. and Arts, vol. xliv, No. 2, p. 269, April, 1843.

† Birds of the Northwest, p. 564, 1874.

‡ Reprinted in Force's Historical Tracts. Tract 5, p. 47.

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me that he took two or three specimens about October 8th, 1875, in the Sound near Milford, Conn., and that he does not consider them particularly rare at that season. Mr. W. W. Coe has an immature specimen taken at Saybrook, December 8th, 1874. Mr. Robert Morris, of this city, tells me that he has killed it late in *July* on the West Haven meadows, and has also seen it, about the same time, and early in August, on the Quinnipiac marshes.

241. Aix sponsa (Linné) Boie. Wood Duck; Summer Duck.

A tolerably common summer resident, breeding in holes in trees. Arrives in March (March 18, 1876, Osborne), frequenting fresh-water ponds and streams, and lays in May, remaining into October. Zadock Thompson, speaking of its occurrence in Vermont, says, "The Wood Duck is one of the most beautiful birds seen in this State, and is one of the very few permanent residents here."*

242. Fuligula marila (Linné) Stephens. Greater Scaup Duck; Broad-bill; Blue-bill.

A rather common winter resident. Taken at Stratford by Linsley. Captain Brooks informs me that they are "plenty at Guilford, Conn." In spring Mr. Thos. Osborne has taken it as late as May 17th, (1876).

243. Fuligula affinis Eyton. Lesser Scaup Duck; Blue-bill.

A common winter resident. Much prized for the table. Remains till the middle or latter part of May (May 17, Osborne). Dr. Wood, of East Windsor Hill, once killed it on the Connecticut river in his vicinity.

244. Fuligula collaris (Donovan) Bonaparte. Ring-necked Duck.

It is not common, but may winter. Linsley took it at Stratford, Conn. Mr. Geo. Bird Grinnell has secured several specimens of this bird, in early spring, on the Sound, in the vicinity of Milford, Conn.

245. Fuligula ferina, var. **Americana** (Eyton) Coues. Red-head; Pochard.

Rare. Lindsley took it at Stratford. The Rev. J. H. Hand writes me that he has taken it at Westbrook, Conn., but it is rare there.

* History of Vermont, p. 109, 1842.

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246. Fuligula vallisneria (Wilson) Stephens. Canvas-back Duck.

Rare. "Stratford," Conn., (Linsley). Mr. Robert Morris, of this city, saw one May 7th, 1876. I am also informed by Rev. J. H. Hand, that it occurs at Westbrook, Conn., where it is "very rare."

247. Bucephala clangula (Linné) Coues. Golden-eye; Whistler.

A common winter resident. Found on the Sound and on fresh-water ponds from November to April or May (April 16th). Captain Brooks writes, from Faulkner's Island, "only see them here in very cold winters."

248. Bucephala Islandica (Gmelin) Baird. Barrow's Golden-eye.

A rare winter visitant from the North. I include this species on the strength of a most typical (male adult) specimen in the cabinet of Mr. John H. Sage, of Portland, Conn. It was purchased by him, November 14th, 1867, from a man who said it was killed on the Sound, and there seems to be no reason for doubting his statement, especially since it has been taken as far South as New York City.* Mr. William Brewster records it from Massachusetts with the following remarks: "I obtained an adult female in the flesh from Cape Cod, December 7th, 1871, which was pronounced by Prof. Baird unquestionably *B. Islandica.* Since then I have seen numbers of females and two fine adult males in the Boston Markets, most of them shot within State limits."†

249. Bucephala albeola (Linné) Baird. Butter-ball; Buffle-head.

A common winter resident. Found both on fresh and salt water. Nov., April 15th.

250. Harelda glacialis (Linné) Leach. Long-tailed Duck; Old Wife; Old Squaw.

A common winter resident; particularly abundant during the migrations. Captain Brooks writes me that they "usually come in October and leave in April or May." This is by far the most abundant species of duck found along our coast, and during the migrations (notably in November) hundreds of thousands of them can be seen on the Sound, covering the water as far as the eye can reach in every direction, and almost deafening one by their constant, and, to my ears, not altogether unpleasant, cackle. They are continually on the

* Coues' Birds of New England, p. 52 (300), 1868.

† Am. Nat., vol. vi, No. 5, pp. 306–7, May, 1872.

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move and, notwithstanding their immense numbers, it is no easy task to approach within gunshot of the flock. Though as strictly maritime as any of our ducks, they have occasionally strayed so far into the interior as Central Ohio (Wheaton). Mr. Grinnell tells me that they occasionally breed here, but these may be wounded birds—"pensioners" as they are commonly called. Dr. Wood has taken it on the Connecticut River above Hartford, but it is rare there.

251. Camptolæmus Labradorius (Gmelin) Gray. Labrador Duck; Pied-Duck.

A very rare winter visitor. Linsley took it at Stratford, Conn. Pennant, in his Arctic Zoölogy, says that this species was "sent from *Connecticut* to Mrs. Blackburn,* in England."

252. Somateria mollissima (Linné) Leach. Eider Duck.

A rare winter visitant along our coast. Linsley states that "one or two Eider Ducks were killed" at Stratford, Conn., by Mr. Lucius Curtis. Mr. Grinnell tells me, that he saw a specimen killed on the Sound, near Milford, Conn., by a gunner (Samuel Brown by name) May 29th, 1877, and that two Eider Ducks, *probably* of this species, were shot there in the fall of 1874.

253. Somateria spectabilis (Linné) Boie. King Eider.

A rare winter visitor, like the last, and also taken at Stratford, Conn., by Linsley, who says of it: "I have obtained here this season two specimens of the King Duck, said never to have been seen here before. They are among the best for the table."† Giraud also states that "an adult male in perfect plumage was shot on Long Island Sound, in the winter of 1839."‡

254. Œdemia Americana (Wilson) Swainson. Black Scoter; Gray Coot.

A tolerably common winter resident, but less so than either of the following. Linsley had it from Stratford. Have seen it early in October (October 4, 1876), and again in November, but the hunters regard it as rather rare. It sometimes visits the Great Lakes in winter, and Dr. Wheaton writes me that one was taken near Columbus, Ohio, in Dec. 1876.

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* Arctic Zoölogy, vol. ii, p. 559, 1785. † Catal. Birds of Conn., p. 270, 1843.
‡ Birds of Long Island, p. 333, 1844.

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255. Œdemia fusca (Linné) Swainson. Velvet Scoter; White-winged Coot.

It is extremely abundant on the Sound during fall and spring, some remaining through the winter. Arrives about the first of October (Oct. 4, 1876) remaining till the middle of May (May 15, Osborne). Thousands of them visit the coast in October and November, to feed on the small shells (chiefly *Mactra lateralis*) which the shallow muddy bottoms furnish in abundance. And in this connection it may be stated that the alimentary canals of the sea ducks afford the conchologist a rich collecting ground, often yielding unexpected treasures. This is the largest of our ducks, and like the other members of the genus, is generally considered unfit for the table, but when properly prepared and well cooked they are by no means bad eating. Though properly marine, it occurs on the Great Lakes in winter. Dr. Wood has taken two specimens on fresh water near East Windsor Hill, Conn.

256. Œdemia perspicillata (Linné) Stephens. Surf Duck; Sea Coot; Scoter.

A common winter resident, being intermediate in numbers between the two foregoing—that is, it is neither abundant nor uncommon. Arrives late in September or early in October, remaining till the middle of April. Captain Brooks writes me that they are "common through fall, winter and spring" in the vicinity of Faulkner's Island, Conn. I saw one on the Sound as late as June 29th (1877), and I am told that a few generally remain all summer, but they are probably "pensioners" (wounded birds) and do not breed.

257. Erismatura rubida (Wilson) Bonaparte. Ruddy Duck.

It is not rare during the migrations. Taken by Linsley at Stratford, Conn. Found both on fresh and salt water. Mr. J. N. Clark, of Saybrook, tells me that they are rather common about the mouth of the Connecticut, but that full plumaged birds are very rare. Dr. Crary tells me that one was shot on the Connecticut river, near Hartford, Conn., in October, 1858.

258. Mergus merganser Linné. Sheldrake; Merganser.

It is common during migrations; some probably wintering. Frequents fresh water lakes and rivers in the interior, and is "found occasionally about Guilford Harbor, Conn." (Capt. Brooks), but is not

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common on salt water. Remains into April (April 17, 1875, male adult shot, Sage).

259. Mergus serrator Linné. Red-breasted Merganser.

A common migrant, wintering on the Sound. Captain Brooks writes me that they are "plenty during winter and spring" about Faulkner's Island, Conn. Also found on fresh water. Remains into April (April 14, 1876, Osborne).

260. Mergus cucullatus Linné. Hooded Merganser.

A winter resident; not common. Linsley obtained two specimens from a fresh water pond near Stratford, Conn., and Captain Brooks has one, killed at Guilford, Conn., on salt water. Mr. W. W. Coe has taken it during the migrations, and I saw one in November, 1875. Dr. Wood has taken it near East Windsor, but finds it rarely. Mr. Sage writes me that one was killed near Middletown, Conn., March 6th, 1876.

Family, SULIDÆ.

261. Sula bassana Linné. Common Gannet; Solon Goose.

A rare winter visitant. Captain Brooks writes me from Faulkner's Island, that he has "only seen two specimens," one of which is now in his collection: "It was killed at Guilford, Conn., in the spring, about ten years ago." Linsley took it at Stratford, Conn., and, concerning its gastronomic proclivities, remarks: "The true Solon Goose killed here, which I presented to the Yale Natural History Society, had in its stomach a bird, and in the stomach of the latter was also a bird—destruction on destruction. Mr. B. Silliman, Jr., and Dr. Whelpley, who opened the stomach, observed this fact, as the former gentleman informed me. It was previously supposed this bird lived wholly on fishes."* Giraud saw a few specimens, killed about Long Island and in the vicinity of New York City.† Mr. Robert Morris tells me that he saw an adult specimen, shot off Branford late in the fall of 1872 or 1873. There is also, in a restaurant in New Haven, a mounted example of the young of the Solon Goose, killed near here a few years ago.

* Am. Jour. Sci. and Arts, vol. xliv, No. 2, p. 271, April, 1843.

† Giraud's Birds of Long Island, p. 345, 1844.

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262. Sula fiber Linné. Booby Gannet.

A rare or accidental visitor from the South. Linsley took it at Guilford, Conn. It has been taken as far north as Massachusetts (in September).*

Family, PHALACROCORACIDÆ.

263. Graculus carbo (Linné) Gray. Common Cormorant; Shag.

A tolerably common winter visitant. Captain Brooks writes me that they are "plenty in April and May" and are sometimes seen in fall. Linsley took it at Stonington, Conn.

264. Graculus dilophus (Swainson) Gray. Double-crested Cormorant.

It occurs along the coast during fall and spring, but usually not in very large numbers, though Captain Brooks informs me that he "saw large flocks of them feeding about Faulkner's Island, Conn., in the month of May, 1876," and that he captured two of them. Linsley had a specimen from Stratford, Conn., and he regarded it as a very rare bird. Mr. W. W. Coe has a specimen in his cabinet which he killed on the Connecticut River, near Middletown, October 29th, 1875. While out duck-hunting at the mouth of the East Haven River, November 13th, 1875, with Mr. Thomas Osborne, we saw a Cormorant which I judged to be of this species. In speaking of the Cormorants, in New England, in 1675, Josselyn observes: "Though I cannot commend them to our curious palats, the *Indians* will eat them when they are fley'd, they take them prettily, they roost in the night upon some Rock that lyes out in the Sea, thither the *Indian* goes in his Birch-*Canow* when the moon shines clear, and when he is come almost to it, he lets his *Canow* drive on of it self, when he is come under the Rock he shoves his boat along till he come just under the *Cormorants* watchman, the rest being asleep, and so soundly do sleep that they will snore like so many Pigs; the *Indian* thrusts up his hand of a sudden, grasping the watchman so hard round about his neck that he cannot cry out; as soon as he hath him in his *Canow* he wrings off his head, and making his *Canow* fast, he clambreth to the top of the Rock, where walking softly he takes them up as he pleaseth, still wringing off their heads; when he hath slain as many as his *Canow* can carry, he gives a shout which awaketh the surviving *Cormorants*, who are gone in an instant."†

* Putnam, in Proceed. Essex Inst., vol. i, p. 221, 1856.

† Josselyn's Two Voyages to New England, p. 102, 1675.

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Family, TACHYPETIDÆ.

265. Tachypetes aquilus (Linné) Vieillot. Frigate Pelican; Man-of-war Bird.

An extremely rare accidental visitor from the South. But one instance of its occurrence in New England has been recorded, and that was published in the Naturalist, by Mr. Grinnell, nearly two years ago: "The occurrence of *Tachypetes aquilus* in Connecticut is not generally known, Long Island being, up to this time, the northernmost locality on record for this bird. A female of this species was killed at Faulkner's Island in this State, in the autumn of 1859, and is now in the collection of Captain Brooks. It was hovering over the island when shot."* I have seen this specimen in Capt. Brooks's Cabinet.

Family, LARIDÆ.

266. Stercorarius parasiticus (Brünn.) Schæff. Richardson's Jaeger.

A rare winter visitor. Linsley gave it from Bridgeport, Conn. I have lately seen a specimen in the cabinet of Mr. John H. Sage, of Portland, Conn., which was killed at that place in the fall of 1875.

267. Stercorarius Buffoni (Boie) Coues. Long-tailed Jaeger.

A rare straggler from the far North. Not previously recorded from Connecticut. I have just received, from Mr. Wm. F. Lane, a beautiful adult specimen of this Larine plunderer, which he shot on the Community Lake at Wallingford, Conn., August 30th, 1873. Mr. Lane writes me that he was out sailing on the lake, with his brother, when they noticed a curious bird, unlike any they had ever before seen. "It was chasing a swallow, which it soon caught, and then lit on the water with the swallow in its mouth, and commenced swimming around and did not seem to be very wild." Mr. Lane then went ashore for his gun, and, on returning, sailed so close to the bird that his brother was obliged to splash the water with an oar in order to make it fly, and as it rose he shot it. He says: "The bird was alone and had been flying around the lake for about an hour when I shot it. I noticed that it was very swift on the wing, also a very fast swimmer. It did not seem at all afraid of anyone."

NOTE.—The Pomarine Jaeger, *Stercorarius pomatorhinus* (Temminck) Vieillot, doubtless occurs as a rare winter visitant.

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* Am. Nat., vol. ix, No. 8, p. 470, Aug., 1875.

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268. Larus marinus Linné. Great Black-backed Gull.

A winter resident; not rare. Linsley records it from Stratford, Conn. During January, February and March, one frequently sees two or three *Larus marinus* flying about the Harbor in company with the common Herring Gulls.

269. Larus argentatus, var. **Smithsonianus** Coues. Herring Gull.

An abundant winter resident. Arrives from the North in October, remaining till April or May. Have seen hundreds of them together in New Haven Harbor in February and March. Mr. Thomas B. Osborne has seen them in June and early September, and a few doubtless spend the summer, but they are probably young birds and do not breed. Dr. Wood occasionally finds it about the Connecticut River above Hartford. On June 29th, 1877, I saw five together near Faulkner's Island, Conn. Four of these were in the young (gray) plumage, while one was white.

270. Larus Delawarensis Ord. Ring-billed Gull.

Not rare. Linsley found it at Stonington, Conn. The young of this species, easily recognized by having "a broad, subterminal band of black" across the tail (Coues), may frequently be seen in winter associated with the foregoing. My attention was first called to it by Mr. E. P. Bicknell, of Riverdale, N. Y.

271. Larus tridactylus Linné. Kittiwake Gull.

Occurs in winter, but is not common. Linsley gave it from Stonington, Conn. Mr. Osborne has seen it from March 8th till April 14th.

272. Larus atricilla Linné. Laughing Gull.

Not common. Possibly a few breed on some of the islands off our coast. Linsley states that they were occasionally killed at Stonington, Conn. Mr. Osborne informs me that he saw one June 1st, 1876.

273. Larus Philadelphia (Ord) Gray. Bonaparte's Gull.

Is tolerably common in fall. Captain Brooks writes me that they are "quite common about Faulkner's Island, Conn., in October and November," and that he occasionally sees them "with Terns at Goose Island, Conn., *in summer.*" Linsley states that he "obtained an

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individual of this beautiful species of gull, August 1st, 1842." (p. 271.) I saw the remains of one that had been killed in November, 1875, near New Haven. Specimens of it are also in the collections of W. W. Coe and J. H. Sage of Portland, Conn., and Mr. Wm. F. Lane sends me a specimen from Wallingford, Conn., stating that eight were seen there in the fall of 1874.

NOTE.—The Burgomaster (*Larus glaucus* Brünnich) may sometimes occur along our coast as a rare winter visitant from the North, but I have, as yet, been unable to procure satisfactory evidence of its presence within our limits, although it has been taken on Long Island.

274. Sterna hirundo Auct. Common Tern; Wilson's Tern; Sea Swallow.

A common summer resident along the coast. Captain Brooks informs me that they breed at Goose Island, Conn., but not so abundantly as the Roseate.

275. Sterna macroura Naumann. Arctic Tern.

A rare visitor to our shores. Mr. J. N. Clark, of Saybrook, Conn., writes me: "I have an undoubted specimen, in the fall plumage of the young (as described by Coues), taken here last season—never captured a mature bird."

276. Sterna Dougalli Mont. (*S. paradisea* of Authors.)* Roseate Tern.

An abundant summer resident. Captain Brooks writes me that they first "make their appearance about the middle of May, and commence laying about the first of June, at Goose Island (one mile west of Faulkner's Island, Conn.), where they breed in great quantities, if not disturbed." Through the kindness of Captain Brooks I have recently (June 29, 1877) visited Goose Island, and have thus been permitted to witness the magnificent aërial evolutions of these beautiful birds, as hundreds of them swept to and fro over our heads, constantly uttering their characteristic cries. They would rise high in the air and immediately dive to the water's edge, then, suddenly turning, would sweep over the island and settle on the large rocks with which it is bordered, always, as the Captain remarked, "keeping their heads to the wind'ard." It is truly a splendid sight, and one well worth going many miles to see. As they cover the rocks, almost

* See Coues' Birds of the Northwest, p. 688 1874.

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hiding them from view, their jet-black caps and pearly mantles contrast nicely with the pure white under parts and bright red legs. The eggs were now hatching and thousands of downy young covered the island. I actually caught an adult female entangled in the weeds, among which their eggs are deposited with little attempt at a nest. Where the weeds were particularly thick, forming dense mats, the eggs were sometimes placed *upon*, as well as under, them; we found several such. Great credit is due Captain Brooks for his watchfulness over this little colony: but for him they would long since have been exterminated.

277. Sterna superciliaris, var. **Antillarum** Coues. Least Tern.

Not very common. Linsley took it at Stratford, Conn. Mr. J. N. Clark, of Saybrook, Conn., tells me that they are sometimes quite abundant there during the migrations.

278. Sterna fuliginosa Gmelin. Sooty Tern.

A rare visitor from the South. The claim of this species to a place among the birds of New England has only recently been established, the only authentic instances of its capture having been published within a year. In a late number of the "Bulletin," Mr. H. A. Purdie says that Mr. J. N. Clark has a specimen in his collection "that last summer flew against the side of the steamboat wharf depot, at Saybrook, Conn. Stunned by the concussion it fell and was picked up. It had been noticed for several days, flying about the mouth of the river, as something unusual."* Professor Sanborn Tenney states that a specimen was killed as far inland as the northwestern corner of Massachusetts ("near the Hoosac river") in September, 1876.† These two are, so far as I am aware, the only recorded instances of its capture in New England. Mr. Frederick T. Jencks, of Providence, R. I., writes me that he took a particularly beautiful specimen at Point Judith, R. I., last fall. Through the kindness of several friends I am enabled to add five Connecticut examples to those given above, thus increasing the total number ascertained to have occurred in New England to eight, six of which were killed in this State. Four of these I have myself seen. The circumstances connected with the capture of these specimens are as follows: Two adult birds killed themselves, last September (1876), by flying against the lighthouse

* Bull. Nutt. Ornith. Club, vol. ii, No. 1, p. 22, Jan., 1877.

† Am. Nat., vol. xi, No. 4, p. 243, April, 1877.

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tower at Faulkner's Island. One of them is now in the collection of Captain O. N. Brooks, of that place; Mr. Elbert Coe, of Stony Creek, Conn., has a mounted specimen which was killed there with a stone, late in the summer (1876); Mr. Norman Elmore, of Granby, Conn., has just sent me, for examination, a bird of this species that was taken in that vicinity, September 20th, 1876. The Rev. J. Howard Hand writes me that it was procured in a singular place and manner: "It was knocked down with a stick by a gentleman who was netting wild pigeons. He first saw it, I think, on the pigeon poles. He got it alive, but of course could not get it to eat, and after keeping it two or three days it died." This and the one killed by Elbert Coe, Esq., are both in the young-of-the-year plumage, as described by Coues.* It is a singular fact that all these specimens were killed last fall, and probably all in September.

279. Hydrochelidon lariformis (Linné) Coues. Black Tern; Short-tailed Tern.

A rare visitor, occurring chiefly in fall. Though essentially an inland species, it is sometimes found along the coast during migrations. Captain Brooks took one specimen near Goose Island, Conn., about twelve years ago—the only one he has ever seen. Mr. George Bird Grinnell informs me that his brother shot a bird of this species, late in August, near Milford, Conn.

NOTE.—Several species of Terns, not mentioned above, doubtless occur within our limits. *Sterna anglica* (*aranea* of Wilson), *S. cantiaca* (*acuflavida* of Cabot), *S. caspia* and *S. regia* are to be looked for as rare visitors from the South, while *S. Forsteri* probably occurs occasionally in fall, and *Xema Sabini* may be met with as a rare straggler from the North.

Family, PROCELLARIIDÆ.

280. Cymochorea leucorrhoa (Vieillot) Coues. Leach's Petrel.

Tolerably common off the coast in summer, but is more frequently seen outside than in the Sound. I am informed by Captain Brooks that it is "occasionally seen during the summer months cruising in the vicinity of Faulkner's Island." Dr. Crary, of Hartford, Conn.,

* Birds of the Northwest, p. 699, 1874.

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informs me that one was shot, October 27th, 1857, by George Meigs, on the Connecticut River, above Hartford. Since writing the above I have twice seen it, on the Sound, in the vicinity of Faulkner's Island, and near New Haven. I am inclined to believe that a few breed on some of the islands off our coast.

281. Oceanites oceanica (Kuhl.) Coues. Wilson's Petrel.

Not common; occurs off the coast in summer. Linsley says that he has seen this species "not only in our Sound, but even west of Stratford, and sitting quietly upon the water," and further states that he once caught a specimen "at sea, by floating about two hundred feet of thread in the air, against which it flew, and thus became entangled and taken."*

282. Puffinus major Faber. Greater Shearwater.

Not rare in winter off the coast, but generally keeps outside the Sound. Linsley found it common about Stonington in the southeast corner of the State. In the Museum of Wesleyan University, at Middletown, is the head of a bird of this species which is said to have been killed at Granby, Conn.

Family, COLYMBIDÆ.

283. Colymbus torquatus Brunn. Loon; Great Northern Diver.

A tolerably common winter resident. Arrives from the North in October, remaining till April or May (April 29, 1876, Osborne). I have a beautiful specimen killed at Branford, Conn., April 23d, 1875, and presented to me by Dr. Wm. H. Hotchkiss. It has been known to breed on a pond at Easthampton, Conn. (W. G. Buell).

284. Colymbus septentrionalis Linné. Red-throated Diver.

A common winter resident. Arrives from the North in October, remaining till May, and Mr. Nichols informs me that he saw one as late as June 2d (1877). Frequently killed by gunners while duck-shooting on the Sound, but, as Captain Brooks writes, "you seldom get one with the red throat."

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* Am. Jour. Sci. Arts, vol. xliv, No. 2, . 272, April, 1843.

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NOTE.—*Colymbus Arcticus* may possibly occur as an extremely rare winter visitor from the far north, but I am unable, as yet, to find sufficient evidence of its capture within our limits, although there is a specimen so labelled in the Museum at Middletown. I cannot believe this to be anything more than an immature *Colymbus septentrionalis*, with the throat tinged with black.

Family, PODICIPIDÆ.

285. Podiceps cristatus (Linné) Latham. Crested Grebe.

Tolerably common during the migrations and in winter. Recorded by Linsley from Stratford, Conn. W. W. Coe and J. H. Sage have each specimens of this species taken here in winter. Dr. Wood, of East Windsor Hill, tells me that he has had but four specimens from that locality, and that they were all caught by hand in winter; having alighted in the snow they were unable to rise.

286. Podiceps griseigena, var. **Holbolli** (Reinh.) Coues. Red-necked Grebe.

A rather rare winter resident. Captain Brooks says he has not seen one for years. Occurs both on fresh and salt water. Linsley took it at Stratford. Mr. J. H. Sage has a specimen which he took at Saybrook, Conn., February 23d, 1875. Dr. Wood has a very handsome specimen, in full plumage, shot near East Windsor some years ago. I am informed by Dr. Crary, of Hartford, Conn., that one was shot in that vicinity, by Jerry Crocker, October 19th, 1860.

287. Podiceps cornutus Latham. Horned Grebe.

A common winter resident. Found on the coast as well as on lakes and ponds in the interior. Arrives in September or October, remaining till May.

288. Podilymbus podiceps (Linné) Lawrence. Pied-billed Dabchick; Hell Diver.

A summer resident; common during the migrations. Found chiefly on fresh water, but is not rare on the Sound. Arrives from the North in September, remaining into November (and a few may winter). Mr. Geo. Bird Grinnell tells me that it breeds within the State.

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Family, ALCIDÆ.

289. Utamania torda (Linné) Leach. Razor-billed Auk.

A rare winter visitor in the Sound.

290. Mergulus alle (Linné) Vieillot. Sea Dove; Dovekie.

A rare winter visitor on our coast. Prof. G. Brown Goode published the first authentic record of the capture of this species within the limits of the State. He states that, at Middletown, during a severe northeast storm, about the middle of November, 1871, "two individuals were captured in full winter plumage, and plump, though with empty stomachs. Their occurrence thirty miles inland is somewhat remarkable. Allen records the capture of a single specimen at Greenfield, Mass., on the Connecticut, and Linsley places the species among the birds of Connecticut on the strength of one captured near Martha's Vineyard,"* Mass. Several others were secured at Middletown, by W. W. Coe and J. H. Sage, during this same storm during which Mr. Goode's were taken, and Mr. Sage has still another, captured there November 25th, 1874, in a gale. One was taken at Saybrook at the same time. Dr. Wood, of East Windsor Hill, showed me a specimen of this species, shot near Portland, Conn., November 10th, 1849. Two other individuals were killed on a pond at Wallingford, Conn., in September, 1874, by Wm. F. Lane, Esq. It is occasionally taken as far south as Egg Harbor, New Jersey.†

291. Lomvia troile (Linné) Brandt. Foolish Guillemot; Murre.

A rare winter visitant in the Sound, though common enough outside. Captain Brooks took one near Faulkner's Island, Conn., "about eight years ago."

NOTE.—The Great Auk (*Alca impennis* Linné), supposed now to be extinct, was formerly, without doubt, a winter visitor to our coast. Aside from the three species of *Alcidæ* given above, as occurring within our limits, several others have been found still farther south and may be looked for in winter off Stonington, Conn., and perhaps even in the Sound. These are: *Fratercula Arctica*, *Uria grylle*, and *Lomvia arra*.

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* Am. Nat., vol. vi, No. 1, p. 49, Jan. 1872.

† Birds of East Penn. and New Jersey. By Wm. P. Turnbull, p. 48, 1869.

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B. *List of the Families of Birds now found in the State of Connecticut, with the number of their representative species.*

		Species.			Species.
1	Turdidæ,	8	25	Picidæ,	8
2	Saxicolidæ,	1	26	Strigidæ.	11
3	Sylviidæ,	3	27	Falconidæ,	15
4	Paridæ,	3	28	Cathartidæ,	1
5	Sittidæ.	2	29	Columbidæ,	2
6	Certhiidæ,	1	30	Tetraonidæ,	2
7	Troglodytidæ,	4	31	Charadriidæ,	6
8	Alaudidæ,	1	32	Hæmatopodidæ,	2
9	Motacillidæ.	1	33	Recurvirostridæ,	1
10	Sylvicolidæ,	34	34	Phalaropodidæ,	2
11	Tanagridæ,	2	35	Scolopacidæ,	24
12	Hirundinidæ,	6	36	Tantalidæ,	2
13	Ampelidæ,	2	37	Ardeidæ,	8
14	Vireonidæ,	5	38	Rallidæ,	9
15	Laniidæ,	2	39	Anatidæ,	32
16	Fringillidæ,	32	40	Sulidæ,	2
17	Icteridæ,	8	41	Phalacrocoracidæ,	2
18	Corvidæ.	3	42	Tachypetidæ,	1
19	Tyrannidæ,	10	43	Laridæ,	14
20	Caprimulgidæ,	2	44	Procellariidæ,	3
21	Cypselidæ,	1	45	Colymbidæ,	2
22	Trochilidæ,	1	46	Podicipidæ,	4
23	Alcedinidæ,	1	47	Alcidæ,	3
24	Cuculidæ,	2			

C. *Special Lists, giving, in tabular form, the residents, summer and winter visitants, migrants, etc.*

(A.) Summer Residents.

1. *Species known to breed.*

1 Turdus migratorius.
2 Turdus mustelinus.
3 Turdus fuscescens.
4 Mimus Carolinensis.
5 Harporhynchus rufus.
6 Sialia sialis.
7 Parus atricapillus.
8 Sitta Carolinensis.
9 Certhia familiaris.
10 Troglodytes aëdon.
11 Telmatodytes palustris.
12 Cistothorus stellaris.*
13 Mniotilta varia.
14 Parula Americana.
15 Helmitherus vermivorus.
16 Helminthophaga pinus.
17 Helminthophaga chrysoptera.*
18 Helminthophaga ruficapilla.
19 Dendrœca æstiva.
20 Dendrœca virens.*
21 Dendrœca cærulescens.*
22 Dendrœca Blackburniæ.*
23 Dendrœca Pennsylvanica.
24 Dendrœca discolor.
25 Dendrœca pinus.
26 Siurus auricapillus.
27 Siurus motacilla.
28 Geothlypis trichas.
29 Icteria virens.
30 Myiodioctes mitratus.
31 Myiodioctes Canadensis.*
32 Setophaga ruticilla.
33 Pyranga rubra.
34 Hirundo horreorum.
35 Tachycineta bicolor.
36 Petrochelidon lunifrons.
37 Cotyle riparia.
38 Progne purpurea.
39 Ampelis cedrorum.
40 Vireo olivaceus.

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* An asterisk placed after a species indicates that it does not breed abundantly.

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41 Vireo gilvus.
42 Vireo flavifrons.*
43 Vireo solitarius.
44 Vireo Noveboracensis.
45 Carpodacus purpureus.
46 Chrysomitris tristis.
47 Passerculus Savanna.
48 Pooecetes gramineus.
49 Coturniculus passerinus.
50 Coturniculus Henslowi.*
51 Ammodromus maritimus.
52 Ammodromus caudacutus.
53 Melospiza palustris.
54 Melospiza melodia.
55 Melospiza Lincolni.*
56 Spizella socialis.
57 Spizella pusilla.
58 Passer domesticus.
59 Euspiza Americana.*
60 Goniaphea Ludoviciana.
61 Cyanospiza cyanea.
62 Pipilo erythrophthalmus.
63 Dolichonyx oryzivorus.
64 Molothrus pecoris.
65 Agelæus phœniceus.
66 Sturnella magna.
67 Icterus spurius.
68 Icterus Baltimore.
69 Quiscalus purpureus.
70 Corvus Americanus.
71 Cyanurus cristatus.
72 Tyrannus Carolinensis.
73 Myiarchus crinitus.
74 Sayornis fuscus.
75 Contopus virens.
76 Empidonax Traillii.
77 Empidonax minimus.
78 Antrostomus vociferus.
79 Chordeiles Virginianus.
80 Chætura pelagica.
81 Trochilus colubris.
82 Ceryle alcyon.
83 Coccyzus erythrophthalmus.
84 Coccyzus Americanus.
85 Picus villosus.
86 Picus pubescens.
87 Melanerpes erythrocephalus.*
88 Colaptes auratus.
89 Bubo Virginianus.
90 Scops asio.
91 Otus vulgaris, var. Wilsonianus.
92 Brachyotus palustris.
93 Syrnium nebulosum.
94 Nyctale Acadica.
95 Circus cyaneus, var. Hudsonius.
96 Accipiter fuscus.
97 Accipiter Cooperi.
98 Falco communis.*
99 Falco sparverius.*
100 Buteo borealis.
101 Buteo lineatus.
102 Buteo Pennsylvanicus.
103 Pandion haliaëtus.
104 Haliaëtus leucocephalus.*
105 Ectopistes migratorius.
106 Zenædura Carolinensis.
107 Bonasa umbellus.
108 Ortyx Virginianus.
109 Ægialitis vocifera.
110 Ægialitis meloda.
111 Philohela minor.
112 Gallinago Wilsoni.*
113 Totanus semipalmatus.*
114 Ereunetes pusillus.*
115 Tringoides macularius.
116 Actiturus Bartramius.
117 Ardea herodias.*
118 Ardea virescens.
119 Nyctiardea grisea, var. nævia.
120 Botaurus minor.
121 Ardetta exilis.
122 Rallus longirostris.
123 Rallus elegans.
124 Rallus Virginianus.
125 Porzana Carolina.
126 Porzana Noveboracensis.*
127 Porzana Jamaicensis.*
128 Gallinula galeata.
129 Anas obscura.
130 Aix sponsa.
131 Harelda glacialis.*
132 Sterna hirundo.
133 Sterna Dougalli.
134 Colymbus torquatus.*
135 Podilymbus podiceps.

2. *Species which probably breed occasionally, but are not known to do so.*

1 Mimus polyglottus.
2 Polioptila cærulea.
3 Lophophanes bicolor.
4 Thryothorus Ludovicianus.
5 Dendrœca cærulea.
6 Dendrœca Dominica.
7 Siurus nævius.
8 Oporornis formosus.
9 Pyranga æstiva.
10 Stelgidopteryx serripennis.
11 Cardinalis Virginianus.
12 Contopus borealis.
13 Empidonax Acadicus.
14 Sphyrapicus varius.
15 Centurus Carolinus.
16 Strix flammea, var. Americana.
17 Falco columbarius.
18 Ægialitis Wilsonia.
19 Totanus solitarius.
20 Sterna superciliaris, var. Antillarum.
21 Cymochorea leucorrhoa.

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(B.) Resident Species.

1 Turdus migratorius.
2 Sialia sialis.
3 Parus atricapillus.*
4 Sitta Carolinensis.*
5 Certhia familiaris.*
6 Ampelis cedrorum.
7 Carpodacus purpureus.
8 Chrysomitris tristis.
9 Melospiza melodia.
10 Spizella socialis.
11 Passer domesticus.*
12 Molothrus pecoris.
13 Agelæus phœniceus.
14 Sturnella magna.
15 Corvus Americanus.
16 Cyanurus cristatus.*
17 Ceryle alcyon.
18 Picus villosus.*
19 Picus pubescens.*
20 Melanerpes erythrocephalus.
21 Colaptes auratus.
22 Bubo Virginianus.*
23 Scops asio.*
24 Otus vulgaris, var. Wilsonianus.*
25 Brachyotus palustris.*
26 Syrnium nebulosum.*
27 Nyctale Acadica.*
28 Falco communis.
29 Falco columbarius.
30 Falco sparverius.
31 Buteo borealis.
32 Buteo lineatus.
33 Buteo Pennsylvanicus.
34 Haliaëtus leucocephalus.
35 Zenædura Carolinensis.
36 Bonasa umbellus.*
37 Ortyx Virginianus.*
38 Philohela minor.
39 Anas obscura.
40 Harelda glacialis.
41 Larus argentatus, var. Smithsonianus.

(C.) Migrants not known to breed.

(Some of these winter.)

1 Turdus Pallasi.
2 Turdus Swainsoni.
2[a] Turdus Swainsoni, var. Aliciæ.
3 Regulus calendula.
4 Regulus satrapa.
5 Anorthura troglodytes, var. hyemalis.
6 Anthus Ludovicianus.
7 Helminthophaga peregrina.
8 Dendrœca coronata.
9 Dendrœca striata.
10 Dendrœca castanea.
11 Dendrœca maculosa.
12 Dendrœca tigrini.
13 Dendrœca palmarum.
14 Siurus nævius.
15 Oporornia agilis.
16 Geothlypis Philadelphia.
17 Myiodioctes pusillus.
18 Passerculus princeps.
19 Junco hyemalis.
20 Spizella monticola.
21 Zonotrichia albicollis.
22 Zonotrichia leucophrys.
23 Passerella iliaca.
24 Scolecophagus ferrugineus.
25 Contopus borealis.
26 Empidonax flaviventris.
27 Sphyrapicus varius.
28 Aquila chrysaëtus.
29 Squatarola Helvetica.
30 Charadrius fulvus, var. Virginicus.
31 Ægialitis semipalmata.
32 Hæmatopus palliatus.
33 Strepsilas interpres.
34 Recurvirostra Americana.
35 Phalaropus fulicarius.
36 Macrorhamphus griseus.
37 Tringa minutilla.
38 Tringa maculata.
39 Tringa fuscicollis.
40 Tringa maritima.
41 Tringa alpina.
42 Tringa subarquata.
43 Tringa canutus.
44 Calidris arenaria.
45 Limosa fedoa.
46 Limosa Hudsonica.
47 Totanus melanoleucus.
48 Totanus flavipes.
49 Totanus solitarius.
50 Tryngites rufescens.
51 Numenius longirostris.
52 Numenius Hudsonicus.
53 Numenius borealis.
54 Fulica Americana.
55 Anser hyperboreus.

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* Those marked with an asterisk are resident *individually*, while the others are represented here, in winter, by individuals which probably breed much farther north.

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56 Branta bernicla.
57 Branta Canadensis.
57a Branta Canadensis, var. Hutchinsii.
58 Anas boschas.
59 Dafila acuta.
60 Chaulelasmus streperus.
61 Mareca Americana.
62 Querquedula Carolinensis.
63 Querquedula discors.
64 Spatula clypeata.
65 Fuligula marila.
66 Fuligula affinis.
67 Fuligula collaris.
68 Fuligula ferina, var. Americana.
69 Fuligula vallisneria.
70 Bucephala clangula.
71 Bucephala Islandica.
72 Bucephala albeola.
73 Œdemia Americana.
74 Œdemia fusca.
75 Œdemia perspicillata.
76 Erismatura rubida.
77 Mergus merganser.
78 Mergus serrator.
79 Mergus cucullatus.
80 Graculus carbo.
81 Graculus dilophus.
82 Larus tridactylus.
83 Larus atricilla.
84 Larus Philadelphia.
85 Sterna macroura.
86 Sterna superciliaris, var. Antillarum.
87 Hydrochelidon lariformis.
88 Colymbus septentrionalis.
89 Podiceps cristatus.
90 Podiceps cornutus.

(D.) Winter residents, more or less regular.*

1 Regulus satrapa.
2 Sitta Canadensis.
3 Anorthura troglodytes, var. hyemalis.
4 Eremophila alpestris.
5 Dendrœca coronata.
6 Collurio borealis.
7 Pinicola enucleator.
8 Plectrophanes nivalis.
9 Spizella monticola.
10 Junco hyemalis.
11 Zonotrichia albicollis.
12 Nyctea Scandiaca.
13 Astur atricapillus.
14 Archibuteo lagopus, var. Sancti-Johannis.
15 Tringa maritima.
16 Branta bernicla.
17 Branta Canadensis.
18 Dafila acuta.
19 Fuligula marila.
20 Fuligula affinis.
21 Bucephala clangula.
22 Bucephala albeola.
23 Harelda glacialis.
24 Œdemia Americana.
25 Œdemia fusca.
26 Œdemia perspicillata.
27 Mergus serrator.
28 Mergus cucullatus.
29 Graculus carbo.
30 Larus marinus.
31 Larus argentatus.
32 Larus Delawarensis.
33 Colymbus torquatus.
34 Colymbus septentrionalis.
35 Podiceps cristatus.
36 Podiceps cornutus.

(E.) Winter visitants, more or less irregular.

1 Lophophanes bicolor.
2 Parus Hudsonicus.
3 Ampelis garrulus.
4 Loxia leucoptera.
5 Loxia curvirostra, var. Americana.
6 Ægiothus linaria.
7 Chrysomitris pinus.
8 Plectrophanes Lapponicus.
9 Passerculus princeps.
10 Zonotrichia leucophrys.
11 Hylotomus pileatus.
12 Picoides Arcticus.
13 Syrnium cinereum.
14 Surnia ulula, var. Hudsonia.
15 Nyctale Tengmalmi, var. Richardsoni.
16 Aquila chrysaëtus.
17 Cygnus Americanus.
18 Anser hyperboreus.
19 Bucephala Islandica.
20 Camptolæmus Labradorius.
21 Somateria mollissima.
22 Somateria spectabilis.
23 Sula bassana.
24 Stercorarius parasiticus.
25 Stercorarius Buffoni.
26 Larus tridactylus.
27 Puffinus major.
28 Podiceps griseigena, var. Holbolli.
29 Utamania torda.
30 Mergulus alle.
31 Lomvia troile.

* This list does not include resident species.

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(F.) Irregular Summer Visitants.

1 Mimus polyglottus.
2 Polioptila cærulea.
3 Lophophanes bicolor.
4 Dendrœca cærulea.
5 Dendrœca Dominica.
6 Oporornis formosus.
7 Pyranga æstiva.
8 Stelgidopteryx serripennis.
9 Euspiza Americana.
10 Cardinalis Virginianus.
11 Corvus ossifragus.
12 Milvulus forficatus.
13 Empidonax Acadicus.
14 Centurus Carolinus.
15 Strix flammea, var. Americana.
16 Cathartes aura.
17 Ægialitis Wilsonia.
18 Ibis falcinellus, var. Ordii.
19 Ibis alba.
20 Ardea egretta.
21 Ardea candidissima.
22 Ardea cærulea.
23 Porzana Jamaicensis.
24 Porphyrio Martinica.
25 Sula fiber.
26 Tachypetes aquilus.
27 Sterna fuliginosa.
28 Hydrochelidon lariformis.
29 Cymochorea leucorrhoa.
30 Oceanites oceanica.

(G.) Rare Accidental Visitors.

1 Mimus polyglottus.
2 Polioptila cærulea.
3 Lophophanes bicolor.
4 Parus Hudsonicus.
5 Oporornis formosus.
6 Pyranga æstiva.
7 Stelgidopteryx serripennis.
8 Ampelis garrulus.
9 Collurio Ludovicianus.
10 Euspiza Americana.*
11 Cardinalis Virginianus.
12 Corvus ossifragus.
13 Milvulus forficatus.
14 Hylotomus pileatus.
15 Picoides Arcticus.
16 Centurus Carolinus.
17 Strix flammea, var. Americana.
18 Syrnium cinereum.
19 Surnia ulula, var. Hudsonia.
20 Nyctale Tengmalmi, var. Richardsoni.
21 Aquila chrysaëtus.
22 Ægialitis Wilsonia.
23 Steganopus Wilsoni.
24 Phalaropus fulicarius.
25 Limosa Hudsonica.
26 Ibis falcinellus, var. Ordii.
27 Ibis alba.
28 Ardea egretta.
29 Ardea candidissima.
30 Ardea cærulea.
31 Porzana Jamaicensis.
32 Porphyrio Martinica.
33 Cygnus Americanus.
34 Bucephala Islandica.
35 Camptolæmus Labradorius.
36 Somateria mollissima.
37 Somateria spectabilis.
38 Sula bassana.
39 Sula fiber.
40 Tachypetes aquilus.
41 Stercorarius parasiticus.
42 Stercorarius Buffoni.
43 Sterna fuliginosa.
44 Utamania torda.
45 Mergulus alle.
46 Lomvia troile.

(H.) Rare and Irregular Migrants.

1 Passerculus princeps?
2 Ægialitis Wilsonia.
3 Hæmatopus palliatus.
4 Steganopus Wilsoni.
5 Phalaropus fulicarius.
6 Tringa subarquata.
7 Limosa fedoa.
8 Limosa Hudsonica.
9 Numenius borealis.
10 Cygnus Americanus.
11 Anser hyperboreus.
12 Anas boschas.
13 Chaulelasmus streperus.
14 Fuligula collaris.
15 Fuligula ferina, var. Americana.
16 Fuligula vallisneria.
17 Bucephala Islandica.
18 Sterna macroura.
19 Hydrochelidon lariformis.

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* Although Linsley gives this species as "very common" at New Haven (in 1842), it has not since been met with, either by myself or any other collector in the State, so far as I am aware. Hence I am forced to regard its occurrence, at present, as purely accidental, and this without doubting Linsley's statement that it was once common. Indeed, I have recently seen two of Linsley's mounted specimens.

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D. *An Analysis of Linsley's "Catalogue."*

In the year 1843, the Rev. James H. Linsley published, in the American Journal of Science and Arts, "*A Catalogue of the Birds of Connecticut, arranged according to their natural families.*"* In this Catalogue Mr. Linsley enumerates 302 species, this number including both those that had already been detected within our limits and those whose presence he thought likely (judging from their occurrence in contiguous States) future investigation might reveal. He likewise included the introduced and domesticated species—such as the California Quail, Pea Cock, Guinea Fowl, the various races of the domestic Pigeon (*Columba livia* Linné), and the common barn-yard fowl, numbering them with our native birds. Many species are given twice, and some even three times, the immature and seasonal plumages having been mistaken for distinct species. There are also a few doubtful forms, and at least two had been exterminated before his paper was written.† Hence it is that a critical examination of this list at once enables us to eliminate 63 species, thus reducing the total number from 302 to 239.‡

(A.) LIST OF THOSE SPECIES GIVEN BY LINSLEY IN HIS "*Catalogue of the Birds of Connecticut,*" CONCERNING THE OCCURRENCE OF WHICH HE PROBABLY HAD SUFFICIENT PROOF.

1 Turdus migratorius.
2 Turdus mustelinus.
3 Turdus Pallasi.
4 Turdus fuscescens.
5 Mimus polyglottus.
6 Mimus Carolinensis.
7 Harporhynchus rufus.
8 Sialia sialis.
9 Regulus calendula.
10 Regulus satrapa.
11 Polioptila cærulea.
12 Lophophanes bicolor.
13 Parus atricapillus.
14 Sitta Carolinensis.
15 Sitta Canadensis.
16 Certhia familiaris.
17 Troglodytes aëdon.
18 Anorthura troglodytes, var. hyemalis.
19 Telmatodytes palustris.
20 Cistothorus stellaris.
21 Eremophila alpestris.
22 Anthus Ludovicianus.
23 Mniotilta varia.
24 Parula Americana.
25 Helmitherus vermivorus.
26 Helminthophaga ruficapilla.
27 Dendrœca æstiva.
28 Dendrœca virens.
29 Dendrœca cærulescens.
30 Dendrœca cærulea.
31 Dendrœca coronata.
32 Dendrœca Blackburniæ.
33 Dendrœca striata.
34 Dendrœca castanea.
35 Dendrœca Pennsylvanica.
36 Dendrœca maculosa.
37 Dendrœca tigrina
38 Dendrœca discolor.
39 Dendrœca palmarum.
40 Dendrœca pinus.
41 Siurus auricapillus.
42 Siurus nævius.
43 Geothlypis trichas.
44 Icteria virens.
45 Myiodioctes mitratus.
46 Myiodioctes Canadensis.
47 Setophaga ruticilla.
48 Pyranga rubra.
49 Pyranga æstiva.
50 Hirundo horreorum.
51 Tachycineta bicolor.
52 Cotyle riparia.
53 Progne purpurea.
54 Ampelis cedrorum.
55 Virea olivaceus.
56 Vireo gilvus.
57 Vireo flavifrons.
58 Vireo solitarius.
59 Vireo Noveboracensis.
60 Collurio borealis.

* Am. Jour. Sci. and Arts, vol. xliv, No. 2, pp. 249–274, April, 1843.

† Namely: the Wild Turkey (*Meleagris gallopavo*) and the Pinnated Grouse or Prairie Chicken (*Cupidonia cupido*).

‡ The collection of birds given by Linsley to the Yale Natural History Society, and kept for many years in the Yale Medical School, has recently been transferred to the Peabody Museum of Yale College.

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61 Pinicola enucleator.
62 Carpodacus purpureus.
63 Loxia curvirostra, var. Americana.
64 Ægiothus linaria.
65 Chrysomitris pinus.
66 Chrysomitris tristis.
67 Plectrophanes nivalis.
68 Passerculus Savanna.
69 Pooecetes gramineus.
70 Coturniculus passerinus.
71 Ammodromus maritimus.
72 Ammodromus caudacutus.
73 Melospiza palustris.
74 Melospiza melodia.
75 Junco hyemalis.
76 Spizella monticola.
77 Spizella socialis.
78 Spizella pusilla.
79 Zonotrichia albicollis.
80 Zonotrichia leucophrys.
81 Passerella iliaca.
82 Euspiza Americana.
83 Goniaphia Ludoviciana.
84 Cyanospiza cyanea.
85 Cardinalis Virginianus.
86 Pipilo erythrophthalmus.
87 Dolichonyx oryzivorus.
88 Molothrus pecoris.
89 Agelæus phœniceus.
90 Sturnella magna.
91 Icterus spurius.
92 Icterus Baltimore.
93 Scolecophagus ferrugineus.
94 Quiscalus purpureus.
95 Corvus Americanus.
96 Corvus ossifragus.
97 Cyanurus cristatus.
98 Tyrannus Carolinensis.
99 Myiarchus crinitus.
100 Sayornis fuscus.
101 Contopus borealis?
102 Contopus virens.
103 Empidonax Traillii.
104 Empidonax minimus ("Acadicus").
105 Antrostomus vociferus.
106 Chordeiles Virginianus.
107 Chætura pelagica.
108 Trochilus colubris.
109 Ceryle alcyon.
110 Coccyzus erythrophthalmus.
111 Coccyzus Americanus.
112 Hylotomus pileatus.
113 Picus villosus.
114 Picus pubescens.
115 Sphyrapicus varius.
116 Centurus Carolinus.
117 Melanerpes erythrocephalus.
118 Colaptes auratus.
119 Strix flammea, var. Americana.
120 Bubo Virginianus.
121 Scops asio.
122 Otus vulgaris, var. Wilsonianus.
123 Brachyotus palustris.
124 Syrnium cinereum.
125 Syrnium nebulosum.
126 Nyctea Scandiaca.
127 Nyctale Acadica.
128 Circus cyaneus, var. Hudsonius.
129 Accipiter fuscus.
130 Falco communis.
131 Falco columbarius.
132 Falco sparverius.
133 Buteo borealis.
134 Buteo lineatus.
135 Buteo Pennsylvanicus.
136 Archibuteo lagopus, var. Sancti-Johannis.
137 Pandion haliaëtus.
138 Haliaëtus leucocephalus.
139 Cathartes aura.
140 Ectopistes migratorius.
141 Zenædura Carolinensis.
142 Bonasa umbellus.
143 Ortyx Virginianus.
144 Squatarola Helvetica.
145 Charadrius fulvus, var. Virginicus.
146 Ægialitis vocifera?
147 Ægialitis Wilsonia.
148 Ægialitis semipalmata.
149 Ægialitis meloda.
150 Hæmatopus palliatus.
151 Strepsilas interpres.
152 Steganopus Wilsoni.
153 Philohela minor.
154 Gallinago Wilsoni.
155 Macrorhamphus griseus.
156 Ereunetes pusillus.
157 Tringa minutilla.
158 Tringa maculata.
159 Tringa fuscicollis.
160 Tringa alpina, var. Americana.
161 Tringa canutus.
162 Calidris arenaria.
163 Limosa fedoa.
164 Limosa Hudsonica.
165 Totanus semipalmatus.
166 Totanus melanoleucus.
167 Totanus flavipes.
168 Totanus solitarius.
169 Tringoides macularius.
170 Actiturus Bartramius.
171 Numenius longirostris.
172 Numenius Hudsonicus.
173 Numenius borealis.
174 Ibis falcinellus, var. Ordii.
175 Ardea herodias.
176 Ardea candidissima.
177 Ardea cærulea.
178 Ardea virescens.
179 Nyctiardea grisea, var. nævia.
180 Botaurus minor.
181 Ardetta exilis.
182 Rallus longirostris.
183 Rallus elegans.
184 Rallus Virginianus.
185 Porzana Carolina.

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186 Porzana Noveboracensis.
187 Fulica Americana.
188 Cygnus Americanus.
189 Anser hyperboreus.
190 Branta bernicla.
191 Branta Canadensis.
192 Branta Canadensis, var. Hutchinsii.
193 Anas boschas.
194 Anas obscura.
195 Dafila acuta.
196 Chaulelasmus streperus.
197 Mareca Americana.
198 Querquedula Carolineusis.
199 Querquedula discors.
200 Spatula clypeata.
201 Aix sponsa.
202 Fuligula marila
203 Fuligula collaris.
204 Fuligula ferina, var. Americana.
205 Fuligula vallisneria.
206 Bucephala clangula.
207 Bucephala albeola.
208 Harelda glacialis.
209 Camptolæmus Labradorius.
210 Somateria mollissima.
211 Somateria spectabilis.
212 Œdemia Americana.
213 Œdemia fusca.
214 Œdemia perspicillata.
215 Erismatura rubida.
216 Mergus merganser.
217 Mergus serrator.
218 Mergus cucullatus.
219 Sula bassana.
220 Sula fiber.
221 Graculus carbo.
222 Graculus dilophus.
223 Stercorarius parasiticus.
224 Larus marinus.
225 Larus argentatus, var. Smithsonianus
226 Larus atricilla.
227 Larus Philadelphia.
228 Sterna hirundo.
229 Sterna superciliaris, var. Antillarum
230 Oceanites oceanica.
231 Puffinus major.
232 Colymbus torquatus.
233 Colymbus septentrionalis.
234 Podiceps cristatus.
235 Podiceps griseigena, var. Holbölli.
236 Podiceps cornutus.
237 Podilymbus podiceps.
238 Utamania torda.
239 Uria grylle.

(B.) LIST OF THOSE SPECIES GIVEN BY LINSLEY IN HIS "*Catalogue of the Birds of Connecticut*," CONCERNING THE OCCURRENCE OF WHICH HE DID NOT HAVE SUFFICIENT PROOF.

1 Helmitherus Swainsoni.
2 Helminthophaga chrysoptera.
3 Dendrœca Dominica.
4 Oporornis agilis.
5 Petrochelidon lunifrons.
6 Ampelis garrulus.
7 Collurio Ludovicianus.
8 Collurio Ludovicianus, var. excubitoroides.
9 Loxia leucoptera.
10 Quiscalus major.
11 Picoides Arcticus.
12 Accipiter Cooperi.
13 Recurvirostra Americana.
14 Lobipes hyperboreus.
15 Phalaropus fulicarius.
16 Tringa maritima.
17 Tringa subarquata.
18 Totanus chloropus.
19 Tryngites rufescens.
20 Ardea egretta.
21 Gallinula galeata.
22 Porphyrio Martinica.
23 Branta leucopsis.
24 Histrionicus torquatus.
25 Larus Delawarensis.
26 Larus tridactylus.
27 Rhynchops nigra.
28 Fulmarus glacialis.
29 Cymochorea leucorrhoa.
30 Puffinus obscurus.
31 Fratercula Arctica.
32 Mergulus alle.
33 Lomvia troile.
34 Lomvia arra.

(C.) LIST OF THOSE "SPECIES" GIVEN BY LINSLEY, IN HIS "*Catalogue of the Birds of Connecticut*," WHICH REPRESENT IMMATURE, ABNORMAL, OR SEASONAL PLUMAGES OF OTHER STATED SPECIES.

Name used by Linsley.	*Modern Equivalents.*
1 Regulus cristatus = tricolor.	Regulus satrapa.
2 Sylvia trochilus = æstiva.	Dendrœca æstiva.
3 Sylvia sphagnosa = Canadensis.	Dendrœca cærulescens.

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Name used by Linsley.	*Modern Equivalents.*
4 Sylvia parus = Blackburniæ.	Dendrœca Blackburniæ.
5 Sylvia autumnalis = castanea.	Dendrœca castanea.
6 Sylvia Roscoe = trichas.	Geothlypis trichas.
7 Fringilla ambigua = Icterus pecoris.	Molothrus pecoris.
8 Strix Scandiaca = Virginica (albino)?	Bubo Virginianus.
9 Falco temerarius = columbarius.	Falco columbarius.
10 Falco buteoides = hyemalis.	Buteo lineatus.
11 Falco chrysaëtos? Linné = leucocephalus.	Haliaëtus leucocephalus.
12 Falco Washingtoniensis = leucocephalus.	Haliaëtus leucocephalus.
13 Limosa Edwardsii? = Hudsonia.	Limosa Hudsonica.
14 Falligula nigra = Americana.	Œdemia Americana.
15 Phalacrocorax graculus = carbo.	Graculus carbo.
16 Larus capistratus = Bonapartii.	Larus Philadelphia.
17 Podiceps minor = Carolinensis.	Podilymbus podiceps.

(D.) LIST OF INTRODUCED AND DOMESTICATED SPECIES GIVEN BY LINSLEY, IN HIS "*Catalogue of the Birds of Connecticut*," WHICH SCARCELY DESERVE TO BE MENTIONED AMONG OUR NATIVE BIRDS.

1 Columba domestica Linné.
2 Pavo cristatus Linné.
3 Numida meleagris Linné.
4 Gallus domesticus Linné.
5 Lophortyx Californicus Bonaparte.
6 Anas moschata Willoughby.

(E.) LIST OF THOSE SPECIES GIVEN BY LINSLEY IN HIS "*Catalogue of the Birds of Connecticut*," WHICH WERE EITHER EXTERMINATED OR DRIVEN BEYOND THE STATE BEFORE HIS "CATALOGUE" WAS WRITTEN.

1 Meleagris gallopavo Linné.
2 Cupidonia Cupido Baird.

(F.) LIST OF THOSE SPECIES GIVEN BY LINSLEY IN HIS "*Catalogue of the Birds of Connecticut*," WHICH WERE INSERTED, PROBABLY, ON ERRONEOUS IDENTIFICATION.

1 Quiscalus major Vieillot.*
2 Empidonax Acadicus (Gmelin) Baird.†
3 Sylvia flava Linné.
4 Larus canus Linné.
5 Larus fuscus Linné.

(G.) LIST OF THOSE SPECIES GIVEN BY LINSLEY IN HIS "*Catalogue of the Birds of Connecticut*," THE IDENTITY OF WHICH I HAVE NOT BEEN ABLE TO DETERMINE.

1 Sylvia auricollis Latham.
2 Sylvia flava Linné.

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* "*Probably a mistake.*" Coues' Birds of the Northwest, p. 204, 1874.

† Probably *E. minimus*.

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E. *List of the Publications, containing Notes on the Birds of New England, to which reference has been made in the foregoing Review.*

A Description of New-England, by Captaine Iohn Smith. Printed at London, 1616. [Tract 1, vol. ii.]

New Englands Trials. Written by Captaine Iohn Smith, sometimes Governour of Virginia, and Admirall of New-England. London. Printed by William Iones. 1622.

New England's Plantation. Or a Short and trve description of the commodities and discommodities of that Countrey.—Written by a reuerend Diuine now there resident. London, Printed by *T. C.* and *R. C.* for *Michael Sparke*, dwelling at the Signe of the *Blue Bible* in *Greene Arbor* in the little *Old Bailey*. 1630. [Reprinted in Peter Force's Historical Tracts, vol. i, Tract 12.]

Gov. Thomas Dudley's Letter to the Countess of Lincoln, March, 1631. Printed in full in Peter Force's Historical Tracts, vol. ii, Tract iv. Washington, 1838.

New English Canaan; Or New Canaan, containing An Abstract of New England. Composed in three Bookes, etc. Written by Thomas Morton, of Cliffords Inn, Gent. Upon ten Yeers Knowledge and Experiment of the Country. Printed by Charles Green. 1632. [Reprinted in Peter Force's Historical Tracts. vol. ii, Tract 5. 1838.]

New Englands Prospect. A true, lively, and experimentall description of that part of *America*, commonly called Nevv England; discovering the state of that Countrie, both as it stands to our new-come *English* Planters; and to the old Native Inhabitants. Laying downe that which may both enrich the knowledge of the mind-travelling Reader, or benefit the future Voyager. By William Wood. Printed at London, etc. 1634.

New-England's Rarities Discovered: in Birds, Beasts, Fishes, Serpents, and Plants of that Country. [Etc.] By John Josselyn, Gent. London. 1672. [Reprinted in Archæologia Americana, vol. iv, pp. 133–238.]

An Account of two Voyages to New-England, etc. By John Josselyn, Gent. The Second Addition. *London.* Printed for G. Widdowes at the Green Dragon in St. Pauls Church-yard, 1675. [Reprinted in Collections of the Massachusetts Historical Society, vol. iii, 3d Series. 1833.]

Travels into North America; containing its Natural History, and a Circumstantial Account of its Plantations and Agriculture in general, etc.. etc. By Peter Kalm. Translated into English, by John Reinhold Forster, F.A.S. Vol. ii. London, 1771.

Arctic Zoölogy. By Thomas Pennant. Vol. ii. 1785.

The History of New-Hampshire. By Jeremy Belknap, A.M. Vol. iii. Boston, 1791.

The Natural and Civil History of Vermont. By Samuel Williams, LL.D. Published according to Act of Congress. Printed at Walpole, New Hampshire, 1794.

A Statistical Account of the County of Middlesex, in Connecticut. By David D. Field. Published by the Connecticut Academy of Arts and Sciences. Middletown, Conn. Printed by Clark & Lyman. April, 1819.

American Ornithology. By Alexander Wilson. Edinburgh, 1831.

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A Manual of the Ornithology of the United States and Canada. By Thomas Nuttall. 1832.

Report on the Geology, Mineralogy, Botany, and Zoology of Massachusetts. Made and published by order of the Government of that State, etc. By Edward Hitchcock (Report on Birds, by Ebenezer Emmons, M.D.). Amherst, 1833.

The Birds of North America. By John James Audubon. 1831–9.

A Report on the Ornithology of Massachusetts. By William B. O. Peabody. 1839.

History of Vermont, Natural, Civil, and Statistical. In three parts, with a new map of the State, and 200 engravings. By Zadock Thompson. Burlington: Chauncey Goodrich, 1842. Also Appendix to same, published in 1853.

A Catalogue of the Birds of Connecticut, arranged according to their natural families. By the Rev. James H. Linsley. [Published in the Am. Jour. Sci. and Arts, vol. xliv, No. 2, pp. 249–74. April, 1843.]

The Birds of Long Island. By J. P. Giraud, Jr. New York, 1844.

The Birds of Essex County, Mass. By F. W. Putnam. [Published in Proceed. Essex Inst., vol. i, pp. 201–31. 1856.]

A List of Birds observed at Grand Menan and at Yarmouth, N. S., from June 16 to July 8, 1856. By Dr. Henry Bryant. [From the Proceedings of the Boston Society of Natural History, vol. vi. March, 1857.]

Birds of North America. By S. F. Baird. [Vol. ix of Pacific Railroad Reports, 1858.]

Catalogue of the Birds found at Norway, Oxford County, Maine. By A. E. Verrill. With a List of the Birds found in Maine not observed at Norway. [From Proceedings of the Essex Institute, vol. iii. 1863.]

Catalogue of the Birds found in the Vicinity of Calais, Maine, and about the Islands at the Mouth of the Bay of Fundy. By George A. Boardman. [Edited by Prof. A. E. Verrill.] [From the Proceedings of the Boston Society of Natural History, vol. ix. Sept., 1862.]

Catalogue of the Birds found at Springfield, Mass., with notes on their Migrations, Habits, &c.; together with a List of those Birds found in the State not yet observed at Springfield. By J. A. Allen. 1864. [From the Proceedings of the Essex Institute, vol. iv, No. 2, pp. 48–98. July, 1864.]

A Descriptive Catalogue of the Birds of Massachusetts. By E. A. Samuels. 1864. [From Massachusetts Agricultural Report. 1863.]

Catalogue of Birds found in the vicinity of Waterville, Kennebec County, Maine. By Charles E. Hamlin. [Printed in the Tenth Annual report of the Secretary of the Maine Board of Agriculture, pp. 168–173. 1865.]

History of Lynn, Essex County, Massachusetts: including Lynnfield, Saugus, Swampscot, and Nahant. By Alonzo Lewis and James R. Newhall. Boston, 1865.

A List of the Birds of New England. By Elliott Coues, Assistant Surgeon U. S. A. [Reprinted from the Proceedings Essex Institute, vol. v.] Salem, Mass. pp. 71. 1868.

The Birds of East Pennsylvania and New Jersey. By William P. Turnbull. 1869.

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A Catalogue of the Birds of Coos County, N. H., and Oxford County, Maine. By C. J. Maynard, with Notes by William Brewster. Oct., 1871. [From the Proceedings of the Boston Society of Natural History, vol. xiv. Oct. 18, 1871. Repaged.]

Key to North American Birds. By Elliott Coues. 1872.

A Partial Catalogue of the Birds of Grand Menan, New Brunswick. By Harold Herrick. [From the Bulletin of the Essex Institute, vol. v, Nos. 2 and 3. 1873.]

The Birds of Florida. By C. J. Maynard. 1873.

A History of North American Birds. By Baird, Brewer and Ridgway. 1874.

Birds of the Northwest. By Elliott Coues. 1874–5.

The Birds of New England and Adjacent States. By Edward A. Samuels. Boston, 1875.

A Catalogue of the Birds of New England. By Thomas M. Brewer. 1875.

The Naturalist's Guide. By C. J. Maynard.

The Land and Game Birds of New England. By H. D. Minot. Salem, 1877.

American Journal of Science and Arts. New Haven.

Proceedings of the Boston Society of Natural History.

Annals of the Lyceum of Natural History. New York.

Proceedings of the Essex Institute.

Bulletin Essex Institute.

Bulletin of the Museum of Comparative Zoölogy, Cambridge.

American Naturalist. Salem.

Bulletin of the Nuttall Ornithological Club. Cambridge, 1876–7.

New England Farmer.

Hartford Times, for 1861.

Rod and Gun = American Sportsman.

Collections of the Massachusetts Historical Society.

Report of the Secretary of the Maine Board of Agriculture.

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II. A List of Writings relating to the Method of Least Squares, with historical and critical Notes. By Mansfield Merriman.

The following list contains the titles of 408 papers, books and parts of books, relating to the Method of Least Squares and the Theory of accidental Errors of Observation, chronologically arranged according to their dates of publication. The first was issued in the year 1722 and the last in 1876. Previous to 1805, the year of Legendre's announcement of the principle of Least Squares, there are 22 titles; since 1805 there is a continual yearly increase in the number of titles,—thus:

From 1805	to 1814	inclusive,	there are	18	titles,
" 1815	" 1824	"	"	30	"
" 1825	" 1834	"	"	32	"
" 1835	" 1844	"	"	45	"
" 1845	" 1854	"	"	63	"
" 1855	" 1864	"	"	71	"
" 1865	" 1874	"	"	95	"

These books and memoirs are in eight languages, and classified according to the place of publication, they fall under twelve countries. It may be interesting to note the number belonging to each, viz:

Countries.		*Languages.*	
Germany	153	German	167
France	78	French	110
Great Britain	56	English	90
United States	34	Latin	16
Belgium	19	Italian	9
Russia	16	Dutch	7
Italy	14	Danish	5
Austria	10	Swedish	4
Switzerland	9		
Holland	7	Total	408
Sweden	7		
Denmark	5		
Total	408		

The 408 titles may be roughly classified as 313 memoirs, 72 books and 23 parts of books. They were written by 193 authors, 127 of whom produced only one book or paper each.

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In preparing this list I have been able to use only the libraries of Yale College, (including that of the Connecticut Academy of Arts and Sciences,) but I think no work in those libraries relating to the subject has been left unconsulted. Out of the total 408 works, I have seen 312; the titles of these and the accompanying notes have been drawn from actual inspection. I have made no attempt to consult the literature of the Russian and Hungarian languages, and with wider library facilities the number of titles in the Italian, Dutch and Scandinavian languages would undoubtedly have been greater. Of course no work of this kind can be regarded as complete.

It has been my aim to record all writings which can be considered as contributions to the science of the Adjustment of Observations, and I think that those marked as actually inspected may be truly so regarded. Many works on Astronomy and Probability which devote but a page or two to the subject, as well as numerous practical papers, in which the Method of Least Squares is used incidentally and briefly, have been left unnoticed; to record all of these would be well nigh impossible, nor would the value of the list be thereby increased. Among the 96 which I have not seen there may possibly be a few that would be rejected after actual inspection.

The following is the arrangement of the list.

At the head of a title is placed the year of publication. In the case of memoirs this often differs from the date of the volume in which they are contained; for instance, Nautical Almanacs are published several years preceding and Transactions of Learned Societies often several years after the date which they bear. When a memoir is published in parts extending over two or more years it is recorded under the date of the first part.

The author's name follows the date of publication. In the index at the end of the list the full names of authors are given, and a distinction is made in the text when two persons of the same surname have written on the subject.

The titles of books, and pamphlets published as books, are printed in *italics*, and the titles of memoirs in ordinary type. Those which I have actually inspected have their titles enclosed in single quotation marks ('——') and these are intended to correspond with the originals in punctuation, spelling and when possible in the use of capitals.

The place of publication of books is given, with references to subsequent editions or translations. The usual terms 4to, 8vo, etc. are added, although they give little idea of the size of a book, and the

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number of pages is noted whenever I have been able to ascertain it. When a single chapter only of a book relates to the subject, the title of that chapter is treated like a memoir.

After the title of a memoir is placed *in italics* the title of the volume containing it. This is abridged in the usual manner, but the words of the title are never transposed. For instance, *Bulletin des Sciences de la Societé Philomathique de Paris* is abridged into *Bull. Soc. Philom. Paris.* In a few instances I have added the place of publication or have prefixed the name of the editor in order to ensure perfect clearness. The number of the volume and the pages which are devoted to the memoir are given; the mention of the year renders it unnecessary to note the various series.

When the work was begun it was intended to make the notes very full so as to give a tolerably complete history of the Method of Least Squares. But as the number of titles began to multiply under research it became evident that the plan would produce a manuscript too voluminous for publication. The notes were hence abbreviated into their present form. The work begun as historical has, I am afraid, ended by being largely bibliographical.

Sometimes the notes give an account of the contents of a memoir or an estimate of its value; sometimes they take the form of a direct quotation from the memoir itself or state the opinion of some subsequent reviewer; and occasionally they offer critical remarks of my own. But always they aim to give such cross references as will enable the student to follow up special lines of investigation and gain the fullest information concerning a particular memoir or book. Brief as the notes are, I hope they will be found at least suggestive by those who use them. To the future historians of mathematical science they will undoubtedly be of very great value.

The mode of cross reference usually adopted is to mention simply the year and author. Thus "1818 BESSEL" refers either to a book published in 1818 by BESSEL or to notes under that heading.

The following table points out some of the most valuable papers on the proofs of the Method of Least Squares:

First publication of the Method	see	1805	LEGENDRE.
First and Second Proofs		1808	ADRAIN.
Third Proof		1809	GAUSS.
Fourth Proof		1810	LAPLACE.
Fifth Proof		1812	LAPLACE.
Theory and Practice compared		1818	BESSEL.
Sixth Proof		1823	GAUSS.

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Seventh Proof	see 1825	IVORY.
Eighth Proof	1826	IVORY.
Proof of the Arithmetical Mean	1832	ENCKE.
Ninth Proof	1837	HAGEN.
Tenth Proof	1838	BESSEL.
LAPLACE'S Proof extended and improved	1844	ELLIS.
Eleventh Proof	1844	DONKIN.
ADRAIN'S Second Proof rediscovered	1850	HERSCHEL.
Twelfth Proof	1856	DONKIN.
On the Arithmetical Mean	1864	DEMORGAN.
[Thirteenth Proof]	1870	CROFTON.
Analysis of several Proofs	1872	GLAISHER.
Proof of the Arithmetical Mean	1875	SCHIAPARELLI.

I have drawn information from every source within my reach. On the Proofs of the Method I have found GLAISHER'S memoir of 1872 of the greatest value, and while working on the early writers TODHUNTER'S *History of Probability* was continually before me. It is here also the place to acknowledge my indebtedness to Prof. H. A. NEWTON of Yale College for valuable suggestions and kind assistance.

1722 COTES. 'Æstimatio errorum in mixta mathesi, per variationes partium trianguli plani et sphærici.' *Opera miscellanea* (appended to *Harmonia mensurarum;* Cantabrigiæ, 4to), pp. 1–22. —Memoir republished, Lemgoviæ, 1768, 8vo.

Only the closing paragraph relates to accidental errors of observation; this gives the following rule: "Sit p locus Objecti alicujus ex Observatione prima definitus, q, r, s ejusdem Objecti loca ex Observationibus subsequentibus; sint insuper P, Q, R, S pondera reciproce proportionalia spatiis Evagationum, per quæ se diffundere possint Errores ex Observationibus singulis prodeuntes, quæque dantur ex datis Errorum Limitibus; & ad puncta p, q, r, s posita intelligantur pondera P, Q, R, S, & inveniatur eorum gravitatis centrum Z: dico punctum Z fore Locum Objecti maxime probabilem, qui pro vero ejus loco tutissime haberi potest."

COTES'S rule only agrees with modern methods when the observations are directly made upon one quantity. See LAPLACE, *Théorie analytique des Probabilités*, third edition, p. cxxxviii, p. 346; and IVORY, *Phil. Mag.*, 1825, Vol. LXV, p. 4.

1749 EULER. *Pièce qui a remporté le prix de l'Academie royale des sciences en* 1748, *sur les inegalities du mouvement de Saturne et de Jupiter.* Paris, 4to.

Contains a method for the combination of linear equations similar to the following.

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1750 MAYER. Abhandlung über die Umwälzung des Mondes um seine Axe. *Kosmographische Nachrichten u. Sammlungen* for 1748, pp. 52–183.

By twenty-seven observations upon the position of a moon spot MAYER obtained twenty-seven equations each containing three unknown quantities. To solve these he added together those nine equations in which the values of the coefficients of one of the unknown quantities were the greatest, then the nine in which these coefficients were the least and lastly the remaining nine; thus obtaining three resulting equations with three unknown quantities.

MAYER's method is given by LALANDE, *Astronomie*, second edition, Vol. III, pp. 418–428. See WOLF, *Handbuch der Math.*, Vol. I, p. 279, Vol. II, p. 199; and also below, 1830 FRANCOEUR.

[1754] KÄSTNER. Om geometriska aberrationer. *Svenska Vetensk. Acad. Handl.* for 1753, p. 126. —German translation in *Schwedische Akad. Abhandl.* for 1753, p. 131.

Treats of errors in Surveying and probably contains nothing of value on the theory of accidental errors. See also the same memoirs for 1768, p. 147 and p. 159.

1755 BOSCOVICH. *De littera expeditione per Pontificam ditionem ad dimetiendos duos meridiani gradus.* Romae, 4to, pp. xxii, 516. —French translation by HUGON entitled *Voyage astronomique et geographique....*; Paris, 1770, 4to.

A method of combination of observations for the determination of the mean ellipticity of the earth from measured arcs of the meridian is here used. The method itself is proved in the French translation; see below 1760. For description of the book see TODHUNTER, *History of Theories of Attraction....*, Vol. I, pp. 305–321, 332–334.

1756 SIMPSON. 'A letter to the Right Honourable George Earl of Macclesfield, President of the Royal Society, on the Advantage of taking the Mean of a number of Observations, in practical Astronomy.' *Phil. Trans. Lond.* for 1755, Vol. XLIX, Pt. I, pp. 82–93. —Reprint, see 1757.

This memoir is interesting and valuable as being the first in which the Theory of Probability is applied to the discussion of errors of observation and in which the idea of a law of facility of error is implied. At the beginning of the letter SIMPSON says that his attention had been called to the subject by the fact that "some persons, of considerable note, have been of opinion, and even publickly maintained, that one single observation, taken with due care, was as much to be relied on as the Mean of a great number."

The letter contains two propositions; the first gives a method of determining the probability that the error of the mean of n observations shall be less than an assigned value, provided it is equally prob-

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able that the error of a single observation may be any one of the quantities, $-v, \ldots -3, -2, -1, 0, 1, 2, 3, \ldots v$. The second gives a method of determining that probability, provided the probabilities of the single errors $-v, -v+1, \ldots -1, 0, 1, \ldots v-1, v$ are proportional to the terms $1, 2, \ldots v, v+1, v, \ldots 2, 1$. This is illustrated by a numerical example. SIMPSON remarks that the advantage of the mean can be shown, whatever series be used to express the chances of the errors.

1757 SIMPSON. 'An Attempt to show the Advantage arising by Taking the Mean of a Number of Observations, in practical Astronomy.' *Miscellaneous Tracts*....(London, 4to), pp. 64–75.

A reprint of the preceding, the opening and closing paragraphs being omitted and nearly four pages of new matter added. We find here the following axioms stated for the first time; 1. that positive and negative errors are equally probable, 2. that there are certain assignable limits within which all errors may be supposed to fall.

In the added matter the case of *continuous* errors is discussed, and the probability that the error of the mean is less than an assigned value found for the case of the second proposition by making v and other quantities in the formulæ infinite. SIMPSON represents the law of facility of error geometrically by the sides of an isosceles triangle and draws a curve to show the increased precision of the mean as compared with single observations. He closes by finding under the same supposition as to the law of facility, the probability that the mean is nearer to the truth than a single observation taken at random. The whole memoir must have been extremely valuable at the time of its publication.

1760 BOSCOVICH. De recentissimis graduum dimensionibus, et figura, ac magnitudine terrae inde derivanda. *Philosophia recentior à* BENEDICTO STAY....(Romae, 3 vols., 8vo), Vol. II, pp. 406–426.

A method of combining discordant observations upon the lengths of degrees of the earth's meridian is here given. The adjustment is effected under the two conditions that the sum of the negative errors shall be equal to the sum of the positive errors, and that each sum shall have the least possible value. The problem was solved by a geometric construction depending upon the properties of the centre of gravity of figures.

The method is also given in the French translation of BOSCOVICH's work of 1755. See LINDENAU in *Zach's Monatliche Correspondenz*, 1806, Vol. XIV, p. 132; TODHUNTER, *Hist. of Theories of Attraction*...., Vol. I, pp. 321–332; and below, 1785 BERNOULLI, 1792 LAPLACE.

1760 LAMBERT. *Photometria sive de mensura et gradibus luminis*..... Augustæ Vindelicorum, 8vo, pp. [xxx], 547.

Contains many remarks on the arithmetical mean and also proposes a method for judging of the precision of the measurements, which

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consists in comparing the mean with a new mean found after rejecting that observation deviating most from the first mean. See *Nova Acta Lipsiae*, 1760, p. 560; also *Lambert's Beyträge*..... Vol. I, p. 426.

1765 LAMBERT. 'Theorie der Zuverlässigkeit der Beobachtungen und Versuche.' *Beyträge zum Gebrauche der Mathematik*....(Berlin, 1765–70, 3 vols., 8vo; second ed., 1792), Vol. I, pp. 424–488.

Contains a method of adjusting simple observations founded on the principle that the algebraic sum of the errors shall be zero. The method is illustrated by the determination of empirical formulæ for the length of the seconds pendulum, the declination of the magnetic needle, etc.

1774 LAPLACE. 'Determiner le milieu que l'on doit prendre entre trois observations données d'un même phénomène.' *Mém. Acad. Paris, par divers savans* [étrangers], Vol. IV, pp. 634–644.

This is Part V of the 'Mémoire sur la probabilité des causes par les évènemens,' which occupies pages 621–656 of the volume. It contains the first attempt to deduce a rule for the combination of observations from the principles of Probability.

LAPLACE begins by saying that the law of probability of errors of observation may be represented by a curve whose equation is $y=\varphi(x)$, x being any error and y its probability; and this curve must have three properties: 1st, it must be symmetrical with reference to the axis of y, since positive and negative errors are equally probable; 2nd, the axis of x must be an asymptote, since the probability of the error ∞ is 0; 3rd, the area of the curve must be unity, since it is certain that an error will be committed.

LAPLACE takes $\varphi(x)$ as $\varphi(x)=\frac{1}{2}me^{-mx}$ (x being regarded as always positive), but his reasons for doing so are slight. With this law he finds the mean of three observations, regarding it as corresponding to an ordinate which divides a curve $u=\varphi(x_1)\varphi(x_2)\varphi(x_3)$ into two equal parts. His result is as follows: Let M_1, M_2 and M_3 be the three measurements, of which M_1 is the least; let M_1+x be the mean; it is required to find x. Put $M_2-M_1=p$ and $M_3-M_2=q$: then x is given by

$$x=p+\frac{1}{m}\log.(1+\tfrac{1}{3}e^{-mp}-\tfrac{1}{3}e^{-mq}),$$

in which m is a constant depending upon the precision of the observation. LAPLACE then shows that this value cannot agree with the rule of the arithmetical mean, and he computes a table for finding x for certain given ratios of q to p. For instance

if $q=0.0\,p$ ..... $x=0.860\,p$,
$q=0.1\,p$ ..... $x=0.894\,p$,
$q=0.2\,p$ ..... $x=0.916\,p$,
$q=0.3\,p$ ..... $x=0.932\,p$,
. . . . . .

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Thus if three measurements of an angle give

$$M_1 = a^\circ\, b'\, 0'', \quad M_2 = a^\circ\, b'\, 40'' \quad \text{and} \quad M_3 = a^\circ\, b'\, 50'',$$

we have $p=40''$, $q=10''$ and $q=0.25p$; then from the table $x=37''$ and the adjusted result is $a^\circ\, b'\, 37''$, while by the usual rule of the average we would have $a^\circ\, b'\, 30''$. By LAPLACE's table the mean will lie nearer to the two observations which most nearly agree, than in the common method.

For remarks on this memoir see TODHUNTER, *History of Probability*, p. 469, and GLAISHER, *Mem. Astron. Soc. Lond.*, 1872, Vol. XXXIX, pp. 121–123.

[1774] LAGRANGE. 'Mémoire sur l'utilité de la méthode de prendre le milieu les résultats de plusieurs observations; dans lequel on examine les avantages de cette méthode par le calcul des probabilités; & où l'on résoud différens problêmes relatifs à cette matière.' *Miscellanea Taurinensia* (*Mél. Soc. Turin*) for 1770–1773, Vol. V, pp. 167–232 of the math. part.

This memoir is a more thorough presentation of the subject treated by SIMPSON in 1757 with much new matter added. LAGRANGE makes no allusion however to previous writings on the subject. The expression "Law of Facility of Error" occurs here for the first time.

For an extended account of the contents of the memoir see TODHUNTER, *History of Probability*, pp. 301–313. See also below 1785 BERNOULLI, 1788 EULER, and 1804 TREMBLEY. In 1850 ENCKE gives a translation of part of the memoir, with comments.

1778 BERNOULLI (Daniel). 'Dijudicatio maxime probabilis plurium observationum discrepantium atque verisimillima inductio inde formanda.' *Acta Acad. Petrop.* for 1777, Pt. I, pp. 3–23 of the memoirs.

The now familiar illustration of a marksman firing at a target is here introduced, and the conclusion drawn that small errors are more probable than large ones, and that the method of taking the arithmetical mean "non sine ratione dubitare potest," since it supposes the observations of equal weight.

DANIEL BERNOULLI takes a circle $y=\sqrt{r^2-x^2}$ as representing the law of facility of error, y being proportional to the probability of the error x, and r a constant. Then if observations give the errors x_1, x_2, x_3 ...., the product $\sqrt{r^2-x_1^2}\,\sqrt{r^2-x_2^2}\,\sqrt{r^2-x_3^2}$ must be a maximum to give the most probable value of the observed quantity. He finds that this value coincides with that given by the rule of the arithmetical mean for one and for two observations, and that it nearly coincides for three when a suitable value is given to r. For a greater number than three his method leads to unmanageable equations. He closes by remarking that the problem is indeterminate. See *Zach's Monatliche Correspondenz*, 1805, Vol. XI, pp. 486–490.

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DANIEL BERNOULLI's method agrees closely with modern theory. See 1778 EULER, 1785 BERNOULLI and TODHUNTER, *History of Probability*, p. 236.

1778 EULER. 'Observationes in praecedentem dissertationem Illustr. BERNOULLI.' *Acta Acad. Petrop.* for 1777, Part I, pp. 24–33 of the memoirs.

EULER considers that DANIEL BERNOULLI is correct in objecting to the arithmetical mean when the observations are of unequal precision, but that he "was quite arbitrary in proposing to make the product of the probabilities a maximum. EULER proposes another method which amounts to making the sum of the fourth powers of the probabilities a maximum."—TODHUNTER, *Hist. of Probability*, p. 237.

1781 LAPLACE. 'Mémoire sur les Probabilités.' *Hist. Acad. Paris* for 1778, pp. 227–332 of the math. part.

Pages 322–332 are devoted to the discussion of the mean to be taken between discordant observations. LAPLACE says that by the expression *mean* or *mean result* an infinite number of things may be understood. Of these the one which is implied in adjusting observations is a value such that the resulting error shall be a minimum, and this corresponds to a value such that the sum of the errors, each multiplied by its probability shall be a minimum. The method of his memoir of 1774 is then substantially repeated.

LAPLACE remarks that if $\pm a$ be the limits of error, the law of facility ought to be $\varphi(x)=\frac{1}{2a}\log_e\frac{a}{x}$. His methods lead to unmanagable equations.

1785 BERNOULLI (Jean, III). 'Milieu.' Art. in the *Encyclopedie Méthodique* (Paris, 4to), Vol. II, pp. 404–409. —Second edition, 1789, entitled *Dictionaire Encycl. des Mathématiques.*

BOSCOVICH's method of 1760 and LAMBERT's of 1765 are referred to, and an account is given of DANIEL BERNOULLI's memoir of 1778, which differs slightly from the memoir itself. An account of LAGRANGE's memoir of 1774 is presented with considerable fullness. See TODHUNTER, *Hist. of Probability*, p. 442.

1788 EULER. 'Eclaircissemens sur le mémoire de Mr DE LA GRANGE inséré dans le Ve volume de *Mélanges de Turin*, concernant la méthode de prendre le milieu entre les résultats de plusieurs observations, &c.' *Nova Acta Acad. Petrop.* for 1785, Vol. III, pp. 289–297 of the memoirs.

The memoir seems to have no value; See TODHUNTER, *Hist. of Probability*, p. 250.

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1789 LEGENDRE. 'Mémoire sur les Opérations trigonométriques, dont les résultats dépendent de la figure de la terre.' *Hist. Acad. Paris* for 1787, pp. 352–383 of the memoirs.

The method of *indeterminate corrections*, afterwards of so much use in the computations of Least Squares is here stated and used, although perhaps not for the first time. See 1820 LEGENDRE.

1792 LAPLACE. 'Sur les degrés mesurés des méridiens, et sur les longueurs observées du pendule.' *Hist. Acad. Paris* for 1789, pp. 18–43 of the memoirs.

The matter of this memoir is mostly reproduced in Sections 39–42 of Chap. V, Book III, of the *Traité de mécanique céleste.*

First a method is presented of determining an elliptic meridian so that the greatest error shall be a minimum; see 1831 CAUCHY. Secondly a method is developed for finding an ellipse subject to the following conditions: "1° que la somme des erreurs soit nulle; 2° que la somme des erreurs prises toutes avec le signe + soit un minimum." LAPLACE mentions that BOSCOVICH (1760) had solved the same problem by a different method. He alludes to the ellipse thus determined as "l'ellipse la plus probable."

1799 KRAMP. 'Table première. Intégrales de $e^{-u}dt$, depuis une valeur quelconque de t jusqu' à t infinie.' and 'Table seconde. Logarithmes des intégrales $\int e^{-u}dt$.' *Analyse des refractions* . . . (Strasbourg, 4to), pp. 195–206.

For description of these tables, afterwards of so great use in the Theory of Errors, see GLAISHER, *Lond. Phil. Mag.*, 1871, Vol. XLII, p. 431.

1804 TREMBLEY. 'Observations sur la méthode de prendre les milieux entre les observations.' *Mémoires Acad. Berlin* for 1801, pp. 29–58 of the math. part.

"Appears to be of no value whatever."—TODHUNTER, *Hist. of Probability*, p. 428.

1804 PRONY. *Recherches physico-mathématiques sur la théorie des eaux courantes.* Paris, 4to.

Contains applications of LAPLACE's methods of 1792 to the deduction of empirical formulæ from discordant observations.

1805 LEGENDRE. '*Nouvelles méthodes pour la détermination des orbites des comètes.*' Paris, 4to, pp. viii, 80. —Second edition, see 1806.

The date 1806 is generally given for this book, in which year a second edition seems to have been issued. The copy before me is plainly dated, "An XIII—1805."

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In the preface LEGENDRE gives an outline of his method for computing orbits. On page viii he says: "Il faut ensiuite, lorsque toutes les conditions du problême sont exprimées convenablement, déterminer les coëfficiens de manière à rendre les erreurs les plus petites qu'il est possible. Pour cet effet, la méthode qui me paroît la plus simple et la plus générale, consiste à rendre *minimum* la somme des quarrés des erreurs. On obtinent ainsi autant d'equations qu'il y a de coëfficiens inconnues; ce qui achève déterminer tous les éléméns de l'orbite.....la méthode dont je viens de parler, et que j'applle *Méthode des moindres quarrés*, peut être d'une grande utilité....."

On page 64 is an application of the method to the solution of three equations with two unknown quantities, in which the now well-known rule for the solution of normal equations is followed. On page 68 and 69 are references to its use. Pages 72–80 constitute an Appendix "Sur la Méthode des moindres quarrés." In this, after having mentioned that it is impossible that the sum of the errors should be zero when the number of given equations exceeds that of the unknown quantities, LEGENDRE says: "De tous les principes ....je pense qu'il n'en est pas de plus général, de plus exact, ni d'une application plus facile que celui .... qui consiste à rendre *minimum* la somme des quarrés des erreurs. Par ce moyen, il s'établit entre les erreurs une sorte d'équilibre qui empêchant les extrêmes de prévaloir, est tres-propre à faire connoître l'état du systême le plus proche de la vérité."

LEGENDRE then proceeds to deduce the rule for the formation of "l'equation du *minimum* par rapport à l'une des inconnues," or as we now say of a normal equation. His notation is the following: in the n equations

$$\begin{aligned} 0 &= a + bx + cy + \dots \\ 0 &= a' + b'x + c'y + \dots \\ 0 &= a'' + b''x + c''y + \dots \\ &\dots\dots\dots\dots \end{aligned}$$

a, b, c...., a', b', c'....are known by observation or theory, and x, y,....are to be determined. By forming the sum of the squares of these equations, differentiating with reference to each unknown separately and placing the derivatives equal to zero, he finds

$$\begin{aligned} 0 &= \int ab + x\int b^2 + y\int bc + \dots \\ 0 &= \int ac + x\int bc + y\int c^2 + \dots \\ &\dots\dots\dots\dots \end{aligned}$$

which are the same in number as the unknown quantities x, y...., and in which

$$\begin{aligned} \int ab &= ab + a'b' + a''b'' + \dots \\ \int b^2 &= b^2 + b'^2 + b''^2 + \dots \\ &\dots\dots\dots\dots \end{aligned}$$

LEGENDRE next demonstrates that the rule of the arithmetical mean is a particular case of his general principle. He then supposes the position of a point in space to be determined by three observations and finds the values of its coördinates given by the method. Noticing their identity with those for the centre of gravity of three points in space, he announces that the sum of the squares of the distances of

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all the molecules of a body from its centre of gravity must be a minimum, and that hence "la méthode des moindres quarrés fait connoître, en quelque sorte, le centre autour duquel viennent se ranger tous les résultats fournis par l'expérience, de manière à s'en écarter le moins qu'il est possible." A numerical example of the application of the method to the determination from five observations of the form of a meridian of the earth closes the book.

The honor of the first publication of the Method of Least Squares belongs to LEGENDRE. Although he failed to claim that his method gives most probable or most advantageous results, yet the remarks above quoted indicate that he fully recognized it as a rule giving a plausible and reasonable mean. See below 1814.

1805 PUISSANT. *Traité de Géodésie, ou Exposition des Méthodes astronomiques et trigonométriques*..... Paris, 4to. —Third edition, see 1842.

This contains an explanation and application of LEGENDRE's Method of Least Squares. See review in *Zach's Monatliche Correspondenz*, Vol. XVI, p. 455.

1806 LEGENDRE. 'Supplément aux Nouvelles Méthodes pour la détermination des orbites des comètes.' Paris, 4to, pp. 55.

The copy of this supplement which is before me is preceded by a title page reading, "*Nouvelles méthodes pour la détermination des orbites des comètes; avec un Supplément*..... A Paris, . .. Année 1806." This title page was probably prefixed to copies of the work of 1805, with the Supplement added, as the date 1806 is usually stated for it. A second Supplement appeared in [1820].

On pages 28 and 43 the Method of Least Squares is used.

1806 LINDENAU. 'Ueber den Gebrauch der Gradmessungen zur Bestimmung der Gestalt der Erde.' *Zach's Monatliche Correspondenz*, Vol. XIV, pp.113–158, etc.

LAPLACE's method of 1792 is stated and used in the discussion of the most probable elliptic meridian. The method is attributed to BOSCOVICH; see 1760.

On pages 138–143 "die....von LEGENDRE vorgeschlagenen Methode, die er *Méthode des moindres quarrés* nennt" is explained and applied in the determination of the elliptic meridian. The date of LEGENDRE's *Nouvelles méthodes*....is given as 1806.

1806 GAUSS. 'II Comet vom Jahr 1805.' *Zach's Monatl. Corres.*, Vol. XIV, pp. 181–186.

On page 184 GAUSS says: "LEGENDRE's Werk....habe ich noch nicht gesehen. Ich hatte mit Fleiss mir deswegen keine Mühe gegeben, um bey der Arbeit an meiner Methode ganz in der Kette meiner eigenen Ideen zu bleiben. Durch ein paar Worte, die DE LA LANDE in der letztern *Historie de l'Astronomie*, 1805, fallen lässt,

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méthode des moindres quarrés, gerathe ich auf die Vermuthung, dass ein Grundsatz, dessen ich mich schon seit zwölf Jahren bey mancherley Rechnungen bedient habe, und den ich auch in meinem Werke mit gebrauchen werde, ob er wol zu meiner Methode eben nicht wesentlich gehört,—dass dieser Grundsatz auch von LEGENDRE benutzt ist."

The work of LEGENDRE here alluded to is the *Nouvelles méthodes* ....1805, and that of GAUSS the *Theoria motus* ...1809, then in preparation. GAUSS mentions some of the advantages of his method for computing orbits but gives no hint of the principle of Least Squares.

1808 BOWDITCH. 'Solution of Mr. PATTERSON's Prize Question for correcting a survey, proposed in No. II. page 42, No. III. page 68, by NATHANIEL BOWDITCH, to whom the Editor has awarded the prize of ten dollars.' *The Analyst or Math. Museum*, Vol. I, pp. 88–92.

The Prize Question was: "In order to find the content of a piece of ground, ...I measured, with a common circumferentor and chain, the bearings and lengths of its several sides,..... But upon casting up the difference of latitude and departure, I discovered....that some error had been contracted in taking the dimensions. Now it is required to compute the area of this enclosure, on the *most probable supposition* of this error."

BOWDITCH's solution depends on several "principles" or hypotheses, the chief of which is "that in measuring the lengths of any lines the errors would probably be in proportion to their lengths." No principles of the Theory of Probability are employed except such as are by common sense implied. His solution coincides with that given by the Method of Least Squares.

This Prize Question undoubtedly led to the following 'Research' by ADRAIN, the Editor of *The Analyst.*

1808 ADRAIN. 'Research concerning the probabilities of the errors which happen in making observations.' *The Analyst or Math. Museum*, Vol. I, pp. 93–109.

This paper seems to have been unknown to mathematicians until 1871 when it was partly reprinted in *Amer. Jour. Sci.*, see 1871 ABBE. It is of great historical interest as containing the first deduction of the law of facility of error

$$\varphi(x) = ce^{-h^2x^2}$$

$\varphi(x)$ being the probability of any error x, and c and h constants depending upon the precision of the measurement. The term "Least Squares" is not used, and ADRAIN seems to have been entirely unacquainted with LEGENDRE's writings.

ADRAIN gives two deductions of this law. The first, occupying pages 93–95 has been reprinted as noted above and need not here be repeated. It depends upon the "self-evident principle" that the true errors of measured quantities are proportional to the quantities them-

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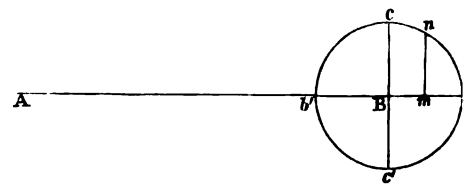
selves. The arbitrary nature of this assumption is shown by GLAISHER, *Mem. Astron. Soc. Lond.*, 1872, Vol. XXXIX, pp. 75–81, where the proof is analyzed and regarded as "very slight and inconclusive."

The second proof occupies pages 96–97, and not having been alluded to in the reprint noted above, I give it in ADRAIN'S own words as *The Analyst* is quite rare:

"Suppose that the length and bearing of AB are to be measured; and that the little equal straight lines Bb, Bc are the equal probable errors, the one $Bb=Bb'$ of the length of AB, and the other $Bc=Bc'$ (perpendicular to the former) of the angle at A, when measured on a circular arc to the radius AB: and let the question be to find such a curve passing through the four points b, c, b', c', which are equally distant from B, that, supposing the measurement to commence at A, the probability of terminating on any point of the curve may be the same as the probability of terminating on any one of the four points b, c, b', c'."



Then follows trivial reasoning which ends by concluding that "the curve must be the simplest possible" and "must consequently be the circumference of a circle having its centre in B." This established, the proof is the following:

"Now let us investigate the probability of the error $Bm=x$, and of $mn=y$. Let X and Y be two similar functions of x and y denoting those probabilities, X', Y', their logarithms, then $X\times Y=$ constant, or $X'+Y'=$ constant, and therefore $\dot{X}'+\dot{Y}'=0$, or $X'\dot{x}+Y'\dot{y}=0$, whence $X'\dot{x}=-X'\dot{y}$. But $x^2+y^2=r^2=Bb^2$, therefore $x\dot{x}=-y\dot{y}$, by which dividing $X'\dot{x}=-Y'\dot{y}$, we have $\frac{X'}{x}=\frac{Y'}{y}$; and therefore, by a fundamental principle of similar functions, the similar functions $\frac{X'}{x}$ and $\frac{Y''}{y}$ must be each a constant quantity: put then $\frac{X'}{x}=n$, and we have $X'\dot{x}=nx\dot{x}$, that is $\dot{X}'=nx\dot{x}$, and the fluent is $X'=c+\frac{nx^2}{2}$; in like manner we find $Y'=c+\frac{ny^2}{2}$ and therefore the probabilities themselves are $e^{c+\frac{nx^2}{2}}$ and $e^{c+\frac{ny^2}{2}}$, in which n ought to be negative, for the probability of x grows less as x grows greater."

I have seen no allusion to this proof by any subsequent writer. It is essentially the same as given in 1850 by HERSCHEL and usually called HERSCHEL'S proof. I regard it as defective in taking "$X\times Y=$ constant," or in considering the probabilities of the x and y deviations as independent. See 1850 ELLIS. See BOOLE'S *Finite Differences* (Cambridge, 1860), pp. 228–229.

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In his first proof ADRAIN had found for the probability of the error x in the observed value a the expression $e^{c+\frac{mx^2}{2a^2}}$, and had shown that the most probable values of the observed quantities $a, b, c, \ldots$ whose errors are $x, y, z, \ldots$ must satisfy the condition

$$\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}+\ldots = \text{a minimum.}$$

This principle he applies to four problems; the first showing that the arithmetical mean is a particular case of the method, the second to determine the most probable position of an observed point in space, which is shown to be "precisely in the centre of gravity of all the given points;" the third "to correct the dead reckoning at sea," and the fourth "to correct a survey."

As remarked by ABBE "we must credit Dr. ADRAIN with the independent invention and application of the most valuable arithmetical process that has been invoked to aid the progress of the exact sciences."—*Amer. Jour. Sci.*, 1871, Vol. I, p. 415.

1809 GAUSS. 'Determinatio orbitæ observationibus quotcunque maxime satisfacientis.' *Theoria motus corporum coelestium....* (Hamburgi, 4to), Lib. II, Sect. III, pp. 205–224. —French translation, see 1855 BERTRAND. —English translation by DAVIS (Boston, 1858, 4to), pp. 249–273. —German trans. by HAASE (Hannover, 1865).

That demonstration of the Method of Least Squares usually called GAUSS's proof or GAUSS's first proof is here presented. Assuming that the arithmetical mean of direct observations is the most probable value of the measured quantity, it deduces that the law of facility of error is given by

$$\varphi(x) = ce^{-h^2x^2}$$

from which the principle of Least Squares at once follows. This proof has been adopted by the majority of books on the subject; see for instance 1832 ENCKE, 1857 DIENGER, 1858 RITTER, 1864 CHAUVENET, 1867 HANSEN and MERRIMAN'S *Elements of the Method of Least Squares* (London, 1877, 8vo).

The demonstration as given by GAUSS contains three defects. 1. It is not recognized that the probability of a definite error x, is an infinitesimal; this is avoided by some later writers. 2. The distinction between *true errors* and *residuals* (or calculated errors) is not sharply drawn; according to GAUSS's reasoning the law $\varphi(x)=ce^{-h^2x^2}$ is not strictly a "law of facility of error" but only a law of distribution of residuals. 3. The rule of the arithmetical mean is assumed. For critical analyses of this proof see below, 1843 REUSCHLE, 1844 ELLIS and 1872 GLAISHER.

Practical features of the method,—the formation of normal equations and the determination of weights and degrees of precision are also discussed in the Section and hints are given regarding its use in astronomy. On page 221 is an attempt to justify the principle of

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Least Squares on the ground that any other process would lead to impracticable calculations.

I quote a historical remark on page 221: "Ceterum principium nostrum, quo jam inde ab anno 1795 usi sumus, nuper etiam a clar. LEGENDRE in opere *Nouvelles méthodes ...comètes*, 1806 prolatum est....."

1810 BESSEL. *Untersuchungen über die scheinbare und wahre Bahn der grossen Cometen von* 1807. Königsberg, 4to.

The Method of Least Squares is used in determining the orbit and is called "den *moindres quarrés*." See *Zach's Monatl. Corres.*, 1810, Vol. XXII, pp. 205–212.

1810 LAPLACE. 'Mémoire sur les approximations des formules qui sont fonctions de très-grands nombres, et sur leur application aux probabilités.' *Mém. Inst. France* for 1809, pp. 353–415, 559–565.

Pages 383–389 and 559–565 are devoted to the Theory of Errors, and the principle of Least Squares is proved "lorsque les résultats entre lesquels on doit prendre un milieu, sont donnés chacun par un très-grand nombre d'observations, quelles que soient d'ailleurs les lois de facilité des erreurs de ces observations."

The matter of these pages is reproduced in the *Théorie analytique des Probabilités*, pp. 329–335, 340–342; see below under 1812 for remarks concerning the proof. See *Bull. Soc. Philom. Paris*, 1810, Vol. II, pp. 132–136 for an account of the contents of the memoir.

1811 GAUSS. 'Disquisito de elementis ellipticis Palladis ex oppositionibus 1803, 1804, 1805, 1807, 1808, 1809.' *Comment. Soc. Göttingen*, Vol. I, 26 pp. —French translation, see 1855 BERTRAND.

Twelve equations involving six unknown corrections to the elements of the orbit are solved by the Method of Least Squares. We here find for the first time the notation

$$[ab]=a'\,b'+a''\,b''+a'''\,b'''+\ldots.$$

and also the algorithm for the solution of normal equations by successive substitution, since universally followed in lengthy computations.

A partial translation of the above with comments is given by ZACH in the *Monatl. Corres.*, 1811, Vol. XXIV, pp. 449–465.

1811 LAPLACE. 'Du milieu qu'il faut choisir entre les résultats des observations.' *Mém. Inst. France* for 1810, Pt. I, pp. 317–347.

Contains "the theory of errors substantially coincident with so much of the same theory as we find in pages 314–328 and 340–342 of the *Théorie ...des Prob.*" —TODHUNTER, *History of Probability*, p. 490. See below 1812.

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An account of the contents of this memoir, with historical remarks on the subject of Least Squares, is given by DELAMBRE, *Bull. Soc. Philom. Paris*, 1811, Vol. II, pp. 262–266, an English translation of which is in *Tilloch's Phil. Mag.*, 1812, Vol. XXXIX, pp. 240–244.

1811 LAPLACE. 'Du milieu qu'il faut choisir entre les résultats d'un grand nombre d'observations.' *Connaissance des Tems* for 1813, pp. 213–223. —German translation in *Zach's Monatl. Corres.*, 1812, Vol. XXV, pp. 105–120.

Contains matter which is reproduced in the *Théorie . . . des Prob.*, 1812, pp. 322–329.

1812 LAPLACE. '*Théorie analytique des Probabilités.*' Paris, 4to. pp. 464. —Third edition, 1820, 4to, pp. cxlii, 506, with Introduction and three Supplements (see 1814, 1815, 1818, 1820). —Fourth edition (Vol. VII of *Oeuvres de Laplace*) Paris, 1847, 4to, pp. cxcv, 691.

". . . . the greater part of the *Théorie des Probabilités* is a reprint of papers in the Memoirs of the Academy, which appear to contain the contents of the first papers on which he set down his processes. These with preliminary chapters, descriptive not of what follows, but of the general methods which he drew from the following parts, make up the whole work."—DE MORGAN, *Theory of Probabilities* in *Encyc. Metrop.*, p. 453. It "is by very much the most difficult mathematical work we have met with."—*Ibid*, p. 418. TODHUNTER in his *History of Probability*, p. 560, ELLIS (1844) and other writers have also testified to the abstruseness of LAPLACE's methods.

The Method of Least Squares is developed in Chap. IV, (pages 304–348) of the second part of the work. The analysis only extends to the case of two unknown quantities or elements, and the number of observations is required to be very large or infinite. Under these restrictions the Method is shown to give most advantageous results, whatever be the law of facility of error provided only that positive and negative errors are equally probable. LAPLACE's definition of *most advantageous results* is the following: " . . . . si l'on multiplie les erreurs possibles d'un élément par leurs probabilités respectives, le système le plus avantageux sera celui dans lequel la somme de ces produits tous pris positivement, est un *minimum*." The results thus obtained are not necessarily the *most probable*.

In the concluding paragraph of Chap. IV and in the opening pages of the First Supplement (see 1815), LAPLACE has given a general account of his method of analysis. These remarks and the table of contents at the end of the volume, give a much clearer idea of the steps of the demonstration than does Chap. IV itself. The principal objection against the validity of the proof is that it requires an infinite or very large number of observations. With this requirement, however, GAUSS's proof of 1809 becomes perfectly logical and the results are the *most probable*, not merely *most advantageous*.

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LAPLACE's proof has been greatly improved by subsequent writers. ELLIS in 1844 extended it to any number of unknown quantities, TODHUNTER in his *History of Probability*, pp. 560–588 supplied a valuable commentary, and GLAISHER in 1872 presented it in a clear and simple form. See also below, 1824 POISSON, 1847 DEMORGAN, 1852 BIENAYMÉ, 1861 AIRY, 1873 LAURENT and 1875 DIENGER.

On pages 318–319 is given what is sometimes called LAPLACE's second proof of the Method of Least Squares. This depends on the definition that "la valeur moyenne de l'erreur a craindre en plus" should be a minimum. The reasoning is similar to GAUSS's proof of 1823; see ELLIS's and GLAISHER's papers quoted below under 1844 and 1872.

[1813] PLANA. Sur divers problêmes de probabilité. *Mém. Acad. Turin* for 1811–12, Vol. XX, pp. 355–408.

1814 CAUCHY. Mémoire sur le système de valeurs qu'il faut attribuer á divers Élemens, determineés par un grand nombre d'observations. [Paris, Lith. MS.]

Probably the same as his memoir of 1831.

1814 LAPLACE. '*Essai philosophique sur les probabilités.*' Paris, 4to. —Sixth edition, Paris, 1840, 8vo, pp. 274. —Introduction to second edition of *Théorie....des Prob.*, pp. cvi; to third edition, pp. cxlii; to fourth edition, pp. v–clxix.

Near the end are some remarks concerning the history of the Theory of Errors of observation, and descriptive of LAPLACE's processes.

1814 LEGENDRE. 'Méthode des moindres quarrés, pour trouver le milieu le plus probable entre les résultats de differentes observations.' *Mém. Inst. France* for 1810, Pt. II, pp. 149–154.

Pages 72–75 of the *Nouvelles méthodes....*1805, are here quoted and reference made to the practical applications of the method given in that work, in order to call attention to LEGENDRE's priority of publication.

1815 BESSEL. 'Ueber den Ort des Polarsterns.' *Berlin. Astron. Jahrb.* for 1818, pp. 233–240.

We here find the first mention of *probable error*. After giving 48 observations on the right ascension of Polaris whose arithmetical mean is $55^m\ 48^s.5104$, BESSEL says: "Der *wahrscheinliche* Fehler einer einzelnen Beobachtung ist, nach den wirklichen vorkommenden Fehlern zu urtheilen $=1^s.067$, und daher der wahrscheinliche Fehler des Endresultats $=0^s.154$. Die Grund dieser Schätzung des wahrscheinlichen Fehlers, beruhen auf der von GAUSS gegebenen Entwickelung der Wahrscheinlichkeit, einen Fehler von gegebener

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Grösse zu begehen; ihre Mittheilung muss ich bis auf eine andere Gelegenheit versparen." See 1816 BESSEL, and also page 196 of the memoir 1816 GAUSS.

1815 LAPLACE. 'Sur l'application du calcul des probabilités à la philosophie naturelle.' *Connaissance des Tems* for 1818, pp. 361–381. —First Supplement to third edition of *Théorie....des Prob.* (Paris, 1820, 4to), pp. 3–26.

This is devoted partly to a general description of LAPLACE's proof of the Method of Least Squares, and partly to the discussion of the probability of results obtained by that method; a numerical example illustrates the use of his formulæ. See TODHUNTER, *Hist. of Probability*, p. 610; also see 1869 TODHUNTER.

1816 BEECK-CALKOEN. Over de Theorie der Gemiddelde Waardij. *Verhandl. Nederland. Inst.*, Vol. II, pp. 1–19.

Treats of LAPLACE's method of adjustment of 1792.

1816 BESSEL. 'Untersuchungen über die Bahn des OLBERSschen Kometen.' *Abhandl. Akad. Berlin* for 1812–13, pp. 117–160 of the math. section.

BESSEL defines the *probable error* as follows: "Ich verstehe unter dieser Benennung die Grenze, die eine Anzahl kleinerer Fehler von einer *gleichen* Anzahl grösserer trennt, so dass es wahrscheinlicher ist, eine Beobachtung innerhalb jeder *weiteren* Grenze von der Wahrheit abirren zu sehen, als ausserhalb derselben." If we designate by $\frac{\Sigma x}{n}$ the mean of the errors all taken positively, by $\frac{\Sigma x^2}{n}$ the mean of the squares of those errors, and by r the probable error of a single observation, his demonstration shows that

$$r = 0.8453\,\frac{\Sigma x}{n} \quad \text{or} \quad r = 0.6745\sqrt{\frac{\Sigma x^2}{n}}.$$

BESSEL does not distinguish between true errors and residuals. These formulæ he uses in finding the probable errors of the elements of the orbits, which are deduced by the help of the Method of Least Squares.

1816 GAUSS. 'Bestimmung der Genauigkeit der Beobachtungen.' *Zeitschr. f. Astron. u. ver. Wiss.*, Vol. I, pp. 185–196. —Also *Gauss Werke*, Vol. IV (Göttingen, 1873, 4to), pp. 109–117. —See 1855 BERTRAND.

This memoir gives three methods for finding the probable error from given observations. The first, which is that usually presented in text-books, finds that r the probable error of an observation of the weight unity, is given (most probably) by

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$$r = 0.6744897\sqrt{\frac{\Sigma x^2}{n}},$$

Σx^2 being the sum of the squares of the errors and n the number of observations, and that it is an even wager that the true value of r lies between

$$\frac{0.6744897\sqrt{\frac{\Sigma x^2}{n}}}{1-\frac{0.4769363}{\sqrt{n}}} \text{ and } \frac{0.6744897\sqrt{\frac{\Sigma x^2}{n}}}{1+\frac{0.4769363}{\sqrt{n}}}$$

In the second method the most probable value of the sum Σx^m is discussed and formulæ for probable error found when m has the values 1, 2, 3, 4, 5 and 6. The second of these, which agrees with the one given above, is shown to be the best. The third method leads to a different and less accurate formula.

Nothing in the investigation shows whether Σx^2 is the sum of the squares of the *true* or of the *computed* errors. By later writers it has been generally taken as referring to the former. See 1816 BESSEL, 1819 YOUNG, 1823 GAUSS, 1856 PETERS, 1866 BÖRSCH.

1818 ADRAIN. 'Investigation of the Figure of the Earth and of the Gravity in different Latitudes.' *Trans. Amer. Phil. Soc.*, Vol. I, pp. 119–135.

A formula for the length of the seconds pendulum is determined by the Method of Least Squares. ADRAIN alludes to the process as having been discovered by himself in 1808. See *Amer. Jour. Sci.*, 1871, Vol. I, p. 415, and *Mem. Astron. Soc. Lond.*, 1872, Vol. XXXIX, p. 78.

1818 BESSEL. '*Fundamenta astronomiæ pro anno MDCCLV deducta ex observationibus viri incomparabilis* JAMES BRADLEY *in specula astronomia Grenovicensi per annos* 1750–1762 *institutis.*' Regiomonti, folio, pp. 325.

In pages 18–21 results of the computations of the mean and probable errors of the declination and right ascension of certain stars as deduced from the observations are given. Three sets of measurements, two of 300 and one of 470, are investigated as a test of the exponential law $\varphi(x)=\frac{h}{\sqrt{\pi}}e^{-h^2x^2}$, the theoretical number of errors between given limits being computed from KRAMP's tables and compared with the actual number of residuals. A close agreement was found, and this may perhaps be called a practical proof of the principle of Least Squares.

Tables of logarithms of $e^{t^2}\int_t^\infty e^{-t^2}dt$ are given; *regula minimorum quadratorum* is several times applied; and in pages 116–123

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methods are given for finding probable errors of quantities indirectly observed.

1818 CAUCHY. Sur la méthode d'erreurs d'un grand nombre d'observations. [Paris], 4to.

1818 LAPLACE. 'Application du calcul des probabilités aux opérations géodésiques.' *Connais. des Tems* for 1820, pp. 422–440.

The matter of this article is reproduced in pages 3–25 of the Second Supplement to the third edition of the *Théorie des.... Prob.* For abstract see *Annal. Chem. et Phys.* 1817, Vol. V, pp. 351–356; also *Zeitschr. für Astron.*, Vol. V, pp. 1–9.

1818 LAPLACE. '*Deuxième supplément à la Théorie Analytique des Probabilités.*' Paris, 4to, pp. 50. —Fourth edition of *Théorie ....*, pp. 569–623.

"LAPLACE shows how the knowledge obtained from measuring a *base of verification* may be used to correct the values of the elements of the triangles of a survey ..... LAPLACE explains a method of treating observations which he calls the *method of situation* and which he considers may in some cases claim to be preferable to the *most advantageous method* explained in his fourth chapter."—TODHUNTER, *History of Probability*, pp. 611–612.

1818 LITTROW. 'Ueber die gerade Aufsteigung der vornehmsten Fixsterne.' *Zeitschr. f. Astron.*, Vol. VI, pp. 1–26.

The theory of Least Squares and probable errors is used.

1818 LINDENAU. 'Versuch einer Bestimmung der wahrscheinlichsten Bahn des Cometen von 1680, mit Rücksicht auf die planetarischen Störungen während der Dauer seiner Sichtbarkeit.' *Zeitschr. f. Astron.*, Vol. VI, pp. 27-120, 129–208.

The elements and their probable errors are found by the help of GAUSS's method of elimination.

1818 PLANA. 'Allgemeine Formel um nach der Methode der kleinsten Quadrate die Verbesserungen von 6 Elemente zu berechnen und zugleich das jeder derselben zukommenden Gewicht zu bestimmen.' *Zeitschr. f. Astron.*, Vol. VI, pp. 249–264.

1819 LAPLACE. 'Application du calcul des probabilités aux opérations géodésiques de la méridienne de France.' *Annal. Chem. et Phys.*, vol. XII, pp. 37–41. —*Connaiss. des Tems*, for 1822, pp. 346–348. —Trans. in *Tilloch's Phil. Mag.*, 1821, Vol. LVIII, pp. 133–136.

A discussion of the length of the meridian between Perpignan and Formentera, two points distant about 460000 metres and joined by

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26 triangles. LAPLACE shows that the probable error of the computed distance is 8.194 metres. The investigation is reproduced in the Third Supplement (1820), pp. 3–7.

1819 LAPLACE. 'Mémoire sur l'application du calcul des probabilités aux observations, et specialement aux opérations du nivellement.' *Annal. Chem. et Phys.*, Vol. XII, pp. 337–341.

On the results of a discussion of the probability of errors in the computed elevation of Paris. The discussion itself is given in the Third Supplement (1820), pp. 16–28.

1819 PAUCKER. '*Ueber die Anwendung der Methode der kleinsten Quadratssumme auf physikalischen Beobachtungen.*' Mitau, 4to, pp. 32.

Published as a Gymnasium "Programm." It contains an application of the Method of Least Squares to the determination of empirical formulæ for the expansion of fluids, the specific gravity of water, and the elasticity of steam. See 1825 MUNCKE.

1819 WALBECK. *Dissertatio de forma et magnitudine telluris, ex dimensis arcubus meridiani definiendis.* Åbo, 8vo.

The first discussion of measurements of *several* arcs of meridians by the Method of Least Squares.

1819 YOUNG. Remarks on the probabilities of error in physical observations, and on the density of the earth, considered, especially with regard to the reduction of experiments on the pendulum.' *Phil. Trans. Lond.* for 1819, pp. 70–95.

Pages 70–83 are devoted to the Theory of Errors. Besides many interesting remarks it contains a method for finding probable errors, supposing that the probabilities of the several errors are proportional to the terms of the series $(1+1)^m$, m being an even number. His result for the probable error of the mean of n observations is

$$R = 0.85 \frac{\Sigma x}{n\sqrt{n}}$$

in which $\frac{\Sigma x}{n}$ denotes "the mean of all the actual errors." YOUNG refers to BESSEL, GAUSS, &c., as having used only the sum of the squares of the errors in determining the probable error, and regards his method as more accurate. See 1816.

1820 BESSEL. 'Bestimmung der geraden Aufsteigungen der 36 MASKELYNEschen Fundamental-Sterne für 1815, auf Königsberger Beobachtungen gegründet.' *Abhandl. Akad. Berlin* for 1818–19, pp. 19–36 of the mathematical part. —*Abhandl. von Bessel* (Leipzig, 1875, 4to), Vol. II, pp. 238–245.

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Contains discussions on the probable errors of the observations. See also *Abhandl. Akad. Berlin* for 1825, pp. 23–35.

[1820] LAPLACE. '*Troisième Supplément à la Thèorie analytique des Probabilités.*' Paris, 4to, pp. 36. —Fourth edition of the *Théorie* . . . . , pp. 624–660.

See 1819. At the end of the Supplement is an investigation of the general case of "observations assujeties à plusieurs sources d'erreurs." See TODHUNTER, *Hist. of Probability*, p. 612.

[1820] LEGENDRE. '*Nouvelles méthodes pour la détermination de l'orbite des comètes. Second supplément.*' Paris, 4to, pp. 80.

In pages 3 and 4 LEGENDRE says: "Jái donné le premier deux méthodes sûres pour obtenir la solution à la fois la plus simple et la plus exacte, savoir: la *méthode des corrections indéterminées* . . . . . , et la *méthode des moindres carrés* qui paraisait alors pour la première fois." See 1789 and 1805.

Pages 79–80 contain a "Note par M * * *" in which the honor of the discovery of the Method of Least Squares is claimed for LEGENDRE on the ground of priority of publication, and in which GAUSS although not mentioned by name receives several sharp hits.

1821 SVANBERG. Om roterande systemers principal-axlar och sannolikaste medel-resultatet af gifna observationer. *Vetensk. Akad. Handl. Stockholm* for 1821, pp. 388–408.

1821 ———. 'Dissertation sur la rercherche du milieu le plus probable, entre les résultats de plusieurs observations ou experiénces.' *Gergonne's Annales de Math.*, Vol. XII, pp. 181–204.

This paper discusses at some length the different methods which may be imagined for finding a mean value, and concludes that the problem is indeterminate because it is impossible to render it independent of the law of facility of error, concerning which there may be "une infinité d'hypothèses." It tries to determine a mean, first supposing that the probability of each given measurement is inversely proportional to the error committed and secondly supposing that that probability is inversely proportional to the square of the error, and concludes that the arithmetical mean can only be used when the observations differ but slightly among themselves.

The paper ends by offering a method for the correction of the arithmetical mean, which amounts to this: First find the average of the measured quantities and compute the residuals. Then take the reciprocal of each residual as the weight of its corresponding observation and find the mean of these weighted observations. Or as weights the reciprocals of the squares of the residuals may be taken. The new mean gives new residuals from which a second approximation may be made, and so on. In a note at the end, the editor (GERGONNE) suggests that this approximation will always tend to one of the given measurements as the mean.

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1822 ENCKE. '*Die Entfernung der Sonne von der Erde aus dem Venusdurchgange von* 1761.' Gotha, 8vo, pp. 179. —Part II entitled '*Der Venusdurchgang von* 1769 . . . .' Gotha, 1824, 8vo, pp. 112.

The Method of Least Squares is applied to the reduction of 149 observations from the transit of 1761 and 106 from that of 1769, and to the determination of probable errors. The most probable distance of the sun from the earth is found to be 20666800 German geographical miles with the probable error of 89150 miles.

1822 GAUSS. 'Anwendung der Wahrscheinlichkeitsrechnung auf eine Aufgabe der practischen Geometrie.' *Astronomische Nachrichten*, Vol. I, col. 81–88. —See 1855 BERTRAND.

The problem is: To determine the position of a point from horizontal angles taken at that point between other points whose position is exactly known. A numerical example is given in which the number of known points is five and the number of angles is six. This is often called POTHENOT's problem; see 1840 GERLING, 1866 SCHOTT.

1823 GAUSS. 'Theoria combinationis observationum erroribus minimis obnoxiæ.' *Comment. Soc. Göttingen*, Vol. V, pp. 33–90. —Also *Gauss Werke*, Vol. IV (Göttingen, 1873, 4to), pp. 1–53. —French trans. see 1855 BERTRAND.

This memoir contains GAUSS's second Proof of the Method of Least Squares. The following quotation from pages 37–38 shows the hypothesis upon which the proof is based: ". . . . integrale $\int xx\, \varphi(x)\, dx$ ab $x = -\infty$ usque ad $x = +\infty$ extensum (seu valor medius quadrati x^2) aptissimum videter ad incertitudinem observationum in genere definiendam et dimetiendam, ita ut e duobus observationum systematibus, quæ quoad errorum facilitatem inter se differunt, eæ præcisione prestare ceuseantur, in quibus integrale $\int xx\, \varphi(x)\, dx$ valorem minorem obtinet." GAUSS does indeed recognize and point out that this is only an arbitrary convention, but he justifies himself in adopting it on the ground that the definition of most advantageous results must be arbitrary, since the question is in its very nature indefinite, and that his definition leads to simple operations. The values of the unknown quantities found by his method he calls "valores maxime plausibiles."

GAUSS's method leads to the rule of Least Squares, whatever be the number of observations or whatever be the law of facility of error provided only that positive and negative errors are equally probable. For analyses of his proof see 1844 ELLIS and 1872 GLAISHER, the former regarding it as valid and the latter as unsatisfactory. In my opinion it is but little more than a begging of the question to assume that the mean of the squares of the errors is a measure of precision. See below 1825 IVORY, 1847 GALLOWAY and 1872 HELMERT.

The memoir contains an extended presentation of the practical features of the method and in this respect is of great value. The algo-

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rithm for the solution of normal equations by the method of substitution (1816), the determination of weights and of formulas for mean error occupy the second part of the memoir. The value of the mean error of an observation of the weight unity being $m = \sqrt{\frac{\Sigma x^2}{n}}$, GAUSS takes Σx^2 as referring to the true errors, and determines $m = \sqrt{\frac{\Sigma v^2}{n-q}}$ as a practical formula, Σv^2 referring to the computed residuals, n being the number of observations and q that of the unknown quantities: the investigation however is not very clear. See 1816 BESSEL and GAUSS, and 1856 BIENAYMÉ.

For GAUSS's own account of the contents of these memoirs see the *Göttingische gelehrte Anzeigen*, Feb. 26, 1821 and Feb. 24, 1823. These reviews are reprinted in Vol. IV. of *Gauss Werke*, pp. 95–104. GAUSS here states that in the year 1797 he found that the determination of the *most probable* values of observed quantities was impossible, unless the law of facility of error was known; and that since 1801 he had used the Method of Least Squares almost daily. See 1830 RIESE.

1824 BERLIN. *Explanatio methodi quadratorum minimorum.* Lundæ, 4to.

1824 FOURIER. 'Règle usuelle pour la recherche des résultats moyens d'un grand nombre d'observations.' *Férussac's Bull. Sci. Math.*, Vol. II, pp. 88–90.

The rule given is expressed by the formula

$$m = 0{\cdot}4769 \frac{\sqrt{\frac{\Sigma a^2}{n} - \left(\frac{\Sigma a}{n}\right)^2}}{\sqrt{2n}}$$

$a_1, a_2, a_3, \ldots$ being the results of the n observations.

1824 POISSON. 'Sur la probabilité des résultats moyens des observations.' *Connaiss. des Tems* for 1827, pp. 273–302; for 1832, pp. 3–22.

These memoirs are a commentary on LAPLACE's fourth Chapter (1812) and seem to form a kind of translation which POISSON made of LAPLACE's investigations for his own satisfaction. A large part of the memoirs are reproduced in his *Recherches* . . ., see 1837. See also 1830 HAUBER, 1847 GALLOWAY and TODHUNTER's *History of Probability*, pp. 560–588. See *Jahrb. Chem. u. Phys.*, Vol. IV, pp. 38–42.

"POISSON confines himself to the case in which *one* element is to be determined from a large number of observations, but he treats this case in a more general manner than LAPLACE had done. LAPLACE had assumed that positive and negative errors were equally

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likely, and that the law of facility of error is the same at every observation; but POISSON makes neither of these assumptions."—TODHUNTER in *Trans. Camb. Phil. Soc.*, 1869, Vol. XI, p. 219.

1824 PUISSANT. 'Application de la methode la plus avantageuse à la determination de l'aplatissement du sphéroïde osculateur en France, par le comparaison d'un arc de méridien avec un arc de parallèle.' *Férussac's Bull. Sci. Math.*, Vol. I, pp. 271–274. —*Connaiss. des Tems* for 1827, pp. 230–232.

A general statement of the method, which seems to have been elsewhere published in detail.

1825 IVORY. 'On the Method of the Least Squares.' *Tilloch's Phil. Mag.*, Vol. LXV, pp. 3–10, 81–86, 161–168.

This paper contains two attempted proofs of the principle of Least Squares by methods independent of the Theory of Probability. The first, in page 5, rests on a vague analogy with the properties of a lever and is in the words ef ELLIS "little more than a *petitio principii* concealed by a metaphor." The second, in pages 6–7, rests on the supposition that "the mean of the sum of the squares of the errors may be taken as a measure of the precision of the observations" which can scarcely be assumed as evident; this is similar to GAUSS's proof of 1823.

Pages 81–86 are devoted to discussing the probability of errors. IVORY makes no distinction between true errors and residuals, and does not recognize that the probability of any definite error must be an infinitessimal. The remaining pages attempt to show that the Method of Least Squares cannot give the most advantageous or probable results unless the law of facility of error is $\varphi(x) = ce^{-h^2x^2}$, and that LAPLACE's demonstration "whatever merit it may have in other respects is neither more or less general than the other solutions of the problem."

These two proofs are examined and exposed by ELLIS in 1844, and the second proof with the criticisms on LAPLACE are analyzed by GLAISHER in 1872. See also 1851 HOSSARD.

1825 MUNCKE. 'Beobachtung.' Art. in *Gehler's Physikalisches Wörterbuch*, second ed. Vol. I, pp. 884–912.

Contains matter from 1823 GAUSS and 1819 PAUCKER.

1826 GAUSS. 'Chronometrische Längenbestimmung.' *Astron. Nachr.*, Vol. V, col. 227–240, 245–248. —See also 1855 BERTRAND.

The Method of Least Squares is used and formulæ for finding mean errors of the results given.

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1826 [FOURIER]. Recherches statistiques sur la ville de Paris et le département de la Seine [par le Compte CHABROL-DE-VOLVIC]. Paris, 4to, Vol. III, in 1826, Vol. IV in 1829.

These volumes contain investigations by FOURIER on weights, probable errors, etc. See note by DEMORGAN in *Lond. Assur. Mag.*, Vol. XIV, p. 89.

1826 IVORY. 'On the Method of the Least Squares.' *Tilloch's Phil. Mag.*, Vol. LXVIII, pp. 161–165.

Contains IVORY's third attempted proof of the Method, which is still more absurd than those of 1825. See ELLIS's analysis in his paper of 1844. See also below, 1830 FRANCOEUR.

1827 ROSENBERGER. 'Ueber die, auf Veranstaltung der französischen Academie, während der Jahre 1736 und 1737 im Schweden vorgenommene Gradmessung.' *Astron. Nachr.*, Vol. VI, col. 1–32.

An application of a method, communicated to the author by BESSEL, for the adjustment of geodetic triangulations by the use of Least Squares. See 1831 HANSEN. See *Abhandl. von Bessel* (Leipzig, 1875), Vol. III, pp. 16–19.

1827 ———. Ueber die Theorie der Zuverlässigkeit der Beobachtungen und Versuche und der von derselben abhängigen Bestimmungen des Mittels aus gegebenen Zahlen. [Berlin].

1828 BESSEL. 'Ueber die Bestimmung des Gesetzes einer periodischen Erscheinung.' *Astron. Nachr.*, Vol. VI, col. 333–348. —*Abhandl. von Bessel* (Leipzig, 1875, 4to), Vol. II, pp. 364–372.

Proposes the periodic function since so much used for discussing recurring phenomena, and illustrates its application to the determination of empirical formulæ. See 1864 SCHOTT.

1828 GAUSS. 'Supplementum theoriæ combinationis observationum erroribus minimis obnoxiæ.' *Comment. Soc. Göttingen*, Vol. VI, pp. 57–98. —Also *Gauss Werke*, Vol. IV, pp. 57–93. —French trans. see 1855 BERTRAND.

This memoir discusses a method for the combination of observations when the observed quantities are not expressed as explicit functions of the unknown quantities to be determined; and when the problem furnishes rigorous equations of condition which the determined values of the unknown quantities must exactly satisfy. The *method of correlatives* for the adjustment of such conditioned observations is given and an algorithm presented for its use. A numerical example involving twenty-four observations subject to thirteen conditions illustrates the use of the formulæ. See the *Gött. gelehrte Anzeigen*, Sept. 25,

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1826, or *Gauss Werke*, Vol. IV, pp. 104–108 for GAUSS's own account of the contents of the memoir. See also 1830 RIESE.

1828 QUETELET. *Instructions populaires sur le calcul des probabilités*, Bruxelles, 18mo.

1830 FRANCOEUR. 'De la composition des formules astronomiques, et de la détermination des constantes; Équations de condition; Méthodes de TOBIE MAYER et des moindres carrés.' *Astronomie pratique* (Paris, 8vo; second ed., 1840), pp. 412–431.

IVORY's proof of 1826 is given as a perfectly valid "démonstration." This is repeated in the second edition.

1830 HANSEN. Commentatio de gradus præcisionis computatione. [Gotha, 4to.]

This was first printed "in einem Program, womit die hiesige Sternwarte [at Gotha] des Jubiläum des würdigen OLBERS gefeiert hat." It contained a method for finding the weights of values determined by the Method of Least Squares, which for a small number of unknown quantities is perhaps shorter than that of GAUSS (1823). See *Astron. Nachr.*, Vol. VIII, col. 462–463, and ENCKE in *Berlin Astron. Jahrb.* for 1835, p. 297.

1830 HAUBER. Ueber die Bestimmung der Genauigkeit der Beobachtungen. *Baumgartner's Zeitschr. f. Phys. Math., etc.*, Vol. VII, pp. 286–314.

1830 HAUBER. Verallgemeinerung der POISSON'schen Untersuchungen über die Wahrscheinlichkeit der mittlern Resultate der Beobachtungen in den "Additions à la *Connaissance des Temps* de 1827." *Baumgartner's Zeitschr.*, Vol. VII, pp. 406–429.

1830 HAUBER. Theorie der mittleren Werthe. *Baumgartner's Zeitschr.*, Vol. VIII, pp. 25–26, 147–179, 295–315, 443–445; Vol. IX. pp. 302–322; Vol. X, pp. 425–457.

[1830] NÜRNBERGER. Betrachtung über die Methode der kleinsten Quadrate.

1830 POISSON. 'Note sur la probabilité du résultat moyen des observations.' *Férussac's Bull. Sci. Math.*, Vol. XIII, pp. 266–277.

On LAPLACE's method of deducing formulæ for probable errors.

1830 POISSON. 'Mémoire sur la proportion des naissances des filles et des garçons.' *Mém. Acad. Paris*, Vol. IX, pp. 239–308.

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The general term of the binomial theorem is shown to take the exponential form ce^{-u^2}, when the number of terms is indefinitely great: See LAPLACE, *Théorie des . . . . Prob.*, Chap. III, and 1837 HAGEN. Formulæ for probability of errors between given limits and for probable errors are also developed. See 1836 POISSON.

1830 RIESE. '[Eine Recension.]' *Jahrb. für wissen. Kritik* for 1830, Pt. I, col. 269–284.

A review of Gauss's memoirs *Theoria combinationis . . . . .*, 1823, and *Supplementum theoriae. . . . .*, 1827, giving an account of their contents and a popular exposition of the subject.

1831 CAUCHY. 'Mémoire sur le système de valeurs qu'il faut attribuer à divers Élémens, déterminés par un grand nombre d'observations, pour que la plus grande de toutes les erreurs, abstraction faite du signe, devienne un minimum.' *Jour. École Poly.*, Vol. XIII (cahier 20), pp. 175–221.

This was perhaps published about 1814, See *Bull. Soc. Philom. Paris*, for 1824, pp. 92–99. The methods of 1760 BOSCOVICH and 1792 LAPLACE, are particular cases of CAUCHY's solution. See LAPLACE's *Théorie . . . des Prob.*, Chap. IV, Art. 24.

1831 DEGEN. 'Recherches sur la parabole, déterminée par la méthode des moindres carrés et qui représenté le moins défectueusement, quil il soit possible, un système quelconque de points donnés dans un plan.' *Mém. Acad. St. Péters. par divers savants*, Vol. I, pp. 13–28.

1831 GRUNERT. 'Berechnung der wahrscheinlichsten Resultate aus gegebenen Beobachtungen. Methode der kleinsten Quadrate.' *Klügel's Mathematisches Wörterbuch*, Vol. V, Art. "Wahrscheinlichkeitsrechnung,' pp. 983–1027.

A clear presentation of the Method of Least Squares according to GAUSS, the proof being that of 1809.

1831 HANSEN. 'Darlegung einer neuen Methode, bei Anwendung der Methode der kleinsten Quadrate, die Gewichte der unbekannten Grössen zu berechnen.' *Astron. Nachr.*, Vol. VIII, col. 463–468.

A simplification of GAUSS's method of 1823 for finding weights. Not the same as 1830 HANSEN. See 1832 ENCKE.

1831 HANSEN. 'Ueber die Anwendung der Wahrscheinlichkeitsrechnung auf geodätische Vermessungen im Allgemeinen, und über die MAUPERTUIS'sche Gradmessung.' *Astron. Nachr.*, Vol. IX, col. 189–220, 237–262.

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The measurements are discussed by GAUSS's method of correlatives (1828) whose algorithm is given in full, and also by a new method of HANSEN. ROSENBERGER had in 1827 examined the same measurements.

1831 LITTROW. Bemerkungen zum practischen Gebrauche der Wahrscheinlichkeitsrechnung. *Baumgartner's Zeitschr. f. Phys.*, Vol. IX, pp. 433–449.

1831 PUISSANT. 'Application du calcul des probabilités à la mesure de la precision d'un grand nivellement trigonométrique.' *Mém. Acad. Paris*, Vol. X, pp. 533–547. —*Connaiss. des Temps* for 1834, pp. 3–17.

A modification of the method of LAPLACE's Third Supplement (1820), illustrated by a practical example.

1832 ENCKE. 'Ueber die Begründung der Methode der kleinsten Quadrate.' *Abhandl. Akad. Berlin* for 1831, pp. 73–78 of the mathematical part.

After brief notices of five proofs of the Method of Least Squares, ENCKE gives the preference to GAUSS's of 1809. To establish this more rigidly he offers a demonstration to show that for direct observations the rule of the arithmetical mean gives the most probable result. This demonstration (in my opinion not a rigorous one) has been followed by many subsequent writers. It is repeated by ENCKE in the article quoted next below, and is particularly stated with confidence by CHAUVENET in 1864. For criticisms see 1843 REUSCHLE and 1872 GLAISHER. See also ENCKE's later opinion, below under 1850.

1832 ENCKE. 'Ueber die Methode der kleinsten Quadrate.' *Berlin. Astron. Jahrbuch* for 1834, pp. 249–312; for 1835, pp. 253–320; for 1836, pp. 253–308. —Republished in *Encke's astronomische Abhandlungen* (Berlin, 1866), Vol. I, Nos. xii, xiii, xiv.

These memoirs form a treatise on the Method of Least Squares, from which many text-books have been compiled.

The first memoir contains the proof of 1809 GAUSS, reinforced by ENCKE's attempted demonstration of the validity of the arithmetical mean, the discussion of weight and probable errors, and two tables of the probability integral $\frac{2}{\sqrt{\pi}}\int e^{-t^2}dt$, the first between the limits 0 and t, and the second between the limits 0 and $0.476936\frac{x}{r}$ (x being any error and r the probable error). These were computed from KRAMP's tables of 1799 as quoted by BESSEL in 1818. See *Lond. Phil. Mag.*, 1871, Vol. XLII, p. 431, et sq. A translation of this first memoir and a reprint of the tables is given in *Taylor's Scientific Memoirs*, 1841, Vol. II, pp. 317–369.

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ENCKE takes $y=\varphi(x)$ as the equation of the curve expressing the probability of error, and regarding x and y as continuous variables recognizes clearly that for a given error $\varphi(x)$ must be an infinitesimal. But strange to say he deduces

$$\varphi(x)=\frac{h}{\sqrt{\pi}}e^{-h^2x^2}$$

in which h is a finite quantity.

The second memoir contains the practical features of the method—GAUSS's algorithm for the solution of normal equations, GAUSS's (1823) and HANSEN's (1830 and 1831) methods of determining weights, etc. The third is devoted to the discussion of conditioned observations. At the time of publication these memoirs must have been of great value to students.

1832 LITTROW. *Die Wahrscheinlichkeitsrechnung und ihre Anwendung auf das wissenschaftliche und practische Leben.* Wien, 8vo.

1832 PUISSANT. 'Deuxième mémoire sur l'application du calcul des probabilités aux mesures géodésiques.' *Mém. Acad. Paris*, Vol. XI, pp. 123–156.

Adjustment of triangulations, determinations of probable errors, etc.

1834 BESSEL. 'Betrachtung über die Methode der Vervielfältigung der Beobachtungen.' *Astron. Nachr.*, Vol. XI, col. 269–290. —*Abhandl. von Bessel* (Leipzig, 1875), Vol. III, pp. 306–317.

This valuable paper deduces rules for the adjustment of angles taken by the method of repetitions, and formulæ for finding their weights and probable errors.

1834 STROOTMAN. *Bevattelijk onderrigt in de Kansrekening, of de leer der waarschijnlijkheden.* Breda, 12mo.

1835 CAUCHY. 'Mémoire sur l'interpolation.' Lith. MS. —Translation in *Lond. Phil. Mag.*, 1836, Vol. VIII, pp. 459–468. —Reprinted in *Liouville's Jour. Math.*, 1837, Vol. II, pp. 193–205; in *Moigno's Leçons de calcul différentiel* (Paris, 1840), pp. 513–526.

When an empirical formula is to be derived from a great number of observation equations, CAUCHY's method may be used as easily, although perhaps with less accuracy than the Method of Least Squares. See below 1853 BIENAYMÉ and CAUCHY, 1842 GRUNERT and 1861 SCHOTT. See an article by BARTLETT in *Amer. Jour. Sci.*, 1862, Vol. XXXIV, pp. 27–33.

1836 POISSON. 'Formules relatives aux probabilités qui dépendent de très grand nombres.' *Comptes Rend. Acad. Paris*, Vol. II, pp. 603–613.

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The general term of the binomial $(p+q)^m$, in which $p+q=1$, is shown to approach the form $\frac{e^{-u^2}}{\sqrt{2\pi mpq}}$, as m indefinitely increases. See 1830 POISSON.

1836 ROUVROY. Ueber die Methode der kleinsten Quadrate. Appendix to his *Mechanik* (Dresden and Leipzig, 8vo).

1837 HAGEN. *Grundzüge der Wahrscheinlichkeits-Rechnung.* Berlin, 8vo. —Second edition, see 1867.

This work contains HAGEN's proof of the Method of Least Squares. It is based upon the following hypothesis: "Der Beobachtungsfehler ist die algebraische Summe einer unendlich grossen Anzahl elementärer Fehler, die alle gleichen Werth haben und eben so leicht positiv, wie negativ sein können." This postulated, the proof consists in finding the general term of the expansion of $(\frac{1}{2}+\frac{1}{2})^{2m}$, m being indefinitely large. The law of facility of error takes the form $\varphi(x)=(\pi m)^{-\frac{1}{2}}e^{-\frac{x^2}{m}}$ from which the principle of Least Squares at once follows.

The algebraic work of HAGEN's method had in a somewhat different form been given by LAPLACE in the *Théorie . . . . des Probabilités* (1812), p. 301, and in the articles 1830 POISSON and 1836 POISSON. HAGEN's method is more elementary and in connection with his original hypothesis forms, I think, one of the best proofs of the Method of Least Squares.

HAGEN's proof is given in the writings 1849 WITTSTEIN, 1850 ENCKE, 1852 DIENGER, and in a modified form 1846 QUETELET, 1865 TAIT, 1866 NATANI and others. Also see PRICE's *Integral Calculus* (Oxford, 1865), pp. 376–379. A discussion between KUMMELL and MERRIMAN concerning this proof is now (Oct., 1877) going on in the *Jour. Franklin Institute;* see Vol. CIV, pp. 173–187, 270–274, et sq.

1837 POISSON. '*Recherches sur la probabilité des jugemens en matière criminelle et en matière civile, précédées des règles générales du calcul des probabilités.*' Paris, 4to, pp. ix, 415. —German trans. by SCHNUSE called *Lehrbuch der Wahrscheinlichkeitsrechnung;* Braunschweig, 1841, 8vo.

The matter of POISSON's previous memoirs on the law of great numbers is reproduced in Chap. III, and of those on the probability of the mean in Chap. IV.

1838 BESSEL and BAEYER. '*Gradmessung in Ost-Preussen und ihre Verbindung mit preussische und russische Dreiecksketten.*' Berlin, 4to, pp. xiv, 452.

A geodetic work of great value, containing many applications of the Method of Least Squares. See extracts in *Abhandl. von Bessel,* (Leipzig, 1875), Vol. III, pp. 82–138.

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1838 BESSEL. 'Untersuchungen über die Wahrscheinlichkeit der Beobachtungsfehler.' *Astron. Nachr.*, Vol. XV, col. 369–404. —*Abhandl. von Bessel* (Leipzig, 1875), Vol. II, pp. 372–391.

"Ich werde nämlich die Entstehungsart der Beobachtungsfehler *aus ihren Ursachen*, zum Grunde des Folgenden machen. Wenn man aufangs die Fehler einer gewissen Beobachtungsart also aus *einer*, auf gegebene Art wirkenden Ursache hervorgehend betrachtet, so wird dadurch ihre jedesmalige Grösse x eine gegebene Function eines Arguments ξ, welches in derselben Art willkührlich ist, wie das Fallen eines Würfels. Aus dem Ausdrucke $x = f\xi$ kann aber der Ausdruck $\varphi(x)$ abgeleitet werden, ..."

BESSEL seems to use the word *Ursache* in the sense of *a source of error.* His first investigation is of a case arising in the measurement of angles, where the error x is related to the *Ursache* ξ by the law $x = a \sin \xi$, every value for ξ between the limits $\pm \frac{1}{2}\pi$ being equally possible. The law of facility of error he finds $\varphi(x) = \pi^{-1}(a^2 - x^2)^{-\frac{1}{2}}$, and the probable error is 2.568 times greater than by the Method of Least Squares. An example where $x = a\xi^2$, which he shows may actually arise, gives also disagreeing results.

In the second part of the investigation we read: "Ich werde nun die Wahrscheinlichkeit eines Fehlers untersuchen, welcher aus der *Zusammenwirkung* mehrer, von einander unabhängiger Ursachen ensteht," each error so arising being considered as equally likely to be positive or negative. The result of the investigation is that the law of facility of error approximates closely to the exponential form $\varphi(x) = ce^{-h^2x^2}$, provided that "viele Ursachen zur Hervorbringen des Beobachtungsfehlers *zusammenwirken*," and "dass unter den, aus den einzelen Ursachen hervorgehenden mittlern Fehlern, keiner die ubrigen beträchtlich übertreffe," and these conditions BESSEL thinks, are present in most observations.

This memoir is very valuable as showing that the exponential law of facility is not to be regarded as an *à priori* rule, free from exception, and as throwing new light on the condition under which it exists. On the whole it may be considered as a new proof of that law and hence of the Method of Least Squares.

1838 BIENAYMÉ. 'Mémoire sur la probabilité des résultats moyens des observations; démonstration directe de la règle de LAPLACE.' *Mém. . . par divers savants, . . Acad. Paris*, Vol. V, pp. 513–558.

The rule of LAPLACE here meant is a method for finding the probability of the error of the mean. The opening pages contain some interesting historical remarks, but the investigation itself is very long and tedious and seems to be of little value.

1838 DEMORGAN. '*Essay on Probabilities*,' London, 12mo. pp. xviii, 306, XL.

This popular book devotes a chapter to methods of finding weights and estimating probabilities of mean results. It contains tables of the error functions.

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1838 DEMORGAN. 'On a Question in the Theory of Probabilities.' *Trans. Camb. Phil. Soc.*, Vol. VI, pp. 423–430.

"The object of this paper is the correction of an oversight made both by LAPLACE and M. POISSON in pages 279 and 209 of their respective works on the Theory of Probabilities."

1838 PUISSANT. 'Sur l'application du calcul des probabilités à la mesure de la precision d'un grand nivellement géodésique independant des distances respectives des stations.' *Comptes Rend. Acad. Paris*, Vol. VII, pp. 1132–1135.

The mean errors of the adjusted elevations are deduced.

1839 BIENAYMÉ. Théorème sur la probabilité des résultats moyens des observations. *Procés-Verb. Soc. Philom. Paris* for 1839, pp. 60–65.

1839 HANSEN. 'Auflösung einer allgemeinen Aufgabe aus der Wahrscheinlichkeitsrechnung.' *Astron. Nachr.*, Vol. XVI, col. 9–16, 27–32.

On the determination of the values and weights of observed quantities subject to conditional equations. A particular case of HANSEN's problem is noted below under 1841.

1839 JAHN. *Die Wahrscheinlichkeitsrechnung und ihre Anwendung auf das wissenschaftliche und praktische Leben.* Leipzig, 8vo, pp 240.

1840 BESSEL. 'Neue Formel von JACOBI, für einen Fall der Anwendung der Methode der kleinsten Quadrate.' *Astron. Nachr.*, Vol. XVII, col. 305–308. —*Abhandl. von Bessel* (Leipzig, 1875), Vol. II. pp. 401–402.

Formulæ for solving normal equations containing *three* unknown quantities and determining at the same time the weights. They are regarded by BESSEL as shortening considerably the numerical work required by previous methods. See below 1873 SEELIGER.

1840 BESSEL. 'Ein Hülfsmittel zur Erleichterung der Anwendung der Methode der kleinsten Quadrate.' *Astron. Nachr.*, Vol. XVII, col. 225–230. —*Abhandl. von Bessel*, Vol. II, pp. 398–401.

Computations arising in the formation of normal equations can be shortened by the use of tables of squares of numbers.

1840 BIENAYMÉ. La constance des résultats moyens. *Procés-Verb. Soc. Philom. Paris* for 1840, pp. 19–26.

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1840 BIENAYMÉ. Principe des probabilités entièrement nouveau. *Procés-Verb. Soc. Philom. Paris* for 1840, pp. 37–43.

1840 BIENAYMÉ. Quelques proprietés des moyens arithmétiques de puissances de quantities positives. *Procés-Verb. Soc. Philom.* for 1840, pp. 67–70.

1840 BRAVI. *Theorica e practica del probabile.* Bergamo, second edition, 2 vols, 8vo.

1840 CLEMENS. 'Ueber die Methode der kleinsten Quadrate. *Tilsit Gymnasium Programm* for 1839–40, pp. . . . . .; for 1848, pp. 1–12.

The first part of this memoir I have not seen. The second discusses the determination of empirical formulæ for periodic observations, the calculation of probable errors and the correction of a field survey.

1840 GERLING. POTHENOT'*sche Aufgabe, in practischer Beziehung dargestellt.* Marburg, 8vo, pp. 32.

Contains mainly the solution of 1822 GAUSS. See 1866 SCHOTT.

1841 BESSEL. 'Ueber einer Fehler in der Berechnung der französischen Gradmessung und seinen Einfluss auf die Bestimmung der Figur der Erde.' *Astron. Nachr.*, Vol. XIX, col. 97–116. —*Abhandl. von Bessel* (Leipzig, 1875), Vol. III, pp. 55–62.

Forty-four observations subject to sixteen conditions are adjusted. The method here employed for the combination of measurements of arcs of meridians has furnished a model for many subsequent investigations.

1841 HANSEN. 'Eine Aufgabe aus der practischen Geodäsie und deren Auflösung.' *Astron. Nachr.*, Vol. XVIII, col. 165–176.

"Die Lage *zweier unbekannten* Punckte durch Hülfe der Lage *zweier bekannten* Punckte zu bestimmen, ohne jene von diesen aus zu beobachten."

1841 HÜLSSE. *Ueber die Berechnung von Beobachtungen durch die Methode der kleinsten Quadratssumme.* Leipzig, 4to.

1841 JACOBI. 'De formatione et proprietatibus Determinantium.' *Crelle's Jour. Math.*, Vol. XXII, pp. 285–318.

Near the end of this memoir are remarks concerning the application of determinants to the solution of equations and to the finding of weights in the Method of Least Squares.

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1841 QUETELET. Ueber das Gesetz des Quadrats der Temperaturen. *Bull. Akad. München* for 1841, col. 261–263.

1842 FRIES. *Versuch einer Kritik der Principien der Wahrscheinlichkeitsrechnung.* Braunschweig, 8vo.

1842 GRUNERT. 'Ueber CAUCHY's Interpolationsmethode.' *Archiv. Math. u. Phys.*, Vol. II, pp. 41–60.
See 1835 CAUCHY.

1842 LITTROW. 'Theorie der kleinsten Quadrate.' *Gehler's Physicalisches Wörterbuch*, Vol. X, Pt. II, Art. 'Wahrscheinlichkeitsrechnung,' pp. 1200–1251.
A clear elementary exposition of the Method and its theory according to 1832 ENCKE.

1842 LOBATSCHEWSKY. 'Probabilité des résultats moyens tirés d'observations répetées.' *Crelle's Jour. Math.*, Vol. XXIV, pp. 164–170.
Serial expressions are deduced for the probability of error in the mean of a limited number of observations.

1842 MERZ. *De theoria probabilitatis adhibita in physicam.* Monachii.

1842 PUISSANT. '*Traité de géodésie, ou exposition des méthodes trigonométriques et astronomiques, applicables à la mesure de la terre, et à la construction du canevas des cartes topographiques.*' Paris, 2 vols, 4to, Vol. I, pp. xvi, 515, and XI pl. Vol. II, pp. xi, 496, xxxii, and III pl. —First edition, see 1805.
The results of many of PUISSANT's previous memoirs are here recorded, and the Method of Least Squares is often used. The work is one of the most valuable treatises on Geodesy extant.

1842 RAMUS. 'Sur une question de probabilité relative aux corrections des hauteurs barométriques.' *Crelle's Jour. Math.*, Vol. XXIV, pp. 80–84.
A determination of the probability "que dans un grand nombre d'observations la difference des résultats moyens des hauteurs barométriques observées et réduites soit comprise dans l'intrevalle λ."

1842 RÖBER. Experiment. Art. in *Handwörterbuch Chemie u. Phys.* (Berlin, 8vo.)
See *Archiv. Math. u. Phys.*, Vol. XI, p. 375.

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1843 ARGELANDER. 'Ueber die Anwendung der Methode der kleinsten Quadrate auf einen besondern Fall.' *Astron. Nachr.*, Vol. XXI, col. 163–168.

Simplifies the formation of normal equations, etc., when the observations are made on the simple sums or differences of the unknown quantities.

1843 BERKHAN. *Ueber die Methode der kleinsten Quadrate.* Blankenburg, 8vo.

1843 COURNOT. '*Exposition de la théorie des chances et des probabilités.*' Paris, 8vo, pp. viii, 448. —German trans., Braunschweig, 1849, 8vo.

Chapters on the theory of means and the adjustment of observations are given. At the end is a table of values of the error function. COURNOT's methods are often quite awkward.

1843 GERLING. '*Die Ausgleichungs-Rechnung der practischen Geometrie, oder die Methode der kleinsten Quadrate mit ihren Anwendungen auf geodätische Aufgaben.*' Hamburg and Gotha, 8vo, pp. xix, 409, and 4 pl.

This book is dedicated to GAUSS. The principle of Least Squares is assumed as the basis of the methods of adjustment. It consists of four parts; the first treats of direct, the second of indirect, and the third of conditioned observations, while the fourth discusses the form and number of conditional equations which need to be considered in the adjustment of triangulations.

The book is fully illustrated with practical examples, and contains the best systematized development of the application of the method to the treatment of simple geodetic measurements that has yet appeared. GERLING issued later (1845, 1855, 1862) some papers supplementary to the work, one of which contains a long list of errata.

1843 REUSCHLE. 'Ueber die Deduction der Methode der kleinsten Quadrate aus Begriffen der Wahrscheinlichkeitsrechnung.' *Crelle's Jour. Math.*, Vol. XXVI, pp. 333–364; Vol. XXVII, pp. 182–184.

This is a critical examination of the deduction of the expression for the probability of an error between given limits and of other points in the "Metaphysik" of the Method. ENCKE's proof of the validity of the arithmetical mean (1832) is also examined and found to be imperfect.

1844. ENCKE. 'Bemerkungen zu der Abhandlung No. 22. Band 26. Heft 4, dieses Journals.' *Crelle's Jour. Math.*, Vol. XXVIII, pp. 213–222.

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A reply to REUSCHLE's criticisms. ENCKE tries to explain that in the two expressions

$$\Sigma_0^a \varphi(x) = 1 \quad \text{and} \quad \int_0^a \varphi(x)\, dx = 1$$

the symbol 1 has different meanings. He also shows that REUSCHLE had failed to understand his proof of the arithmetical mean.

1844 DONKIN. An Essay on the Theory of the Combination of Observations. *Trans. Ashmolean Soc.* (Oxford. 8vo). —French abridgment, 'Sur la théorie....' in *Liouville's Jour. Math.* 1855, Vol. XV, pp. 297–322.

DONKIN attempts to establish "une espèce de Statique métaphysique sur des preuves de la même force que celles qu'on emploie en déduisant, à priori, les lois de la Statique ordinare." The word "force" is taken to mean "tout motif qui nous porte à altérer la valeur attribuée à une quantité," and to these "forces" the principles of centre of gravity of bodies, of virtual velocities, etc., are applied, and the usual rules for the adjustment of observations by means of normal equations, weights, mean errors, etc., are deduced. No law of facility of error enters into the discussion.

DONKIN's reasoning does not always seem to me clear or rigorous.

1844 ELLIS. 'On the Method of Least Squares.' *Trans. Camb. Phil. Soc.*, Vol. VIII, pp. 204–219. —Also *Ellis's Mathematical and other Writings* (Cambridge, 1863, 8vo), pp. 12–37.

In this paper it is attempted "to bring the different modes in which the subject has been presented into juxtaposition, as that the relations which they bear to one another may be clearly apprehended."

ELLIS first takes up GAUSS's proof of 1809. He considers that GAUSS is not justified in assuming that the rule of the arithmetical mean gives the most probable values, and he shows that besides mere convenience no satisfactory reason can be assigned why it should be so regarded. His remarks on this point are extremely valuable and sound. See 1872 GLAISHER.

LAPLACE's demonstration is taken up and presented in a different but greatly simplified form, extended to the case of any number of unknown elements. GAUSS's second proof of 1823 is also analyzed and the conclusion arrived at that "nothing can be simpler or more satisfactory." Lastly IVORY's three proofs (1825–6) are discussed and their illogical character clearly exposed. The paper is one of the most valuable in the theoretical literature of the subject.

1844 JACOBI. 'Ueber eine neue Auflösungsart der bei der Methode der kleinsten Quadrate vorkommenden lineären Gleichungen.' *Astron. Nachr.*, Vol. XXII, col. 297–306.

An abridged method for the solution of certain forms of normal equations.

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[1844] QUETELET. Sur l'appréciations des documents statistiques et en particular sur l'appréciations des moyennes. *Bull. statistique de Belgique*, Vol. II.

1845 BESSEL. 'Auszug aus einem Schreiben....' *Astron. Nachr.*, Vol. XXIII, col. 1–4.

On the probable errors of interpolations in logarithmic tables.

1845 FISCHER. Die Theorie der Beobachtungsfehler und ihre Ausgleichung durch die Methode der kleinsten Quadrate. Pt. I of his *Lehrbuch der höheren Geodäsie;* Darmstadt, 8vo.

1845 GERLING. 'Nachträge zur Ausgleichungs-Rechnung.' *Archiv. Math. u. Phys.*, Vol. VI, pp. 141–146, 375–378.

Three supplements to GERLING's book (1843). The first and second treat of the determination of points by angle measurements and the third of the precision of chain measurements.

1846 BRAVAIS. 'Analyse mathématique sur les probabilités des erreurs de situation d'un point.' *Mém. . . par divers savans . . Inst. France*, Vol. IX, pp. 255–332.

Observations being made on the coördinates of a point the probability that the apparent and true places are separated by a given distance is investigated, as also is the "valeur de la crainte mathématique de l'erreur" which is shown to be represented by the surface of a certain ellipsoid. The method requires the observations to be numerous.

In determining the probability that a point in a plane is located on an elementary area $dy.dx$, BRAVAIS takes the probabilities of the x and y deflections as independent. See 1850 HERSCHEL.

1846 GALLOWAY. 'On the application of the Method of Least Squares to the Determination of the most probable Errors in a Portion of the Ordnance Survey of England.' *Mem. Astron. Soc. Lond.*, Vol. XV, pp. 23–69.

A discussion of a triangulation including ten stations at which thirty-five angles are observed subject to nineteen conditional equations. The methods of adjustment, of determining weights, of forming the conditional equations and all the steps of the process are given at length with great clearness. See *Month. Notices Astron. Soc.*, 1843, Vol. V, pp. 262–264 for a full account of contents of the memoir.

1846 QUETELET. '*Lettres à S. A. R. le Duc régnant de Saxe-Cobourg et Gotha, sur la Théorie des Probabilités appliquée aux sciences morales et politiques.*' Bruxelles, large 8vo, pp. iv, 450. —English trans. by DOWNES; London, 1849, 8vo.

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These letters are of so elementary a character that S. A. R. is informed of the meaning of the signs + and —. They contain however a valuable popular exposition of the Theory of means and of the laws of error. For review of the book see 1850 HERSCHEL.

At the end of the book is an appendix containing many valuable "Notes." In pages 375–380 a table of the terms of the binomial $(\frac{1}{2}+\frac{1}{2})^{999}$ for eighty terms on each side of the middle term is given and the method of its computation explained: these have since been called QUETELET's numbers, see 1869 GALTON. In pages 384–387 it is shown that the general term of the binomial $(\frac{1}{2}+\frac{1}{2})^m$ approaches the exponential form $ce^{-h^2x^2}$ as m indefinitely increases: this is similar to HAGEN's investigation of 1837.

In pages 412–424 are printed three suggestive letters from BRAVAIS, in which not only doubts are expressed as to the *à priori* necessity of the exponential law of facility of error, but examples are given to show that it is not universally true *à posteriori*. BRAVAIS' view is that every cause of partial error gives rise to a distinct curve of facility and that the combination of these approaches the exponential form as a limit, partly because of the necessary law that positive and negative errors are equally likely, and partly because the combination itself must tend toward the binomial form. He alludes to HAGEN's proof as not sufficiently rigorous.

1847 DEMORGAN. 'Theory of Probabilities.' *Encycl. of Pure Math.* (*Encycl. Metrop.*), Pt. II, pp. 393–490.

A great part of this work is translated and adapted from LAPLACE's *Théorie . . . . des Prob.*, 1812, enriched by comments. The Method of Least Squares is treated at considerable length according to LAPLACE's method. At the end are given valuable tables, those of 1799 KRAMP and 1832 ENCKE, and also factorial tables.

1847 GALLOWAY. 'Probabilities.' *Encycl. Brittanica*, seventh ed., Vol. XVIII, pp. 591–639. Eighth edition, Vol. XVIII, pp. 588–636. —Also separately, Edinburgh, 1848, 8vo.

POISSON's analysis (1824) of LAPLACE's method is given, and also GAUSS's proof of 1823.

1848 MATZKA. 'Beweis des obersten Grundsatzes der Methode der kleinsten Quadrate.' *Archiv. Math. u. Phys.*, Vol. XI, pp. 369–377.

A suggestive article. Let x be the true value of a quantity for which observations give the values $a, b, c, \ldots$, then

$$x=f(a, b, c, \ldots)$$

and also we must have

$$x-m=f(a-m, b-m, c-m, \ldots).$$

Applying TAYLOR's theorem, MATZKA deduces for x

$$x=\frac{ha+ib+kc+\ldots}{h+i+k+\ldots}$$

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in which, h, i, k, ... are positive quantities whose values cannot be determined. If the observations are of equal weight and infinite in number the rule of the arithmetical mean is shown to follow.

1848 MEYER. Résolution d'un problême du calcul des probabilités. *Bull. Acad. Belgique*, Vol. XV, Pt. II, pp. 508–512.

1848 MEYER. 'Mémoire sur l'application du calcul des probabilités aux operations du nivellement topographique.' *Mém. Acad. Belgique*, Vol. XXI, 25 pp.

An excellent practical paper.

1848 WOLF. Bestimmung mittlerer Längen und Gewichte. *Mittheil. Gesell. Bern* for 1848, pp. 238–243.

1849 BAEYER. *Die Küstenvermessung und ihre Verbindung mit der Berliner Grundlinie.* Berlin, 4to.

A valuable work for geodetic engineers.

1849 GÖTZE. 'Ueber die Bestimmung der Bedingungsgleichungen bei der Aufsuchungen der wahrscheinlichsten Elemente eines Himmelskörpers aus einem vollständigen Systeme von geocentrischen Beobachtungen.' *Astron. Nachr.*, Ergänzungs Heft, col. 159–234, 239–240.

1849 PAUCKER. Der Ausgleichungsbau und der mittlere Fehler der Beobachtung. *Arbeit Kurland. Gesell.*, Vol. VII, pp. 91–131.

1849 WITTSTEIN. 'Die Methode der kleinsten Quadrate.' Appendix to *Wittstein's Lehrbuch der Differential-und-Integralrechnung* (Hannover, 1868–9, 8vo), Vol. II, pp. 343–442.

Contains a very clear exposition of HAGEN's demonstration (1837) of the law of facility of error and an excellent elementary presentation of the practical features of the method.

1849 WOLF. Note zur Methode der kleinsten Quadrate. *Mittheil. Gesell. Bern* for 1849, pp. 140–144.

1849 WOLF. Versuche zur Vergleichung der Erfahrungswahrscheinlichkeit mit der mathematischen Wahrscheinlichkeit. *Mittheil. Gesell. Bern* for 1849, pp. 97–100, 183–185; for 1850, pp. 85–88, 209–212; for 1851, pp. 17–36.

See also 1853 WOLF. I regret that I have been unable to see these articles.

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1850 BACHE. 'Comparison of the Results obtained in Geodesy by the Application of the Theory of Least Squares.' *Proc. Amer. Assoc.* for 1849, pp. 102–105.

A statement of probable errors of measurements of angles in U. S. Coast Survey triangulations. At the end of the paper, which seems to be a brief abstract of the original, there are some remarks by PEIRCE, GOULD and HENRY, which probably were incorrectly reported.

1850 ENCKE. 'Ueber die Anwendung der Wahrscheinlichkeits-Rechnung auf Beobachtungen.' *Berlin Astron. Jahrb.* for 1853, pp. 310–351.

The object of this paper is to establish greater confidence in the practice of taking the arithmetical mean and in the validity of the exponential law of facility of error. Six of the ten problems of LAGRANGE's memoir of 1774 are translated and a few comments added. HAGEN's demonstration of 1837 is also given in full and spoken of in very favorable terms. ENCKE alludes to the use of the "Erfahrungssatz des Prinzips des arithmetischen Mittels" in his memoir of 1832 and says, "so blieb doch immer eine willkührliche Annahme übrig." At the end of the article is an attempt to explain why $\int_{-\infty}^{+\infty} \varphi(x)dx=1$, when $\varphi(x)$ is the probability of the error x.

1850 GUY. 'On the Relative Value of Averages derived from different Observations.' *Jour. Statis. Soc.*, Vol. VIII, pp. 30–45.

The observations discussed are statistical facts.

1850 HERSCHEL. 'QUETELET on Probabilities.' *Edinburgh Rev.*, Vol. XCII, pp. 1–57. —*Herschel's Essays* (London, 1857, 8vo), pp. 365–465.

This paper contains in a popular form another proof of the Method of Least Squares. Supposing a stone dropped with the intention that it shall hit a mark on a horizontal plane, the reasoning assumes that the deflections from rectangular axes through the mark are independent; and deduces the exponential form $ce^{-h^2x^2}$ for the law of deviation or error. From this the Method of Least Squares at once follows. This proof was put into algebraic language by ELLIS (see below) and the unwarrantable character of the assumption clearly pointed out. See above 1846 BRAVAIS, and below 1857 BOOLE, 1867 THOMPSON and TAIT, 1872 SCHLÖMILCH, and particularly 1872 GLAISHER. See also 1808 ADRAIN, where this proof was first given.

1850 ELLIS. 'Remarks on an alleged Proof of the "Method of Least Squares," contained in a late number of the *Edinburgh Review*.' *Lond. Phil. Mag.*, Vol. XXXVII, pp. 321–328, 462. —Also *Ellis's Mathematical Writings* (Cambridge, 1863, 8vo), pp. 51–62.

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HERSCHEL's proof is discussed and regarded as unsatisfactory. LAPLACE's method is also explained and defended. The paper is very interesting and valuable. See GLAISHER, *Mem. Astron. Soc. Lond.*, 1872, Vol. XXXIX, pp. 112–113; also below 1851 DONKIN.

1850 HOSSARD and POUDRA. 'Seconde Théorème de minimum pour un système de droite dans l'espace.' *Nouv. Ann. Math.*, Vol. IX, pp. 241–242.

On the centre of gravity of a system of points. See also Vol. VII, p. 407 and 454.

1850 PAUCKER. Der mittlere Fehler zweiter Ordnung. *Arbeit Kürland. Gesell.*, Vol. VIII, pp. 104–112.

1850 VERDAM. '*Verhandeling over de methode der kleinste quadraten.*' Groningen, Vol. I, 4to, pp. xxi, 409.

I have not seen Vol. II of this work and find no direct reference to its publication: it was probably published about 1853. Vol. I is devoted to the practical features of the method and Vol. II to the theory.

Vol. I assumes the principle of Least Squares and develops the methods of adjustments and comparison by probable errors, weights, etc. Numerous well chosen practical examples are given worked out in detail. The work abounds in historical information, and is the most complete text-book on the subject which has come to my notice.

1851 AIRY. 'On a Question of Probabilities which occurs in the use of a fixed collimator for the Verification of the Constancy of Position of an Azimuth Circle.' *Rep. Brit. Assoc.* for 1850, pp. 1–2.

Is it justifiable to use the Theory of Errors in finding the probable errors of quantities which are partly results of observation and partly deductions of formulæ?

1851 AIRY. 'On the Weights to be given to the separate Results for Terrestrial Longitudes, determined by Observations of Transits of the Moon and Fixed Stars.' *Mem. Astron. Soc. Lond.*, Vol. XIX, pp. 213–229.

Weight is defined as "the reciprocal of the square of the probable error." The paper discusses nine practical cases.

1851 DENZLER. 'Ueber den Fundamentalsatz der Methode der kleinsten Quadrate.' *Mittheil. Gesell. Zürich*, Vol. II, pp. 110–118.

Appears to be of little importance.

1851 DONKIN. 'On certain Questions relating to the Theory of Probabilities.' *Lond. Phil. Mag.*, Vol. I, pp. 353–368, 458–466; Vol. II, pp. 55–66.

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Part III of this paper offers some critical remarks on the Theory of Least Squares with particular reference to ELLIS's paper of 1850. HERSCHEL's proof, it is said, "should be treated with respect." The Method of Least Squares may be used, if for no other reason, because "it is a very *good* method," as shown by GAUSS's proof of 1823.

1851 HOSSARD. 'Note sur la méthode des moindres carrés.' *Nouv. Annal. Math.*, Vol. X, pp. 456–460.

IVORY's first proof (1825) is here rediscovered under a slightly different form.

1851 PAUCKER. Uebereinstimmung der ausgeglichenen Ursachen mit den durch BESSEL's Verfahren gefundenen. *Arbeit Kürland. Gesell.*, Vol. IX, pp. 170–183.

1851 PAUCKER. Einfluss der Gewichte auf die Ausgleichung. *Arbeit Kürland. Gesell.*, Vol. IX, pp. 183–193.

The substance of this and the preceding article is given in the following.

1851 PAUCKER. 'Zur Theorie der kleinsten Quadrate.' *Bull. phys. math. Acad. St. Péters.*, Vol. IX, col. 113–125; Vol. X, col. 33–43, 233–238. —*Mèl. math. Acad. St. Péters.*, Vol. I, pp. 188–204, 333–346, 433–439.

Contains new methods of computation, tests of accuracy, etc., which appear to be of little value.

1852 BIENAYMÉ. 'Mémoire sur la probabilité des erreurs d'apres la méthode des moindres carrés.' *Liouville's Jour. Math.*, Vol. XVII, pp. 33–78. —*Mém.. par divers savans.. Inst. France*, Vol. XV, pp. 615–663.

After some interesting critical remarks, LAPLACE's analysis (1812) is given considerably simplified. According to BIENAYMÉ's investigation the formulæ for probable error ordinarily used are only correct for one unknown quantity. For two, three and four unknown quantities, he finds that the probable errors should be respectively 1.746, 2.281 and 2.716 times larger than those given by the usual formulæ. His expression for the probability that an error is included between given limits differs sensibly for several unknown quantities from the common probability integral, particularly for limits but little removed from $x=0$. See 1873 WREDE.

See *Comptes Rendus Acad. Paris*, Vol. XXXIV, pp. 90–92, or *Liouville's Jour. Math.*, Vol. XVII, pp. 31–32 for a report on this memoir. See also MEYER's *Calcul des Probabilités*, pp. 377–408.

1852 BIVER. *Théorie des moindres carrés établie par l'analyse pure.* Bruxelles, 8vo.

Probably similar to his memoir of 1853.

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1852 DIENGER. 'Ueber die Ausgleichung der Beobachtungsfehler.' *Archiv. Math. u. Phys.*, Vol. XVIII, pp. 149–193; Vol. XIX, pp. 211–227.

HAGEN'S demonstration (1837) is followed. The article forms an almost complete elementary treatise on the Method of Least Squares.

In the Supplement HAGEN'S proof is abandoned, as resting on a questionable hypothesis and GAUSS'S first proof is given in its place. DIENGER appreciates clearly the defects of GAUSS'S method, for he requires the number of observations to be infinite in order that the value given by the arithmetical mean shall coincide with the *true* value of the measured quantity.

1852 DIENGER. 'Ueber die Bestimmung des Gewichts der nach der Methode der kleinsten Quadrate erhaltenen wahrscheinlichsten Werthe der Unbekannten, wenn Bedingungsgleichungen vorhanden sind.' *Archiv. Math. u. Phys.*, Vol. XIX, pp. 197–202.

Contains valuable practical formulæ for the computer.

1852 HARTNER. *Handbuch der niedern Geodäsie, nebst einem Anhange über die Elemente der Markscheidekunst.* Wien, 8vo. —Second edition, 1856, 8vo, pp. xvi, 611. —Fourth edition, 1872, 8vo.

See 1863 BÖRSCH.

1852 LIAGRE. 'Sur la valeur la plus probable d'un côte géodésique commun à deux triangulations.' *Bull. Acad. Belgique*, Vol. XIX, Pt. I, pp. 513–534.

A clear exposition and solution of the problem. An example from 1849 BAEYER is discussed.

1852 LIAGRE. 'Sur la loi de répartition des hauteurs barométriques, par rapport à la hauteur moyenne.' *Bull. Acad. Belgique*, Vol. XIX, Pt. II, pp. 502–514.

The law is shown to agree with the exponential law of facility of error.

1852 PEIRCE (B.) 'Criterion for the Rejection of doubtful observations.' *Gould's Astron. Jour.*, Vol. II, pp. 161–163.

This Criterion, founded on a principle of the Theory of Probability, proposes a method for determining by successive approximation, whether or not a suspected observation may be rejected. Tables are needed for its application: for these see below 1855 GOULD and 1864 CHAUVENET.

It is a fatal objection to this criterion that its use involves a contradiction of reasoning. The arithmetical mean, for instance, can only be used when the observations are all of equal weight, and the rejection of an observation which deviates considerably from the

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mean asserts that the weights of the several values are not equal. See below 1856 AIRY and WINLOCK, 1868 STONE and particularly 1872 and 1873 GLAISHER.

The criterion has been used to some extent in the U. S. Coast Survey office, but has elsewhere, I believe, found no acceptance.

1852 QUETELET. 'Sur quelques proprietés curieuses qui presèntent les résultats d'une série d'observations, faites dans la vue de déterminer une constant, lorsque les chances de rencontrer des écarts en plus et en moins soint égales et indépendantes les unes des autres.' *Bull. Acad. Belgique*, Vol. XIX, Pt. II, pp. 303–317.

An interesting investigation, illustrated by a discussion of meteorological observations.

1852 WOLF. Beitrag zur Lehre von der Wahrscheinlichkeit. *Mittheil. Gesell. Bern* for 1852, pp. 133–134.

1853 BIVER. 'Théorie analytique des moindres carrés.' *Liouville's Jour. Math.*, Vol. XVIII, pp. 169–200.

The principle of the arithmetical mean is proved according to 1832 ENCKE. The term "risque de erreur" is given to the function $A+B\Sigma x^2+C\Sigma x^4 D+\Sigma x^6+\ldots$ and it is shown that this becomes a minimum when Σx^2 is a minimum, and this condition is regarded as furnishing "les valeurs les plus plausibles des inconnues." Formulæ for weights and mean errors are also developed.

1853 CAUCHY. 'Mémoire sur l'evaluation d'inconnues déterminées par un grand nombre d'equations approximatives du premier degré.' *Comptes Rendus Acad. Paris*, Vol. XXVI, pp. 1114–1122.

It is maintained that the method of interpolation (1835 CAUCHY) can be used for determining several unknown quantities from a redundant number of equations, with results nearly as accurate as by the Method of Least Squares.

1853 BIENAYMÉ. 'Remarque sur les differences qui distinguent l'interpolation de M. CAUCHY de la méthode des moindres carrés, et qui assurent la supériorite de cette méthode.' *Comptes Rendus Acad. Paris*, Vol. XXVII, pp. 5–13. —*Liouville's Jour. Math.*, Vol. XVIII, pp. 299–308.

It is maintained that the two methods differ "complétement," and that even a contradiction exists. CAUCHY's method, it is said, is only a modification of the ordinary process of elimination, which assures no especial degree of probability to the results and which requires in practice as many operations as the Method of Least Squares. See below CAUCHY.

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Further remarks by BIENAYMÉ referring to this discussion are given in pages 68–69, 197, 206 of Vol. XXXVII of the *Comptes Rendus.*

1853 CAUCHY. 'Mémoire sur l'interpolation, ou Remarques sur les Remarques de M. JULES BIENAYMÉ.' *Comptes Rendus Acad. Paris*, Vol. XXXVII, pp. 64–68.

Gives an extract from the memoir of 1835, and maintains that in many investigations the method of interpolation is preferable to that of Least Squares.

1853 CAUCHY. 'Sur la nouvelle méthode d'interpolation comparée à la méthode des moindres carrés.' *Comptes Rendus Acad. Paris*, Vol. XXXVII, pp. 100–109.

The new method is claimed to be often the shortest, and the Method of Least Squares is said to give most probable results only under certain conditions.

1853 CAUCHY. 'Mémoire sur les coefficients limitateurs ou restricteurs.' *Comptes Rendus Acad. Paris*, Vol. XXXVII, pp. 150–162.

In the latter part of the article the "restricteurs" are applied to the theory of Least Squares, and it is concluded that that Method furnishes most probable results only when the law of facility of error is the same for all the errors, when no limits can be assigned to the magnitude of an error, and when the probability of an error x is proportional to $e^{-h^2x^2}$.

1853 CAUCHY. 'Sur les résultats moyens d'observations de même nature, et sur les résultats les plus probables.' *Comptes Rendus Acad. Paris*, Vol. XXXVII, pp. 198–206.

The conclusions of the preceding article are confirmed.

1853 CAUCHY. 'Sur la probabilité des erreurs qui affectent des résultats moyens d'observations de même nature.' *Comptes Rendus Acad. Paris*, Vol. XXXVII, pp. 264–272.

Shows that the most probable values may sometimes differ from those found by the Method of Least Squares.

1853 BIENAYMÉ. 'Considerations à l'appui de la découverte de LAPLACE sur la loi des probabilités dans la méthode des moindres carrés.' *Comptes Rendus Acad. Paris*, Vol. XXXVII, pp. 309–324. —*Liouville's Jour. Math.*, 1867, Vol. XII, pp. 158–176.

An answer and review of some of CAUCHY's articles: also maintains that the mean of the sum of the squares of the errors is under all circumstances a measure of the precision of the observations.

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1853 CAUCHY. 'Sur la plus grande erreur à craindre dans un résultat moyen, et sur le système de facteurs qui rend cette plus grande erreur un minimum.' *Comptes Rendus Acad. Paris*, Vol. XXXVII, pp. 326–334.

The system of factors is often very different from that given by the Method of Least Squares.

1853 CAUCHY. 'Mémoire sur les résultats moyens d'un très grand nombre d'observations.' *Comptes Rendus Acad. Paris*, Vol. XXXVII, pp. 381–385.

An abstract only is given. The result seems to be that the mean is worthy of great confidence.

1853 GRUNERT. 'Elementare Betrachtungen über die Bildung der Bedingungsgleichungen aus gegebenen Beobachtungen.' *Archiv Math. u. Phys.*, Vol. XXI, pp. 453–486.

"Bedingungsgleichung" is not here used in its usual sense. The paper contains an investigation of the value to be taken for x when $A-Bx=0$, and many values of A and B are given by observation. A certain form for x is shown to involve the principle of Least Squares.

1853 LIAGRE. *Calcul des probabilités et la théorie des erreurs, avec des applications aux sciences d'observation en general, et à la géodésie en particular.* Bruxelles, 8vo.

This is a standard work on the subject.

1853 LIAGRE. 'Sur l'erreur probable d'un passage observé à la lunnette méridienne de l'Observatoire royal de Bruxelles.' *Bull. Acad. Belgique*, Vol. XX, Pt. II, pp. 303–312.

1853 QUETELET. *Théorie des probabilités.* Bruxelles, 12mo, pp. 104.

1853 WOLF. Versuche zur Vergleichung der mathemathischen Wahrscheinlichkeit. *Mittheil. Gesett. Bern* for 1853, pp. 23–28.

See also 1849 WOLF.

1853 ———. *Tafeln zur Berechnung der Wahrscheinlichkeit des Vorkommens von Beobachtungsfehler.* Berlin, lith. MS., 4to, pp. 11.

1854 D'ARREST. 'Beitrag zur Methode der kleinsten Quadrate.' *Bericht. Sächsich. Gesell.*, 1854, pp. 133–136. —*Astron. Nachr.*, Vol. XLI, col. 35–40.

Points out six new geometrical properties of the probability curve, and shows how its equation may be derived from a certain mechanical idea.

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1854 HÉLIE. *Mémoire sur la probabilité du tir des projectiles de l'artillerie navale.* Paris, 4to. —Second ed., 1856, 8vo.

1855 BACHE. 'Comparison of the reduction of horizontal angles by the methods of dependant directions and of dependant angular quantities, by the method of least squares.' *Rep. Coast Survey U. S.* for 1854, pp. 63*–70.*

The first method is shown to possess a slight advantage.

1855 BERTRAND. '*Méthode des moindres carrés. Mémoires sur la combinaison des observations, par* CH. FR. GAUSS. Paris, 8vo, pp. 167.

This volume renders quite accessible the Latin memoirs of GAUSS. It contains translations of pages 208–220 of the *Theoria motus* ..... 1809, pages 20–26 of *Disquisito de* ..... *Palladis*, 1811, and the whole of the memoirs of 1816, 1822, 1823, 1826 and 1827. The memoirs of 1823 and 1827 form the bulk of the book, the others being added at the end as "Notes."

1855 BERTRAND. 'Sur la méthode des moindres carrés.' *Comptes Rendus Acad. Paris*, Vol. XL, pp. 1190–1192.

Historical and critical remarks made on presenting a copy of the above book to the Paris Academy.

1855 GERLING. 'Ueber die Schätzung des mittlern Fehlers directer Beobachtungen.' *Archiv Math. u. Phys.*, Vol. XXV, pp. 219–222.

Contains three pages of corrections and errata to his book (1843).

1855 GOULD. 'Report ..... containing directions and tables for the use of PEIRCE's Criterion for the Rejection of Doubtful Observations.' *Rep. Coast Survey U. S.* for 1854, pp. 131*–138*.

Reprinted with a different title in *Gould's Astron. Jour.*, 1855, Vol. IV, pp. 81–87.

1855 HILGARD. 'Discussion of the probable error of observation with a twenty-six inch portable transit .....' *Rep. Coast Sur. U. S.*, for 1854, p. 121*.

The probable error is given as $\pm 0.072^s$.

1855 LIAGRE. 'Sur la probabilité de l'existence d'une cause d'erreur regulière dans une série d'observations.' *Bull. Acad. Belgique*, Vol. XXII, Pt. II, pp. 9–13, 15–54.

A valuable theoretical discussion illustrated with practical examples.

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1855 LLOYD. On the mean results of observations. *Trans. Irish Acad.*, Vol. XXII, pp. 61–73.

See abstract in *Proc. Irish Acad.*, Vol. IV, pp. 180–183.

1855 PEIRCE (B.). 'Report upon the Determination of Longitude by Moon Culminations.' *Rep. Coast Sur. U. S.* for 1854, pp. 109*–120*.

"The small errors which are beyond the limits of human perception, are not distributed according to the mode recognized by the Method of Least Squares, but either with the uniformity which is the ordinary characteristic of matters of chance, or more frequently in some arbitrary form dependant upon individual peculiarities . . . . ."

1855 SCHOTT. 'Adjustment of horizontal angles of a triangulation.' *Rep. Coast Sur. U. S.*, for 1854, pp. 70*–86*.

This and many of SCHOTT's following papers are very valuable, but they are not usually clear except to those who already understand the subject.

1855 SCHOTT. 'Probable error of observation derived from observations of horizontal angles at any single station, and depending on directions.' *Rep. Coast Sur. U. S.* for 1854, pp. 86*–95*.

A discussion of 350 measurements taken at eleven stations.

1856 AIRY. 'Letter from . . . [Remarks on PEIRCE's Criterion.]' *Gould's Astron. Jour.*, Vol. IV, pp. 137–138.

The Criterion is strongly opposed. ". . . . the whole theory is defective in its foundations and illusory in its results." It must be said, however, that some of AIRY's objections are not supported by very good logic.

1856 WINLOCK. 'On Professor AIRY's objections to PEIRCE's Criterion.' *Gould's Astron. Jour.*, Vol. IV, pp. 145–147.

AIRY's objections are taken up in detail; some of them are shown to apply equally well to the Method of Least Squares.

1856 PETERS. 'Ueber die Bestimmung des wahrscheinlichen Fehlers einer Beobachtung aus den Abweichungen der Beobachtungen von ihrem arithmetischen Mittel.' *Astron. Nachr.*, Vol. XLIV, col. 29–32. —Trans. in *Rep. Coast Sur. U. S.* for 1856, pp. 307–308.

Let Σx be the sum of the residual errors all taken positive, and n the number of direct observations of equal weight. Then PETERS' result is, that r, the probable error of a single observation is,

$$r = 0.845347 \frac{\Sigma x}{\sqrt{n(n-1)}}.$$

See on this formula 1869 LÜROTH, and 1875 HELMERT.

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1856 SCHOTT. 'Solution of Normal Equations by indirect Elimination.' *Rep. Coast Sur. U. S.* for 1855, pp. 255–264.

Several methods are given and illustrated by numerical examples. The paper is of great value to a computer.

1856 SCHOTT. 'Report . . . . . of a discussion of the secular variation in the magnetic declination on the Atlantic and part of the Gulf Coast of the United States.' *Rep. Coast Survey U. S.* for 1855, pp. 306–337.

The process of deducing empirical formulæ for declination from observations by the Method of Least Squares is explained and illustrated, as also that of finding the probable errors of the constants which enter into such formulæ and of the computed results. Formulæ are deduced for fourteen stations. See also *Rep. Coast Survey* for 1855, p. 306, and for 1859, p. 296.

1856 VORLÄNDER. 'Ueber die Genauigkeit der Längenmessungen mit der Messkette auf verschiedenen Bodenarten.' *Zeitschr. Math. u. Phys.*, Vol. I, pp. 142–159.

The precision is regarded as inversely proportional to the length of the line. Tables are given showing results for different kinds of ground. See 1863 BÖRSCH.

1857 ANDRÄ. 'Fehlerbestimmung bei der Auflösung der POTHENOT'schen Aufgabe mit dem Messtische.' *Astron. Nachr.*, Vol. XLVII, col. 193–202.

Points for which the probabilities of error are equal have an ellipse as locus. The most probable ellipse is assumed to be given by the Method of Least Squares. See *Bull. math. phys. Acad. St. Pétersb.*, Vol. VII, p. 145.

1857 D'ARREST. 'Schreiben . . . . [über die Berechnung der Gewichte].' *Astron. Nachr.*, Vol. XLVII, col. 17–20.

Contains formulæ for the easy determination of the relative weights of three unknown quantities, and also remarks concerning the geometric signification of weights and their connection with determinants.

1857 BABINET and HOUSEL. '*Calculs pratiques appliqués aux sciences d'observation.*' Paris, 8vo, pp. xvi, 388.

Mostly devoted to the theory of numerical approximations and interpolation formulæ. Two pages are given to the Method of Least Squares.

1857 BAUR. ——————. *Programm des Stuttgarter Polytechnikursus* für 1857.

An article on determination of weights, etc.

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1857 BELLAVITIS. Considerazioni sulla theoria della probabilita. *Atti Istit. Venezia* for 1856–7, pp. 299–321.

1857 BOND. 'On the use of Equivalent Factors in the Method of Least Squares.' *Mem. Amer. Acad.*, Vol. VI, pp. 179–212. —Also privately printed, Cambridge, 1858, 4to, pp. 36.

It is shown that it is unnecessary to adhere in numerical computations to the strict letter of the Method of Least Squares, and that its application ".....requires the use of such numbers only, in the arithmetical processes peculiar to it and characteristic of the Method, as may be designated by one of the numerals 0, 1, 2,....9, or of the fractions $\frac{1}{2}$, $\frac{1}{3}$,....$\frac{1}{9}$, or by a product of one of these numbers by an integral power of 10."

1857 BOOLE. On the Application of the Theory of Probabilities to the question of the Combination of Testimonies or Judgments. *Trans. Soc. Edinb.*, Vol. XXI, pp. 597–652.

In the first part of this memoir the rule of the arithmetical mean is discussed. "The result of BOOLE's investigation is that if n observations $p_1, p_2, \ldots p_n$ be made upon the same quantity, then the most probable value of that quantity is a certain linear function of p_1, p_2, $\ldots p_n$; this BOOLE demonstrates by his Calculus of Logic, and the analysis is of so peculiar a character that....I feel scarcely qualified to express a decided opinion on its merits. ....the [final] result takes the form of the arithmetic mean."—GLAISHER, *Mem. Astron. Soc. Lond.*, 1872, Vol. XXXIX, p. 124.

In the latter part of the paper HERSCHEL's demonstration is reproduced and defended against the arguments of ELLIS; see 1850. See GLAISHER's paper, just quoted, pp. 115.

1857 DIENGER. '*Ausgleichung der Beobachtungsfehler nach der Methode der kleinsten Quadratssummen. Mit zahlreichen Anwendung, namentlich auf geodätische Messungen.*' Braunschweig, 8vo, pp. viii, 168.

An excellent elementary text-book. GAUSS's proof of 1809 is followed, with the improvement that the probability of a definite error is an infinitesimal. See 1852 DIENGER. Among the practical questions treated is the theory of repetitions in angle measurements.

1857 DONKIN. 'On an Analogy relating to the Theory of Probabilities, and on the Principle of the Method of Least Squares.' *Quart. Jour. Math.*, Vol. I, pp. 152–162.

Donkin observes that if two observations of an unknown quantity give $x=a$ and $x=b$, then the most probable value of x is $\frac{1}{2}(a+b)$, but that we cannot regard the arithmetical mean of more than two observations as most probable. Taking x to represent the true value of the unknown quantity DONKIN says: "....it appears a natural and

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obvious assumption (though I do not pretend that it is *not* an assumption) that *the probability that x is between x and $x+dx$ must be expressible in the form* $\psi\left(x-\frac{a+b}{2}\right)dx$." From this the exponential law of facility of error is deduced.

For an analysis of this reasoning see GLAISHER, *Mem. Astron. Soc. Lond.*, Vol. XXXIX, p. 118, who classes it among the proofs of the Method.

1857 LIAGRE. Sur la mesure de précision des observations méridiennes faites à l'Observatoire royal de Bruxelles. *Bull. Acad. Belgique*, Vol. III, pp. 330–338.

1857 PETZVAL. 'Fortsetzung des Berichtes über optische Untersuchungen. [Ausgleichungstheorie].' *Sitzungsber. math-nat. Acad. Wien*, Vol. XXIV, pp. 129–144.

PETZVAL concludes that the Method of Least Squares is entirely inapplicable in Optics. He proposes "die Methode der numerisch gleichen Maxima und Mimmia," which consists in making the sum of the $2m$ powers a minimum, m being a variable which tends toward infinity as a limit. The development and application of this method is to constitute the First Part of Vol. III of his work on Optics. This method was mentioned by LAPLACE in the *Théorie....des Probabilités*, p. 345.

1857 SAWITSCH. *Die Anwendung der Wahrscheinlichkeitstheorie auf die Berechnung der Beobachtungen und geodätische Messungen, oder die Methode der kleinsten Quadrate.* (Russian). St. Petersburg, 8vo. —German edition, Leipzig, 1863, 8vo.

1857 VORLÄNDER. 'Ueber das geodätische Vorwärts-Einschneiden.' *Zeitschr. Math. u. Phys.*, Vol. II, pp. 299–316.

A practical application of the Method of Least Squares.

1857 WREDE. 'Några anmärkningar rörande minsta quadratmethodens tillämpning.' *Öfvers. Acad. Förhandl. Stockholm*, Vol. XIV, pp. 73–81.

On the error ellipse.

1857 ZECH. *Einladung zur akademischen Feier des Geburtstags des Königs von Württemberg, nebst einer Abhandlung zur Methode der kleinsten Quadrate.* Tübingen, 4to.

The first exhaustive discussion of the adjustment of indirect observations subject to conditional equations. See JORDAN, *Elemente der Vermessungskunde* (Stuttgart, 1877, 8vo), p. 6.

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1858 BOUNIAKOWSKY. 'Sur un instrument destiné à faciliter l'application numérique de la méthode des moindres carrés, et à contrôler les résultats obtenus par cette méthode.' *Bull. phys. math. Acad. St. Péters.*, Vol. XVII, col. 289–298. —*Mél. math. Acad. St. Péters.*, Vol. II, pp. 602–614.

A machine for calculating the sums $a_1^2 + a_2^2, + a_3^2 + \ldots.$, and $a_1b_1 + a_2b_2 + a_3b_3 + \ldots$

1858 CASORATI. 'Intorno ad alcuni puncti della theoria dei minimi quadrati.' *Annali di Math.*, Vol. I, pp. 329–343.

Discussion by the use of determinants, etc.

1858 CLARKE. '*Ordnance Trigonometrical Survey of Great Britain and Ireland. Account of the Observations and Calculations of the Principal Triangulations and of the Figure, Dimensions and mean Specific Gravity of the Earth as derived therefrom.*' London, 4to, pp. xvii, 782, with an Atlas of 28 Plates.

The whole triangulation is adjusted by the Method of Least Squares. The method of correlatives is explained at length and illustrated for a case involving seventy-four observations subject to thirty-nine conditional equations. An inspection of this book will give students an idea of the stupendous calculations which men of science undertake and execute.

1858 DIDION. *Calcul des probabilités appliqué au tir des projectiles.* Paris, 8vo.

For an exposition of this subject see SONNET'S *Dictionaire des mathématiques appliquées*, (Paris, 1867), pp. 1103–1108.

1858 GRUNERT. 'Drei Grössen x, y, z, deren Summe die gegebene Grösse s ist, sind durch Messung bestimmt worden, und man habe dadurch für diese drei Grössen respective die Werthe a, b, c erhalten. Da diese Werthe mit Beobachtungsfehler behaftet sind, und ihre Summe also im Allgemeinen nicht genau s ist, so soll man dieselben so verbessern, dass die verbesserten Werthe genau die Summe s geben, und die Summe der Quadrate der Verbesserungen ein Minimum ist.' *Archiv. Math. u. Phys.*, Vol. XXXI, pp. 480–481.

1858 JULLIEN. 'Mémoire sur la probabilité des erreurs dans la somme ou dans la moyenne de plusieurs observations.' *Annali di Math.*, Vol. I, pp. 76–88, 149–155, 227–237.

Errors subject to the law of facility $\varphi(x) =$ constant are particularly discussed.

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1858 KOHLER. '*Die Landesvermessung des Königsreichs Württemberg.*' Stuttgart, 8vo, pp. xii, 428, and 3 pl.

In the Appendix particularly are applications of the Method of Least Squares.

1858 RITTER. '*Manual théoretique de l'application de la méthode des moindres carrés au calcul des observations.*' Paris, 8vo, pp. 80.

An excellent little text-book, in which GAUSS's first demonstration is followed, with the improvement that the probability of a single error is an infinitesimal.

1858 SCHINDLER. *Ueber Fehler bei der Berechnung eines ebenen Dreiecks.* Prag, 4to.

1858 SCHOTT. 'Account of the method and formulæ for the determination of the astronomical latitude by means of the zenith telescope as used in the survey of the coast of the United States.' *Rep. Coast Survey U. S.* for 1857, pp. 324–334.

Weights and probable errors are found. The notation used on p. 333 is unusual and uncouth.

1858 TCHÉBYCHEFF. 'Sur les fractions continues.' *Liouville's Jour. Math.*, Vol. III, pp. 289–325.

Translated from the Russian by BIENAYMÉ: treats of interpolation by the Method of Least Squares.

1858 TCHÉBYCHEFF. 'Sur l'interpolation des valeurs fournis par les observations.' *Bull. phys. math. Acad. St. Péters.*, Vol. XVI, col. 353–357. —*Mél. math. Acad. St. Péters.*, Vol. II, pp. 345–351.

1858 VORLÄNDER. 'Zur praktischen Geometrie.' *Zeitschr. Math. u. Phys.*, Vol. III, pp. 189–193.

On the adjustment of a quadrilateral whose sides and diagonals are measured.

1858 VORLÄNDER. 'Bemerkungen über das numerische Eliminiren bei geodatischen Operationen.' *Zeitschr. Math. u. Phys.*, Vol. III, pp. 16–22.

On methods of abridging the computations.

1858 GERLING. 'Bemerkungen über das indirecte Eliminiren bei geodätischen Arbeiten.' *Zeitschr. Math. u. Phys.*, Vol. III, pp. 377–382.

Referring to the preceding article of VORLÄNDER.

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1858 VORLÄNDER. *Ausgleichung der Fehler polygonometrischer Messungen.* Leipzig, 8vo, pp. 55.

Besides the Method of Least Squares shorter approximate processes are given.

1858 DIENGER. 'Ueber die Ermittelung des wahrscheinlichen Fehlers bei Längenmessungen.' *Archiv. Math. u. Phys.*, Vol. XXXI, pp. 225–228.

Opposes VORLÄNDER's method given in the preceding.

1859 TCHÉBYCHEFF. 'Sur l'interpolation dans le cas d'un grand nombre de données fournies par les observations.' *Mém. Acad. St. Péters.*, Vol. I, No. 5, 81 pp.

1859 TCHÉBYCHEFF. 'Sur l'interpolation par la méthode des moindres carrés.' *Mém. Acad. St. Péters.*, Vol. I, No. 15, 24 pp.

[1860] BAEYER. *Wissenschaftliche Begründung der Rechnungsmethoden des Centralbureaus der europäischen Gradmessung: I. Die Methode der kleinsten Quadrate. II. Die Anwendung derselben auf Geodäsie.* [Lith. MSS.?] 4to.

1860 DEDAKIND. 'Ueber die Bestimmung der Präcision einer Beobachtungsmethode nach der Methode der kleinsten Quadrate.' *Vierteljahrs. Gesell. Zürich*, Vol. V, pp. 76–83.

The most probable value of the measure of precision h is found to be $\sqrt{\frac{n-1}{2\Sigma x^2}}$ and not $\sqrt{\frac{n}{2\Sigma x^2}}$. See 1866 BÖRSCH.

1860 HULTMANN. *Sur les moindres carrés.* Stockholm, 4to.

1861 AIRY. '*On the Algebraical and Numerical Theory of Errors of Observations and the Combination of Observations.*' Cambridge and London, 8vo, pp. xvi, 103. —Second ed., 1875.

Only LAPLACE's *Théorie analytique des Probabilités* was consulted in preparing this book, and as a consequence it is unreadable except by those already thoroughly acquainted with the subject.

1861 BORCHARDT. 'Ueber Interpolation nach der Methode der kleinsten Quadrate.' *Crelle's Jour. Math.*, Vol. LVIII, pp. 270–272.

On TCHÉBYCHEF's method; see 1858 and 1859.

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1861 GERLING. Notiz in Betreff der Prioritäts-Verhältnisse in Beziehung auf die Methode der kleinsten Quadrate. *Nachr. Gesell. Göttingen* for 1861, pp. 273–275.

1861 PETERS. *Ueber die Bestimmung des Längenunterschieds zwischen Altona und Schwerin.* Altona, 4to.

Contains among other matter a detailed history of the discovery of the personal equation. See 1866 RADAU.

1861 SCHOTT. 'Account of CAUCHY's interpolation formula.' *Rep. Coast Sur. U. S.* for 1860, pp. 392–396.

This is a free translation of CAUCHY's article of 1835. The method is illustrated by an example.

1861 WINCKLER. 'Ueber den mittlern Fehler der Kettenmessungen.' *Zeitschr. Math. u. Phys.*, Vol. VI, pp. 109–119.

An excellent practical paper. The mean error is found to be proportional to the square root of the length of the line.

1862 GERLING. 'Ueber Genauigkeit der Functionen bedingter Beobachtungen.' *Archiv. Math. u. Phys.*, Vol. XXXVIII, pp. 379–381.

Contains additions and corrections to his book (1843).

1862 SEIDEL. Ueber eine Anwendung der Wahrscheinlichkeitsrechnung bezüglich auf der Schwankungen in den Durchsichtigkeitsverhältnissen der Luft. *Sitzungsber. Baiersch. Acad.*, Vol. II, pp. 320–349.

1863 BÖRSCH. 'Ueber die Genauigkeit der Winkel- und Linien-Messungen.' *Zeitschr. Math. u. Phys.*, Vol. VIII, pp. 321–341.

It is concluded that the precision of angle measurements is proportional to the square root of the number of single observations, or to the number of repetitions; and that the precision of linear measurements is inversely proportional to the square root of the length of the line. The articles of VORLÄNDER (see 1856) are discussed as also is HARTNER's (1852) treatment of this subject.

1863 FREEDEN. '*Die Praxis der Methode der kleinsten Quadrate für die Bedürfnisse der Anfänger bearbeitet.*—Erster Theil: Elementare Darstellung der Methode nebst Sammlung vollständig berechneter physikalischer, meteorologischer, geodätischer und astronomischer Aufgaben, welche auf lineäre und transcendente Gleichungen führen.' Braunschweig, 8vo, pp. viii, 114.

An excellent little book, although some of the examples are rather long for a beginner. The principle of Least Squares is assumed.

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1863 KURZ. 'Ueber der Methode der kleinsten Quadrate.' *Vierteljahrsschr. Gesell. Zürich*, Vol. VIII, pp. 225–240.

On the various methods of forming and solving normal equations, of determining weights, etc.

1863 LOBATTO. Over de waarschijnlijkheid van gemiddelde uitkomsten uit een groot aantal waarnemingen. *Archief. Wisk. Genoots*, Vol. II, pp. 96–127.

1864 CHAUVENET. 'Method of Least Squares.' Appendix to *Manual of Spherical and Practical Astronomy* (Philadelphia, 8vo), Vol. II, pp. 469–566. —Also as separate issue under title '*A Treatise on the Method of Least Squares*,....' Philadelphia, 1868.

This is mainly an abridgment of ENCKE's memoirs of 1832. ENCKE's demonstration of the rule of the arithmetical mean is in particular set forth with confidence. The reasoning showing that if $\varphi(x)$ is the probability of the error x, $\varphi(x)dx$ is the rigorous probability that an error falls between x and $x+dx$ is very illogical. At the end are valuable tables, two of the probability integral, and others for using PEIRCE's criterion which is given nearly in the words of its author; see 1852 and 1856. CHAUVENET adds an approximate criterion for the rejection of *one* doubtful observation, which is derived "directly from the fundamental formula upon which the whole theory of the Method of Least Squares is based."

1864 CHRISTOFFEL. '*Bestimmung einer Oberfläche durch lokale Messungen.* [Berlin], 4to.

1864 DEMORGAN. 'On the Theory of the Errors of Observation.' *Trans. Camb. Phil. Soc.*, Vol. X, pp. 409–427.

This is a very valuable contribution to the theory of the arithmetical mean. It is shown that the average "is not merely the *mean value of all the given values:* it is also the *mean supposition of all possible suppositions* as to the mode of obtaining that value," but that "the average is the most probable result only so long as we *know* nothing of the law of facility of error." See GLAISHER, *Mem. Astron. Soc. Lond.*, 1872, Vol. XXXIX, p. 90.

DEMORGAN suggests the name "*critical error*" instead of *probable error.* The entire paper, like all of DEMORGAN's writings, is very interesting and suggestive.

1864 SCHOTT. 'Development of BESSEL's function for the effect of periodic forces, for durations of periods frequently occurring in meteorological and magnetical investigations; with examples.' *Rep. Coast Survey U. S.* for 1862, pp. 232–235.

See above 1828 BESSEL.

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1864 WOOLHOUSE. 'On Interpolation, Summation, and the Adjustment of Numerical Tables.' *Lond. Assurance Mag.*, Vol. XI, pp. 61–88, 300–332; Vol. XII, pp. 136–175.

1865 BRÜNNOW. 'The Method of Least Squares.' *Spherical Astronomy* (First English from the second German edition, London and New York, 8vo), pp. 40–60.

A very elementary sketch of the Method.

1865 GOOSS. '*Begründung der Methode der kleinsten Quadrate.*' Kreutznach, 8vo, pp. 32.

A doctor's thesis. Contains a deduction of the law $y = ce^{-h^2x^2}$ from the axioms that the curve is symmetrical, that it has the axis of x for an asymtote, that the equation must be a simple one, etc. The discussion is not very satisfactory.

1865 TAIT. 'On the Law of Frequency of Error.' *Trans. Soc. Edinb.*, Vol. XXIV, 7 pp.

The principle of the investigation is that an error arising from any source may be compared to the deviation from the most probable result of the number of white or black balls obtained by a great number of drawings from a bag containing equal numbers of white and black balls. The idea and the algebraic work is nearly the same as QUETELET's investigation of 1846. See 1872 GLAISHER.

1865 TODHUNTER. '*A History of the Mathematical Theory of Probability from the time of* PASCAL *to that of* LAPLACE.' Cambridge and London, 8vo, pp. xvi, 624.

This work is invaluable to all students of the Theory of Probability and I have to acknowledge my great indebtedness to it in preparing the early part of this list. None but those who have undertaken such historical researches can form an idea of the immense amount of labor which must have been done in preparing a work like this of TODHUNTER.

TODHUNTER's analyses of the memoirs of LAGRANGE and LAPLACE are full and clear, and his commentary on LAPLACE's proof of the Method of Least Squares greatly simplifies the tedious investigations of the *Théorie analytique des Probabilités.* An account of GAUSS's proof of 1809 is not given.

1866 BÖRSCH. 'Ueber die mittlern Fehler der Resultate aus trigonometrischen Messungen.' *Archiv. Math. u. Phys.*, Vol. XLVI, pp. 40–44.

Σx^2 being the sum of the squares of the residual errors and n the number of direct observations, the mean error has been taken as

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$\sqrt{\frac{\Sigma x^2}{n}}$ and $\sqrt{\frac{\Sigma x^2}{n-1}}$. The paper compares the results given by these two formulæ and accords the preference to the first. See 1816 BESSEL, 1816 GAUSS, 1823 GAUSS and 1860 DEDAKIND.

1866 NATANI. 'Quadrate (Methode der kleinsten).' *Hoffmann's Mathematisches Wörterbuch*, Vol. V, pp. 16–33.

GAUSS's proof of 1808 and HAGEN's of 1837 are given, and free use is made of WITTSTEIN's work of 1849. The familiar equation expressing the law of facility of error appears here under the strange notation $\varphi(\gamma)=Ce^{-a^2\gamma^2}$.

1866 RADAU. 'Ueber die persönlichen Gleichungen bei Beobachtungen derselben Erscheinungen durch verschiedene Beobachter.' *Reper. f. phys. Technik*, Vol. I, pp. 202–218, 306–321; Vol. II, pp. 115–156.

This is a translation from the *Moniteur scientifique* for 1865. It gives a detailed history and discussion of the subject of personal equation. See 1861 PETERS.

1866 SCHOTT. 'The problem of determining a position by angles observed upon a number of given stations. Solution of GAUSS, with example.' *Rep. Coast Survey U. S.* for 1864, pp. 116–119.

The method is taken from GERLING's book, see 1840.

1866 SCHOTT. 'Report on the method of reduction, and results of the connection of the Epping base line with the primary triangulation in the Eastern States.' *Rep. Coast Survey U. S.* for 1864, pp. 120–144.

A valuable practical paper containing determinations of probable errors of observations, and the discussion of a case of adjustment in solving thirty-five normal equations and fifty-eight equations of correlatives.

1866 WINCKLER. 'Allgemeine Sätze zur Theorie der unregelmässigen Beobachtungsfehler.' *Sitzungsber. Akad. Wein*, Vol. LIII, Pt. II, pp. 6–41.

An investigation of relations between mean values of powers of errors and expressions for probability of errors.

1867 ANDRÄ. *Den Danske Gradmaaling.* Vol. I, Kopenhagen, 4to, pp. 608. Vol. II in 1873, pp. 504.

The adjustment of indirect observations subject to conditional equations is fully treated.

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1867 HAGEN. '*Grundzüge der Wahrscheinlichkeits-Rechnung.*' Second edition, Berlin, large 8vo, pp. x, 187.

This is rewritten from the edition of 1837. The proof of the law of facility is given substantially the same as before. The distinction between true and computed errors is not however clearly drawn.

The book is an excellent one for students and engineers, the greater part being of a practical character.

1867 HANSEN. 'Von der Methode der kleinsten Quadrate im Allgemeinen und in ihrer Anwendung auf die Geodäsie.' *Abhandl. Sächsisch. Gesell.*, Vol. XIII, pp. 571–806. —Also separately, Leipzig, 8vo, pp. 236.

In the opening pages the law of facility $\varphi(x)=h\pi^{-\frac{1}{2}}e^{-h^2x^2}$ is deduced by GAUSS's method of 1809, x being regarded as the residual or computed error. The remarks on page 797 concerning probable errors seem to be true and valuable. The book is particularly full in the treatment of conditioned observations, and is a valuable one for geodetic engineers. See 1868 and 1869 for supplements to the work.

1867 SCHOTT. 'Results of the primary triangulation of the coast of New England, from the north-eastern boundary to the vicinity of New York.' *Report Coast Survey U. S.* for 1865, pp. 187–203.

A continuation of SCHOTT's articles of 1855 and 1866, devoted mainly to the discussion of the probable errors of the linear and angular measurements of the triangulation. A comparison is also given of the measured lengths of three base lines with the lengths as computed through the triangulation. The paper is a very valuable one.

1867 THOMPSON and TAIT. 'Experience.' Chap. III, of their *Treatise on Natural Philosophy*, (Oxford, 8vo), Vol. I, pp. 303–320.

HERSCHEL's proof (1850) is given and spoken of as "simple and apparently satisfactory." See below 1872 SCHLÖMILCH.

1867 TCHÉBYCHEFF. 'Des valeurs moyens.' *Rec. Sci. Math.*, Vol. II. —*Liouville's Jour. Math.*, Vol. XII, pp. 177–184.

On "Espérances mathématiques," their arithmetical means, etc.

1868 FRISIANI. 'Sulle più vantaggiosa combinazione delle osservazioni.' *Mem. Istit. Lombardo*, Vol. II, pp. 1–21.

The principle of Least Squares is proved, assuming that the arithmetical mean gives the most probable result.

1868 HANSEN. 'Fortgesetzte geodätische Untersuchungen, bestehend in zehn Supplementen zur Abhandlung von der Methode der

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kleinsten Quadrate im Allgemeinen und in ihrer Anwendung auf die Geodäsie.' *Abhandl. Sächsisch. Gesell.*, Vol. XIV, pp. 1–184.

For review see *Jahrb. Fortschr. Math*, Vol. I, p. 388.

1868 HELMERT. 'Studien über rationelle Vermessungen im Gebiete der höheren Geodäsie.' *Zeitschr. Math. u. Phys.*, Vol. XIII, pp. 73–120, 163–186.

See *Jahrb. Fortschr. Math.*, Vol. I, p. 389.

1868 HENCKE. *Ueber die Methode der kleinsten Quadrate.* Leipzig, 8vo.

A doctor's dissertation containing historical and critical information relating to Least Squares. I regret that I have been unable to see a copy of it.

1868 LEBOULENGÉ. 'Études de ballistique expérimentale.' *Mém. Acad. Belgique*, Vol. XX, pp. 1–94.

Contains applications of the Method of Least Squares.

1868 MILLER-HAUENFELS. '*Höhere Markscheidekunst. Praktisch-theoretische Anleitung beim Markscheiden die vermeidlichen Fehler zu umgehen, die unvermeidlichen aber in einfacher und streng wissenschaftlicher Weise zu verbessern.*' Wien, 8vo, pp. xii, 291.

A valuable book for mining engineers. In the first or practical part processes and their applications are given, while the proofs follow in the second part. An attempt is made to show that the arithmetical mean is the most probable result by the theory of combinations, all true errors being taken as equal. The term *absolute weight* is introduced for h^2. GAUSS's first proof of the Method of Least Squares is given.

1868 SCHIAPARELLI. 'Sul principio della media arithmetica nel calcolo dei resultati delle osservationi.' *Instit. Lombard. Rendiconti*, Vol. I, pp. 771–778.

Contains a new demonstration of the validity of the arithmetical mean. See 1875.

1868 STONE. 'On the Rejection of Discordant Observations.' *Month. Not. Astron. Soc. Lond.*, Vol. XXVIII, pp. 165–168.

PEIRCE's and CHAUVENET's criteria (see 1852 and 1864) are regarded as troublesome to use and as based on an erroneous principle, and a criterion is proposed, which embodies, in the opinion of the author, the true grounds on which the judgment rests when rejecting discordant observations or mistakes. See below 1873 GLAISHER and STONE.

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1869 DORNA. 'Sulla media arithmetica vel calcolo di compensazione.' *Atti. Accad. Torino*, Vol. IV, pp. 757–763.

On ENCKE's demonstration of the arithmetical mean, and on SCHIAPARELLI's article of 1868.

1869 FAÀ DE BRUNO. '*Traité élémentaire du calcul des erreurs, avec des tables stéréotypées, ouvrage utile à ceux qui cultivent les sciences d'observation.*' Paris, 8vo, pp. vii, 72, XLV.

The tables are the best part of this *ouvrage*, but in that giving the values of sin^2 there is at least one dangerous error. This was one of the first books on the Method of Least Squares which I read, and I take this opportunity to warn young students against it. The text is full of typographical and other errors and the subject is presented neither clearly or fully. The list of literature at the end does not contain the names of LEGENDRE, IVORY, ENCKE, BESSEL, ELLIS, or HERSCHEL, gives only one work by HANSEN, and does not mention GAUSS's *Theoria motus*..... The book deserves a speedy oblivion.

1869 GALTON. '*Hereditary Genius; an Inquiry into its Laws and Consequences.*' London, 8vo. —Amer. edition, 1870.

The exponential law of error is used in dividing mankind into grades of intellect. QUETELET's numbers (1846) are employed for this purpose and are given in the appendix.

1869 HANSEN. 'Entwickelung eines neuen veränderten Verfahrens zur Ausgleichung eines Dreiecksnetzes mit besonderer Betrachtung des Falles in welchem gewisse Winkel voraus bestimmte Werthe bekommen sollen.' *Abhandl. Sächs. Gesell.*, Vol. XIV, pp. 185–287.

1869 HELMERT. 'Beiträge zur Theorie der Ausgleichung trigonometrischer Netze.' *Zeitschr. Math. u. Phys.*, Vol. XIV, pp. 174–208.

See *Jahrb. Fortschr. Math.*, Vol. II, p. 835.

1869 JORDAN. 'Ueber die Bestimmung der Genauigkeit mehrfach wiederholter Beobachtungen *einer* Unbekannten.' *Astron. Nachr.*, Vol. LXXIV, col. 209–226.

Contains a method for finding probable errors from the $\frac{1}{2}n(n-1)$ differences between n observations taken two by two. See below ANDRÄ.

1869 ANDRÄ. 'Schreiben....[om Bestemmelsen af den sandsynnlige Feil ved Hjaelp af Iagttagelsernes Diffentser].' *Astron. Nachr.*, Vol. LXXIV, col. 283–284.

Objects to JORDAN's method on the ground that the differences are not independent. See 1872 for continuation of this discussion.

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1869 LÜROTH. 'Bemerkung über die Bestimmung des wahrscheinlichen Fehlers.' *Astron. Nachr.*, Vol. LXXIII, col. 187–190.

If n be the number of observations and q that of the unknown quantities, the probable error of a single observation is found to be

$$r = 0.8453 \frac{\Sigma x}{\sqrt{n(n-q)}}.$$

This is an extension of the formula given by PETERS in 1856. See 1876 HELMERT.

1869 ROGERS. 'On the Variability of Personal Equation in Transit Observations.' *Amer. Jour. Sci.*, Vol. XLVII, pp. 297–307.

A discussion of interesting experiments.

1869 TODHUNTER. 'On the Method of Least Squares.' *Trans. Camb. Phil. Soc.*, Vol. XI, pp. 219–238.

On page 9 of the First Supplement (1815) or on page 539 of the national edition of the *Théorie....des Prob.*, LAPLACE gave, without demonstration, a certain formula. "The primary object of this communication is to demonstrate the result which as I have stated LAPLACE merely enunciated..... A secondary object of the communication is to develop LAPLACE's own process of investigating the method of Least Squares; some of the results which he obtained for the case of *two* elements are here demonstrated to hold for the case of any number of elements."

1869 WATSON. 'Method of Least Squares, Theory of the Combination of Observations, and Determination of the most probable system of elements from a series of observations.' Chap. VII of his *Theoretical Astronomy* (Philadelphia, 8vo), pp. 360–425.

An elementary sketch of the subject according to GAUSS and ENCKE.

[1869] THIELE. Undersogelse af Omlobsbevaegelsen i Dobelstjerne.

"THIELE hat gezeigt dass der wahrscheinlichste Werthe bei durch Schätzung ermittelnden Doppelsterndistanzen das geometrische Mittel ist."—HELMERT, *Ausgleichungsrechnung*, p. 95.

1870 CROFTON. 'On the Proof of the Law of Errors of Observations.' *Phil. Trans. London* for 1870, pp. 175–188.

The object of this paper is to determine the law of facility of error on the hypothesis that an error arises from the joint operation of a large number of small sources of error, positive and negative errors *not* being equally probable. The investigation is not very clear.

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1870 LAZARUS. 'On some problems in the Theory of Probabilities.' *Lond. Assurance Mag.*, Vol. XV, pp. 244–257.

This is translated from the German. It treats of the general term of the binomial $(\frac{1}{2}+\frac{1}{2})^m$ when m is very large.

1870 NEOVIUS. *Lärobok i minsta qvadrat-methoden.* Åbo, 8vo, pp. 109.

For review see *Bull. Math. et Astron.*, Vol. II, pp. 134–136.

1870 SANTINI. Compendiato esposizione del modo più vantaggiosa di resolvere una serie di equazioni lineari, risultanti da operationi tutti ugualmente probabili, per la determinazione degli elementi di una proposta teorica. *Mem. Istit. Veneto.*, Vol. XIV.

1870 TCHÉBYCHEFF. 'Formule d'interpolation par la méthode des moindres carrés.' *Mém. Couronnés Acad. Belgique*, Vol. XXI, (Appendix to "N. Majewski, Mémoire....'), 9 pp.

See *Jahrb. Fortschr. Math.*, Vol. II, p. 116.

1870 WOLF. '*Handbuch der Mathematik, Physik, Geodäsie und Astronomie.*' Zürich, 2 vols. 8vo. Vol. I (1870), pp. xii, 492; Vol. II (1872), pp. viii, 459.

This valuable work of reference contains a brief sketch of the history of the Method of Least Squares, with a short development of its theory according to 1832 ENCKE.

1871 ABBE. 'A Historical Note on the Method of Least Squares.' *Amer. Jour. Sci.*, Vol. I, pp. 411–415.

Points out that the Method was independently discovered and published by ADRAIN in 1808, and reprints a portion of the original investigation. Interesting biographical notes relating to ADRAIN are also given.

1871 FRANKE. '*Die Dreiecksnetze vierter Ordnung, als Grundlagen geodätischer Detail-Aufnahmen zu technischer oder staatswirthschaftlichen Zwecken.*' München, 8vo, pp. xii, 261.

Numerous examples of adjustment are given. The theory and practice of the subject are presented in different chapters. It is an excellent book.

1871 GLAISHER. 'Tables of the Error-function.' *Lond. Phil. Mag.*, Vol. XLII, pp. 431–436.

An account of KRAMP'S, BESSEL'S, ENCKE'S and other tables of the values of the probability integral, with a new table of values from $x=3.00$ to $x=4.50$.

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1871 JORDAN. Ueber die Genauigkeit einfacher geodätischer Punktbestimmungen. *Zeitschr. für Math. u. Phys.*, Vol. XVI, pp. 397–413.

1871 KLINKERFUES. 'Die Berechnung einer Bahn aus einer grösseren Zahl von Beobachtungen nach der Methode der kleinsten Quadrate.' *Theoretische Astronomie* (Braunschweig, 8vo), pp. 328–379.

An elementary sketch of the Method of Least Squares.

1871 MINDING. 'Zur Theorie der kleinsten Quadrate.' *Bull. Acad. St. Péters.*, Vol. XVI, col. 305–308.

On the solution of normal equations, determination of weights, etc.

1871 PETERS. 'Quadrate der Zahlen von 1 bis 10,000.' *Astron. Tafeln und Formeln* (Leipzig, 8vo), pp. 151–169.

Deserves a place here as an aid in the Method of Least Squares on account of its great convenience, being arranged like logarithms.

1871 SCHOTT. 'Method of adjustment of the secondary triangulation of Long Island Sound.' *Rep. Coast Sur. U. S.* for 1868, pp. 140–146.

1871 ZACHARIAE. '*De mindste Qvadraters Methode.*' Nyborg, 8vo, pp. viii, 234.

This is an excellent text-book. See review in *Jahrb. Fortschr. Math.*, Vol. III, p. 95.

1872 DIENGER. 'Ueber einen Satz der Wahrscheinlichkeitsrechnung und damit zusammenhängende bestimmte Integrale.' *Abhandl. Böhmis. Gesell.*, Vol. V, 44 pp.

See *Jahrb. Fortschr. Math.*, Vol. IV, p. 89.

1872 GLAISHER. 'Remarks on certain portions of LAPLACE'S Proof of the Method of Least Squares.' *Lond. Phil. Mag.*, Vol. XLIII, pp. 194–201.

The matter of this paper is mostly included in the following.

1872 GLAISHER. 'On the Law of Facility of Errors of Observations, and on the Method of Least Squares.' *Mem. Astron. Soc. Lond.*, Vol. XXXIX, pp. 75–124.

This is perhaps the most valuable of all the theoretical memoirs on our list, presenting as it does clear critical analyses of the principal proofs of the law $\varphi(x)=ce^{-h^2x^2}$ and of the Method of Least Squares. It has been of great value to me in preparing this list.

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ADRAIN's first proof is examined at length and its reasoning shown to be defective. Then are analysed in order: 1. GAUSS's first proof, including ENCKE's, DEMORGAN's and ELLIS's remarks on the arithmetical mean; 2. LAPLACE's method, POISSON's and ELLIS's simplifications and IVORY's criticisms; 3. GAUSS's second demonstration; 4. HERSCHEL's proof, with ELLIS's and BOOLE's criticisms thereon; 5. TAIT's and similar proofs; 6. DONKIN's proof of 1857. By means of the index at the end of this list the reader may refer back to these papers, where I have often quoted GLAISHER's remarks.

It is considered unproved that the arithmetical mean gives the most probable result. GAUSS's second proof is regarded as resting upon an arbitrary assumption, which practically assumes the point to be proved. LAPLACE's method is considered as giving the only correct and philosophical analysis of the question, and this GLAISHER shows leads directly to the exponential law of facility, provided that the sources of error are very great in number and that positive and negative errors are equally likely. "TAIT's proof" is found insufficient. The proofs of 1837 HAGEN, 1838 BESSEL, 1844 DONKIN and 1870 CROFTON are not discussed.

PEIRCE's criterion for the rejection of doubtful observations is regarded as "destitute of scientific precision." "....under no circumstances have we a right to say an observation has no weight, though it may be better to give it none than to give it as much as the best." The method of assigning weights in such cases is hinted at; see below 1873 GLAISHER.

For accounts of the contents of the memoir see *Monthly Notices*, Vol. XXXII, p. 241, and *Jahrb. Fortschr. Math.*, Vol. IV, p. 92.

1872 HELMERT. '*Die Ausgleichungsrechnung nach der Methode der kleinsten Quadrate mit Anwendungen auf die Geodäsie und die Theorie der Messinstrumente*,' Leipzig, 8vo, pp. xi, 348.

The exponential law is regarded as a law proved by experience. The arithmetical mean is said to be the *most plausible* value. Both the first and second proofs of GAUSS are given, and the second is regarded as better and more general.

While the theoretical part of the book is not satisfactory, the practical part renders it valuable for geodetic engineers. Conditioned observations in particular are well treated.

1872 HILGARD. 'An application of an Exponential Function.' *Proc. Amer. Assoc.* for 1871, pp. 61–63.

A certain statute relating to errors in coinage is discussed.

1872 HOPKINSON. On the calculation of empirical formulæ. *Messenger Math.*, Vol. II, pp. 65–67.

A method less accurate than Least Squares.

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1872 JORDAN. 'Vergleichung der Genauigkeit verschiedener Gradmessungen.' *Astron. Nachr.*, Vol. LXXX, col. 17–22.

The mean errors of 17 angle measurements and 21 base line measurements are given, the latter for a line one kilometer in length. The greatest mean error of a base line measurement is 63.2 mm. and the least 0.12 mm., the first being measured in 1739 and the second in 1860. This is one of those papers in which the results of long continued research and labor are expressed in a few lines.

1872 JORDAN. 'Ueber die Bestimmung des mittleren Fehlers durch Wiederholung der Beobachtungen.' *Astron. Nachr.*, Vol. LXXIX, col. 219–222.

On the method of deducing probable errors from the $\frac{1}{2}n(n-1)$ differences of n measurements, given by him in 1872. See next article.

1872 ANDRÄ. 'Ueber die Bestimmung des wahrscheinlichen Fehlers durch die gegebenen Differenzen von m gleich genauen Beobachtungen einer Unbekannten.' *Astron. Nachr.*, Vol. LXXIX, col. 257–272.

An investigation of JORDAN's method leading to somewhat different results. See *Jahrb. Fortschr. Math.*, Vol. IV, p. 577, and *Bull. Math. et Astron.*, Vol. IX, pp. 27. See below 1876 HELMERT.

1872 JORDAN. 'Ueber die Bestimmung des Gewichts einer durch die Methode der kleinsten Quadrate bestimmten Unbekannten.' *Zeitschr. Math. u. Phys.*, Vol. XVII, pp. 350–352.

A simplification of demonstrations of two methods.

1872 LORENZ. Udjevning af Jagttagelses fyl. *Tidsskr. for Math.*, Vol. II, pp. 1– . . , 97, 125, 162.

See *Jahrb. Fortschr. Math.*, Vol. IV, p. 94. The same Journal contains articles by ZACHARIAE.

1872 RUMPEN. '*Ueber den Zusammenhang der von* GAUSS *begründeten Methode der kleinsten Quadrate mit der algebraischen Theorie der quadratischen Formen.*' Bonn, 1872, 8vo, pp. 40.

A doctor's thesis. The conditions for minimum squares, etc., discussed by help of determinants.

1872 SCHLÖMILCH. 'Ueber die Bestimmung der Wahrscheinlichkeit eines Beobachtungsfehlers.' *Zeitschr. Math. u. Phys.*, Vol. XVII, pp. 87–88.

HERSCHEL's proof is taken from 1869 THOMPSON and TAIT, and pronounced "einfache und anschauliche."

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1872 WOOLHOUSE. 'On the Philosophy of Statistics.' *Comp. to Almanac* for 1872, pp. 5–22. —*Lond. Assur. Mag.*, XVII, pp. 26–30.
A popular article copied largely from QUETELET.

1872 ZACHARIAE. 'Note betreffend die Bestimmung des mittleren Fehlers.' *Astron. Nachr.*, Vol. LXXX, col. 67–70.
Offers without demonstration a new formula for the mean error of a base line measured in several portions. See 1873 HELMERT and JORDAN.

1873 BROWN. 'On the Application of the Binomial Law to Statistical Enquiries, illustrated by the Law of the Growth of Man at different ages.' *Lond. Assur. Mag.*, Vol. XVII, pp. 340–351.

1873 CROOKES. 'On the Probability of Error in Experimental Research.' *Quart. Jour. Sci.*, Vol. III, pp. 1–13.
Contains some formulæ for weights taken from 1838 DEMORGAN, with an application to determining the probability of error in the atomic weight of thallium.

1873 DEFOREST. 'On some Methods of Interpolation applicable to the graduation of irregular Series, such as Tables of Mortality, &c. &c.' *Rep. Smithsonian Inst.* for 1871, pp. 273–339; for 1873, pp. 319–355.
For continuation see below under 1876.

1873 HELMERT. 'Bestimmung des mittlern Fehlers der Längen messungen aus der Differenzen von Doppelmessungen.' *Astron. Nachr.*, Vol. LXXXI, col. 49–52.
A comparison of the common formulæ with the one given by ZACHARIAE in 1872, showing that the latter is less accurate.

1873 JORDAN. 'Ueber die Berechnung der mittleren Fehlers einer Basismessung.' *Astron. Nachr.*, Vol. LXXXI, col. 51–56.
Also a criticism on ZACHARIAE's formula.

1873 ZACHARIAE. 'Ueber die Bestimmung des mittleren Fehlers einer in meheren Theilen doppelt gemessenen Grundlinie.' *Astron. Nachr.*, Vol. LXXXI, col. 225–228.
Defends his formula against the above. See *Jahrb. Fortschr. Math.*, Vol. V, p. 127.

1873 JAHN. *Sur la probabilité du tir et la méthode des moindres carrés.* Paris, 8vo.

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1873 JORDAN. 'Verallgemeinerung eines Satzes der Methode der kleinsten Quadrate.' *Zeitschr. Math. u. Phys.*, Vol. XVIII, pp. 116–120.

Concerning the solution of normal equations.

1873 JOUFRET. *Sur la methode des moindres carrés et ses applications du tir.* Paris, 8vo.

1873 LAURENT. '*Traité du calcul des probabilités.*' Paris, 8vo. pp. xii, 268.

This is intended as an introduction to the study of LAPLACE's *Théorie.... des Prob.* At the end is the best list of literature on the Method of Least Squares which I have seen.

1873 NEWCOMB. 'A mechanical Representation of a familiar Problem.' *Monthly Notices Astron. Soc. Lond.*, Vol. XXXIII, pp. 573–574.

"Given at several epochs, observed values of a quantity which varies uniformly with the time, to find by Least Squares the most probable values of the two constants which fix its value at any time." The analogy of the question with one of equilibrium in mechanics is pointed out. The solution of a system of linear equations by Least Squares may be represented in a similar way.

1873 PEIRCE (C. S.) 'On the theory of errors of observations.' *Rep. Coast Sur. U. S.* for 1870, pp. 200–224.

Gives an account of some interesting experiments "made to study the distribution of errors in the observation of a phenomena not seen coming on, as in the case of a transit, but sudden as in the case of the emersion of a star from behind the moon." The results are given graphically and show a decided approximation to the exponential law of facility.

In the ten pages of introduction new ideas are offered concerning a notation, "suggested by the study of the logic of relations."

1873 SEELIGER. 'Ueber die JACOBI'sche Auflösung eines Systems von Normalgleichungen mit drei Unbekannten.' *Astron. Nachr.*, Vol. LXXXII, col. 249–252.

See above 1840 BESSEL.

1873 STONE. 'On the most Probable Result which can be derived from a number of direct Determinations of Assumed Equal Values.' *Month. Not. Astron. Soc. Lond.*, Vol. XXXIII, pp. 570–572.

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Shows that the arithmetical mean is the most probable results for $n+1$ observations, provided it is the most probable result for n observations, and as it is undoubtedly such for $n=2$, " ... it can be shown to be generally true."

1873 WREDE. 'Några anmärkningar rörande minste qvradtmethoden.' *Öfversigt Förhandl. Acad. Stockholm*, Vol. XXX, No. 8, pp. 3–34; No. 10, pp. 21–26.

The probable error is said to be not always 0.6745 of the mean error but depends upon the number of unknown quantities involved. See 1852 BIENAYMÉ. BESSEL's investigation of 1838 is also discussed.

1873 GLAISHER. 'On the Rejection of Discordant Observations.' *Monthly Notices Astron. Soc. Lond.*, Vol. XXXIII, pp. 391–402.

It is here clearly pointed out how inconsistent is the rejection of discordant observations by a criterion founded on the supposition of the validity of the arithmetical mean. The idea first advanced by DEMORGAN (*Encyc. Metrop.*, 1847) that the mean is only an approximate value to be used in weighting the observations from which a new mean is to be deduced, and so on, is here developed to a certain extent. See 1821 ——. The criterion given by STONE in 1868 is examined and pronounced untrustworthy and wrong.

1873 STONE. 'On the Rejection of Discordant Observations.' *Monthly Notices*, Vol. XXXIV, pp. 9–15.

A reply to the preceding in which GLAISHER's arguments are examined at length and the validity of the criterion maintained. GLAISHER's method for weighting observations is also discussed and regarded "as mathematically unsound."

1874 GLAISHER. 'Note on a paper by Mr. STONE, "On the Rejection of Discordant Observations."' *Monthly Notices*, Vol. XXXIV, p. 251.

1874 STONE. 'Note on a Discussion relating to the Rejection of Discordant Observations.' *Monthly Not.*, Vol. XXXV, pp. 107–108.

1874 CANTOR. '*Historische Notizen über die Wahrscheinlichkeitsrechnung.*' Halle, 8vo, pp. 8.

This is of no value. GAUSS alone is mentioned in connection with Least Squares.

1874 FECHNER. 'Ueber die Bestimmung des wahrscheinlichen Fehlers eines Beobachtungsmittels durch die Summe der einfachen Abweichungen.' *Poggendorff's Annal. Phys.*, Jubelband, pp. 66–81.

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This is an extract from a memoir published in *Abhandl. Sächs. Gesell;* see below 1875. FECHNER deduces the formula

$$r_0 = \frac{1.195502}{\sqrt{2n - 0.8548}} \cdot \frac{\Sigma x}{n}$$

n being the number of observations and Σx the sum of the residuals all taken positively. See 1876 HELMERT.

1874 GLAISHER. 'On the Solution of the Equations in the Method of Least Squares.' *Month. Not. Astron. Soc. Lond.*, Vol. XXXIV, pp. 311–334.

A critical discussion of GAUSS's, BESSEL's and JACOBI's methods for solving equations and determining weights.—See *Jahrb. Fortschr. Math.*, Vol. VI, p. 145.

1874 HANSEN. 'Von der Bestimmung der Theilungsfehler eines gradlinigen Maassstabes.' *Abhandl. Sächs. Gesell.*, Vol. XV, pp. 525–668.

1874 JEVONS. '*The Principles of Science: a Treatise on Logic and Scientific Method.*' London, 8vo. Vol. I, pp. xvi, 463; Vol. II, pp. vii, 480.

Chapters on 'The Method of Means,' 'The Law of Error,' 'Theory of Approximation,' etc., give valuable remarks concerning the fundamental principles of the Method of Least Squares.

1874 JORDAN. *Taschenbuch der praktischen Geometrie. Eine Sammlung von Resultaten der höheren und niederen Vermessungskunde.* Stuttgart, 8vo, pp. xi, 416.

In Part I the Method of Least Squares is presented. See review in *Schlömilch's Lit. Zeit.*, Vol. XVIII, pp. 33–40. The book is now (1877) being republished under the title '*Elemente der Vermessungskunde;*' and Part 1, pp. 1–136, contains a valuable treatise on Least Squares.

1874 MEYER. 'Calcul des Probabilités de A. MEYER, publie sur les manuscrits de l'auteur par F. FOLIE.' *Mém. Soc. Liége*, Vol. VI, No. 2, x+446 pp. —Also published under title *Cours de Calcul.... fait à la Universite de Liége....* Bruxelles, 8vo.

This is a comprehensive and valuable work. In the parts relating to the theory of observations the proofs of 1809 GAUSS and 1837 HAGEN are given, as also the investigations of 1852 BIENAYMÉ.

1874 POWALKY. 'On the Combination of the different Results of various Series of Observations.' *Month. Not. Astron. Soc. Lond.*, Vol. XXXIV, pp. 476–479.

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1874 SEIDEL. 'Ueber die Berechnung der wahrscheinlichsten Werthe solcher Unbekannten zwischen welchen Bedingungs-Gleichung bestehen.' *Astron. Nachr.*, Vol. LXXXIV, col. 193–210.

The conditional equations are regarded as having infinite weights. This way of consideration appears to lead to a new method of solution.

1874 SEIDEL. Ueber ein Verfahren, die Gleichungen, auf welche die Methode der kleinsten Quadrate führt, so wie lineäre Gleichungen überhaupt, durch successive Annährung aufzulösen. *Abhandl. Akad. München* for 1874, pp. ....

See *Jahrb. Fortschr. Math.*, Vol. VI, p. 147.

1875 AIRY (W.). 'On the Probable Errors of Levelling; with Rules for the Treatment of Accumulated Errors.' *Proc. Inst. Civ. Engs.* for 1875, pp. .... —*Engineering News*, Vol. IV, pp. 77–78, 84–85, 92, 98–99, 104–105.

A very valuable practical paper.

1875 BAEYER. 'Ueber Fehlerbestimmung und Ausgleichung eines geometrischen Nivellements.' *Astron. Nachr.*, Vol. LXXXVI, col. 177–188.

BESSEL's method for adjusting a triangulation is applied to a connected system of levels.

1875 BELLATI. Intorno ad un modo di semplificare in alcuni casi l'applicazione del metodo dei minimi quadrati al calcolo delle costanti empiriche. *Atti Istit. Veneto*, Vol. I.

1865 BIENAYMÉ. 'Application d'un théorème nouveau du Calcul des probabilités.' *Comptes Rendus Acad. Paris*, Vol. LXXXI, pp. 417–423. —*Bull. Math. et Astron.*, Vol. IX, pp. 219–225.

If a series of observations be arranged in the order of the measurements, there are certain maxima and minima whose probable number and position are given by the theorem. On pp. 458–459, 491–492 of this volume of the *Comptes Rendus* are remarks by BERTRAND on the theorem.

1875 DIENGER. 'Die LAPLACE'sche Methode der Ausgleichung von Beobachtungsfehler bei zahlreichen Beobactungen.' *Denkschr. Akad. Wien*, Vol. XXXIV, 42 pp.

The method is extended to the case of several unknown quantities. See *Repertorium Math.*, Vol. I, p. 241.

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1875 FAYE. 'Note accompagnant la presentation d'une Notice autographiée sur la méthode des moindres carrés.' *Comptes Rendus, Paris.*, Vol. LXXX, pp. 352–356.

The exponential law of error is regarded as an empirical law established by experience.

[1875] FECHNER. Ueber den Ausgangswerth der kleinsten Abweichungssumme, dessen Bestimmung und Verwendung. *Abhandl. Sächs. Gesell.*, Vol. [XVI].

See title 1874 FECHNER.

1875 FRANKE. 'Die trigonometrische Punktbestimmung im Netz-Auschluss, mit besonderer Rücksicht auf eine rationelle Fehler-Ausgleichung.' München, 8vo, pp. viii, 69.

Reference is here made to articles by TULLA, JORDAN and others on a graphical method of adjustment, whose titles I regret not to be able to give. See *Monatsbl. Badisch. Geometervereins* for 1875.

1875 GALTON. 'Statistics by Intercomparison, with Remarks on the Law of Frequency of Error.' *Lond. Phil. Mag.*, Vol. XLIX, pp. 33–46.

If all the men of a tribe were arranged in a row according to their heights, the middle man would have the *mean* height.

The curve $y = ce^{-h^2x^2}$ is called an "ogive" and it is regarded as more likely to be approximately true of a statistical series than any other that can be specified *à priori*.

1875 HELMERT. 'Ueber die Formel für den Durchschnittsfehler.' *Astron. Nachr.*, Vol. LXXXV, col. 353–366.

The formula given in 1856 by PETERS is discussed, and shown to be correct only for direct observations. A new formula for probable error is proposed. See 1869 LÜROTH and 1876 HELMERT.

1875 LAURENT. 'Sur la méthode des moindres carrés.' *Liouville's Jour. Math.*, Vol. I, pp. 75–80.

A discussion of 1444 observations to deduce an empirical law of error. The result is that the exponential law represents closely the probabilities of error.

1875 MEES. Ueber die Berechnung des wahrscheinlichen Fehlers einer endlichen Zahl von Beobachtungen. *Zeitschr. Math. u. Phys.*, XX, pp. 145–152.

GAUSS's method (1816) is considered incorrect.

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1875 HELMERT. Ueber die Berechnung des wahrscheinlichen Fehlers aus einer endlichen Anzahl wahrer Beobachtungsfehler. *Zeitschr. Math. u. Phys.*, Vol. XX, pp. 300–303.

A reply to MEES's article above.

1875 NATANI. '*Methode der kleinsten Quadrate. Mit den Hülfsatzen aus der Analysis und Wahrscheinlichkeitsrechnung nebst einem Anhange über die ballistische Linie.*' Berlin, 8vo, pp. 42.

HERSCHEL's proof is given. The numerical examples concern the probability of striking a target.

1875 SCHIAPARELLI. 'Sur le principe de la moyenne arithmétique.' *Astron. Nachr.*, Vol. LXXXVII, col. 55–58.

A demonstration that the arithmetical mean of direct observations gives "le seul résultat plausible et conciliable avec les exigences pratiques de la question." See 1848 MATZKA, 1868 SCHIAPARELLI and 1876 STONE.

1875 SCHOLS. Over de theorie der fouten in de ruimte en in het platte vlak. *Verhandl. Akad. Amster.*, Vol. XV.

See *Jahrb. Fortschr. Math.*, Vol. VII, p. 114.

1875 SCHOLS. De interpolatie-formule von TCHÉBYCHEF volgens de methode der kleinste vierkanten. *Versl. Akad. Amsterdam*, Vol. IX, pp. 301–311.

1875 SCHOTT. 'Determination of weights to be given to observations for determining time with portable transit instruments, recorded by the chronographic method.' *Rep. Coast Sur. U. S.* for 1872, pp. 222–226.

1875 TILLY. Note sur le principe de la moyenne arithmetique et sur son application à la théorie mathématique des erreurs. *Nouv. Corres. Math.*, Vol. I, pp. 137–147.

See *Bull. Math. et Astron.*, Vol. XII, p. 146.

1875 TILLY. Théorie mathématique des erreurs. *Ballistique*, (Bruxelles, 8vo), pp. 155-225.

The methods of GAUSS, HAGEN and LAPLACE compared. See review in *Jahrb. Fortschr. Math.*, Vol. VII, p. 108.

1875 VAN GEER. Over het gebruick von determinanter by de methode der kleinste kwadrater. *Nieuw Arch.*, Vol. I, pp. 179–188.

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1876 BAUERNFEIND. 'Methode der kleinsten Quadrate.' *Elemente der Vermessungskunde* (Stuttgart, fifth edition, 2 vols., 8vo), Vol. II, pp. 4–30.

An elementary sketch of the method.

1876 CHAMBERS (C.) and CHAMBERS (F.). 'On the Mathematical Expression of Observations of Complex Periodical Phenomena; and on Planetary Influence on the Earth's Magnetism.' *Phil. Trans. Lond.*, Vol. CLXV, pp. 361–402.

1876 DEFOREST. '*Interpolation and Adjustment of Series.*' New Haven, 8vo, pp. 52.

A supplement to his memoirs of 1873. Besides other valuable matter, methods for finding probable errors of adjusted terms are given.

1876 FERRERO. *Esposizione del metodo dei minimi quadrati.* Firenze, 8vo, pp. 234.

1876 HAGEN. '*Untersuchungen über die gleichförmige Bewegung des Wassers.*' Berlin, 8vo, pp. 104.

All known observations on the mean velocity of rivers are discussed by the Method of Least Squares, and the most probable law and formula for mean velocity are deduced.

1876 HELMERT. 'Die Genauigkeit der Formel von PETERS zur Berechnung des wahrscheinlichen Fehlers directer Beobachtungen gleicher Genauigkeit.' *Astron. Nachr.*, Vol. LXXXVIII, col. 113–132.

Simplifications are given of HELMERT's formula of 1875, and the formulæ of FECHNER, JORDAN and ANDRÄ (see 1869–1874) are discussed.

1867 HELMERT. Ueber die Wahrscheinlichkeit der Potenzsummen der Beobachtungsfehler und über einige damit im Zusammenhang stehende Frage. *Zeitschr. Math. u. Phys.*, Vol. XX, pp. 192–218.

1876 KUMMELL. 'New Investigation of the Law of Errors of Observation.' *The Analyst*, Vol. III, pp. 133–140, 165–171.

HAGEN's proof of 1837 is given abbreviated and improved, and the usual rules for normal equations and probable errors are deduced. The probability to commit no error at all is regarded as an absolute constant. See *Jour. Franklin Inst.*, 1877, Vol. CIV, pp. 270–274.

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1876 LÜROTH. 'Vergleichung von zwei Werthen des wahrscheinlichen Fehlers.' *Astron. Nachr.*, Vol. LXXXVII, col. 209–220.

The usual formula is compared with a new formula and shown to give larger values.

1876 SAFFORD. 'On the Method of Least Squares.' *Proc. Amer. Acad.*, Vol. XI, pp. 193–201.

"The main object of this paper is to give rules for good observing derived from this theory." Hints for abbreviating computations are added.

1876 SKINNER. '*Principles of Approximate Computations.*' New York, 12mo, pp. v, 98.

Presents simple rules for conducting computations involving approximate quantities, in such a manner as to require the fewest figures and to show at once the degree of accuracy of the result.

1876 STONE. 'Sur le principe de la Moyenne arithmétique. *Astron. Nachr.*, LXXXVIII, col. 61–64.

Points out that some of the assumptions of SCHIAPARELLI'S proof of 1875 agree with those of his own proof of 1873. The article is in English.

1876 VENN. '*The Logic of Chance. An Essay on the foundations and province of the Theory of Probability, with special reference to its logical bearings and its applications in Moral and Mental Science.*' London, second edition, 8vo, pp. xxvii, 488.

VENN'S views are: First, almost any regular and symmetrical method of treating the errors of observation will tend to approximate indefinitely toward the truth as the number of observations is indefinitely multiplied, and this whatever be the law of facility; secondly, the Method of Least Squares is the best method (upon the reasonably probable supposition of the universality of the exponential law), that is, it approximates quicker to the truth as the number of observations is increased than any other method; but its superiority over other reasonable methods is small in comparison with their common superiority over single observations.

'*Jahrbuch über die Fortschritte der Mathematik.*' Berlin, 8vo. One vol. of about 750 pages appears yearly in 3 parts.

This invaluable publication has been of great use to me in preparing the above list for the years 1868–75. Vol. VIII (not yet issued) embracing the literature for 1876, will undoubtedly contain the titles of some writings on the Method of Least Squares which are not given here.

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ERRATA.

Page 172, line 21 from foot, *for* Remarks *read* 'Remarks.
" 173, " 11 " top, " Jái " J'ai.
" 179, " 8 " foot, " unhekannten *read* unbekannten.

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ADDENDA.

1876 RÜDIGER. *Die Methode der kleinsten Quadrate abgeleitet aus der Wahrscheinlichkeitslehre, und ihre Anwendung auf naturwissenschaftlichen Messungen.* Frankfurt an der Oder, 8vo, pp. 48.

A doctor's dissertation.

1876 ———. '*Zusammenstellung der Literatur der Gradmessungs-Arbeiten.*' Berlin, 4to, pp. 32.

This is drawn up by commissioners of the states and countries belonging to the European International Geodetic Survey. It contains references to about 380 writings on Geodesy, 200 of which are German, 64 English, 50 Italian and 24 French. The English and Italian literature is well presented, the German and French is not. Coming from such a source, this list should have been a great deal better.

The work is received just as this sheet goes to press, and the following are the additions which it renders necessary to the preceding pages:

The work by BAEYER recorded on page 206 is a lithographed manuscript issued in 1867 or 1868. Two other parts on Geodesy were also published.

The following articles, all relating to the adjustment of geodetic triangulations, should properly have been mentioned in my list: 1865 SCHIAPARELLI, *Istit Lombard. Rend.*, Vol. II, pp. 348–359; Vol. III, pp. 27–41. 1867 WITTSTEIN, *Astron. Nachr.*, Vol. LXIX, col. 289–298; 321–328. 1868 PRONDZYNSKI and BÖRSCH, *Astron. Nachr.*, Vol. LXXI, col. 145–154; 265–268; Vol. LXXV, col. 87–90.

A lithographed manuscript by BAEYER, entitled *Untersuchungen über die Ausgleichung nach Winkel- und Seitengleichungen*, was published in 1871.

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III.—Ventral Fins of Ganoids. By James K. Thacher.

Es erscheint als eine allgemeine Regel, dass homodyname, ungleichartig entwickelte Körpertheile an den hinteren Abschnitten indifferenter erscheinen als an den vorderen, dass besonders die Gliedmaassen des Schultergürtels viel reichere Umgestaltungen eingehen als jene des Beckengürtels, an denen sich das ursprüngliche Verhalten länger und vollständiger bewahrt.—*Gegenbaur, Jen. Zeitschr.*, Bd. v, p. 417.

In February last I published a paper* to show that the limbs with their girdles were derived from a series of similar simple parallel rays, and that they were a specialization of the continuous lateral folds or fins evidenced in embryos, which were with some probability homologous with the lateral folds or metapleura of the adult *Amphioxus*. The following short paper is designed to do little more than corroborate the views there stated.

In Pl. LIX, fig. 64 of that previous paper, an *Acipenser* ventral is exhibited. The ventral of another individual of the same species, *Acipenser brevirostris*, is figured here, Pl. I, fig. 1. Comparing these two we find certain differences. In the first place the number of rays is not the same, being seven in fig. 64 and eight in fig. 1. But we see that the orad ray of fig. 64 is very broad, as it is also in fig. 65, which exhibits the other ventral of the same specimen, and in the text we read "the breadth and outline of *c* (the orad ray) raises a suspicion of its double character." This suspicion, then, is confirmed by the eight rays of fig. 1, where the orad ray is slender like the rest. But there are other differences, and these, aside from insignificant differences in the terminal segmentations, lie in the manner in which the originally separate rays have united with one another. In fig. 64 the aborad three rays are all separate and complete, while in fig. 1 there is quite a different state of things. Here the proximal part of 8 has united with the adjacent part of 7; 7 is free from 6. But the proximal segment of the latter, which is free in fig. 64, is here, in fig. 1, joined to 5. But 5 is free from 4 and 4 from 3 in fig. 1, as is not the case in figs. 64 and 65.

The part *bd* is a large flat piece of cartilage of about the thickness of the rays. The process *b* bends slightly downward (ventrad) and is a little foreshortened in the figure. It approaches its fellow on the other side, but does not unite with it.

* Median and Paired Fins. Trans. Conn. Acad., Vol. iii, 1877.

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There is represented in fig. 1 a peculiar foramen, which either did not exist or was overlooked in the specimen figured in fig. 64. This long tunnel enters the substance of the cartilage from the dorsal side at the place marked *f*, and running in the middle of the cartilage, it pursues the course marked by the dotted line to emerge between the rays 2 and 3.

Its remarkable position, boring the substance of the cartilage to no purpose, and the still more remarkable parallelism between its course and the line of division between rays 3 and 4, are sufficient reasons for regarding it as an indication of an early separation between these two rods 2 and 3. The second foramen *g* proceeds almost directly through the cartilage.

Now if we construct a fin in which all the rays are separate which are separate in either of the two fins figured, and in which the separation between 2 and 3, which is indicated in the way just spoken of, is retained, we shall have a form of fin represented in Pl. I, fig. 2. From evidence then obtained within the limits of this single species, it is proved that there must have been fins of this form possessed by some of the ancestors of these individuals; fins in which each of the eight simple rays maintained its independence and simplicity, except so far as regards the concrescence of the proximal segments 1 and 2, and the process *a* rising from the orad side of the first ray.

Scaphirhynchus, as is well known, is the nearest living relative of *Acipenser*, and the two are very similar to one another. The dorsal side of a left ventral of this fish is exhibited in Pl. II, fig. 7. We have here seven rays, but the orad one is so wide as to raise the belief that it has been formed by the union of two. In this the betipping fails. There is the same iliac process *a* growing up from the edge of the fin, but the chief difference lies in the prolongation of the pubic part,* and the degree of concrescence, which is greater than it is in either of the specimens of *Acipenser* described. As is evident, this fin throws no light on the general question here under discussion. It is a mere modification of the *Acipenser* fin. And to this we return.

We had arrived at a form, Pl. I, fig. 2, consisting of eight simple parallel rays, except that the two orad rays have united in their proximal joints, and that there is the iliac process *a*, and that the

* In the figure (Pl. II, fig. 7,) the pubic part, which bends a little ventrad when in position, is here flattened out and not foreshortened. It should be noted that the aborad ray, 7, lies dorsad of the next ray, thus partially concealing it in the figure. This is what we find as a general rule in the ventrals of Elasmobranchs, and the conformation is important in building the intromittent organs of the males.

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distal segmentation is wanting on the first ray. If it were not for the process *a*, if here in *Acipenser* we had nothing but the flat piece *b d*, there could, I believe, be no hesitation on the part of anyone in regarding this piece *b d* as formed by concrescence from the rays 1 and 2. But the piece *a* raises an indefinite suspicion. One is led to think that it may possibly be the remains of something, and that despite the apparent similarity, the orad part of the fin may have something more in it than the aborad, and that the part *a* may not be a simple original upgrowth of cartilage dorsad from the orad side of the first ray.

To assure ourselves that it is this last and nothing else we turn to *Polyodon*. A view of the ventral surface of a right ventral is exhibited in Pl. I, fig. 3. Here the number of rays, thirteen, is greater than it was in *Acipenser* and *Scaphirhynchus*. The segmentation into three parts is to be noted, as also the absence of the distal joint in the orad ray, agreeing with what we have found so generally in dorsal, anal and ventral fins. There has been some concrescence, but not very much, as seen in the proximals 12–13 and 1–2–3–4–5. The shortening of ray 7 is worthy of remark. The rays differ a little from those that form the ventrals of *Acipenser* and *Scaphirhynchus*, in that in these latter the rays continue thick to their proximal ends, and the median edge of the fin is quite heavy, whereas in *Polyodon* the proximal ends of the rays become very thin; so that while the distal end of the proximal segment is thick and round, the proximal end runs out into a thin blade. Thus also, as would be expected, the large piece *b d* becomes, as it approaches the median line, a thin lamina of cartilage. This fin lies very nearly all in one plane and there is consequently no foreshortening of any of the parts.

When we turn the fin over so that its dorsal side is presented to view, we see that the structure of the skeleton differs from what it previously appeared to be in a very striking, and as we shall see, significant way. The rods are not simple rods, but from the orad side of the distal part of the proximal segment of *each* ray, a blade-like process runs dorsad, approaching, consequently, a direction at right angles to the plane of the fin, and lying in the intermuscular septa of the fin. One of the rays from the middle of the fin is exhibited in Pl. I, fig. 3, A. The side of the ray is presented, and the process *a'* has been turned about its line of junction with the ray so as to lie in the plane of the paper, and not at right angles to it, as it naturally would. This process *a'* is quite thin and flexible in the case of all the rays of *Polyodon*, while the ray itself at the point where it

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sends off this process is thick and heavy. It is, moreover, a real process, a continuation of the cartilage of the ray, and not even articulated with it. The processes are highest in the middle of the fin and diminish in size aborad until they hardly appear on the last two or three rays. Toward the orad extremity of the fin they remain very well developed, though not quite so long as they are in the middle of the fin. One rises at the point marked *a* fig. 3, Pl. I, and others from the plate *b d* at points which would have been the orad sides of the distal ends of the proximal segments 2, 3, 4 and 5 had the proximal segments of those rays not united more or less completely to form the large plate *b d*.

If now we compare this fin of *Polyodon* with that of *Acipenser* and *Scaphirhynchus*, we find that in each there is a process growing dorsad from the place marked *a* in *Polyodon*, Pl. I, fig. 3, and the corresponding points in *Acipenser* and *Scaphirhynchus*, Pl. 1, figs. 1 and 2, Pl. II, fig. 3, *a*. There is no difference between the two processes except that of size. The correspondence between the two is so exact that had the species been very different from one another there could have been no doubt of the homology of the process in the one with that in the other, but this is made doubly sure by the fact of the close similarity of *Polyodon* and the Acipenserids.

Again, there can be no doubt of the correspondence, homodynamism, of the first process in *Polyodon* and those that belong to the other rods. They all spring from exactly the same place and differ in size only. We have in *Polyodon* a foot with thirteen toes *where each toe has its own separate ilium.*

It has thus become apparent that the iliac process in the Acipenserids is not a vestige of anything more complex and important, but simply what it was stated to be, a process from the ray, and that the orad part of the fin does not include anything essentially distinct from what appears in the hinder part. We are thus brought to a choice between a series of simple and of forked rays as that from which the Gnathostome limb has been derived. There does not, however, seem to be any room for doubt as to which of these the original form was. That the simpler structure should have been the first produced and that the more complex should have been developed out of this is certainly more consonant with our general experience of organic change than the reverse sequence would have been. Moreover, I have proved that in the similar median fins the original skeletal elements were perfectly simple rods developed in the freely flapping membrane of the fin, and the analogical inference from these to the

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paired fins is a strong one, for it is something more than a mere analogy, being supported by causal considerations.

Again, the wide-spread absence of this dorsal process is inconsistent with its constituting a part of each ray of the Protognathostomi or any of their ancestors. Although Chimæra has a large iliac process from the orad part of the fin, the Sharks proper, where we might expect to find the skeleton in no very specialized condition, have nothing which seems to represent a dorsal process of any of the rays. Neither do the Holostean Ganoids, nor most, not to say all, of the Teleosts. Among the Batoidei we do have dorsal processes from the pelvic cartilage. Their position renders their strict correspondence with the iliac process of the *Acipenser* fin somewhat improbable, while it suggests the possibility that they may represent some of the processes of the aborad rays of the *Polyodon* fin. Even if this should be the case the absence of these processes in the Sharks would force us to regard the conformation in the Rays and in the Holocephali as having arisen independently of that in *Polydon*. With this possible exception and the exception of the *Polydon* ventrals, no ventral fin or limb anywhere shows signs of any forking aborad of the first ray. Nor is there any forking observed in pectoral and median fins except so far as this may be represented in the scapula. It is easier to believe that these comparatively few instances were late, and some of them independent, developments, than to believe that there has been an annihilation, so nearly complete, of dorsal processes which were universal in the beginning.

Thus it seems to me that the order of events, as far as regards the Chondrostean Ganoids, was like this: The simple ray with the three segments was the early form. An upgrowth of cartilage dorsad from the proximal segment of the first ray took place in the Acipenserids. In *Polyodon* this was imitated by the hinder rays. This is an instance of the development of likeness out of unlikeness in homodynamous parts, a kind of change which must have frequently occurred in the history of stocks. It is the opposite of differentiation, and it does not consist in mere reversion to primitive similarity, but in the attainment of similarity in any way. It is exemplified in a department of phenomena resembling somewhat those of biology in the formation of French plurals by the addition of *s*, though the Latin had formed the plurals of the same words in very diverse ways. Thus, too, the close similarity between the fore and hind limb in various forms now living, has doubtless in many cases been preceded by a greater dissimilarity in the direct ancestral lines.

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The other Ganoids do not offer us much of interest in their ventral fins, exhibiting as they do only modifications by concrescence, and showing no advance either in the direction of the Dipnoans or the Stapedifera. Figure 1, Plate II, represents the pelvic girdle and rays of *Polypterus*. The fact that the two pubic parts abut on one another without uniting, though fastened together by tough connective tissue, that there is no iliac process and that the whole structure lies in one horizontal plane should be stated. If this fin is compared with that of *Acipenser* and *Scaphirhynchus* there can be no doubt, I think, that the solid pubic pieces in *Polypterus* represent the basal segments of rays, and therefore correspond to the orad proximal parts formed of united rays in *Polyodon* and the Acipenserids. Some one or more of the four rays in *Polypterus* may be composed of more than a single ray. This is not easy to determine, but there is no doubt that they correspond to the middle and terminal parts of the complete tri-segmented ray, and that the terminal segmentations have disappeared as they have in the orad rays of the Chondrostei, and so very generally in the orad rays of Elasmobranch ventral and median fins.

The fins of *Lepidosteus* and *Amia* are not remarkable for much beside their similarity. They are represented in Pl. II, figs. 2 and 3. Comparing them with the fins of *Polypterus* we see that they seem to offer an extension of that same process of reduction which we noticed in that fish. The parts lettered *b d* certainly are homologues of one another in all three. But the middle and terminal segments of the primitive ray, represented by the four well-developed rays of *Polypterus*, are here reduced to the insignificant parts at the extremities of the main portions *b d*. The median one of these small pieces is somewhat produced and ossified. The pubic parts overlap one another but do not join, being merely fastened together by connective tissue. And both in *Lepidosteus* and in *Amia* the overlapping was in the same way, that is, the extremity of the right pubis was dorsad of that of the left. There is no iliac process in either of these forms.

We thus see that in these living Holostean Ganoids the only modification is the reduction in the number of the rays, (and this may be only apparent) and the concrescence especially of the proximal segments. They offer us no advance toward the Dipnoan or Stapediferal limb-skeleton. As to the light which they throw backward on the earlier history, the separation of the pubic part of one side from that of the other and the absence of an iliac process is to be noticed. Otherwise they are uninteresting forms.

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Before reviewing the results obtained, I wish to call attention again to the shark ventrals, and to insist on the comparison between them and the Chondrostean ventrals. If the reader will place the figures 1, 2, 3 and 7, side by side with figures 67 and 69 of my paper of last February, or with any of the figures of shark ventrals given by Gegenbaur, there can be no question as to the way in which they are related to one another, and the way in which the latter were formed. If we compare fig. 67 for example, with fig. 3, Pl. I, there can be no hesitation in recognizing the corresponding parts of each; in seeing that the solid "basale metapterygii" of 67 answers to the hinder part of the row of proximals, for the most part separate, of fig. 3, and that the part *b d* in the latter figure represents, more or less exactly, one half the pelvic girdle in the former. The iliac part or parts are not developed in the shark. Thus the most essential difference between the Chondrostean and Elasmobranch, as regards their ventral fins, would be obliterated by that process of concrescence, whose absolute insignificance is shown by the varying degrees in which it shows itself, here in the sturgeon ventrals and elsewhere. I think I may regard the formation of the shark ventrals from a series of three jointed rays in the way exhibited diagrammatically in my former paper, as established fully and finally. It will moreover be observed, that this method of forming the sharks' ventrals, resulting as it did in a "uniserial archipterygium," is incompetent to form a "biserial archipterygium," and that consequently we have in the shark ventral no approach to the *Ceratodus* fin.

The Relations of the Groups of Gnathostomes and the Modification of the Fin-skeleton in those Groups.

Views regarding the history of any part or organ determine to a certain extent the views which are held regarding the genealogical connection of the groups in which that part or organ is exhibited; and conversely genealogic connections already established may throw, and usually do throw, a certain weight of evidence for or against a view of the development of an organ. And inasmuch as from the history of one part light is thrown upon the genealogic connection of the groups, and as from this genealogic connection inferences can be drawn respecting the history of other parts, it is evident that in this way the grounds for accepting or rejecting a view of the history of one organ may become the grounds for accepting or rejecting a view proposed for the history of another organ; and be the weight of this evidence great or small, there is no way to bring our belief in one

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sequence to bear on that in a different sequence, except by the interposition of a genealogical table more or less complete. And this possibility of making one view confirm another arises, of course, from the fact that, although we may consider the histories of heart, of brain, of skull, or of limb, each by itself, still each heart and limb was only a part of an animal in which were all the other parts.

I therefore here introduce the following views respecting the affinities of the groups involved in this discussion, views in which there is little more original than an independent judgment. I give them without the proof, which is extremely bulky for its weight.

The Gnathostomes and Stapedifera* are genetic† groups.

The Gnathostomes minus the Elasmobranchs are a genetic group. The nasal fossa is bridged in all the members of this group, with very few and insignificant exceptions.‡ It is never bridged in the Elasmobranchs. We will call the members of this group then Gephyrrhina (*Γέφυρα* = a bridge).

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* Stapedifera = Amniota + Amphibia. In my paper of last February I referred incidentally to the stapes as formed from the proximal part of the hyoid arch. This was a mistake, and my attention was called to it by the following passage from Wiedersheim:—. . . . . "so ist es beinahe traditionell geworden, die bei der ungleich besser studirten Entwickelungsgeschichte der Anuren über den Schallzuleitungsapparat gewonnenen Resultate ohne Weiteres auch auf die Urodelen auszudenen. Darauf beruht die in die verschiedensten Abhandlungen und Lehrbücher übergegangene Behauptung: 'das Operculum der Urodelen hat sich vom Hyoidbogen abgeschnürt.'" (R. Wiedersheim, Kopfskelet der Urodelen, Morph. Jahrb. Band, III, 1877.) Wiedersheim proceeds to show that the stapes or "operculum" of the Urodela is cut out of the cartilage of the periotic capsule as a disk filling the fenestra ovalis, from which a boss, more or less prolonged into a rod in the various species, arises by continuous spreading of the cartilage. He refers to Parker as having arrived at the same results. But it appears that this is only half of the story. Those invaluable memoirs of Parker on the skull, for which every living anatomist must bear a feeling of personal gratitude, seem to establish beyond all reasonable doubt that the stapes of mammals and what corresponds to it in the Stapedifera amammalia is never developed from the hyoid arch, but always from the periotic capsule. Moreover, it appears that in the Batrachia, or tailless Amphibia, and in the Amniota amammalia this periotic stapes connects itself with the proximal part of the hyoid arch to form the columella; and it seems to be proved that this proximal part of the hyoid arch may sometimes originate, ontogenetically, neither in continuity with the distal portion nor at the same time with it.

† By a genetic group I mean an assemblage of animals, or plants, which are more nearly related to one another than any of them is to anything outside the group. Thus the latest common ancestors of any two members of any genetic group would be later than the latest common ancestors of any form in the group and any form outside.

‡ The Chromides, except *Symphysodon*, and the Labroidei ctenoidei or Pomacentridæ, have only one external opening on each side. There are also some curious modifications of the olfactory apparatus among the Tetrodontina.

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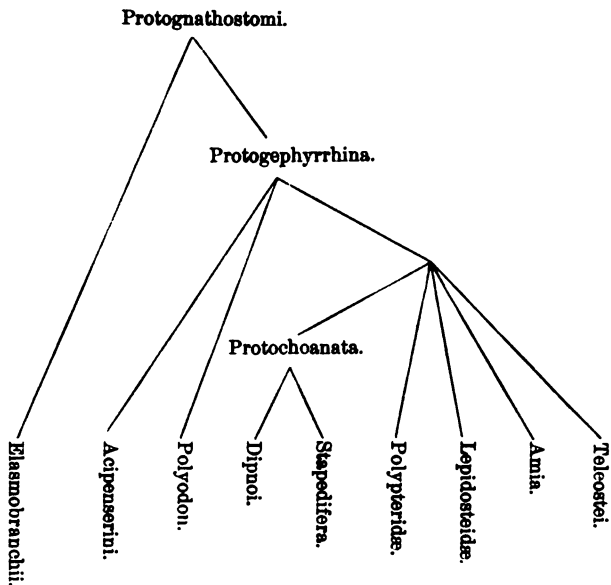
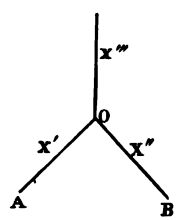
The Stapedifera together with the Dipnoi constitute a genetic group. They have posterior nares, or choanæ, opening within the mouth. We may then call them Choanata.

With a less degree of certainty, I should regard the Elasmobranchii, Teleostei, and Dipnoi, as genetic groups: as also a group formed by withdrawing the Chondrostei from the Gephyrrhina. These relations are exhibited in the following table.



It will be noticed that in two places in this table the division is into more than two lines, into three in one case and into five in the other. I do not intend to indicate that any such multiplicate fission really took place, but merely that it is not yet evident exactly how these sub-groups are related to one another. The relation of the genetic groups A, B, and C may be that exhibited by making C's line come in at X′, X″ or X‴. But while it is practically impossible that X should coincide with O, it is very easy for X to get so near O as to be indistinguishable from it, and this will depend principally on the amount of change that has taken place between X and O, and on the completeness of our knowledge of the results of that change as now exhibited in living or fossil forms. To represent the lines as



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diverging from O seems a fit way of symbolizing the fact that the point of union of C with the other lines came so near O that in our present ignorance we can not distinguish it from O. And this, I take it, has been the practice hitherto.

From this examination of the Ganoid ventrals have been obtained the following results, for the most part merely confirmatory of views previously expressed.

I. The hind limbs with their arches have been derived from a series of simple rays divided into three segments.

II. The "Archipterygium" never existed outside of the Choanate line, and probably never outside the Dipnoan.

III. While the two former results are well established and may be regarded as final, the views which we must form respecting the number of rays, the amount of concrescence between adjacent rays, the union of the two pubes and the development of the iliac process are much more indefinite and uncertain. It seems probable that there was a reduction in the number of the rays, in going from the Protognathostomi to the Protogephyrrhina, and perhaps a still farther reduction in the line leading off toward the Teleosts. It is not unlikely that the concrescence in each of the ancestral groups just mentioned lay between that of *Scaphirhynchus* and *Polyodon*. The union of the two pubes is found in all Elasmobranchiates, except the Holocephali, in some Teleosts, and in the Dipnoi. Whether this slight development was independent in the different groups where it occurred or not seems at present impossible to say. An iliac process is found in Chimæra, in the Rays, in the Chondrostei, apparently in the Dipnoi, and in the Stapedifera; there seems to be the same doubt about the history of this. But it is not very important. The essential fact seems to be this—that the line from the Protognathostomi to the Protostapedifera was in such a state that no very extraordinary set of circumstances was sufficient to evoke an iliac process.

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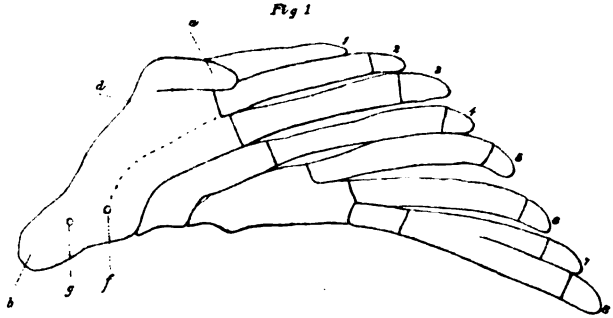
Fig 1



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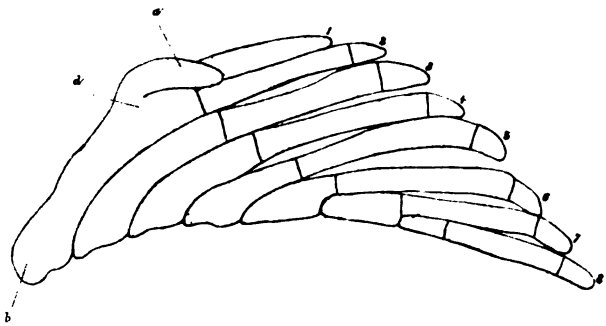

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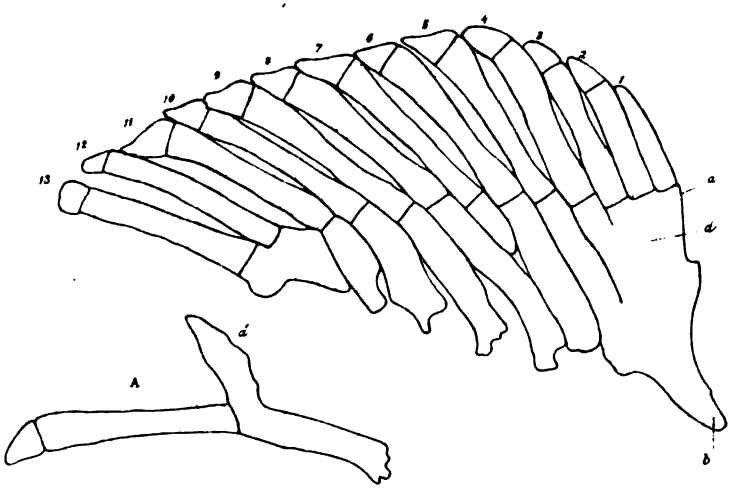

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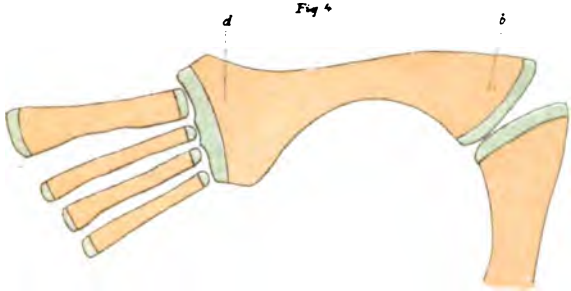
Fig 4



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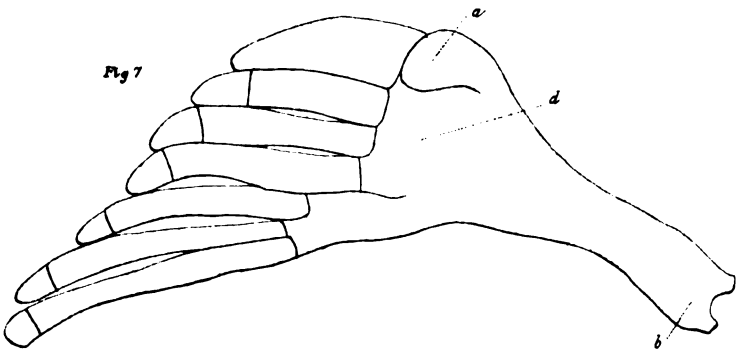

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1880, July 18.
C. F. G.
the Author.

IV. Some Interesting New Diptera. By S. W. Williston.

Rhynchocephalus Sackenii, n. sp. ♀.

Black with light yellowish hair. Head broader than thorax, brownish black. Front broad, thinly blackish haired on the vertex; the lower part, the face, cheeks, and posterior orbits, with thick, bushy, yellowish white hair, becoming nearly white below. Antennæ short, reddishyellow, base of first joint infuscated, two first joints subquadrate, third circular. Style of three joints, first joint short, yellowish, second joint twice as long, basal half infuscated, third as long as two first, fuscous. Proboscis reaching the hind coxæ, labium black, other parts, with the slender minute palpi, luteous. Dorsum of thorax and scutellum brownish black with yellowish hair; pleuræ and pectus with longer, bushy, grayish white hair. Abdomen short and broad, black; second segment above, and all the segments upon their sides, with yellowish hair, somewhat intermixed with black at the incisures; third and remaining segments above with sparse hairs and thick yellowish tomentum, wanting upon their anterior borders, giving the abdomen a slightly fasciated appearance. Venter with whitish pile. Lamellæ of the ovipositor slender, black, luteous at extreme base, about as long as intermediate femora. Feet luteous. Femora white tomentose, with tufts of hair on their undersides near the coxæ; anterior and middle pairs, for their basal two-thirds, and posterior, except extreme tips, black. Anterior and middle tarsi infuscated, posterior more so, blackish. Wings hyaline; neuration as in the figure:

Adventitious oblique vein but slightly arcuated, terminating beyond the middle of the apical half, not continued to posterior border, so that the third and fifth posterior cells are not completely separated; both cross veins obsolete. Three submarginal cells; first and second open, slender. First posterior open, second closed, the brief petiole terminating in the end of the costal vein before the tip of the wing, fourth (third of Osten-Sacken) closed, as usual; third a little shorter

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than fifth. Long. corp. 9^{mm}; long. al. 9^{mm}. Olympia, Washington Territory,—H. K. Morrison.

The present species is a most interesting addition to our fauna. Hitherto only one, possibly two, species of this family and belonging to the short-proboscid division, have been described from North America. The family is subtropical, and the occurrence of the present, the first North American representative of the rhynchocephalous division (*Nemistrinini*), from such high latitude is remarkable, and, moreover, is a striking example of the resemblance pointed out by Baron Osten-Sacken between the Western Asiatic and our Pacific faunæ. Of the genus *Rhynchocephalus*, Schiner considered *R. Tauscherii*, from the region south of the Black Sea, as the type, and which, evidently, is the only known species, beside the present, that will ultimately remain in the genus, so diverse are the other two species, from the same regions, hitherto placed with it. To *R. Tauscherii* (Meig. Sys. Besch., vol. vi, pl. 66, fig. 6) the present species shows a very close resemblance, the length of proboscis and color of the head being the chief differences.

Of the *Nemistrinini*, a fossil species has been described by Dr. Scudder from the Tertiary shales of Florissant, forming the type of a new genus (*Palembolus*, Bul. U. S. Geol. Sur., vol. iv, No. 2, p. 526). So far as I can judge, the genus is nearly related to *Rhynchocephalus*. I would differ somewhat from Dr. Scudder's excellent interpretation of this, the most intricate neuration among diptera. It is not the anterior, but the posterior, intercalary vein that is present, as in many *Midasidæ*. The vein which he describes as the anterior intercalary, is composed of the anterior basal transverse and the fifth longitudinal. Dr. Loew defined the family as having both intercalary veins, but I can find no evidence of the anterior one.

Silvius pollinosus, n. sp. ♀.

Small, black, thickly grayish white pollinose. Head black, shining, concealed beneath dense gray pollen, ocelli very distinct, area not denuded. Frontal callosity black, shining, doubly rounded above, doubly concave and pointed below. Face with three shining black spots, the middle one smaller, sometimes obsolete. Antennæ fuscous, second joint less than half as long as first, basal joint grayish pollinose and black pilose, base of all the joints somewhat luteous, the third especially so on the inner and upper sides, quinque-annulate, terminal portion black. Palpi reddish, proboscis black. Tho-

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rax black and shining where denuded of the gray pollen; three indistinct lighter stripes of the color of front, abbreviated posteriorly. Abdomen thickly pollinose, the blackish, opaque ground-color with narrow whitish posterior borders to all the segments; two series of small, median, more or less indistinct, brownish spots on the anterior parts of second, third, and fourth segments; tip thinly whitish pilose. Legs reddish; ends of all the femora and tibiæ, anterior tarsi, middle, and posterior tarsi, except the larger part of basal joint, black. Wings pubescent, hyaline; stigma brown, reaching across the marginal cell, veins at bases of second submarginal, first, second, and fourth posterior cells, with narrow blackish clouds; also a number (4–8) of minute brown spots on all the veins beyond the discal cell. Long. corp. 8^{mm}. Kansas,—E. W. Guild.

Agrees closely in structure—except the pubescent wings—with *S. gigantulus* Lw., the only other North American species, but is very different in coloration.

Chrysops discalis, n. sp. ♀.

Large. Front black, with gray pollen, ocellar area not denuded, callosity small, shining-black, oval. Face yellow, with four small black spots, the upper pair broadly, the lower narrowly separated. Antennæ: basal joints fuscous, white pubescent, first joint yellow on inner side, third joint black. Palpi the color of the face, white pubescent Thorax with four cinereous stripes, leaving black intervals, of which the median is narrow. Scutellum cinereous. Abdomen: first segment black, with a small yellow spot on each side of the scutellum; second segment yellowish cinereous with four black spots, the outer ones smaller and posterior, the median ones larger, subquadrate, anterior, narrowly interrupted; third segment with the four spots subequal, indistinct, posterior part and all the remaining segments blackish, with cinereous pollen and yellowish incisures. Venter reddish yellow with three series of black spots, median ones broad and triangular with the base in front, outer ones small, oval; sixth and seventh segments chiefly black with reddish yellow posterior margins. Legs reddish yellow; extreme tips of all the femora, anterior tarsi and tips of other tarsi black, tips of anterior tibiæ infuscated. Wings with pale brownish design. End of first basal cell, all of second and third basal cells, anal angle, large spots in the discal and fifth posterior cell and usual crossband, hyaline. Apical portion grayish, with a distinct brown spot at the furcation of the third longi-

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tudinal, covering a stump of a vein. Four specimens. Como, Wyoming, June 20. Long. corp. 10–11^{mm}.

Belongs in the small Mediterranean group with hyaline discal cell. Easily distinguishable from *C. fulvaster* O. S., of the same region, the only other known American species of this group, by its large size, black frontal and facial callosities and the greater hyalinity of the wings. From *C. quadrivittatus* Say, it is also evidently different.

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V. On the species of Pinnixa inhabiting the New England Coast, with remarks on their early stages. By S. I. Smith.

In the Report of the U. S. Commissioner of Fish and Fisheries for 1871–2, I have figured a species of *Pinnixa* and recorded its occurrence in Vineyard and Long Island Sounds, but referred it incorrectly to *P. cylindrica* (Say sp.) At that time I had examined only three specimens, and all of them were of the species figured. In 1874 and 1875 additional specimens of *Pinnixa* were collected by Fish Commission parties in Fisher's Island Sound, Vineyard Sound, and Buzzard's Bay, but I did not examine them carefully, supposing the specimens to be of the same species as those previously examined. During these seasons, however, I had some opportunities for studying the early stages of the genus.

In 1874 zoeæ of *Pinnixa* were obtained in abundance from the eggs, but I was not able to rear them to later stages. In 1875 zoeæ were again obtained from the eggs and many zoeæ in later stages were taken at the surface. Among those obtained at the surface there were two forms which at first sight looked like two stages of the same species, but on closer examination, they both appeared, judging from the advanced condition of the cephalothoracic and abdominal legs, to be in the last zoea-stage. The larger and much the more abundant of these two forms, the one designated in my notes as the "long-spined zoea," agrees with the zoea of *Pinnixa*, from Newport, Rhode Island, figured by Mr. Faxon (Bull. Mus. Comp. Zool., vol. v, plate 4, figs. 5–15, 1879). The other form, designated in my notes as the "short-spined zoea," is considerably smaller and has very much shorter dorsal and rostral spines, even proportionally shorter than in the first stage of the zoea of *Pinnixa chætopterana* figured by Mr. Faxon (plate 4, fig. 1), so that the distance from the tip of the dorsal to the tip of the rostral spine is only about half as great as in the long-spined form. Except in these characters I was unable to find any differences whatever and, though then aware of the occurrence of one species only on the New England coast, I supposed that the forms represented two species of *Pinnixa*.

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A number of individuals of each form were examined and carefully reared in separate vessels. Two individuals of the long-spined form lived through the last zoea molt and came out in the early stage of the adult form, as well described by Mr. Faxon. Of the short-spined form a single individual was reared through the final zoea molt, and came out a megalops. I was not able during the season of 1875 to rear either the megalops or the young crab through another molt, or to repeat the former observations.

Though these observations were very remarkable, I wished to complete and confirm them, and, if possible, to determine to what particular species the two forms of zoea belonged, and so the publication of the observations was postponed; but no opportunity for completing them occurred, and in 1878 Mr. Faxon, though wholly ignorant of my observations, fortunately repeated the observations upon the long-spined zoea and published the results, together with a brief statement of my observations, and gave figures of the zoeæ which I obtained from the adult *Pinnixa* at Noank, Conn., in 1874. Mr. Faxon identified the adult *Pinnixa*, like those from which the zoeæ were obtained, with *P. chætopterana* of Stimpson, and, in his paper referred to, is the first to record the occurrence of this species on the New England coast.

During the summer of 1879 Prof. H. E. Webster obtained a large number of specimens of *Pinnixa* (which was before not known north of Cape Cod) at Wellfleet, on Cape Cod Bay, and sent them to me for identification. While examining these I have reëxamined the specimens previously obtained on the New England coast, in all between 60 and 70 specimens, and I find there are two quite distinct species among them. As indicated by these collections, by far the most common species is *P. chætopterana* Stimpson, and among the specimens taken by Prof. Webster in Cape Cod Bay, this is the only species represented. From Vineyard Sound, Buzzard's Bay, and Long Island Sound, however, there are a few specimens of a different species, the one which I have figured and referred to as *P. cylindrica* (Say sp.) It is apparently not Say's species, however, but the species described by Stimpson as *P. Sayana*. Stimpson appears to have had only males and his specimens were dredged in 6 fathoms, off the mouth of Beaufort Harbor, North Carolina. The distinctive features of these two species are pointed out further on.

Having ascertained that the adults of two species of *Pinnixa* inhabit the New England coast, it is a matter of considerable interest to determine whether either or both of these are the same as either

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or both of the species which Mr. Faxon and I have observed to differ so widely in regard to the megalops-stage.

The young in the first zoea-stage figured by Mr. Faxon (pl. 4, figs. 1–4) were obtained directly from the eggs of *P. chætopterana* and are of course unquestionably of that species. That the later stages figured by Mr. Faxon (pl. 4, figs. 5–15, and pl. 5) do not belong to the same species as the young in the first zoea-stage, is shown by the difference in the length of the dorsal and rostral spines in the first and last zoea-stages, and by the presence, in the zoeæ in the first stage, of the central lobe of the posterior margin of the telson, while it is wholly wanting in the zoeæ in the last stage. In the several species of Brachyura in which I have examined a series of zoeæ in different stages, the dorsal and rostral spines are proportionally not much if at all longer in the first than in the last stage, and there is often a considerable decrease in the length of these spines in passing from next to the last to the last stage: *Pinnixa* is probably not an exception to this rule. Moreover, among the zoeæ of *Pinnixa* taken in Vineyard Sound in 1875, there are a few specimens of the long-spined form which are evidently in the penultimate zoea-stage, and they agree fully with the numerous specimens in the last stage in the length of the spines and in the form of the telson.

On the other hand the short-spined zoeæ which I observed to pass into a megalops-stage agree with the first stage of the zoeæ from the eggs of *P. chætopterana*, not only in the length of the spines but also perfectly in the form of the telson, the median lobe of the posterior margin being developed precisely as in the zoeæ of the first stage and as figured by Mr. Faxon (pl. 4, fig. 2, *b*). This is sufficient evidence, I think, to show that the short-spined zoeæ upon which my observations were based are the young of *P. chætopterana*, and that the long-spined form, which both Mr. Faxon and I observed to pass directly from the zoea to the adult form, must belong to some other species. The fact that these long-spined zoeæ were very common both at Newport and in Vineyard Sound would seem to indicate a probability that they belong to some species regularly inhabiting the coast of Southern New England, and consequently that they are likely to prove the young of *P. Sayana*,—but this is only a probability. That both forms of the early stages belong to species of the same genus (as the genera of the group to which they belong are now understood) there can be no reasonable doubt, since one of the forms has now been traced from the eggs of a well-known species to the megalops-stage, and the other from the later zoea-stages to the early, but generically characteristic, stages of the adult form.

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The following systematic account of the two species of *Pinnixa* above referred to gives the obvious specific characters of the adults and the principal bibliography of each species.

Pinnixa chætopterana Stimpson.

Pinnixa cylindrica Stimpson, Annals Lyceum Nat. Hist. New York, vii, p. 68 (22) 1859 (partial desc.; South Carolina, in tubes of *Chætopterus*) (*Non* White nec Say sp.)

Pinnixa chætopterana Stimpson, op. cit., vii, p. 235 (107), 1860 (desc.; syn.; same loc.)

Kingsley, Proc. Acad. Nat. Sci. Philadelphia, 1878, p. 324 (9), 1878 (North Carolina); op. cit., 1879, p. 402, 1880 (Virginia, Florida).

Faxon, Bulletin Mus. Comp. Zool. Cambridge, v, p. 263, pl. 4, figs. 1–4 (zoea). pl. 5, figs. 8, 9 (chelipeds), 1879 (Long I. Sd. and Buzzard's Bay).

The carapax is very broad and broader in the males than in the females, adult males and large females being two and a fourth times as broad as long. There is a distinct and minutely tubercular ridge along the anterior inferior edge of the branchial region just over the bases of the chelipeds; and above, and nearly parallel with this, there is a denticulated carina, or line of minute denticles, across the swollen branchial region, on a line from the base of the third ambulatory leg toward the eye, but not reaching the lateral margin of the carapax nor crossing the cervical suture, though there is a slight elevation on the hepatic region opposite the carina. This line of denticles is better marked in the females than in the males. The transverse crest upon the cardiac region is conspicuous in both sexes: in the female it is marked by a slender but sharp carina interrupted for about a third its length in the middle; while in the male the carina is even more broadly interrupted in the middle and projects each side in a very prominent, transversely elongated, dentiform protuberance.

In the male the chelipeds are much stouter than in the female: the propodus is nearly smooth, as long as the carapax and somewhat swollen in the middle; the prehensile edge is terminal, but projects distally considerably at the inferior angle, is nearly transverse, and armed with a triangular tooth near the base of the dactylus and usually with a minute one near the tip. The dactylus is stout, very strongly curved and the prehensile edge is nearly or quite smooth. In several of the males examined one of the chelipeds is a little smaller than the other and in all respects like the chelipeds of the female, but these were probably all cases of reproduced limbs.

In the female the chelipeds are smaller and proportionally less stout and more compressed than in the male; the propodus is shorter than the length of the carapax, and the digital portion is oblique, longer than in the male, and its tip is obliquely truncated so as to

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leave an angular prominence near the middle of the prehensile edge. The dactylus is much longer and less curved than in the male, and its prehensile edge is armed with a small tooth about a third of the way from the base to the tip, and often with minute additional teeth either side of the principal one.

The first pair of ambulatory legs are slender and scarcely longer than the chelipeds; the second pair are a little longer and considerably stouter, but still slender; the dactyli in both pairs are rather slender, and in the female nearly as long as the upper edge of the propodus, but in adult males apparently a little shorter. The third pair are very large, about equally stout in the two sexes, and in adults about as long as the breadth of the carapax, but in young somewhat longer; the merus is about as long as the carapax and about half as broad as long; the upper edge is angular and usually minutely denticulated distally, there is a slight transverse groove at the distal end, the sides are smooth and rounded, but the inferior edge projects in a thin and conspicuously denticulated carina; there is a similar but much less conspicuous carina upon the propodus and also on the ischium; and there is a slight crest upon the upper edge of the carpus and propodus. The posterior ambulatory legs are short, reaching beyond the merus of the third pair, but they are much stouter than the second pair and the merus is carinated and grooved as in the third pair, though much less conspicuously.

The following measurements of the carapax and one of the third ambulatory legs in a number of specimens show the proportions of these parts of the animal more fully than the description.

No.	Sex.	Carapax:			Third ambulatory leg:							
		Length.	Breadth.	Ratio.	Length.	Merus.		Propodus.		Dactylus.	
5	♂	3·9mm	8·7mm	1 : 2·23	9·4mm	4·0 × 2·0mm		2·0 × 1·4mm		1·2 × 0·3mm	
3	"	3·9	8·8	1 : 2·26	9·5	4·0	2·0	2·0	1·4	1·2	0·4
1	"	4·0	9·1	1 : 2·27	9·7	4·1	2·1	2·0	1·5	1·3	0·4
3	"	4·1	9·3	1 : 2·26	9·8	4·2	2·1	2·0	1·5	1·3	0·4
3	"	4·4	10·0	1 : 2·27	10·5	4·4	2·2	2·2	1·6	1·4	0·5
3	"	4·5	10·2	1 : 2·27	11·2	4·6	2·3	2·0	1·5	1·4	0·5
5	"	4·7	11·0	1 : 2 34	11·0	4·7	2·5	2·2	1·8	1·5	0·5
3	"	4·9	11·5	1 : 2·35	11·0	5·0	2·4	2·3	1·8	1·5	0·6
4	♀?	2·3	4·7	1 : 2·04	6·0	2·3	0·9	1·3	0·6	1·0	0·2
3	♀	2·6	5·5	1 : 2·12	6·5	2·6	1·2	1·5	0·9	1·0	0·3
3	"	3·0	6·3	1 : 2·10	7·4	2·8	1·3	1·7	1·0	1·1	0·3
5	"	3·2	7·0	1 : 2·19	7·7	3·2	1·7	1·6	1·2	1·1	0·3
5	"	4·6	10·0	1 : 2·17	10·5	4·1	2·4	2·0	1·5	1·3	0·5
3	"	4·6	10·3	1 : 2·24	10·4	4·1	2·2	2·0	1·4	1·4	0·5
5	"	5·0	11·4	1 : 2·28	11·0	4·7	2·4	2·3	1·8	1·6	0·5
3	"	5·3	12·0	1 : 2·26	12·2	5·0	2·6	2·5	1·7	1·6	0·5

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Specimens Examined.

No.	Locality.	Fath.	Bot.	When coll'd.	Rec'd from.	No. specimens: ♂	♀	with eggs	Dry or Alc.
1	Stony Creek, L. I. Sd.	shore	M., S.	May 15, '71	S. I. Smith	1			Alc.
2	Noank, Fisher's I. Sd.			Aug., 1874	U. S. F. Com.		5	2	"
3	Buzzard's Bay	dredg.		Aug., 1875	"	19	13	1	"
4	Vineyard Sound	surface		Aug. 25, '75	"	yng.	2		"
5	Wellfleet, C. Cod B.	shore		1879	H. E. Webster	10	9	1	"

No. 5 were all found in the tubes of *Amphitrite ornata* Verrill (Leidy sp.)

Pinnixa Sayana Stimpson.

Pinnixa Sayana Stimpson, Annals Lyceum Nat. Hist. New York, vii, p. 236 (108), 1860 (desc. of ♂; North Carolina).

Kingsley, Proc. Acad. Nat. Sci. Philadelphia, 1878, p. 323 (8), 1878.

Pinnixa cylindrica Smith, Invertebrate Animals of Vineyard Sound, Report U. S. Comm. Fish and Fisheries, part i, p. 546 (252), pl. 1, fig. 1, 1874 (Vineyard Sd. and Long I. Sd.) (*Non* Say sp. *nec* Stimpson).

? *Pinnixa* sp. Faxon, Bulletin Mus. Comp. Zool. Cambridge, v, p. 263, pl. 4, figs. 5–15, pl. 5, figs. 1–7, 1879 (early stages; Newport, Rhode Island).

The carapax is narrower than in *P. chætopterana*, but still nearly, or even somewhat more than, twice as broad as long. There is a tuberculose ridge along the anterior inferior margin as in *P. chætopterana*, and above and nearly parallel with it a sharp denticulated carina extending from the base of third ambulatory leg across the branchial region and across the cervical suture to the hepatic region, being most conspicuous at the cervical suture, and separating the dorsal from the nearly perpendicular antero-lateral border of the carapax. In the female there is no carina on the cardiac region, only an obtusely angular ridge separating the flat dorsal from the inclined posterior dorsal region; while in the male (as already described by Stimpson) the ridge is marked by a very slender, but acute, carina not interrupted in the middle.

The chelipeds in the one male examined are unequal, the smaller being in all respects like the chelipeds of the female, while the other (apparently of the normal form for the male) is very much as in the male *P. chætopterana*, though apparently a little smaller in proportion. The tooth near the base of the digital portion of the propodus is inconspicuous but still clearly discernible.

The first and second pairs of ambulatory legs are long and very slender, the first pair being longer than the chelipeds, and the second considerably longer than the breadth of the carapax. The third pair are only a little longer than the second and proportionally consider-

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ably stouter, but very much less stout than in *P. chætopterana:* the merus is about three times as long as broad, the posterior surface is somewhat rough and granular and there is a minutely tubercular and granular carina on the lower edge, but there is no well marked groove across the distal end; there are two lines of granules along the lower edge of the propodus but no real carina. The posterior legs are very much like the third pair except that they are much smaller, reaching only to the distal ends of the carpi of the third pair. The dactyli of all the ambulatory legs are long, slender and nearly straight.

Measurements.

No.	Sex.	Carapax.			Third ambulatory leg:							
		Length.	Breadth.	Ratio.	Length.	Merus.		Propodus.		Dactylus.		
5	♂	2·8mm	5·4mm	1 : 1·93	8·0mm	3·2 × 0·9mm		1·7 × 0·6mm		1·5 × 0·2mm		
5	♀	2·7	5·2	1 : 1·94	7·6	2·9	0·8	1·4	0·4	1·4	0·2	
5	"	2·9	5·6	1 : 1·93	7·2	2·7	0·8	1·4	0·5	1·3	0·2	
5	"	3·3	6·7	1 : 2·03	8·2	3·2	1·0	1·6	0·6	1·4	0·3	
1	"	3·4	6·8	1 : 2·00	8·7	3·3	1·1	1·7	0·7	1·5	0·3	
4	"	3·7	7·4	1 : 2·00	9·1	3·8	1·2	1·8	0·7	1·6	0·3	
2	"	3·7	7·7	1 : 2·08	9·0	3·6	1·2	1·9	0·8	1·6	0·3	
3	"	4·3	8·7	1 : 2·02	10·1	4·0	1·4	2·0	1·0	1·7	0·3	

Specimens Examined.

No.	Locality.	Fath.	Bot.	When coll'd.	Rec'd from.	No. specimens:			Dry or Alc.
						♂	♀	with eggs.	
1	New Haven, L. I. Sd.	dredged			A. E. Verrill		1	0	Alc.
2	" "			1869	A. D. Chidsey		1	0	"
3	" "	shore		May 22, '72	A. E. Verrill		1	0	"
4	Vineyard Sound			1871	U. S. F. Com.		1	0	"
5	Buzzard's Bay	dredged	M.	1875	"	1	3	1	"

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Occasional occurrence of tropical and sub-tropical species of Decapod Crustacea on the Coast of New England. By S. I. Smith.

Some years ago I called attention to the occurrence of the young of a southern species of *Ocypode* on the south shore of Long Island and the coast of southern New England, and more recently Mr. Faxon has recorded the similar occurrence, at Newport, Rhode Island, of *Polyonyx macrocheles* and, incidentally, of *Calappa marmorata*. Since my first observations were published a considerable number of new observations have been made and additional species have been found to occur in a similar way, so that it seemed desirable to bring together all the observations with the view of calling special attention to the subject. This is attempted in the following list which includes all the tropical and sub-tropical species which are known to me to occur, in any stage of the adult form, on the New England coast. The pelagic species which I have mentioned as found near the borders of the Gulf Stream, off St. George's Banks (these Transactions, vol. iii, p. 26; vol. v, pp. 120–122), are doubtless all regular inhabitants of the Gulf Stream even much north of New England, and all probably occur occasionally on the coast of southern New England, but only those which have actually been observed in the bays and sounds along the coast itself are included in the list. Of the species included, *Nautilograpsus minutus*, *Neptunus Sayi*, and *Latreutes ensiferus* belong to the pelagic fauna of the Gulf Stream. The others are all southern species which either drift north in the free-swimming early stages and partially complete their development on our coast, like *Ocypode quadrata* and *Polyonyx macrocheles*, or are accidentally carried north in some stage of the adult form and survive for a short time in summer, like *Pachygrapsus transversus*. I have not intended to include any species which could, in any sense, be regarded as properly belonging to the fauna of the New England coast.

Ocypode quadrata J. C. Fabricius.

Cancer quadratus J. C. Fabricius, Entomologia Systematica, ii, p. 439, 1793 (diagnosis; "Habitat in Jamaica Mus. Dom. Banks.")

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Ocypode quadrata J. C. Fabricius (continued).

Herbst, Krabben und Krebse, i, p. 257, 1782 (= Fabricius).*

Ocypode quadrata J. C. Fabricius, Supplementum Entomol. System., p. 347, 1798.

Bosc, Hist. nat. Crust., i, pp. 194, 198, 1802 (*Ocypoda;* = Fabricius).

Latreille, Hist. nat. Crust. Ins., vi, p. 49, 1803 (=Fabricius).

Ocypoda albicans Bosc, Hist. nat. Crust., i, p. 196, pl. 4, fig. 1, 1802 (Carolina coast; the figure is very bad and probably not based on the specimens described in the text, the eye-stalks being prolonged as in *O. ceratophthalma.*)

Latreille, Hist. nat. Crust. Ins., vi, p. 48, 1803 (= Bosc).

Ocypode arenarius Say, Jour. Acad. Nat. Sci. Philadelphia, i, p. 69, 1817.

M.-Edwards, Hist. nat. Crust., ii, p. 44, pl. 19, figs. 13, 14, 1837 (*Ocypoda arenaria*); Ann. Sci. nat., III, xviii, p. 143 (107), 1852 (*Ocypode arenaria*).

White, List Crust. British Mus., p. 34, 1847 (*Ocypode arenaria*).

Gibbes, Proc. Amer. Assoc. Adv. Sci., 3d meeting, p. 180 (16), 1850 (*arenaria*).

Coues, Proc. Acad. Nat. Sci. Philadelphia, 1871, p. 122 (*arenaria;* North Carolina; habits).

Von Martens, Archiv für Naturgesch., xxxviii, p. 103, 1872 (*arenaria*).

Smith, Amer. Jour. Sci., III, vi, p. 67, 1873 (*Monolepis inermis*=megalops-stage); Inverteb. Vineyard Sd., Report U. S. Fish Comm., i, p. 545 (251), 534 (240), 1874 (*Ocypoda arenaria*).

Kingsley, Proc. Acad. Nat. Sci. Philadelphia, 1878, p. 322 (7), 1878 (*Ocypoda arenaria*); op. cit., 1879, p. 400, 1880.

Ocypoda rhombea M.-Edwards, Hist. nat. Crust., ii, p. 46, 1837 ("Antilles et Brésil"); Ann. Sci. nat., III, xviii, p. 143 (107), 1852 (*Ocypode*).

White, List Crust. British Mus., p. 35, 1847 (*Ocypode;* Jamaica).

Dana, U. S. Expl. Exped., Crust., p. 322, pl. 19, fig. 8, 1852 (Brazil).

Monolepis inermis Say, Jour. Acad. Nat. Sci. Philadelphia, i, p. 157, 1817.

M.-Edwards, Hist. nat. Crust., ii, p. 264, 1837.

White, List Crust. British Mus., p. 65, 1847.

Gibbes, Proc. Amer. Assoc. Adv. Sci., 3d meeting, p. 192 (28), 1851.

Cancer arenarius Catesby, Nat. Hist. Carolina, ii, pl. 35, 1743.

In 1873 I called attention to the occurrence of the young of this species on the coast of New England and Long Island, and suggested its relation to the *Monolepis inermis* of Say. Since then a great number of the megalops have been taken in Vineyard Sound by Mr. V. N. Edwards, and, early in September, 1875, I took several specimens swimming at the surface in the same region. One of the latter specimens, after being kept alive for some days, buried itself in the sand preparatory to molting, but died before this was fully accomplished. The molting was sufficiently advanced, however, to show

* The sign of equality (=), as here introduced after a reference, is used to indicate that the description, mention, or whatever account of the species may be given in the work referred to, is apparently based wholly on the previous account given by the author whose name follows the sign.

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clearly the form of the young *Ocypode* within the megalops skin and to establish unequivocally my former conclusion that Say's *Monolepis inermis* is really the megalops-stage of this species of *Ocypode*.

I have examined a very large series of specimens of the adult form of this species from Brazil; St. Thomas; Aspinwall; Nassau, New Providence; Texas; Key West, and Sarasota Bay, Florida; the Bermudas; North Carolina; New Jersey; and the south shore of Long Island: and, between specimens of the same size, I can find no differences which could be regarded as specific, though young specimens differ much from adults, and these differences have probably led to the admission of the two nominal species, *arenaria* and *rhombea*.

This species is evidently the *O. quadrata* of Fabricius: the original description of *Cancer quadratus* agrees in every respect, and the comparison with *C. ceratophthalmus* ("Affinis C. ceratophtalmo at oculi simplices") and the habitat ought to leave no doubt whatever on this point. DeHaan, however, refers Fabricius' species to a Japan *Sesarma* (*Grapsus* (*Pachysoma*) *quadratus* DeHaan, Fauna Japonica, p. 62, pl. 8, fig. 3), although he had already (op. cit., p. 29) retained it in the genus *Ocypode*. Succeeding authors have failed to restore Fabricius' species to its proper place. Fabricius' *Ocypode rhombea*, of which the habitat was unknown, is certainly not this species, nor of the genus *Ocypode* in the modern sense, as the first line of the diagnosis, "thorace laeuiusculo utrinque unidentato," plainly shows. If ante-Linnean names, when occasionally binomial, are to be adopted, then Catesby's *arenarius* may be retained.

When writing the report on the crustacea of Vineyard Sound, I had not been able to consult Fabricius' works, and stated that "the Brazilian species, usually called *rhombea* appears to be identical with ours, and if it is really the *rhombea* of Fabricius, his name should undoubtedly be retained." This remark is quoted verbatim in order to correct Mr. Kingsley's statement (Proc. Acad. Nat. Sci. Philadelphia, 1878, p. 322 (7), 1878) that I say "that our form seems to be identical with the Brazilian one, which is known as *O. rhombea* Fabricius, in which case the name *rhombea* will hold."

Grapsus pictus Lamarck.

Cancer grapsus Linné, Systema Naturæ, ed. xii, i, p. 1048, 1767; Amœnit. Acad., 2d ed., iv, p. 252, pl. 3, fig. 10, 1788.

Herbst, Krabben und Krebse, i, p. 115, 1782.

J. C. Fabricius, Systema Entomologiæ, p. 406, 1775; Entomologia System., ii, p. 438, 1793; Suppl. Entom. System., p. 342, 1798.

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Grapsus pictus Lamarck (continued).

Grapsus pictus Lamarck. Systéme Animaux sans Vertèb., p. 150, 1801.
Bosc, Hist. nat. Crust., i, p. 202, 1802.
Latreille, Hist. nat. Crust. Ins., vi, p. 69, 1803.
Desmarest, Consider. Général. Crust., p. 130, pl. 16, fig. 1, 1825.
M.-Edwards, Hist. Nat. Crust., ii, p. 86, 1837 (Antilles); Règne animal de Cuvier, 3me édit., pl. 22, fig. 1.
Gibbes, Proc. Amer. Assoc. Adv. Sci., 3d meeting, p. 181 (17) 1850 (Florida).
Dana, U. S. Expl. Expd., Crust., p. 336, 1852 (Madeira, Cape Verds, Peru, Paumotu Archipelago, Sandwich Is.)
Saussure, Revue Mag. Zool., II, v, p. 362 (9), 1853 (Mazatlan).
Nicolet, in Gay, Hist. de Chile, Zool., iii, p. 166, 1854 (= *G. ornatus* M.-Edwards).
Stimpson, Jour. Boston Soc. Nat. Hist., vi, p. 466 (26), 1857 (=*G. pictus* De Saussure, but supposed not to be that of Latreille).
Streets, Proc. Acad. Nat. Sci. Philadelphia, 1871, p. 240 (no description; Isthmus of Panama).
Miers, Proc. Zool. Soc. London, 1877, p. 73 (Galapagos Is.; >*G. altifrons* Stimp.)
Von Martens, Archiv. Naturgesch., xxxviii, 1872, p. 106 (Cuba).
Hilgendorf, Monatsb. Akad. Wissensch. Berlin, 1878, p. 807 (Mozambique).

Grapsus (*Goniopsis*) *pictus* DeHaan, Fauna Japonica, Crust., p. 33, 1835.

Grapsus strigosus Brullé, in Barker-Webb et Berthelot, Hist. nat. Iles Canaries, Entomologie, p. 15, 1835 (no description; =*G. Webbi*, teste M.-Edwards).
? Stimpson, Jour. Boston Soc. Nat. Hist., vi, p. 446 (26), 1857 (= *Goniopsis strigosus* White, in part).
? Hoffmann, Faune de Madagascar, Crust., p. 20, pl. 5, fig. 31, 1874.

Goniopsis picta Krauss, Südafrikan. Crust., p. 46, 1843.

? *Goniopsis strigosus* White, List. Crust. British Mus., p. 40, 1847 (in part, probably specimens *d* and *j*, Gulf of California).

Grapsus maculatus M.-Edwards, Ann. Sci. nat., III, xx, p. 167 (133), pl. 6 (=pl. 22, Règne animal de Cuvier, Crust.), 1853 (Antilles).
Saussure, Crust. Mexique et Antilles (Mém. Soc. Phys. Hist. nat. Genève, xiv), p. 32, 1858 (*G. maculatus*?).
Stimpson, Ann. Lyceum Nat. Hist. New York, vii, p. 229 (101), 1860 (Florida).
Kingsley, Proc. Acad. Nat. Sci. Philadelphia, 1879, p. 401 (Santa Cruz, Tahiti).
Hoffmann, Faune de Madagascar, Crust., p. 21, pl. 6, figs. 36–38, 1874.

Grapsus maculatus, var. *Pharaonis*, A. M.-Edwards, Nouv. Arch. Mus. Paris, ix, p. 285, 1873.

Grapsus Pharaonis M.-Edwards, Ann. Sci. nat., III, xx, p. 168 (134), 1853 (Red Sea).
Heller, Sitzungsb. mathem.-naturw. Classe Akad. Wissensch., xliii, p. 362, 1861.
Hoffmann, Faune de Madagascar, Crust., p. 20, pl. 5, figs. 32–35, 1874.

Grapsus ornatus M.-Edwards, Ann. Sci. nat., III, xx, p. 168 (134), 1853 (Chili).

Grapsus Webbi M.-Edwards, Ann. Sci. nat., III, xx, p. 167 (133), 1853 (= *G. strigosus* Brullé; Canary Is.)
Stimpson, Proc. Acad. Nat. Sci. Philadelphia, 1858, p. 102 (48).

Grapsus altifrons Stimpson. Ann. Lyceum Nat. Hist. New York, vii, p. 230 (102), 1860 (Cape St. Lucas.).

Pagurus maculatus Catesby, Nat. Hist. Carolina, ii, pl. 36, 1743 (Florida).

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The only specimen I have seen from the New England coast is a small one taken alive, in 1877, by Mr. J. M. Blake, in Provincetown Harbor, Cape Cod Bay, from a whaler just in from sea. This specimen differs so much from the adult *G. pictus* of Florida and the West Indies that it might readily be taken for a different species. The carapax is very much narrower proportionally than in adults, and the branchial regions are less swollen. The front and epistome are more like *G. strigosus* than the adult *pictus:* the front is not perpendicular as in the adult but very oblique, the median and lateral protogastric lobes being much less angular in front and much back of the frontal margin itself; the relative proportions of these lobes, however, and the other characters of the areolation of the carapax agree well with adult specimens. A series of young specimens of *G. pictus* from the coast of Brazil shows, by direct gradations in the form of the front and the relative proportions of the carapax, that the small specimen from Provincetown is an immature individual of this abundant tropical species, which, as far as I know, has not before been recorded from the western side of the Atlantic north of Florida and the Barmudas.

In the adult condition this species appears to vary slightly in the height of the front and the character of the frontal lobes, and considerably in coloration, but I can see no reason for regarding the form, from Chili, named *ornatus* by Milne-Edwards, or that, from the Gulf of California, called *altifrons* by Stimpson, as distinct species. I have examined a considerable number of specimens of the typical *pictus* from Bermuda and from Key West, Florida, two large specimens of the *ornatus* from Callao, Peru, and two large specimens of the *altifrons* from La Paz, Lower California; and I find no characters whatever in the form of the carapax, or in the proportions or armament of the chilipeds or ambulatory legs, by which these supposed species can be distinguished. In the coloration of these specimens there is an apparent difference between those from the Atlantic and those from the Pacific, but still not sufficient, I think, to distinguish them even as geographical color varieties. All the specimens appear to have the same pattern of coloration, but in those from the Gulf of California and Peru the red very much predominates, while in Atlantic specimens the yellow markings appear to occupy a larger proportion of the surface. The Atlantic specimens vary much in this respect, however, and the coloration of some of those from Bermuda approaches very nearly that of the specimens from the west coast of America, and I have no doubt that a large series of specimens from

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different localities would show a complete gradation between the extremes of coloration.

The following table of measurements shows the variation in the proportions of the carapax due to age, and the complete correspondence in the proportions of specimens of similar size from Bermuda and Key West, on the one hand, and Lower California and Peru, on the other. In order to show more clearly the relative proportions of the carapax, the measurements in the last three divisions of the table are given first in millimeters and then in the unit of the length of the carapax. In regard to the height of the front, which is not given in the table, it may be mentioned that it is absolutely higher in the largest specimen from the Bermudas than in either of the specimens from La Paz.

Locality; sex.		Length of Carapax.	Breadth of Carapax:					
			Greatest.		At anterior angles.		Of front.	
a		10·7mm	13·7mm	1·26 lgth.	11·5mm	1·07 lgth.	5·0mm	·45 lgth.
b		11·7	14·3	1·22	12·1	1·03	5·3	·45
b	♂	14·5	17·0	1·17	13·8	0·95	6·6	·45
b	♂	16·2	18·9	1·16	14·8	0·91	7·1	·44
b	♀	18·5	21·3	1·15	16·8	0·91	8·0	·43
b	♀	21·0	24·0	1·14	18·4	0·88	8·8	·42
c	♂	37·2	40·5	1·09	29·5	0·79	14·7	·40
d	♂	43·0	47·3	1·10	34·6	0·80	17·0	·40
d	♂	57·0	61·4	1·08	43·0	0·77	22·5	·39
c	♂	67·0	73·8	1·10	47·7	0·71	26·8	·40
e	♂	60·0	65·4	1·09	43·0	0·70	24·0	·40
e	♂	65·0	72·5	1·11	46·0	0·71	26·0	·40
f	♀	59·0	64·0	1·08	41·5	0·70	22·8	·39

a, Provincetown, Mass. *b*, Brazil (C. F. Hartt). *c*, Bermuda (G. Brown Goode). *d*, Key West, Fla. (received from Dr. Harrison Allen). *e*, La Paz, Lower California (J. Pedersen). *f*, Callao, Peru (F. H. Bradley).

The geographical distribution of the species is, perhaps, sufficiently indicated by the localities given above in the synonymy.

Pachygrapsus transversus Stimpson.

Grapsus transversus Gibbes, Proc. Amer. Assoc. Adv. Sci., 3d meeting, p. 181 (17), 1850 (Florida; said to belong to "*Pachygrapsus* if that be adopted.")

Pachygrapsus transversus Stimpson, Ann. Lyc. Nat. Hist. New York, vii, p. 64 (18), 1859; Amer. Jour. Sci., II, xxvii, p. 446, 1859; Ann. Lyc. Nat. Hist. New York, x, p. 113, 1871.

Smith, Report Peabody Acad. Sci. Salem, 1869, p. 91, 1871 (Pacific coast Central America).

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Pachygrapsus transversus Stimpson (continued).

Kingsley, Proc. Boston Soc. Nat. Hist., xx, p. 158, 1879; Proc. Acad. Nat. Sci. Philadelphia, 1879, p. 400, 1880 (includes specimens from west coast Central America, but does not make *P. socius* a synonym).

Goniograpsus innotatus Dana, Proc. Acad. Nat. Sci. Philadelphia, 1851, p. 249 (3), 1851 (South America); Crust. U. S. Expl. Exped, p. 345, pl, 21, fig. 9, 1852.

Leptograpsus rugulosus M.-Edwards, Ann. Sci. nat., III, xx, p. 172 (138), 1853 (Brazil).

Pachygrapsus innotatus Stimpson, Proc. Acad. Nat. Sci. Philadelphia, 1858, p. 102 (48), 1858 (Madeira).

Metepograpsus mineatus Saussure, Crust. Mexique et Antilles (Mém. Soc. Phys. Hist. nat. Genève, xiv), p. 28, pl. 2, fig. 17, 1858.

Metopograpsus dubius Saussure, ibid., p. 29, pl. 2, fig. 16, 1858.

Pachygrapsus intermedius Heller, Zool. Bot. Verein Verhandl. Wien, xii, 1862, p. 521 (Brazil); Reise der Novara, Crust., p. 44, 1865.

Pachygrapsus socius Stimpson, Ann. Lyc. Nat. Hist. New York, x, p. 114, 1871 (Cape St. Lucas, Panama, Peru).

Grapsus (*Leptograpsus*) *rugulosus* von Martens, Archiv für Naturgesch., xxxviii, p. 108, 1872.

Grapsus (*Leptograpsus*) *mineatus* von Martens, ibid., p. 109, 1872.

Pachygrapsus rugulosus Smith, Trans. Conn. Acad., ii, p. 37, 1869 (=*Leptograpsus rugulosus* M.-Edwards).

Goniograpsus (*Pachygrapsus*) *transversus* Lockington, Proc. California Acad. Sci., 1876, p. — (9), 1877 (=*Pachygrapsus transversus* Smith).

Four adult specimens of this species were taken alive from the bottom of a whaler in Provincetown Harbor, September 3, 1879. As far as I am aware, it has not before been recorded north of Florida on our coast. I have also examined a large number of specimens from the following Atlantic Ocean localities: Bermudas (G. Brown Goode, J. M. Jones); Key West (Dr. Harrison Allen); Aspinwall (F. H. Bradley); Brazil (C. F. Hartt). From the Pacific coast I have examined specimens from Gulf of Fonseca (J. A. McNiel); Acajutla and Panama (F. H. Bradley); Paita, Peru (F. H. Bradley, James Orton).

I can find no characters whatever in Dana's figures or description for distinguishing his *innotatus* from Gibbes' *transversus*. Stimpson says the *transversus* is "scarcely to be distinguished from *P. innotatus*, but the carapax is somewhat more convex." In all of the five specimens which I have examined from Brazil, the carapax is slightly less convex than in the majority of those from Florida and Bermuda, but some of the specimens from these latter localities are as little convex as, or even less convex than, the Brazilian specimens. All the unfaded specimens which I have seen from the west coast of

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America agree well with Stimpson's description of *P. socius* in wanting the patch of dark color usually present at the bases of the propodal digits of the chelipeds of Atlantic Ocean specimens; but in a large series of Atlantic specimens before me there is very great variation in the extent and intensity of the color upon the chelipeds and other parts of the animal, and some of the specimens from Bermuda, and one of those from Brazil want wholly the dark spot upon the chelipeds. The only other characters Stimpson points out for distinguishing the *socius* are: "the carapax is somewhat narrower, less convex, and more strongly striated; and the frontal region is more depressed and expanded." The impossibility of distinguishing Pacific specimens by the narrowness of the carapax or breadth of the front is fully shown in the table of measurements beyond. The convexity and striation of the carapax vary considerably in Atlantic specimens, and in those examined more than enough to include all the Pacific specimens seen.

Several of the specimens from Bermuda have the branchial regions of the carapax much more convex and swollen than is usual, and the convexity is not always equal on the two sides of the carapax. In at least a part of these specimens this irregularity in the form of the branchial region is due to the presence of a Bopyroid parasite within the branchial cavity, and I think it very probable that Saussure's *mineatus* was based on similar parasited specimens.

Heller compares his *intermedius* with Dana's *innotatus* and *simplex* and says that *innotatus* differs from his species only in having the carpus, as well as the hand, smooth above. Heller had apparently seen no specimens of *innotatus*, which is described as having the carpus "smooth" above (although the figure shows indication of slight corrugation), and depended on this one supposed difference for distinguishing the species. There is, as might be expected, considerable variation in the amount of corrugation upon the carpus, some specimens having the carpus very nearly or quite smooth to the naked eye, though in specimens of considerable size it is usually very distinctly corrugated, and I have no doubt Heller would have identified his specimens with the *innotatus* had he had Dana's specimens for comparison. There is certainly nothing in Heller's description to distinguish his species from ordinary specimens of *transversus* from Bermuda and Florida.

The following measurements of the carapax in a large number of specimens were made with special reference to determining the difference in proportions between Atlantic and Pacific specimens. In

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order to give the readiest means of comparison, the breadth and the breadth of the front are given first in millimeters and then, in the same divisions of the table, in units of the length of the carapax.

Locality.	Sex.	Length.	Breadth.		Front.	
Bermuda,	yng.	3·6mm	5·0mm	1·39 lgth.	2·6mm	·72 lgth.
"	"	4·0	5·5	1·37	2·8	·70
"	"	5·1	7·0	1·37	3·8	·74
"	♀	5·6	7·8	1·39	4·3	·77
"	"	7·4	10·4	1·40	5·7	·77
"	"	8·2	11·1	1·35	5·8	·71
Paita, Peru,	"	8·5	11·0	1·30	5·9	·69
Acajutla,	"	9·8	13·1	1·34	7·2	·73
Provincetown,	"	10·1	13·2	1·31	7·2	·71
Florida,	"	11·0	15·0	1·36	8·3	·76
"	"	11·4	15·9	1·40	8·8	·77
Brazil,	"	11·5	15·6	1·36	8·6	·75
Acajutla,	"	13·7	18·2	1·33	9·8	·71
Bermuda,	"	14·2	19·0	1·34	10·8	·76
Provincetown,	♂	6·0	7·9	1·30	4·1	·68
Florida,	"	7·0	9·3	1·33	5·0	·71
Provincetown,	"	7·5	10·0	1·33	5·0	·67
"	"	9·0	11·5	1·28	6·3	·70
Bermuda,	"	9·4	12·8	1·36	6·9	·73
Paita, Peru,	"	10·0	12·8	1·28	7·0	·70
Bermuda,	"	10·5	13·7	1·30	7·6	·72
Panama,	"	10·5	13·9	1·32	7·6	·72
Bermuda,	"	10·7	14·1	1·32	7·8	·73
Panama,	"	10·7	14·2	1·32	7·7	·72
Brazil,	"	11·0	14·3	1·30	7·6	·69
Bermuda,	"	12·4	16·4	1·32	9·0	·73
Florida,	"	12·5	16·3	1·30	8·9	·71
Bermuda,	"	13·1	17·2	1·31	9·6	·73
Panama,	"	13·8	17·4	1·26	9·8	·71
Brazil,	"	14·5	18·2	1·25	10·0	·69
Panama,	"	15·2	19·4	1·27	10·6	·70
Brazil,	"	15·3	19·7	1·29	10·6	·69
Bermuda,	♀	9·2	13·0	1·41	7·1	·77

The limits of variation as shown in the table are found in Atlantic specimens. Considering the considerable variation in the proportions of specimens of about the same size, the differences in this respect between the smallest and the largest specimens measured is surprisingly small. The carapax appears to grow proportionally very slightly narrower with increasing size, as I have noticed in a few other species of Grapsoidea, though the reverse is usually the case in nearly all groups of Brachyura, and is what we should naturally expect from the increase, during the early growth of the animal, in the functional importance of the branchiæ.

The measurements given in the last line in the table are of a parasited specimen with one of the branchial regions considerably distorted.

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Nautilograpsus minutus Milne-Edwards.

A large male of this common pelagic species of the Gulf Stream was taken at Woods' Holl, Vineyard Sound, September 11, 1877, by V. N. Edwards. Length of carapax, 15·1^{mm}; greatest breadth, 14·8; breadth of front between bases of eyes, 9·7; breadth between anterior angles, 13·0.

It has been recorded from the English coast by White, Bell, and others; the Medeterranean, Heller; Cape St. Lucas, Stimpson; Indian Ocean, Milne-Edwards.

Pinnotheres ostreum Say is brought north in considerable numbers in oysters from the Chesapeake and, very likely, occasionally survives for a considerable time in oysters planted on the New England coast, but, as far as I can learn, it is never found in native New England oysters or in those which have been planted here for any considerable time. It may, therefore, properly have claim to a place in the list. I have never seen the male on the New England coast. The figure, purporting to represent the male of this species, which I have given in the Report on the Invertebrate Animals of Vineyard Sound, is in reality the figure of the male of *Pinnotheres maculatus.*

Neptunus Sayi Stimpson (ex Gibbes).

Young specimens of this common pelagic species of the Gulf Stream were several times taken in Vineyard Sound during August and September, 1875. None of the specimens observed were more than 12 to 15^{mm} across the carapax. The megalops of this, or of some closely allied species was taken at the same time.

Calappa marmorata Fabricius.

While at Woods' Holl, with the party of the U. S. Fish Commission, in the summer of 1875, I was surprised at the occurrence of a very remarkable megalops evidently an early stage of some species of *Calappa*, though the genus was at the time not known to me to occur, on our coast, north of the Carolinas. Before leaving Woods' Holl, however, Mr. V. N. Edwards informed me that a small crab, unlike any before known to him, had been found the previous season, under stones at low water, on Ram Island near Woods' Holl, by Mr. Benja. Smith. The specimen was brought to me by Mr. Edwards and proved to be a young *Calappa marmorata*, with the carapax 22^{mm} long. The shores of the same island and other similar localities were carefully searched, during September, 1875, without finding additional specimens. In 1878, however, Mr. Edwards forwarded

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another specimen found at Woods' Holl. These two specimens are very nearly of the same size, the last one being very slightly the larger.

These young specimens, though having most of the essential specific characters of the adult *Calappa marmorata*, yet differ so much that they might readily be mistaken for a distinct species when compared with large specimens. The carapax is much narrower than in the adult, the breadth, including the lateral teeth, being only about a fourth greater than the length. The dorsal surface is more convex and more tuberculose, the tubercles being proportionally much larger and more prominent, giving the carapax a much rougher appearance. In fact the carapax resembles that of *C. convexa* of the west coast of America more than it does the adult *marmorata*, except that it has not the crenulated transverse carinæ above the posterior margin and the similar carinæ upon the teeth of the postero-lateral margin.

The *C. convexa* is sufficiently distinguished from the *marmorata* by these crenulated carinæ, though the proportions of the carapax, which are given by Stimpson as distinctive, are wholly inadequate to distinguish it, as the following table of measurements will show. Stimpson evidently compared small specimens of *convexa* with large ones of *marmorata* without making sufficient allowance for the changes in the proportions of the carapax with increasing size.

In the last division of the following table, the breadth of the carapax is given just in front of the three most posterior teeth of the lateral margin, and the breadth given in each of the last two divisions of the table is given first in millimeters and then in the length of the carapax as a unit.

Locality.	Dry, Alc.	Sex.	Length of Carapax.	Breadth of carapax: Including teeth.		Breadth of carapax: In front of teeth.	
C. marmorata.							
Vineyard Sd.,	Dry		22·0mm	27·0mm	1·23 lgth.	—mm	— lgth.
" "	Alc.	♂	22·5	28·4	1·26	25·9	1·15
Bermuda,	"	♀	34·3	46·5	1·35	39·8	1·16
Key West,	"	♀	47·1	66·0	1·40	57·7	1·22
Egmont Key,	"	♀	54·0	78·0	1·44	70·6	1·31
"	"	♂	58·0	84·9	1·46	75·5	1·30
Key West,	Dry	♂	69·0	106·3	1·54	92·0	1·33
Brazil,	Alc.	♂	73·5	111·0	1·59	95·2	1·30
C. convexa.							
La Paz, L. C.,	Dry	♀	54·0	77·2	1·43	70·0	1·30
"	"	♂	77·0	115·6	1·50	101·2	1·31
"	"	♂	86·0	135·0	1·57	119·0	1·38
"	"	♂	99·0	156·0	1·58	140·0	1·40

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The megalops-stage of *C. marmorata* is so peculiar that I insert the following description. The length of the carapax is $3{\cdot}0^{mm}$; the breadth between the antero-lateral angles, 1·8; greatest breadth, 2·1. The lateral margins of the carapax are considerably expanded over the branchial regions, and the dorsal surface is smooth and strongly convex. The front is broad between the bases of the eyes, tapers to an acute-triangular tip, and is very strongly deflexed so that the terminal portion is perpendicular and the tip on a level with the sternum; the terminal portion is also transversely concave in front, so that, when seen from above, it projects only a little in front of the eyes, and shows a slight emargination in the middle. The eyes are large, the diameter being equal to more than half the breadth of the front between their bases, and project considerably beyond the sides of the carapax. The antennæ are very slender and scarcely longer than the breadth of the front between the bases of the eyes. The external maxillipeds have already assumed the form characteristic of the adult Calappidæ. The chelipeds are short and very stout, and in general form approach strongly those of the adult, but are smooth and unarmed with tubercles or spines, except upon the prehensile edges of the pincers. The merus and carpus are stout and swollen; the body of the propodus is stout and broadly expanded above distally, but the upper edge is smooth and rounded; the digital portion is long, slender, bent downward at nearly a right angle to the rest of the propodus, incurved at the very slender tip, and its prehensile edge slightly tubercular. The dactylus is slender and curved like the propodus, so that the tips of the digits cross when the pincers are closed. The prehensile edge of the dactylus in both chelipeds is armed like the digital portion of the propodus, and in one of the chelipeds there is in addition a stout tooth near the base corresponding to the similar tooth at the base of the dactylus in one of the chelipeds of the adult. The ambulatory legs are small and slender, and the dactylus in the posterior pair is armed at the tip with the three long setæ usually characteristic of these appendages in the megalops-stage. The abdomen is small in proportion to the rest of the animal, tapers very slightly distally, and is strongly convex above. The postero-lateral angles of the second to the fifth segment are prolonged downward into obtuse teeth. The sixth segment is very short, being less than half as long as broad. The telson is as brood as the sixth segment, but shorter than broad and with the posterior margin nearly semicircular. The appendages of the second to the fifth segment of the abdomen are of the usual form and

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structure, with large outer and small inner lamellæ. The uropods are small, with a very short base, and a single broad oval lamella which reaches only a little beyond the tip of the telson.

When the megalops is at rest, the abdomen is curved beneath the body, and the chelipeds and ambulatory legs are folded very compactly beneath the carapax.

The specimens examined were all taken at the surface of the water in the evening, August 26 and 27, and September 2, 1875.

Polyonyx macrocheles Stimpson (ex Gibbes).

Mr. Faxon (Bull. Mus. Comp. Zool., Cambridge, v, p. 256, 1879) states that the adult of this species has been detected once, by Alexander Agassiz, under stones on the shore at Newport, R. I. Mr. Faxon also says: on several warm days in August, 1878, the zoeæ of *Porcellana* [*P. macrocheles*] swarmed in the streaks of smooth water on the edge of the tidal currents at the mouth of Narragansett Bay." Alexander Agassiz had previously observed the zoea of this species at Newport (Proc. Boston Soc. Nat. Hist., x, p. 222, 1866). I have never observed any stage of the species in Vineyard Sound or elsewhere on the New England coast. The adult appears to be not uncommon as far north as Beaufort, North Carolina.

Petrolisthes armatus Stimpson (ex Gibbes). A small specimen of this species was found in a bottle with other invertebrata collected, by Prof. Verrill and a party of students, at Stony Creek, on Long Island Sound, near New Haven, in the autumn of 1867 or 1868. At the time it was brought in, I suspected, without any good reason, that a stray specimen had been accidentally taken out on the excursion in one of the bottles and, in this way, got mixed with the specimens collected; but now I have little doubt that it was an erratic specimen from much further south. As far as I am aware, the species is otherwise not known north of Florida and the Bermudas.

Latreutes ensiferus Stimpson (ex M.-Edwards).

Several small specimens of this Gulf Stream species were taken at the surface in Vineyard Sound, during August and September, 1875.

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Penæus Brasiliensis Latreille.

I have never seen specimens of this species from the New England coast, but Professor Verrill tells me that the live specimens of a large *Penæus*, in all probability this species, on exhibition at the New York Aquarium in the autumn of 1877, were said, by the Superintendent of the Aquarium, to have been taken on the Connecticut shore of Long Island Sound. Dr. Stimpson (Annals Lyceum Nat. Hist. New York, x, p. 232, 1871) had already recorded its occurrence in a fresh-water creek near Somers' Point, New Jersey, and in the Croton River, Sing Sing, New York, so that there can be little doubt that it occasionally extends to the shores of Long Island Sound. It is apparently common from North Carolina! to Brazil!; it is reported from the west coast of Africa by Miers (Proc. Zool. Soc. London, 1878, p. 299), and from the west coast of Central America by Kingsley (Proc. Acad. Nat. Sci. Philadelphia, 1879, p. 427, 1880).

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VII. On the Amphipodus genera, Cerapus, Unciola, and Lepidactylis, described by Thomas Say. By S. I. Smith.

The genus Cerapus.

Say's description of the structure and habits of his *Cerapus tubularis*, though incomplete and, in regard to some of the structural details, incorrect, certainly indicates a remarkable amphipod very unlike any of the other species which have been referred to the genus. Though described by Say in 1817 from specimens found "amongst fuci on the sea beach at Egg-Harbor [New Jersey] in considerable numbers," it seems to have remained unknown for more than fifty years. It was searched for in vain at the original locality by Professor Verrill and myself in April, 1871, but in June of the same year I dredged a few specimens in Vineyard Sound. These specimens were unfortunately all females, while Say's description and figures were based on the male only, so that I did not feel entirely certain of their identity with Say's species. In the Report on the Invertebrate Animals of Vineyard Sound, I inserted the species under Say's name, with a query, and, after briefly describing the coloration and habits, remarked that, "in the structure of the caudal appendages, our specimens are quite different from the species usually referred to *Cerapus*, but I have not thought best to make any changes in the nomenclature of any of the species until the discovery of the male shall make it certain whether our specimens belong to the species described by Say." In 1874 a considerable number of specimens were taken, in the towing-net, on the evenings of July 17th and 21st, in Noank Harbor, Connecticut, by Professor Verrill, though I was unable to discover a single specimen in the same locality a few days later. Among these specimens collected by Professor Verrill, there are fortunately an abundance of males which agree with Say's description and figure and leave no doubt whatever that the species is the same as Say's. The following description and figures are taken from these specimens.

The general form of the animal is like *Ericthonius difformis* but much more slender, the body being broad, depressed, very slender, and, in the male, tapering slightly and continuously from the head to the telson. The head is shorter than the first and second segments

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of the peræon taken together, and in the male about a fourth as long as the entire peræon, but shorter in the female on account of the elongation of the middle segments of the peræon in that sex. There is a slight dorsal carina on the anterior part of the head, and the anterior margin projects in a slight rostrum in the middle, and at the sides in slightly prominent lobes in which the small black eyes are situated. In the lateral margin below the eye there is a large emargination over the base of the antenna as in the allied genera. In the male the seven segments of the peræon are about equal in length, but in the female the first and last are shorter and the fourth and fifth longer than any of the others, the fourth and fifth being each about twice as long as either the first or last. The epimera are all long and low: those of the first four segments are small and inconspicuous; the fifth (Plate IIa, figure 4, *a*) is much larger and projects in a conspicuous, broad and rounded lobe in front of the articulation of the limb; the sixth and seventh are similar in form but successively much smaller.

In the male, the antennulæ and antennæ are approximately equal in length. The antennulæ are slightly more than half as long as the rest of the animal: the first segment of the peduncle is very stout, but compressed laterally and the lower edge raised into a carina which is prominent near the base, where the dorso-ventral diameter is more than half the length of the segment; the second and third segments are sub-equal in length and diameter and each is slightly longer than the first; the flagellum is about as long as the third segment of the peduncle and is composed of three segments, of which the first is rather longer than the second and third taken together. All the segments are furnished with long setiform hairs below and with much fewer and shorter ones above. The antennæ are a very little shorter than the antennulæ and scarcely, if at all, stouter; the third segment of the peduncle is no longer than broad; the fourth is about as long as the second of the antennula; the fifth is a little longer than the fourth; the flagellum is scarcely as long as the fifth segment of the peduncle and is composed of three segments, of which the first is considerably longer than the second and third together. All the segments beyond the third are furnished with setiform hairs very much as in the antennulæ. In the female, the antennulæ and antennæ are proportionally more than a fourth shorter than in the male, the difference resulting mostly from the shortening of the flagella and the distal segments of the peduncles.

The mandibles are nearly as in the *Ericthonius difformis*. The

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palpus is slender, and the third segment is about four times as long as broad and a little shorter, but scarcely longer than, the second. The maxillæ are nearly as in *E. difformis:* in the first pair the inner lobe is small and tipped with one or two setæ only;* in the second pair the two lobes are nearly equally developed. The maxillipeds are essentially as in *E. difformis.*

The gnathopods have essentially the same structure as in *Ericthonius.* The first (Plate IIa, figure 1) are alike in both sexes: the merus is narrow, armed distally with numerous setæ, and its anterior margin articulates with the proximal half of one edge of the triangular carpus; the carpus is nearly as broad as long, the posterior edge is armed with numerous setæ and projects considerably beyond the broad articulation with the propodus; the propodus is as long, but somewhat narrower than the carpus, approximately oval in outline, and thickly armed along the convex posterior edge with setæ and small spines; the dactylus is stout, slightly curved and apparently not capable of complete adduction to the edge of the propodus. The second gnathopods in the female are like the first except that the dactylus is, perhaps, slightly longer. In the adult male, however, the second gnathopods (figure 2) are enormously developed, as in the males of *Ericthonius.* The basis is much stouter but scarcely longer than in the first pair. The ischium and merus are of nearly the same form and size as in the first pair, though the merus is slightly longer. The carpus is more than twice as long and broad as in the first pair: the posterior edge is convex in outline, armed with a few small fasciculi of setæ, and projects distally in a long and stout spiniform tooth; the distal end is very broad, the anterior half only is occupied by the articulation with the propodus, and the edge of the posterior part projects in a prominent obtuse tooth near the base of the propodus, while between this tooth and the great tooth of the posterior angle there is a deep rounded emargination, the border of which is armed with one or two small spines and numerous stout setæ. The propodus is nearly as long as the carpus, about twice as long as broad, slightly curved, and the posterior, or prehen-

* Boeck, Christiania Videnskabs-Selskabs Forhandlinger, 1870, p. 246 (166), and Scandinav. og Arktiske Amphipoder, p. 598, describes the inner lobe of the first maxilla in the Podocerinæ, as destitute of setæ ("lamina interiore parvula, ovali, setis destituta"). This is not characteristic of all the species of the group, however, and will not serve as a character for distinguishing *Cerapus* (as here restricted) from the Podocerinæ; for Boeck himself, in the latter of the works above quoted, plate 28, figure 3f, figures setæ upon this lobe of the first maxilla of *Cerapus* [*Ericthonius*] *abditus*, and they are certainly present in *Ericthonius difformis.*

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sile, edge irregular in outline, minutely dentate and armed with setæ. The dactylus is a little shorter than the propodus, stout, but much narrower at base than the distal end of the propodus, regularly curved, and the prehensile edge minutely serrate and armed with a very few minute setæ.

The first and second peræopods are nearly alike and differ only slightly in the different sexes, and, as in the allied genera, are modified for tube-building, the bases being very large and their interiors almost wholly occupied with the cement-glands. In the male the basis in the first pair (Plate IIa, figure 3) is articulated at the anterior angle of the broad and truncated proximal end, while the posterior angle of the same end is prominent and angular. In the second pair the basis is of similar form, but considerably broader in the middle, and the posterior angle of the distal end does not project and is broadly rounded. In both pairs the ischium is a little longer than broad; the merus is of about the same length but broader than long and with both edges convex in outline; the carpus is scarcely as long as the merus and nearly square; the propodus is narrower but nearly a half longer than the carpus; the dactylus is shorter than the propodus and only slightly curved. In the female the basis in the first pair is proportionally broader than in the male, and the posterior angle of the distal end projects in a rounded lobe separated from the rest of the posterior edge by a marked emargination. In the second pair the basis is only somewhat broader and more oval in outline than in the male.

The third, fourth and fifth peræopods are alike in the two sexes and are reversed and turned upward above the back, with the hooked dactyli directed upward and outward. The third pair (Plate IIa, figure 4, *a*, *b*) are very small, being only a little longer than the basis in the third pair, and in the female scarcely longer than the segment to which they belong. The basis makes more than a third of the entire length, is as long as the ischium and merus together and nearly as broad as long. The ischium is very short and broader than long. The body of the merus is ovoid with the proximal end truncate, and has a long, narrow and spatulate process projecting from the posterior edge near the articulation with the ischium and tipped with one short and three long, plumose setæ. The carpus is articulated upon the middle of the posterior side of the merus and near the base of the spatulate process, to which it approaches in size and form; it is tipped with a single seta and the terminal portion is roughened with very minute scale-like teeth pointing distally. The

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propodus is about as long and nearly as wide as the body of the merus, is articulated close to the base of the carpus, and armed with a single minute seta on each edge near the distal end. The dactylus is very short and stout, with the solid chitinous tip turned sharply backward and upward in an acute hook and armed with a sharp tooth on the outside at the point of curvature. The fourth and fifth peræopods are slender and of nearly the same form as in *Ericthonius difformis.* The fourth are about once and a half as long as the third, the basis is much broader than the other segments and about a fourth longer than broad; the ischium is small and nearly square; the merus is twice as long as the ischium and very slightly broader; the carpus is not quite as long as the merus but of about the same width; the propodus is about as long as the merus but much narrower; the dactylus is stout and hooked very much as in the third pair; the distal angles of the merus and carpus are armed with long setæ which are longest and most numerous on the posterior distal angle of the carpus, which projects in a small lobe. The fifth pair are a little longer and proportionately a little more slender than the fourth pair, but do not differ in other respects.

There are, apparently, only three pairs of branchial lamellæ, a single lamella being borne upon each coxa of the first, second and third pairs of peræopods.

Of ovigerous lamellæ there are also only three pairs and these are borne upon the coxæ of the second pair of gnathopods and of the first and second peræopods.

The first segment of the pleon is slightly shorter than the seventh segment of the peræon, and the second and third are successively still shorter. The three pairs of pleopods (Plate IIa, figure 5, *a*, *b*, *c*), in marked contrast to those of most Amphipoda, differ remarkably among themselves in the relative proportions of the outer and inner lamellæ, and diminish greatly in size from before backward. This is undoubtedly an adaptation to the position of the animal in its closely fitting tube, with the posterior segments of the pleon bent closely beneath the anterior segments, leaving very little room for the action of the pleopods. In the first pair of pleopods (figure 5, *a*) the base is about as long as the base of the first uropod and more than half as broad as long; the inner lamella is a half longer than the base, narrow, and both margins are furnished with long, plumose setæ; the outer lamella is very little shorter than the inner, narrow distally, but broadly expanding proximally till the breadth is equal to half the length, and then suddenly narrowing to the base; the distal two-

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thirds of the inner and the whole of the outer margin are furnished with plumose setæ which are longest at the tip of the lamella and very thickly crowded on the proximal part of the outer margin. The second pair (figure 5, *b*) are very much smaller: the base is little more than half as long as in the first pair; the outer lamella is less than two-thirds as long as in the first pair, ovate, half as broad as long, and both margins are furnished with plumose setæ; the inner lamella is rudimentary, about half as long as the outer and furnished with only two small setæ at the tip and two or three more on the distal part of the inner margin. The third pair (figure 5, *c*) are still smaller: the outer ramus is about two-thirds as long as in the second pair, ovate, and margined with a few plumose setæ distally; the inner is about half as long as the outer, and has only two or three setæ near the tip.

The fourth, fifth and sixth segments of the pleon are much shorter than the first three. As seen from above (Plate IIa, figure 6), the fourth segment is nearly as long as broad; the fifth is a little narrower and not half as long as the fourth, being between three and four times as broad as long; the sixth segment is slightly narrower than the fifth, nearly twice as broad as long, and appears to be partially consolidated with the fifth.

The first pair of uropods (Plate IIa, figure 6) are well developed: the bases are stout and reach to the base of the telson; the outer ramus is narrow, shorter than the peduncle, minutely serrate and sparcely setigerous along the outer edge, and tipped with a slender spine; the inner ramus is smaller than the outer, about three-fourths as long, and tipped with a spine, but the edges unarmed. The second uropods are small, slender and uniramous: the base is about as stout as the outer ramus of the first pair, does not quite reach the base of the telson, and is unarmed; the terminal segment is very short, stout and hooked, and very similar in structure to the terminal segments of the third uropods and the dactyli of the third, fourth and fifth peræopods. The third uropods are like the second, but the bases are much stouter and each is armed with a short seta on the inner edge. The telson is about twice as broad as long and bilobed at the extremity, the lobes being broadly rounded and armed with two or three transverse rows of hooked spines above.

In life, a large portion of the animal appears almost black from the crowding together of numerous specks of very dark purplish pigment. The first segment of the peduncle of the antennula is nearly all colored in this way and there is a broad band of the same character

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at the distal end of each of the other segments of the peduncle. The proximal segments of the antenna are also dark-colored, and there is a broad band of color at the distal end of the fourth segment. Other parts of the peduncles of the antennulæ and antennæ are semi-translucent, and so are the flagella. The head and the whole anterior and middle portions of the body of the animal and the epimera are more or less colored in the same way, as are the gnathopods and the bases of the first and second peræopods; but the distal portions of these peræopods, the whole of the third, fourth and fifth pairs, and the caudal appendages are semi-translucent and nearly or quite devoid of pigment.

Two adult specimens give the following measurements:

	Male.	Female.
Length from front of head to tip of telson,	4·2^{mm}	4·4^{mm}
Length from front of head to second segment of pleon in natural position of rest,	3·5	3 6
Length of attennula,	2·4	1·7
Length of tube,	5·4	5·5
Diameter of tube at middle,	·80	·85

In the largest specimens seen the tubes are about 7^{mm} long and 0·9^{mm} in diameter, while in the smallest they are only 2^{mm} long and 0·45^{mm} in diameter.

The tubes of all the specimens seen are black externally, thin, and very regularly cylindrical, except that they are usually slightly enlarged at one or at both ends. Within they are smoothly lined with a layer of cement, while externally they are covered, to a great extent at least, with minute, elongated pellets, apparently the excrement of the animal,* arranged transversely to the tube and closely

* In several allied species of Amphipoda, the excrement enters largely into the composition of the tube. In 1874 I watched carefully the process of constructing the tubes in several species of Amphipoda. *Microdeutopus grandimanus* (*M. minax* Smith) was a particularly favorable subject for observation. When captured and placed in a small zoophyte trough with small, branching algæ, the individuals almost always proceeded at once to construct a tube, and could very readily be observed under the microscope. A few slender branches of the alga were pulled toward each other by means of the antennæ and gnathopods, and fastened by threads of cement spun from branch to branch by the first and second pairs of peræopods. The branches were not usually at once brought near enough together to serve as the frame-work of the tube, but were gradually brought together by pulling them in and fastening them a little at a time, until they were brought into the proper position, where they were firmly held by means of a thick net-work of fine threads of cement spun from branch to branch. After the tube had assumed very nearly its completed form, it was still usually nothing but a transparent net-work of cement threads woven among the

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crowded together; there appear, however, to be other materials, probably minute fragments of algæ, hydroids, etc., mixed with the pellets. The tube is apparently never attached, but is carried about by the animal, very much after the manner of the larvæ of some species of Phryganeidæ, as described by Say. It is very difficult to force the living animal from its tube, and it probably never quits the tube voluntarily. The ordinary position of the animal when at rest is with the head only protruding from one end of the tube, the antennulæ stretched out in front and diverging at about a right angle, while the antennæ are held out each side at right angles to the tube. The antennulæ and antennæ are the only appendages which are ordinarily used in locomotion, and by means of these alone the animal appears to move about with its tube with the same ease and rapidity as the species of *Podocerus* and *Corophium* do when unencumbered.

As noticed by Say, the animal turns about within its tube very readily, and uses either end of it indifferently as the front. If the tube catch in any way while the animal is moving about, or if it be held fast by forceps, the head is protruded first from one and then from the other end of the tube in quick succession, and the antennulæ and antennæ are thrust along the outside of the tube to discover the

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branches of the alga, though occasionally a branch of the alga was bitten off and added to the frame-work; but very soon the animal began to work bits of excrement and bits of alga into the net. In this case the pellets of excrement, as passed, were taken in the gnathopods and maxillipeds, and apparently also by the maxillæ and mandibles, and broken into minute fragments and worked through the web, upon the outside of which they seemed to adhere partially by the viscosity of the cement threads and partially by the tangle of threads over them. Excrement and bits of alga were thus worked into the wall of the tube until the whole animal was protected from view, while, during the whole process, the spinning of cement over the inside of the tube was kept up. When spinning the cement threads within the tube, the animal was held in place on the ventral side by the second pair of gnathopods and the caudal appendages, the latter being curved beneath the anterior portion of the pleon, and on the dorsal side by the third, fourth and fifth pairs of peræopods extended and turned up over the back, with the dactyli turned outward into the web. The spinning was done wholly with the first and second peræopods, the tips of which were touched from point to point over the inside of the skeleton tube in a way that recalled strongly the movements of the hands in playing upon a piano. The cement adhered at once at the points touched and spun out between them in uniform delicate threads. The threads seemed to harden very quickly after they were spun and did not seem, even from the first, to adhere to the animal itself. In one case in which the entire construction of the tube was watched, the work was apparently very nearly or quite completed in little more than half an hour. In a species of *Amphithoë*, in which the construction of the tube was watched, the process was very similar, though less cement and more foreign material seemed to enter into the structure.

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obstruction. While thus held fast, fully half the body is often protruded from the tube. In turning about within the tube, the terminal segments of the pleon are thrust forward beneath the anterior segments and the peræon pulled back over them, and then the peræon is folded back upon itself in the same way, but the antennulæ, antennæ and head are never first bent beneath the peræon. The structure of the telson, second and third uropods, and of a part of the peræopods is well adapted to the performing of this evolution; the hooks of the second and third uropods and of the telson holding the tail fast to the side of the tube, while the third, fourth and fifth peræopods, holding on with their opposing hooked tips, pull the posterior part of the peræon back over the pleon, and then the first gnathopods help to complete the evolution.

The tubes are usually kept quite free from foreign growths, but among the specimens taken in 1874, there are two individuals in tubes to which are attached the egg capsules of *Tritia trivittata.*

I was not able to discover how the diameter of the tube is enlarged to accommodate the growing animal, but it is probably accomplished by building on a larger portion at one end and pulling to pieces the other end until the whole tube is reconstructed.

As the preceding description shows, this type species of the genus *Cerapus* is generically distinct from the species referred to that genus in recent works, and for which Milne-Edwards' genus *Ericthonius* may properly be retained as explained further on. Say's species is apparently not congeneric with any described species and the genus cannot properly be placed in any of the numerous subfamilies defined by Boeck, though it is probably most nearly allied to his Podocerinæ. I therefore propose the following new subfamily to be placed next Podocerinæ.

Cerapinæ.

The single known genus differs from the Podercerinæ and allied groups in the following characters. There are only three pairs of branchial lamellæ, which are borne on the third, fourth and fifth segments of the peræon, and only three pairs of ovigerous lamellæ, which are borne on the second, third and fourth segments. The second and third pleopods are much smaller than the first, and their inner lamellæ are rudimentary or very small. The second and third uropods are uniramus and nearly alike, the distal segment in each being short and terminating in a hooked point.

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The only known species inhabits unattached, portable tubes, and, as in many allied genera, has large cement glands in the bases of the first and second peræopods.

Cerapus Say.

Say, Jour. Acad. Nat. Sci. Philadelphia, i, p. 49, 1817 (*tubularis* the only species.)
Desmarest, Dictionnaire Sci. nat., xxviii, p. 358, 1823 (=Say).
Latreille, in Cuvier, Règne animal, iv, p. 122, 1829 (=Say.)
Milne-Edwards, Ann. Sci. nat., Paris, xx, p. 383, 1830 (=Say); Hist. nat. Crust., iii, p. 60, 1840 (in part.)
Dana, Amer. Jour. Sci., II, viii, p. 139, 1849, and xiv, p. 309, 1852 (=Say); Crust. United States Expl. Expd., p. 832, 1853 (=Say).
Bate, Catalogue Amphip. Crust. British Mus., p. 262, 1860 (in part).

Cerapus tubularis Say.

Say, Jour. Acad. Nat. Sci. Philadelphia, i, p. 50, pl. 4, figs. 7–9, 1817.
Desmarest, Dictionnaire Sci. nat., xxviii, p. 358, pl. 46, figs. 2a–2e (after Say), 1823 (=Say); Consid. génér. Crust., p. 271, pl. 46, figs. 2a–2e (=last), 1825 (=Say).
Audouin, Précis d'Entomologie, pl. 28, figs. 5–7 (after Say), 1828 (=Say).
Guérin, Iconographie du Règne animal, Crust., pl. 27, fig. 4, (after Say).
Milne-Edwards, Règne animal de Cuvier, 3me édit, pl. 61, fig. 5–5c (after Say), 1837; Hist. nat. Crust., iii, p. 60, pl. 29, fig. 15 (after Say), 1840 (=Say).
Bate, Catalogue Amphip. Crust. British. Mus., p. 262, pl. 45, fig. 1 (after Say), 1862 (=Say).
Smith, Inverteb. Animals Vineyard Sound, Report U. S. Comm. Fish and Fisheries, part i, p. 565 (271), 1874.

Plate IIa.

Great Egg-Harbor, New Jersey (Say). Vineyard Sound, among masses of a large compound ascidian (*Amouroucium pellucidum*), in 8 to 10 fathoms, off Nobska Point, June 27, 1871; several females, some of them carrying eggs. Vineyard Sound, 1875; one specimen. Taken in the towing net, among eel-grass (Zostera), in Noank Harbor, Connecticut, July 17 and 21, 1874 (A. E. Verrill); males, females, and young, many of the females carrying eggs.

The genus *Cerapus* being restricted to the type species and removed from the Podocerinæ, as above, Milne-Edwards' *Ericthonius* appears to be the proper name to restore and retain for the group of species referred to *Cerapus* by recent authors.

Ericthonius.

Ericthonius M.-Edwards, Ann. Sci. nat., Paris, xx, p. 382, 1830 (*difformis* the only species); Hist. nat. Crust., iii, p. 59, 1840 (*difformis* only).
Dana. Amer. Jour. Sci., II, viii, p. 138, 1849, and xiv, p. 309, 1852; U. S. Expl. Expd., Crust., p. 832, 1853 (*Erichthonius*).

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Ericthonius—(continued).

Bate, Report British Assoc. Adv. Sci., 1855, p. 59, 1856 (*Erichthonius*); Ann. Mag. Nat. Hist., II, xix, 149 (17), 1857.

White, Popular Hist. British Crust., p. 196, 1857.

Liljeborg, Œfversigt Vetenskaps-Akad. Förhandlingar, Stockholm, 1855, p. 131.

Costa, Amfipodi Napoli, Mem. Accad. Sci. Napoli, i, pp. 175, 227, 1856.

Bruzelius, Scandinav. Amphipoda Gammaridea (Vetenskaps-Akad. Handlingar, Stockholm, iii), p. 17, 1859 (*Erichtonius*).

Cerapodina M.-Edwards, Hist. nat. Crust., iii, p. 62, 1840 (*abdita* (= *Cerapus abditus* Templeton) the only species).

Dana, Amer. Jour. Sci., II, viii, p. 139, 1849, and xiv, p. 309, 1852; U. S. Expl. Expd., Crust., p. 832, 1853.

Podocerus Kröyer, Naturh. Tidsskrift, iv, p. 163, 1840 (in part).

Dercothoe Dana, Amer. Jour. Sci., II, xiv, p. 313, 1852; U. S. Expl. Expd., Crust., p. 911, 1853.

Pyctilus Dana, Amer. Jour. Sci., II, xiv, p. 313, 1852; U. S. Expl. Expd., Crust., p. 911, 1853.

Cerapus Bate, Catalogue Amphip. Crust. British Museum, p. 262, 1860 (in part).

Bate and Westwood, British Sessile-eyed Crust., i, p. 452, 1863.

Czerniavski, Materialia ad Zoographiam Ponticam comparatam, p. 97, 1868.

Boeck, Christiania Videnskabs-Selskabs Forhandlinger, 1870, p. 250 (170); Scandinav. og arktiske Amphip., p. 611, 1876.

The synonymy of *E. difformis*, the type of the genus, is apparently still in confusion, and as it is a common species upon the coast of northern New England, I subjoin the following account of it:

Ericthonius difformis Milne-Edwards.

Ericthonius difformis M.-Edwards, Ann. Sci. nat., Paris, xx, p. 382, 1830; Hist nat. Crust., iii, p. 60, pl. 29, figs. 12, 13, 1840.

Liljeborg, Œfversigt Vetenskaps-Akad. Förhandlingar, Stockholm, 1855, p. 129.

? Costa, Amfipodi Napoli, Mem. Accad. Sci. Napoli, i, p. 228, 1856.

Bruzilius, Scandinav. Amphipoda Gammaridea (Vetenskaps-Akad. Handlingar, Stockholm, iii), p. 17, 1859 (*Ericthonius*).

Goes, Œfversigt Vetenskaps-Akad. Förhandlingar, Stockholm, 1865, p. 552 (*Ericthonius*).

Podocerus Leachii Kröyer, Naturh. Tidsskrift, iv, p. 163, 1840 ("Femina hujus speciei forma pedum secundi paris Podocerus est, mas vero Erichthonius").

Cerapus rubricornis Stimpson, Inverteb. Grand Manan, p. 46, pl. 3, fig. 33 (caudal appendages), 1853.

Bate, Catalogue Amphip. Crust., British Museum, p. 265, pl. 45, fig. 4 (young ♂), 1862 (Bay of Fundy).

Packard, Memoirs Boston Soc. Nat. Hist., i, p. 297, 1867 (*C. rubiformis*).

Smith, Inverteb. Animals Vineyard Sound, Report U. S. Comm. Fish and Fisheries, part i, p. 565 (271), pl. 4, fig. 18, 1874.

Smith and Harger, Trans. Conn. Acad., iii, pp. 5, 19, 1874.

Podocerus punctatus Bate, Ann. Mag. Nat. Hist., II, xix, p. 148 (17), 1857 ("*P. punctatus* (Edwards, MS. Brit. Mus.)").

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Ericthonius difformis Milne-Edwards—(continued).

Dercothoë (*Cerapus*) *punctatus* Bate, Catalogue Amphip. Crust. British Museum, p. 260, p. 44, fig. 10, 1862 (♀).

Dercothoë (*Cerapus* ♀) *punctatus* Bate and Westwood, British Sessile-eyed Crust., i, p. 461 (fig.), 1863 (♀). ["Milne-Edwards, MS. in Brit. Mus." is quoted as authority for the name as here used.]

? Czerniavski, Materialia ad Zoographiam Ponticam comparatam; p. 96, 1868.

Cerapus difformis Bate, Catalogue Amphip. Crust. British Museum, p. 265, pl. 45, fig. 5, 1862.

Bate and Westwood, op. cit., p. 457 (figs.), 1863.

Norman, Report British Assoc. Adv. Sci., 1868, p. 283, 1869.

Metzger, Jahresbericht der Comm. zur wissensch. Untersuchung der deutschen Meere für 1872–3, Nordsee, p. 278, 1875.

Boeck, Christiania Videnskabs-Selskabs Forhandlinger, 1870, p. 250 (170); Scandinav. og arktiske Amphip., p. 615, 1876.

Meinert, Naturh. Tidsskrift, III, xi, p. 157, 1877.

Cerapus Leachii Bate, Catalogue Amphip. Crust. British Museum, p. 268, 1862 (= Kröyer).

Cerapus Hunteri Bate, Catalogue Amphip. Crust. British Museum, p. 264, pl. 45, fig. 3, 1862 (habitat unknown).

Boeck, Christiania Videnskabs-Selskabs Forhandlinger, 1870, p. 252 (172), 1871 (Scandinavia); Scandinav. og arktiske Amphip., p. 618, pl. 28, fig. 5, 1876.

Vineyard Sound!, 1871, not common. Off Cape Cod!, 29 fathoms, 1879. Stellwagen's Bank! and Jeffrey's Ledge!, 1873. George's Banks!, 50 fathoms, sand and shells, and 45 fathoms, coarse sand, 1872. Casco Bay!, 9 to 34 fathoms, 1873; common. Bay of Fundy!, low-water mark to 50 fathoms, abundant, especially on hard bottoms, 1868, 1870, 1872, 1876; and, in 1872, off Head Harbor, 77 fathoms, mud and stones; and off White Head, Grand Menan, 97 to 105 fathoms, gravel and stones. Halifax, Nova Scotia!, 16 fathoms, rocky, 1877. Straits of Belle Isle, Labrador, (Packard). Spitzbergen, (Goës). Scandinavia, (Kröyer, Liljeborg, Boeck, et al.) British Isles!, (Norman). Brittany, France, (M.-Edwards). It is reported from the Mediterranean by Costa, and from the Black Sea by Czerniavski, (as Bate's *Dercothoe* (*Cerapus* ♀) *punctatus*), but these identifications are, perhaps, erroneous.

The form of the second pair of gnathopods of the adult male varies very much in different specimens. When farthest differentiated from the form characteristic of the female and young, the carpus is triangular, and the spine of the inferior margin very long and separated from the propodal articulation by a deep, triangular sinus, while the propodus is slender, nearly cylindrical, and without prominences on the inferior edge. This is the form which I have figured in the work above referred to, and is apparently very near that figured by Milne-

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Edwards as the original *E. difformis.* It is probably the form characteristic of the old males, though the individuals are often smaller than those with the second gnathopods in the less differentiated condition, in which the body of the carpus is more nearly quadrangular in outline, the spine arising near the propodal articulation, and the propodus shorter and stouter than in the other form and with two conspicuous prominences on the inferior edge. This is the form described as *Cerapus Hunteri*, by Bate and by Boeck. In the large series of specimens before me, there is every gradation between these two forms, and between the latter and the female. In one case, an individual had one of the second pair of gnathopods in the most differentiated form, while the other was somewhat smaller and in the less differentiated form—evidently having been lost and reproduced. The only European specimens which I have examined were received from the Rev. Mr. Norman, and are from the Northumberland coast, England, and among these all the adult males agree better with the *Hunteri* than with the more differentiated form.

The habit of the animal and the character of the tubes are well described by Stimpson. In life, the proximal parts of the flagella of the antennulæ and antennæ are bright red, and the color usually remains for some time in specimens preserved in alcohol.

The genus Unciola.

Unciola Say.

Unciola Say, Jour. Acad. Nat. Sci. Philadelphia, i, p. 388, 1818 (*irrorata* the only sp.)

Milne-Edwards, Ann. Sci. nat., Paris, xx, p. 383, 1830 (=Say).

Dana, Amer. Jour. Sci., II, viii, p. 139, 1849, and xiv, p. 309 (>*Glauconome*); U. S. Expl. Expd. Crust., p. 832, 1853.

Bate, Catalogue Amphip. Crust. British Museum, p. 279, 1862 (>*Glauconome*).

Glauconome Kröyer, Naturh. Tidsskrift, II, i, pp. 491–501 (*leucopis* the only sp.)

Boeck, Christiania Videnskabs-Selskabs Forhandlinger, 1870, p. 259 (179); Scandinaviske og arktiske Amphip., p. 636, 1876.

"*Cyrthopium* Danielssen, Beretning om en zoologisk Reise, 1857, Nyt Magaz. Naturvidensk., xi, p. 8, 1861," (Boeck).

Kröyer's name *Glauconome* is not only more recent than Say's *Unciola*, but it had been used at least twice before Kröyer employed it, once in mollusks and once in polyps.

Unciola irrorata Say.

Unciola irrorata Say, Jour. Acad. Nat. Sci. Philadelphia, i, p. 389, 1818.

Milne-Edwards, Hist. nat. Crust., iii, p. 69, 1840 (=Say).

Stimpson, Marine Invertebrata of Grand Manan, p. 45, 1853.

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Unciola irrorata Say—(continued).

Bate, Catalogue Amphip. Crust. British Mus., p. 279, 1862 (description compiled from Say).

Verrill, Inverteb. Animals Vineyard Sound, Report U. S. Comm. Fish and Fisheries, part i, p, 340 (46), 1874.

Smith, in Verrill, op. cit., p. 567 (273), pl. 4. fig. 19, 1874.

Glauconome leucopis Kröyer, Naturh. Tidsskrift, II, i, p. 491, pl. 7, fig. 2, 1845; in Gaimard, Voyage en Scandinavie, Crust., pl. 19, fig. 1, 1849.

Goes, Œfversigt Vetenskaps-Akad. Förhandlingar, Stockholm, 1865, p. 533.

Boeck, Christiania Videnskabs-Selskabs Forhandlinger, 1870, p. 259 (179); Scandinaviske og arktiske Amphip., p. 636, 1876.

G. O. Sars, Archiv Mathem. Naturvid., Kristiania, ii, p. 360, 1876.

Norman, Proc. Royal Soc. London, xxv, p. 208, 1876.

Unciola leucopes [*-is*] Bate, Catalogue Amphip. Crust. British Mus., p. 279, pl. 47, fig. 3, 1862 (description and fig. after Kröyer).

" *Cyrthopium Darwini* Danielssen, Beretning om en Zoologisk Reise, p. 8," (Boeck.)

This is one of the most abundant of all New England Amphipoda, being found in greater or less abundance in a very large proportion of the dredgings from the shallowest water down, at least, to 400 fathoms, and from all kinds of bottom, though in less abundance in mud than among sand and shells. I have collected it at Great Egg Harbor, New Jersey, where Say's original specimens were obtained, and at various points along the New England coast from Connecticut to the Bay of Fundy. It was dredged by Mr. Harger and myself while on board the Bache in 1872, on St. George's Banks, and in 430 fathoms east of these Banks, north latitude 41° 25′, west longitude 65°, 42·3′. It was obtained in abundance in and off Halifax Harbor, Nova Scotia, by the U. S. Fish Commission, in 1877; and I have examined specimens dredged in the Gulf of St. Lawrence by Mr. Whiteaves, and on the coast of Labrador by Prof. Packard. Kröyer's specimens of *Glauconome leucopis* were from Greenland, and Norman reports it, under the same name, as taken in 100 fathoms in Davis Strait by the Valorous Expedition. Boeck records the same species from Spitzbergen and Norway, and G. O. Sars reports numerous specimens, obtained by the Norwegian Expedition in 1876, from 412, 417, and 520 fathoms, in the area of cold water off the west coast of Norway.

In life, the body of the animal above is usually bright red more or less mottled, especially upon the sides, with white; the outer surfaces of the larger gnathopods are broadly marked with bright red, and the antennulæ and antennæ are annulated with the same color. In some individuals, especially from muddy bottoms, the red is nearly all wanting; in others the red appears, to the naked eye, to

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be uniformily diffused, giving the animal a pale red tint; in still others the red is largely replaced by brown. The animal apparently does not construct tubes for itself, though often found in the tubes of other Amphipoda and in the tubes of Annelida. In the Bay of Fundy I have found it abundantly in small holes in sandy mud near low-water mark.

The species described by Say and that described by Kröyer are not only congeneric, as suggested by Dana, but apparently specifically identical. Specimens from the coast of New England agree in every particular with Boeck's descriptions and with Kröyer's descriptions and figures, except in the details referred to beyond. Say describes the second gnathopods as "adactyle" and the third pair of uropods as simple, depressed and concealed by the others, and he failed to distinguish the very short sixth segment of the pleon from the telson, describing the two together as the terminal segment of the "tail." These are very natural errors, considering the minuteness of the appendages. In all other respects Say's description is correct. Kröyer, on the other hand, incorrectly describes and figures the third uropods as bi-ramus, mistaking the terminal lobe on the inner side of the base for a second ramus. Boeck, who had access to Kröyer's types, states that Kröyer's figure of the third uropod is incorrect but does not mention the corresponding error in the description of the species. White (Catalogue Crust. British Museum, p. 90, 1847) records, as in the British Museum, specimens of *Unciola irrorata* received from Say; Bate, however, (Catalogue Amphip. Crust. British Museum, p. 279, 1862) states that he has seen no specimens, but refers Kröyer's species to *Unciola*, although he in part misquotes and in part misunderstands Say's generic description. Bate appears to have drawn his description of Kröyer's species from the original figure and generic diagnosis and not from the very full description of the species, for he says that no reference to the telson is made either by Say or by Kröyer and that it is not impossible that the genus is synonymous with "*Microdentopus*," although the telson is described by Kröyer, in the description just referred to, and is figured in the Voyage en Scandinavie.

The genus Lepidactylis.

It has long been known to me that Say's genus *Lepidactylis* was congeneric with *Sulcator* of Bate, as suggested by Dana in 1852; but Bate's description and figures not appearing fully to warrant the assumption of the identity of the European and American species,

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and not at the time having access to the figures of Slabber's *Oniscus arenarius*, upon which Latreille's genus *Pterygocera* was based, I, in 1874, recorded, in the Report on the Invertebrate Animals of Vineyard Sound, Say's species under his generic and specific names and made no attempt to straighten the generic or specific synonymy. A richly illustrated memoir, by Carl Bovallius, on the European *Sulcator* (*Pterygocera*), and British specimens received from the Rev. A. M. Norman, have recently afforded an opportunity for comparing the American and European forms. On comparing New England specimens of Say's species with Bovallius' numerous and excellent figures, and with the British specimens, I can find no characters by which the American can be distinguished from the European form. Say's generic name should therefore be retained in place of Latreille's *Pterygocera*, though Slabber's specific name (fortunately a more appropriate designation) takes precedence of Say's *dytiscus*. The following synonymy of the genus and species sufficiently explains these changes in nomenclature. Except under *Lepidactylis*, I give only the earliest reference under each name, as this part of the synonymy is pretty fully given by Bovallius.

Pterygocerinæ Bovallius.

Lepidactylis Say.

Oniscus Slabber, Natuurkundige Verlustigingen, p. 92, pl. 11, figs. 3, 4, 1778.

Lepidactylis Say, Jour. Acad. Nat. Sci. Philadelphia, i, p. 379, 1818 (*dytiscus* the only species).

Desmarest, Dictionnaire Sci. nat, xxviii, p. 358, 1823 (=Say); Consid. génér. Crust., p. 272, 1825 (=Say).

M.-Edwards, Ann. Sci. nat., Paris, xx, p. 397, 1830 (=Say).

Dana, Amer. Jour. Sci., II, viii, p. 138, 1849, and xiv, p. 313, 1852 (>*Bellia* Bate); U. S. Expl. Expd., Crust., p. 912, 1853.

Bate, Catalogue Amphip. Crust. British Museum, p. 111, 1862 (=Say).

Pterygocera Latreille, Encyclopédie méthodique, x, pp. 121, 236, 1825 (teste Bovallius); in Cuvier, Règne animal, 2e édit., iv, p. 124, 1829.

Bellia Bate, Ann. Mag. Nat. Hist., II, vii, p. 318, 1851.

Sulcator Bate, Ann. Mag. Nat. Hist., II, xiii, p. 504, 1854 (in place of *Bellia* preoccupied).

According to Bovallius, Statius Müller, in a German translation of Slabber's work above referred to, in 1781 proposed the generic name *Haustorius* for Slabber's species; but, as Bovallius says, the name is an adjective, has never come into use, and is properly rejected.

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Lepidactylis arenarius.

Oniscus arenatius Slabber, op. cit., 1778 (corrected to *arenarius* in errata).

Lepidactylis dytiscus Say, loc. cit., p. 380, 1818.

Bate, Catalogue Amphip. Crust. British Museum, p. 112, 1862 (=Say).

Smith, Inverteb. Animals Vineyard Sound, Report U. S. Comm. Fish and Fisheries, part i, p. 556 (262), 1874.

Pterygocera arenaria Latreille, Encyclopédie méthodique, 1825 (based on Slabber's species; teste Bovallius).

Bovallius, Notes on Pterygocera arenaria Slabber (Bihang till Svenska Vetenskaps-Akad. Handlingar, iv, no. 8), pp. 1–27, pls. 1–4, 1878.

Bellia arenaria Bate, Ann. Mag. Nat. Hist., II, vii, p. 318, pl. 9, figs. 1–8, 1851 (gen. et sp. nov.)

Sulcator arenarius Bate, Ann. Mag. Nat. Hist., II, xiii, p. 504, 1854.

Sulcator arenatius Boeck, Christiania Videnskabs-Selskabs Forhandlinger, 1870, p. 137 (57).

Coast of Georgia (Say). New Haven!, Connecticut, sandy shores, at low water, not rare, and also dredged in shallow water (A. E. Verrill, S. I. Smith, et al.) Vineyard Sound!, 5 to 10 fathoms, sandy and shelly bottoms, 1871, 1875. Smith's Point!, Beverly, Massachusetts, sand, at low water, August 25, 1878 (J. H. Emerton). These specimens collected by Mr. Emerton are the only ones I have seen from north of Cape Cod. On the European coast, it is reported from the British Isles! (Norman, Bate), the coast of Holland (Slabber), and Scandinavia (Boeck, Bovallius).

EXPLANATION OF PLATE IIa.

Cerapus tubularis.

All the figures are enlarged seventy diameters, except 4, *b*, which is enlarged one hundred and forty diameters.

FIGURE 1.—First gnathopod and epimeron of the right side of an adult male.

FIGURE 2.—Second gnathopod and epimeron of the right side of the same specimen

FIGURE 3.—First peræopod and epimeron of the right side of the same specimen.

FIGURE 4.—*a*, Third peræopod and epimeron of the right side of the same specimen; *b*, distal portion of the same still more enlarged.

FIGURE 5.—Pleopods of the right side of an adult female: *a*, first; *b*, second; *c*, third.

FIGURE 6.—Extremity of the pleon of the same specimen, dorsal view, showing uropods and telson.

NEW HAVEN, July 1, 1880.

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VIII.—New England Annelida.

Part I.—Historical Sketch, with Annoted Lists of the Species hitherto Recorded. By A. E. Verrill.

Previous to the last ten years very little had been published in this country concerning the marine annelids of our coast, and very few of our species had been recorded, either by foreign or American writers.

A large proportion of the New England species are now known to be identical with those of Northern Europe, therefore descriptions of such species are to be found in nearly all foreign works relating to the European annelids, from the days of Linné down to the present time.

It is not my intention to enumerate, at this time, all the European works which must be consulted by a student of our annelids, for such a list would be nearly coëxtensive with the entire literature of the Annelida. Among the more important of the European works, containing descriptions and figures of many of our Annelids, are those of O. Fabricius;[1] Œrsted;[2] M. Sars;[3] Grube;[4] Malmgren;[5] Quatrefages;[6] Ehlers;[7] G. O. Sars;[8] Malm;[9] Theel.[10]

[1] Otho Fabricius, Fauna Groenlandica, pp. 279–315. Hafniæ et Lipsiæ, 1780.
The Latin descriptions in this work are given with considerable detail, and are, in all respects, much better than most of those published in the last century, or even much later. Many of his species are common on the New England coast.

[2] A. S. Œrsted, Grönlands Annulata Dorsibranchiata [eight plates]. Kjobenhavn, 1843.
Annulatorum Danicorum Conspectus, Fasc. I, Maricolæ [plates]. Hafniæ, 1843.

[3] M. Sars, Fauna Littoralis Norvegiæ, i, 1846; ii, 1856.
Beretning om en i Sommeren 1849 foretagen Zoologisk Reise i Lofoten og Finmarken [pp. 1–91; annelids, pp. 76–91].<Nyt. Magaz. Naturvid., vi, 1850.
Bidrag til Kundskaben om Norges Annelider, fjerde afhandling.<Vidensk. Forhandl. Christiania, 1861.
Geologiske og Zoologiske Jagttagelser austillede paa en Reise i en Deel af Trondhjems i Sommeren 1862. 1863.
Also various other papers in the publications of learned societies, 1847–1864.

[4] Adolph Eduard Grube, Die Familien der Anneliden, pp. 163. Berlin, 1851.
Annulata Semperiana, Beiträge zur Kenntniss der Anneliden Fauna der Philippinen, 4to, pp. 300, pl. 15.=Mémoirs de L'académie Impériale des Sciences de St. Petersbourg, VII, vol. xxv, No. 8, 1878.

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Of these, only Ehlers and Quatrefages have actually described or mentioned specimens from the New England Coast.

Of the southern species, which extend northward to the New England coast, some were described by Bosc and other early writers.

The important works of Audouin and Milne-Edwards;[11] of Claparède;[12] of Kinberg,[13] and others, although containing but few of our species, are intimately connected with the history of our anne-

Also many papers in the Archiv für Naturgeschichte; Müller's Archives; Proceedings of the Vienna Academy, 1866–1877; and in the publications of other learned societies.

[5] A. J. MALMGREN, Nordiska Hafs-Annulata.<Ofvers. K. Vet.-Akad. Förh., pp. 51–110, pl. 8–15; pp. 355–410, pl. 18–29. 1865.

Annulater Polychæta Spetsbergiæ, Grœlandiæ, Islandiæ et Scandinaviæ hactenus cognita, pp. 1–127, pl. 1–14. Helsingforsiæ, 1867.

[6] A. DE QUATREFAGES, Histoire naturelle des Annelés marins et d'eau douce, vols. i, ii, with atlas. Paris, 1865. [Published in the autumn of 1866, t. Mg'n].

[7] ERNST EHLERS, Die Borstenwurmur (Annelida Chætopoda), 4to, pp. 748, 24 plates. Leipzig, 1864–8.

[8] G. O. SARS, Diagnoser af nye Annelider fra Christianiafjorden.<Vidensk.-Selsk. Forhandl. Christiania, 1871.

Bidrag til Kundskaben om Dyrelivet paa vore Havbanker.<The same. 1872.

Also other papers in the same serial, and elsewhere.

[9] A. W. Malm, Annulater i hafvet utmed Sverges vestkust och omkring Göteborg.< Göteborgs Kongl. Vetensk. Vitter. Samh. Handlingar, xiv [pp. 69–105, pl. 1]. 1874.

[10] H. J. THÉEL, Les Annélides Polychètes des mers de la Nouvelle-Zemble, 4to, pp. 75, pl. 1–4.<Kongl. Svenska Vet.-Akad. Handlingar, xvi. Stockholm, 1879.

Études sur les Géphyriens inermes des Mers de la Scandinavie, du Spitzberg et du Grœnland [pp. 1–27, pl. 1–4].<K. Svenska Vet. Akad. Handl., iii, No. 6, 1875.

[11] AUDOUIN AND H. MILNE-EDWARDS, Recherches pour servir a l'Histoire naturelle du littoral de la France, vol. ii; Annelides [by Milne-Edwards]=Classification des Annélides et Description de celles qui habitent les côtes de la France.<Annales des Sciences naturelles, xxvii [pp. 337–447, pl. 7–15], 1832; xxviii [pp. 187–247, pl. 9, 10], 1833; xxix [pp. 195–269, 388–412, pl. 13–18], 1833; xxx [pp. 411–425, pl. 22], 1833.

[12] ÉDOUARD CLAPARÈDE, Etudes anatomiques sur les Annélides, Turbellariés, Opalines et Grégarines observées dans les Hébrides.<Mem. de la Soc. de Phys., et d'Hist. nat. de Genève, vol. xvi, 1861.

Glanures Zootomiques parmi les Annélides de Port-Vendres (Pyrénées Orientales). <Mémoires de la Société de Physique et d'Hist. nat. de Genève, vol. xvii [4to, pp. 463–600, pl. 1–8], 1864.

Les Annelides Chétopodes du golfe de Naples. Genève, 1868, 1 vol., in 4to.<The same, vol. xix, xx, Genève, 1868. 1869.

Supplément aux Annélides Chétopodes, etc. Genève, 1870, in 4°.<The same, vol. xx [pp. 365–542, pl. 1–14], 1870.

Recherches sur la Structure des Annélides Sédentaires.=The same, vol. xxii [pp. 200, pl. 1–15], Genève, 1873.

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lids, for in them many of our genera were established, and numerous species from southern Europe, closely allied to our own, were described and illustrated in great detail.

The following lists are arranged, as nearly as possible, in chronological order. Species when recorded for the first time, as from the northeastern coast of America, are printed in italics, unless indeterminable by me. The names placed in the last column as the equivalents of the original names, are those used by the writer in the Check-list of Marine Intervebrates,[14] edition of 1879, unless otherwise indicated. Those names that have been since changed are enclosed in brackets.

Although a considerable number of changes in the nomenclature of the annelids, included in the first edition of the Check-list, have become necessary or desirable, and may be adopted in the second, or systematic part of the present paper, I have thought it best to introduce only some of the more important ones, or those that relate to the more common species, in the following lists, the Check-list being still kept, as a convenient standard of reference, for the various synonymous names that have formerly been used for those species included in it.

The principal changes which I have here introduced in the synonymical lists are as follows:

Cistenides to be changed to *Pectinaria; Anthostoma* to *Scoloplos; Rhynchobolus dibranchiatus* to become the type of the new genus *Euglycera*, herein established (see p. 295).

The common Phascolosoma cæmentarium appears to be identical with *P. strombi* (*Phascolion strombi* Theel) of Europe.

The earliest notices of any of our annelids are to be found in the conchological works of Gould and others, but such species as were mentioned by them are mostly those that form more or less solid tubes, and as their notes and descriptions usually refer only to these tubes, it is seldom possible to identify, with any certainty, the species mentioned by them.

For greater convenience, I have also included, in the lists, the small number of Gephyreans that have been recorded from our coasts. The leeches are omitted.

[13] J. G. H. Kinberg, Annulata Nova.<Stockholm, Akad. Ofversigt, xxi, 1865, pp. 559–574; xxii, 1866, pp. 167–179, 239–258; xxiii, 1867, pp. 97–103, 337–357. Kongliga Svenska Fregatten Eugenies Resa, omkring jorden, Zoology, I, [4to, pp. 1–32, pl. 1–8].<Kong. Svenska Vetenskaps-Akad. Stockholm, 1867.

[14] A. E. Verrill, Preliminary Check-list of the Marine Invertebrata of the Atlantic Coast, from Cape Cod to the Gulf of St. Lawrence. Author's edition, New Haven, June, 1879.

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1842.—Augustus Gould. *Report on the Invertebrata of Massachusetts* [*first edition*]. *Boston*, 1841.

Certain tube-making species are recorded (pp. 7–11), with descriptive notes. A few other species are mentioned by name (p. 343).

List of Species.

Pectinaria Belgica, p. 7, fig. 1.	=[Pectinaria Gouldii V.]
Amphitrite ventilabrum, p. 7.	=Sabella, several species, undeterminable.
Spirorbis nautiloides, p. 8, fig. 3.	
Spirorbis spirillum, p. 8.	=Spirorbis borealis Daud.
Spirorbis sinistrorsa, p. 9, fig. 4.	=Spirorbis lucidus.
Serpula vermicularis, p. 10.	=?Hydroides dianthus V.
Aphrodita aculeata, p. 343.	=Aphrodite aculeata L.
Polynoë squamata, p. 343.	=Lepidonotus squamatus.
Nereis margaritacea, p. 343.	=?Nereis pelagica.
Phyllodoce viridis Johnst., p. 343.	Undeterminable.
Phyllodoce lamelligera Johnst., p. 343.	Undeterminable.

The last five are accompanied neither by descriptions nor by notes on habits, and cannot be determined, except conjecturally, but there can be little doubt as to the identity of the first three of these.

1852.—L. F. de Pourtales. *On the Gephyrea of the Atlantic Coast of the United States.*<*Proc. Amer. Assoc. Adv. Science, for* 1851, [vol. v, pp. 39–42]. 1852.

Echiurus chrysacanthophorus (Couth.) P., p. 39.
Sipunculus Gouldii, sp. nov., p. 40. =Phascolosoma Gouldii.
Phascolosoma Bernhardus, p. 41. =[Phascolion strombi (Mont.) Theel.]=Phascolosoma cæmentarium Quartr.*

1853.—Wm. Stimpson. *Synopsis of the Marine Invertebrata of Grand Manan* [*pp.* 29–36].<*Smithsonian Contributions to Knowledge, vol. vi. Washington*, 1853.

In this work, Dr. Stimpson published a list of the species collected by him in the Bay of Fundy. His list, though exceedingly incom-

* This common and very generally distributed species is, without much doubt, identical with the common shell-inhabiting species of northern Europe—the *Phascolion Strombi* Theel. This species has had a large number of specific names, under three genera. It is the *Strombi* Mont., *dentalii* Gray, *bernhardus* Forbes, *granulatum* Leuck., *capitatus* Rathke, *concharum* Œrst., *cæmentarium* Quatr., *hamulatum* Packard. Perhaps the *Phascolosoma tubicola*, described by me, is only a variety of the same species.

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plete, added many species and contained all that was known, for many years afterwards, concerning the annelids of that prolific region. The species enumerated by him are included in the following list. In the identification of many of the following species I had, formerly, the personal assistance of Dr. Stimpson; in company with him, I also dredged many of the species in the same region, in 1861.*

List of Species.

Sipunculus Bernhardus, p. 28.	=[Phascolion strombi Theel.]
Sternaspis fossor, sp. nov., p. 29.	
Spirorbis spirillum Lam., p. 29.	=Spirorbis borealis.
Spirorbis nautiloides Lam., p. 29.	=Spirorbis Stimpsoni Ver.
Spirorbis vitrea (Fabr.) St., p. 29.	=Spirorbis vitreus.
Spirorbis porrecta (Fabr.), p. 29.	=Spirorbis lucidus Mörch.
Spirorbis quadrangularis, sp. nov., p. 29.	
Spirorbis granulata (Müll.), p. 29.	?=Spirorbis cancellatus.
Vermilia serrula St., sp. nov., p. 29.	
Protula media St., sp. nov., p. 30.	
Sabella pavonina Sav., p. 30.	?=Sabella pavonia Mgn.
Sabella zonalis St., p. 30.	?=Sabella neglecta.
Pectinaria Grœnlandica(?) Grube, p. 30.	=[Pectinaria granulata.]
Lumara flava St., g. et sp. nov., p. 31.	=Thelepus cincinnatus.
Terebella brunnea St., sp nov., p. 31.	=Amphitrite brunnea.
Terebella cirrata Cuv., p. 31.	=Amphitrite cirrata.
Clymene lumbricalis (Fabr.) Sars, p. 31.	=Nicomache lumbricalis.
Arenicola piscatorum Cuv., p. 31.	=Arenicola marina.
Siphonostomum asperum, sp. nov., p. 31.	=Trophonia aspera V.
Tecturella fluccida St., gen. et sp. nov., p. 32, fig. 21.	=Flabelligera affinis Sars.
Brada granosa St., gen. et sp. nov., p. 32.	
Brada sublævis St., gen. et sp. nov., p. 32.	
Ophelia glabra St., sp. nov., p. 33.	=Ophelia limacina Rathke.
Aricia quadricuspis(?) Grube, p. 33.	=[Naineis quadricuspida.]
Glycera capitata Œrsted, p. 33.	=Rhynchobolus capitatus.
Glycera viridescens St., sp. nov., p. 33.	=Goniada maculata.
Phyllodoce Grœnlandica Œrs., p. 33.	
Nephthys ciliata Müll., p. 33.	
Nephthys ingens St., sp. nov., p. 33. Description insufficient.	?=Nephthys paradoxa Malm.

* I also dredged, in that vicinity, in 1859, 1864, 1865, 1868, 1870, 1872, but have not published the very numerous additions to the list of annelids.

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Nereis abyssicola St., sp. nov., p. 33.
Nereis iris St., sp. nov., p. 33.
Nereis denticulata, sp. nov., p. 33, fig. 23.=Nereis pelagica Linné.
Nereis grandis St., sp. nov., p. 34, fig. 24.=Nereis virens Sars.
Enonella bicarinata, gen. et sp. nov., p. 34. Not seen by me.
Eunice Œrstedii St., sp. nov., p, 34. =Leodice vivida, young.
Eunice vivida St., sp. nov., p. 35, fig. 26. =Leodice vivida V.
Onuphis Eschrichtii, p. 35. =Nothria conchylega Mgn.
Cryptonota citrina St., gen. et sp. nov., p. 36, fig. 27. =Spinther citrinus V.
Euphrosyne borealis Œrs., p. 36.
Pholoë tecta St., sp. nov., p. 36. =Pholoë minuta, var.
Lepidonote cirrata Œrs., p. 36. =Harmothoë imbricata.
Lepidonote punctata Œrs., p. 36. =Lepidonotus squamatus.
Lepidonote scabra Œrs., p. 36. =Eunoa Œrstedii Mgn.
Aphrodite aculeata Baster, p. 36.

1855.—Joseph Leidy. *Marine Invertebrate Fauna of the Coasts of Rhode Island and New Jersey.*<*Journal Acad. Nat. Science Philadelphia, II, vol. iii* [*pp.* 144–148, 4*to, two plates; separate copies, pp.* 12–16], 1855.

Dr. Leidy, in this work, gave a brief list of Annelids observed by him, and described a number of new and interesting forms.

List of Species.

Naraganseta coralii L., gen. et sp. nov., p. 144 (12), pl. 11, figs. 46–48. =[Dodecaceria coralii V.]
Sabella oculifera L., sp. nov., p. 145 (13), pl. 11, figs. 55–61. =Potamilla reniformis.
Clymene urceolatus L., sp. nov., p. 145. Not seen by me.
Clymene torquatus L., sp. nov., p. 146. =Clymenella torquata V.
Pectinaria auricoma Grube, p. 146. =[Pectinaria Gouldii V.]*
Terebella ornata L., sp. nov., p. 146, pl. 11, figs. 44, 45. =Amphitrite ornata V.
Spirorbis spirillum Lam., p. 146. =Spirorbis borealis.

* Some of the divisions of the genus *Pectinaria*, proposed by Malmgren as genera, do not seem to me well founded; at least, I cannot regard them as of generic value. Among these is the group named *Cistenides*, which does not seem worthy of separation, even as a sub-genus, from *Pectinaria*. The species hitherto named by me *Cistenides Gouldii* should, therefore, be called *Pectinaria Gouldii* V.

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Torquea eximia L., gen. et sp. nov., p. 146, pl. 11, figs. 51, 52. =Polycirrus eximius V.
Cirrhatulus fragilis L., sp. nov., p. 147, pl. 11, figs. 39–43. =Cirrhinereis fragilis V.
Lumbriconereis splendida Bv., p. 147. =Arabella opalina V.
Eunice sanguinea Mont., p. 147. =Marphysa Leidyi Quatr.
Glycera Americana L., sp. nov., p. 147, pl. 11, figs. 49, 50. =Rhynchobolus Americanus V.
Nereis denticulata St., p. 148. =Nereis pelagica L. and N. virens.
Siphonostomum affine L., sp. nov., p. 148. =Trophonia affinis V.
Lepidonote armadillo (Bosc) Leidy, p. 148, pl. 11, fig. 54. =Lepidonotus squamatus.
Sigalion Mathildæ (*non* Aud. and Edw.), p. 148, pl. 11, fig. 53. =Sthenelais picta V.
Ophelia simplex L., sp. nov., p. 148.
Lumbriculus tenuis, sp. nov., p. 148, pl. 11, fig. 64. Not seen by me.

1860.—Alexander Agassiz. *On the Young Stages of a few Annelids.* <*Annals Lyceum Natural History, New York, vol. viii,* [*pp.* 303–343, *pl.* 6–11, *figs.* 1–58], *June*, 1860.

List of Species.

Spirorbis spirillum Gould (*non* Pagenst.), p. 318, figs. 20–25. =Spirorbis borealis.
Terebella fulgida L. Agassiz, MSS., p. 320, fig. 19, 19*a*. Undetermined. Perhaps =Thelepus cincinnatus or Lepræa rubra.
Polydora ciliatum Clap. (?), p. 330, figs. 26–38.
Nerine coniocephala Johnst. (?), p. 333, figs. 39–45. =Spio, sp.
Phyllodoce maculata Œrsted, p. 333, figs. 46–55.

The last three cannot be positively identified without a reëxamination of the specimens, which are not, at present, accessible to me, for they were sent to Europe, for study, many years ago.

In addition to these, Mr. Agassiz described and figured, in the same paper, some unknown annelid larvæ; one which he supposed to be the larva of a Nemertean (? *Nareda* Girard), and considered the same as "Lovén's annelid larva," which, in the light of subsequent observations, is an annelid, probably identical with, or allied to *Polygordius;* and also the larval forms of *Planaria angulata.* The latter is not the *Planaria angulata* of Fabricius, which was a Nemertean, but is a true Planarian (the *Bdelloura candida* Leidy).

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1862.—ALEXANDER AGASSIZ. *On Alternate Generation in Annelids, and the Embryology of Autolytus cornutus.* <*Journal Boston Society of Natural History, Vol. vii,* [*pp.* 392–409, *pl.* 9–11], *July*, 1862.

Autolytus cornutus, sp. nov., p. 390, pl. 9–11. Massachusetts Bay, Buzzard's Bay.

Autolytus longosetosus A. Ag., p. 404, [21], Male=?A. longisetosus; female probably=A. Alexandri Mgn. Massachusetts Bay.

Mr. Agassiz states that the form that he supposed to be the female of A. longisetosus has "no less than ten" anterior segments, without long setæ. Among the numerous specimens of Autolytus collected at Salem, Mass., by Mr. J. H. Emerton, there are both males and females of what I consider the true *A. longisetosus* (pl. 12, figs. 10, 10*a*, 10*c*, ♂.) These have, *in both sexes, six short anterior segments*, lacking long dorsal setæ, and bearing only short ventral setæ; the post-buccal segment bears very long dorsal cirri (*dc*), but no setæ.

In the same lots, collected in early spring, there is a very distinct species, which has, in both sexes, *fourteen anterior segments*, with short ventral setæ only, including the first post-buccal segment, which bears a pair of very large dorsal cirri, and a small cluster of setæ directly beneath them; the second segment is also very much compressed, and its setæ are more or less completely concealed by the large cirri, in a dorsal view (plate 12, figs. 8–8*c*). These are, perhaps, the *A. Alexandri* Mgn., and they may prove to be the sexual forms of my *Stephanosyllis ornata.* If so, it would confirm the separation of *Stephanosyllis*, as a genus distinct from *Autolytus.*

In the sexual forms that I consider identical with *Autolytus cornutus* A. Ag., I have constantly found *six* anterior setigerous segments *in both sexes* (pl. 12, fig. 4, ♂; fig. 6 ♀, ventral views), but in the male the first fascicle is concealed, in a dorsal view, by the large dorsal cirrus of the first post-buccal segment; this first segment bears no setæ, in this and the several allied species, which I consider typical *Autolyti* (pl. 12, figs. 5, 6, 9–9*b*, 10, 10*a*), but in this species it has a small papilliform ventral cirrus, not mentioned by Mr. Agassiz. He also failed to notice the ventral tentacular cirrus of the female, though he figures and describes the corresponding cirrus of the male. Although this organ is small, and often seen with difficulty, especially when the body is much compressed, as represented in the figures by Mr. Agassiz, it is quite as large as in the male, and often projects beyond the sides of the head.

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Mr. Agassiz stated that in his *A. cornutus* the male had only *five* anterior setigerous segments, while the female had six. It seems possible, however, that he overlooked the first, very short, compressed segment, with its small fascicle of setæ, which is entirely concealed, in a dorsal view, by the large dorsal cirri of the preceding segment. At least, I have never been able to find a male, of this species, with only five anterior setigerous segments.

1863.—A. S. PACKARD, JR. *A list of Animals dredged near Caribou Island, Southern Labrador, during July and August*, 1860. < *Canadian Naturalist and Geologist, vol. vii*, [*pp.* 401–429, 2 *plates*].

The lists of annelids and other invertebrates, in this article, contain many typographical errors, in the names of the species. Throughout the article "feet" was printed, instead of fathoms, for the depths. Some important transpositions also occur in the lists.

List of Species.

Pectinaria Eschrichtii, pp. 403, 418.	=[Pectinaria granulata.]
Onuphis Eschrichtii, p. 403.	=Nothria conchylega.
Vermilia serrula, pp. 403, 418.	
Spirorbis cancellata, pp. 403, 418. (cancellatus)	
Spirorbis vitrea, pp. 403, 417. (vitreus)	
Sipunculus, sp. nov., p. 417.	=[Phascolion strombi Theel.]
Spirorbis spirillum, p. 417.	=Spirorbis borealis.
Spirorbis nautiloides, p. 417.	?=Spirorbis Stimpsoni V.
Spirorbis porrecta, p. 417. (porrectus)	?=Spirorbis lucidus.
Spirorbis glomerata, p. 418. (glomeratus)	
Spirorbis quadrangularis, p. 418.	
Terebella, n. sp., p. 418.	
Siphonostomum plumosum, p. 418.	=Trophonia plumosa.
Cirrhatulus, n. sp., p. 418.	=Cirratulus cirratus.
Nephthys cœca, p. 418.	Doubtful.
Heteroneis arctica, p. 418. (Heteronereis)	=Nereis, sp.
Eteone, sp., p. 418.	
Nereis pelagica, p. 418.	
Nereis, n. sp., pp. 418, 424.	?=Nereis pelagica.
Lepidonote cirrata, pp. 418, 424.	=Harmothoe imbricata.
Lepidonote punctata, pp. 418, 424.	=Lepidonotus squamatus.

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1863.—Wm. Stimpson. *Synopsis of the Marine Invertebrata collected by the late Arctic Expedition, under Dr. I. I. Hayes.* < *Proceedings of the Academy of Natural Sciences of Philadelphia, vol. xv,* [*pp.* 138–142,] *May,* 1863.

All the annelida recorded are from the western coast of Greenland.

List of Species.

Lepidonote cirrata, p. 140.	=Harmothoe imbricata.
Lepidonote punctata, p. 140.	=Lepidonotus squamatus.
Onuphis conchilega, p. 140.	=Nothria conchylega.
Nereis pelagica, p. 140.	
Nephthys cæca, p. 140.	
Phyllodoce Grœnlandica, p. 140.	
Scoloplos quadricuspida, p. 140.	=[Naineis quadricuspida.]
Cirratulus borealis, p. 140.	=Cirratulus cirratus.
Ammotrypane limacina, p. 140.	=Ammotrypane aulogaster.
Siphonostomum plumosum, p. 140.	=Trophonia plumosa.
Tecturella flaccida, p. 140.	=Flabelligera affinis.
Brada inhabilis, p. 140.	
Terebella cincinnata, 141.	=Thelepus cincinnatus.
Terebella cirrata, p. 141.	=Amphitrite cirrata.
Pectinaria Eschrichtii, p. 141.	
Spirorbis nautiloides, p. 141.	
Priapulus caudatus, p. 141.	

1866.—A. de Quatrefages. *Histoire Naturelle des Annelés. Paris,* 1865 [1866]. (Suites a Buffon).

In this general work, largely compiled, Quatrefages included most of the species mentioned by Stimpson and Leidy, and translated (sometimes erroneously) more or less of their descriptions, with some changes in the names. But the only species from our coast that he seems to have personally examined is the common Nereis virens Sars (=*N. grandis* Stimpson), which he redescribed under the name of Nereis Yankiana (i, p. 553, pl. 17, figs. 7, 8), and Phascolosoma cæmentarium, sp. nov.=[Phascolion strombi Theel.]

1867.—A. S. Packard, Jr. *Observations on the Glacial Phenomena of Labrador and Maine, with a view of the recent Invertebrate Fauna of Labrador.* < *Memoirs of the Boston Society of Natural History, vol. i,* [*4to, pp.* 210–303, *pl.* 7, 8].

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This paper includes the species enumerated by Dr. Packard in his preliminary paper, noticed above, with some additions. Although many of the numerous typographical and other errors of that article were corrected in this later and more complete one, the latter also contains various orthographical errors, even in the names of the species, not only among the annelids but in the lists of species of the other groups. Some of these very obvious errors are indicated in the following list. Many of the species in this list I have personally examined.

List of Species.

Syrinx ?, sp. nov., p. 290., pl. 8, fig. 10. Undeterminable.
Phascolosoma hamulatum, sp. nov., pl. 8, fig. 8, p. 290. =[Phascolion strombi Theel.]
Spirobis vitrea, p. 291. (vitreus)
Spirorbis sinistrorsa, p. 292. (sinistrorsus)
Spirorbis porrecta. (porrectus) =Spirorbis lucidus.
Spirorbis cuncellata. (cancellatus)
Spirorbis granulata. (granulatus)
Spirorbis spirillum. =Spirorbis borealis.
Vermilia serrula.
Amphitrite cirrata.
Amphitrite ? sp.
Ampharete Grubei.
Cistenides granulata, p. 292. (granulatus)=[Pectinaria granulata.]
Praxilla Mulleri, p. 293. (Doubtful.) ?=(Praxillella Mulleri V.]
Nicomache lumbricalis.
Spiochætopteras typicus. (Spiochætopterus)
Tubes only. Very doubtful.
Arenicola piscatorum. =Arenicola marina.
Siphonostomum asperum. =Trophonia aspera.
Siphonostomum plumosum. =Trophonia plumosa.
Cinatulus cirrata. (Cirratulus) =Cirratulus cirratus.
Heteronereis arctica ? Œrs. =Nereis, sp.
Nephthys longisetosa. Doubtful.
Two species are confounded in the synonymy of this species.
Nephthys cæca, p. 294. Doubtful.
Eteone cylindrica. Doubtful.
Phyllodice grœnlandica. (Phyllodoce)
Onuphis Eschrichtii. =Nothria conchylega.
Nereis pelagica.
Nereis, sp. ?=Nereis pelagica.
Pholoë minuta.

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Harmothoë imbricata.

Lepidonotus squatmaus, p. 295. (squamatus)

The *Pontobdella? livida*, p. 291, pl. 8, fig. 9, is undoubtedly a Nemertean—perhaps a *Cerebratulus* or *Micrura*, but is indeterminable.

1868.—Ernst Ehlers. *Die Borstenwürmer* (*Annelida Chætopoda*), [*pp.* 1–268, *pl.* 1–11, *publ.* 1864; *pp.* 269–748, *pl.* 12–24, 1868]. *Leipzig*, 1864–1868.

Ehlers published in 1868 the second part of his large and elaborate work on Annelida. He had received from the Museum of Comparative Zoology, Cambridge, a collection of New England annelids, and in this part of his book he described, in detail, a number of our common annelids and illustrated several of them. To one of the most common species, his *Glycera dibranchiata*, he devoted thirty-two pages of text and numerous figures, partly anatomical. He described its anatomy with considerable detail. The following are the species particularly mentioned by him as from the New England coast.

List of Species.

Nereis pelagica, p. 511, pl. 20, figs. 11–20. Labrador to Nahant, Mass.

Nereis virens, p. 559–563, pl. 22, figs. 29–32.

Nereis limbata, sp. nov., pp. 567–570.

Nephthys cæca, p. 588, pl. 23, figs. 10–34. Eastport to Nahant.

Nephthys bucera, sp. nov., pp. 617–619, pl. 23, fig. 8. Massachusetts Bay.

Nephthys discors, sp. nov., pp. 626–629, pl. 23, figs. 39, 40. Eastport, Me.

Nephthys ciliata, p. 629, pl. 23, fig. 36. Edgartown, Mass.

Nephthys picta, sp. nov., pp. 632–635, pl. 23, figs. 9, 35. Vineyard Sd. to S. Carolina.

Glycera capitata, p. 648. =Rhynchobolus capitatus Clap.

Glycera Americana Leidy, pp. 668–670, pl. 23, figs. 43–46. =Rhynchobolus Americanus V.

Glycera dibranchiata, sp. nov., pp. 670–702, pl. 24, figs. 1, 3–8, 10–28. Mass. Bay, New Jersey.=[Euglycera dibranchiata V.]*

* This species differs so much from the typical species of *Glycera* and *Rhynchobolus* (whether the latter be distinct or not), that I propose to establish a new genus (*Euglycera*) for it. *Euglycera* may be distinguished by possessing two ligulate branchiæ to each branchiferous parapodial appendage; one of these is connected, at base, with the dorsal side of the parapodia; the other, with the ventral side. Proboscis and jaws are as in *Rhynchobolus*.

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1871.—A. E. Verrill. *Marine Fauna of Eastport, Maine.* < *Bulletin of the Essex Institute, iii, pp.* 2–6, *Salem, Mass., Jan.* 1871.

Aphrodite aculeata, p. 4.
Nereis grandis St., p. 6. =Nereis virens.
Arenicola piscatorum, p. 6. =Arenicola marina.
Myxicola Steenstrupii, p. 6.
Thelphusa [error typ.] circinnata (Lumara flava St.), pp. 3, 6. =Thelepus cincinnatus.
Torquea ?, p. 6. =[Polycirrus phosphoreus V., 1879.]
Eunice, sp. p. 6. =Leodice vivida.
Lumbriconereis, sp., p. 6. =Lumbrinereis fragilis.

1872–3.—A. E. Verrill. *Brief Contributions to Zoology from the Museum of Yale College, No. XXIII.—Results of Recent Dredging Expeditions on the Coast of New England.* [Nos. 1 and 2.] < *American Journal of Science and Arts, vol. v,* [*pp.* 1–16, *January*, 1873 (author's ed. issued Dec. 13, 1872); *pp.* 98–106, *February*, 1873 (author's ed., Jan. 18.)]

These papers include Annelids dredged in the Bay of Fundy, in 1872, by the party of the U. S. Fish Commission, under the direction of the writer; those dredged at George's Banks and off Nova Scotia, etc., by Messrs. S. I. Smith and O. Harger, on the Coast Survey steamer "Bache," in 1872; and those dredged in the Gulf of Maine, by Messrs. A. S. Packard and C. Cooke, on the same steamer.

List of Species.

Hermione hystrix (?), p. 98. =Lætmatonice armata V.
Antinoë Sarsii, p. 101. (Sarsi.)
Nephthys discors, p. 103.
Nephthys ingens, p. 103, (? *non* Stimp.) =Nephthys incisa Mgn.
Nereis pelagica, pp. 101, 104.
Leodice vivida, pp. 9, 101, 104.
Nothria conchylega, pp. 9, 102, 104. (conchilega.)
Nothria opalina, sp. nov., pp. 98, 102.
Lumbriconereis fragilis, p. 98. =Lumbrinereis fragilis.
Goniada maculata, pp. 98, 103 (descr.)
Ammotrypane aulogaster, p. 101. =Ammotrypane fimbriata V.

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Eumenia crassa, p. 101.
Scalibregma inflatum, p. 98.
Trophonia aspera, p. 98.
Sternaspis fossor, p. 101.
Scolecolepis cirrata, p. 98. =[Scololepis cirrata].
Chætozone setosa, p. 103.
Notomastus latericius, p. 101.
Maldane Sarsii, pp. 99, 103. (Sarsi Mgn.)
Nicomache lumbricalis, p. 101.
Praxilla gracilis Mgn., p. 101. =[Praxillella gracilis V.]*
Praxilla prætermissa Mgn., pp. 101, 103.
=[Praxillella prætermissa V.]
Praxilla torquata, p. 101. =Clymenella torquata V.
Ampharete Finmarchica, p. 101.
Amphicteis Gunneri, p. 101.
Samytha sexcirrata, p. 101.
Samythella elongata, gen. et sp. nov., pp. 98, 103.
Amage auricula, p. 98.
Melinna cristata, pp. 98, 103 (descr.)
Amphitrite cirrata, p. 101.
Pista cristata, p. 98.
Terebellides Stroemi, p. 99. (*Strömii* Sars.)
Thelepus cincinnatus (=Lumara flava St.), p. 104.
Aphlebina, sp., p. 101. =[Polycirrus phosphoreus V., 1879.]
Phascolosoma borealis, p. 102, descr. (boreale). =[Phascolosoma eremita Sars.]
Phascolosoma cæmentarium, pp. 99, 102. =[Phascolion strombi.]
Phascolosoma tubicola, sp. nov., pp. 99, 102. =[Phascolion tubicola.]
Chætoderma nitidula, p. 102, (nitidulum).

1873.—W. C. M'Intosh, *in Whiteaves. Report of a Second* [1872] *Deep-sea Dredging Expedition to the Gulf of St. Lawrence. Montreal*, 1873. *A report to the Minister of Marine and Fisheries for the Dominion of Canada.*

List of Species, p. 14.

Eunoa nodosa, 125 fath.
Ephesia gracilis, 125 fath.
Nothria conchylega, 125 fath.
Ammotrypane aulogaster, 100–125 fath. ?=Ammotrypane fimbriata [V.

* The name, *Praxilla*, Mgn. (1865), having been previously employed for a genus of birds by Reichenbach (1853), I propose to substitute *Praxillella*, for this group.

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Trophonia plumosa, 100–125 fath.
Sabella pavonia, 125 fath.
Goniada maculata, 100–112 fath.
Amphictene auricoma (tube), 100–112 fath. ?=[Pectinaria granulata.]
Terebellides Strœmii, 100–112 fath. (Strömii Sars.)
Thelepus circinatus, 100–112 fath. (cincinnatus) = T. cincinnatus.
Praxilla gracilis, 100–112 fath. =[Praxillella gracilis V.]
Lumbrinereis fragilis, 200 fath.
Balanoglossus, sp.

In the same paper, p. 14, two species of Gephyrea are recorded, on the authority of A. E. VERRILL:

Phascolosoma boreale Kef. =[Phascolosoma eremita Sars.]
Phascolosoma Œrstedii Kef. (?). =[Phascolosoma margaritaceum Sars.]

1873–4.—A. E. VERRILL. *Report upon the Invertebrate Animals of Vineyard Sound and the Adjacent waters, with an account of the Physical Characters of the Region* [*Crustacea by S. I. Smith and O. Harger*].< *U. S. Commission of Fish and Fisheries, Part I. Report on the Condition of the Sea Fisheries of the South Coast of New England in* 1871 *and* 1872, *by Spencer F. Baird, Commissioner* [*pp.* 295–747, 38 *plates and a map*], *Washington*, 1873.
A separate edition was also published by A. E. Verrill and S. I. Smith, with original pagination on the inner margins, and new pagination on the outer margins, but without the chart. 1874.

This work included all the species known at that date from the region between Cape Cod and southern New Jersey. Many additional species have subsequently been discovered, especially in the deeper waters, at a greater distance from the coast, and also among the small free-swimming species of Syllidæ.

Extensive dredgings have subsequently been carried on, within these limits, by the U. S. Fish Commission, during the seasons of 1874, 1875, 1880, and 1881. The results of these explorations have not yet been fully reported upon.

In referring to the pages of this report the numbers refer to the original pagination, except those enclosed in brackets, which refer to the author's edition. The first reference, in each case, is to the systematic list of species.

The volume containing this report is dated 1873, when it was printed, but this part of the volume was not actually issued until early in 1874.

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List of Species.

Aphrodita aculeata Linné, pp. 580 [286], 507.

Lepidonotus squamatus, pp. 581, [287], 320, 332, 392, 397, 410, 422, pl. 10, figs. 40, 41.

Lepidonotus sublevis V., sp. nov., pp. 581, [287], 320, 332, 397, 410, 422, pl. 10, fig. 42.

Lepidonotus angustus V., sp. nov., pp. 581, [287], 494. =Lepidonotus squamatus, var.

Harmothoë imbricata, pp. 582, [288], 321, 332, 392, 397, 410, 422.

Sthenelais picta V., sp. nov., pp. 582, [288], 348, 364, 422, 428, 501.

Nepthys ingens Stimp., pp. 583, [289], 431, 434, 507, 521, pl. 12, figs. 59, 60. =Nephthys incisa Mgn.

Nephthys picta Ehl., pp. 583, [289], 348, 364, 422, 428, pl. 12, fig. 7.

Nephthys bucera Ehl., pp. 583, [289], 416, 422, pl. 12, fig. 58.

Nephthys ciliata, pp. 583, [289].

Eumidia Americana V., sp. nov., pp. 584, [290], 494. (Eumida.)

Eumidia vivida V., sp. nov., pp. 584, [290]. (Eumida.)

Eumidia papillosa V., sp. nov., pp. 584, [290]. (Eumida.)

Eulalia pistacia V., sp. nov., pp. 584, [290].

Eulalia granulosa V., sp. nov., pp. 585, [291].

Eulalia annulata V., sp. nov., pp. 585, [291].

Eulalia gracilis V., sp. nov., pp. 586, [292].

Phyllodoce gracilis V., sp. nov., pp. 586, [292], 494.

Phyllodoce catenula V., sp. nov., pp. 587, [293], 494.

Eteone robusta V., sp. nov., pp. 588, [294], 746.

Eteone limicola V., sp. nov., pp. 588, [294].

Eteone setosa V., sp. nov., pp. 588, [294].

Eteone, sp., pp. 589, [295].

Podarke obscura V., sp. nov., pp. 589, [295], 319, 332, 382, 392, 410, 440, 453, pl. 12, fig. 61.

Autolytus cornutus, pp. 590, [296], 392, 397, 410, 422, 440, 452, 494, pl. 13, figs. 65, 66.

Autolytus, sp., pp. 590, [296], 398. =Proceræa ornata V.

Autolytus, sp., pp. 590, [296].

Proceræa ornata V., sp. nov., pp. 746, [452].

Syllis, sp., p. 590, [296]. =Syllis pallida V., 1875.

Gattiola, sp., pp. 590, [296], 453.

Nereis virens, pp. 590, [296], 317, 341, 367, 371, 377, 440, 453, 455, 463, 468, 487, 514, 519, pl. 11, figs. 47–50.

Nereis limbata, pp. 590, [296], 318, 341, 371, 377, 382, 392, 422, 440, 453, 463, 516, pl. 11, fig. 51.

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Nereis pelagica, pp. 591, [297], 319, 397, 410, 422, 428, 434, 453, pl. 11, figs. 52–55.

Nereis fucata Aud. and Edw., pp. 591, [297], 494.

Nereis, sp., p. 591, [297].

Nectonereis megalops, gen. et sp. nov., pp. 592, [298], 440, 453, pl. 12, figs. 62, 63. =[Nereis megalops V., 1879.]

Diopatra cuprea, pp. 593, [299], 346, 364, 371, 377, 422, 431, 434, pl. 13, figs. 67, 68.

Marphysa Leidyi, pp. 593, [299], 319, 332, 347, 364, 410, 422, 434, 517, pl. 12, fig. 64.

Lycidice Americana, sp. nov., pp. 593, [299], 508.

Lumbriconereis fragilis, pp. 594, [300], 501, 507. (Lumbrinereis).

Lumbriconereis opalina, sp. nov., pp. 594, [300], 320, 352, 342, 364, 371, 377, 397, 410, 422, 428, 434, 468, pl. 13, figs. 69, 70. =Arabella opalina V.

Lumbriconereis tenuis, sp. nov. pp. 594, [300], 320, 332, 342, 364, 371, 377, 422, 463, (Lumbrinereis).

Ninoë nigripes, sp. nov., pp. 595, [301], 508.

Staurocephalus pallidus, sp. nov., pp. 595, [301], 348, 364.

Rhynchobolus Americanus, pp. 596, [302], 332, 342, 364, 371, 377, 428, 434, 453, 453, pl. 10, figs. 45, 46.

Rhynchobolus dibranchiatus, pp 596, [302], 332, 341, 364, 371, 377, 428, 431, 434, 463, pl. 10, figs. 43, 44. =[Euglycera dibranchiata V.]

Eone gracilis, sp. nov., pp. 596, [302]. =[Goniada gracilis V., 1879.]

Aricia ornata, sp. nov., pp. 596, [302], 344, 365.

Anthostoma robustum, sp. nov., pp. 597, [303], 343, 348, 365, 428, pl. 14, fig. 76. =[Scoloplos robustus V.]

Anthostoma fragile, sp. nov., pp. 598, [304], 344, 365. =[Scoloplos fragilis V.]

Anthostoma acutum, sp. nov., pp. 599, [305], 416, 422, 428, 501, 508. =[Scoloplos acutus V.]

Nerine agilis, sp. nov., pp. 600, [306], 346, 365, 490.

Scolecolepis viridis, sp. nov., pp. 600, [306], 345, 364, 453, 463 (Scololepis.

Scolecolepis tenuis, sp. nov., pp. 601, [307], 345, 364 (Scololepis).

Scolecolepis cirrata, pp. 602, [308], 416, 422, 428, 501, 507 (Scololepis).

Spio setosa, sp. nov., pp. 602, [308], 344, 365, 453, pl. 14, fig. 77.

Spio robusta, sp. nov., pp. 603, [309], 345, 365.

Polydora ciliatum ?, pp. 603, [309], 345, 364, 453, pl. 16, fig. 78. =[Polydora littorea V., sp. nov., (pl. 18, fig. 10).]

Ophelia simplex, pp. 603, [309], 319, 332, 410.

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Travisia carnea, sp. nov., pp. 604, [310], 431, 434, 508.
Ammotrypane fimbriata, sp. nov., pp. 604, [310], 508, pl. 15, fig. 79.
Scalibregma brevicauda, sp. nov., pp. 605, [311], 416, 422.
Trophonia affinis, pp. 605, [311], 432, 434, 507, pl. 14, fig. 75.
Brada setosa, sp. nov., pp. 606, [312], 431, 434, 508.
Sternaspis fossor, pp. 606, [312], 507, pl. 14, fig. 74.
Cirratulus grandis, sp. nov., pp. 606, [312], 319, 332, 348, 364, 371, 377, 422, 468, pl. 15, figs. 80, 81.
Cirratulus tenuis, sp. nov., pp. 607, [313], 416, 422.
Cirrhinereis fragilis, pp. 607, [313], 332, 397, 410, 422.
Naraganseta coralii, pp. 607, [313], 397, 410, 422, 494.
=[Dodecaceria coralii V.]
Clymenella torquata, gen. nov., pp. 608, [314], 343, 365, 422, 428, pl. 14, figs. 71–73.
Nicomache dispar, sp. nov., pp. 608, [314], 508.
Maldane elongata, sp. nov., pp. 609, [315], 343, 365, 371, 377.
Rhodine attenuata, sp. nov , pp. 609, [315], 508.
Notomastus luridus, sp. nov., pp. 610, [316], 342, 365, 371, 377.
Notomastus filiformis, sp. nov., pp. 611, [317], 342, 365, 371, 377.
Sabellaria vulgaris, sp. nov., pp. 611, [317], 321, 332, 349, 365, 392, 397, 410, 422, 428, 426, 476, pl. 17, figs. 88, 88a.
Cistenides Gouldii, sp. nov., pp. 612, [318], 323, 332, 349, 365, 371, 377, 422, 428, 434, pl. 17, figs. 87, 87a. =[Pectinaria Gouldii V.]
Ampharete gracilis, pp. 612, [318], 508, pl. 16, fig. 83.
Ampharete setosa, sp. nov., pp. 612, [318], 416, 422, 432, 434.
Amage pusilla, sp. nov., pp. 613, [319].
Melinna cristata, pp. 613, [319], 432, 434, 507.
Terebellides Stroëmi, pp. 613, [319], 507. (Strömii Sars.)
Amphitrite ornata, pp. 613, [319], 320, 321, 332, 348, 365, 377, 382, 422, 428, 453, pl. 16, fig. 82.
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Scionopsis palmata, gen. et sp. nov., pp. 614, [320], 321, 332, 397, 410, 476.
Lepræa rubra, sp. nov., pp. 615, [321], 382, 392, 453.
Polycirrus eximius, pp. 616, [322], 320, 332, 371, 377, 382, 392, 410, 422, 434, 453, 468, pl. 16, fig. 85.
Chætobranchus sanguineus, gen. et sp. nov., pp. 616, [322], 320, 332, 371, 377, 434, 468. =Enoplobranchus sanguineus V.
Potamilla oculifera, pp. 617, [323], 322, 332, 382, 392, 397, 410, 422, 476, pl. 17, fig. 86. =Potamilla reniformis.

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Sabella microphthalma, sp. nov., pp. 618, [324], 323, 332, 392, 397, 410, 422.
Euchone elegans, sp. nov., pp. 618, [324], 432, 434, 508, pl. 16, fig. 84.
Fabricia Leidyi, sp. nov., pp. 619, [325], 323, 332, 397, 410, 422. =Fabricia stellaris Bv.
Serpula dianthus, sp. nov., pp. 620, [326], 322, 332, 392, 397, 410, 416, 422, 426, 428, 476. =Hydroides dianthus V.
Serpula dianthus, var. citrina, pp. 620, [326]. =Hydroides dianthus V., var.
Spirorbis borealis, pp. 621, [327].
Spirorbis lucidus, pp. 622, [328].
Clitellio irrorata, sp. nov., pp. 623, [328], 324, 332, 365, 463.
Lumbriculus tenuis Leidy, pp. 623, [329]. Not seen by me.
Halodrillus littoralis, sp. nov., pp. 623, [329], 324, 332, 338, 365, 463.
Enchytræus triventralopectinatus Minor, pp. 624, [330]. Not seen by me. Doubtful.

1873–4.—A. E. Verrill. *Brief Contributions to Zoölogy from the Museum of Yale College. Nos.* XXV—XXIX. *Results of recent Dredging Expeditions on the Coast of New England. Nos.* 3—7. *From the American Journal of Science and Arts, vol. vi, Dec.*, 1873 *to vol. vii, May*, 1874. [Five plates.]

These papers contain accounts of the dredgings in 1873, in Casco Bay, Maine, and the adjacent waters, by the U. S. Fish Commission, and in the deeper parts of the Gulf of Maine, by the party on the "Bache."

List of Species.

Cryptonota citrina, p. 502. =Spinther citrinus V.
Euphrosyne borealis, pp. 41, 502.
Aphrodita aculeata, pp. 439, 45, 411.
Lætmonice filicornis? pp. 411, 499, 502. =Lætmatonice armata V.
Lepidonotus squamatus, pp. 41, 43.
Eunoa Œrstedi, pp. 41, 43, 411, 502. (Oerstedi Mgn.)
Eunoa nodosa, p. 502.
Nychia cirrosa, p. 439. (see errata, p. 46.) =Eunoa Œrstedi.
Harmothoë imbricata, pp. 39, 43, 45, 411, 502.
Nychia Amondseni, pp. 407, 411.
Antinoë Sarsii, p. 411, 502.
Enipo gracilis, sp. nov., pp. 407, 411, (pl. 6, fig. 4.)
Pholoë minuta, pp. 439, 45, 411.

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Leanira tetragona, pp. 407, 411.
Nephthys ingens, pp. 45, 411, 502. =Nephthys incisa Mgn.
Nephthys ciliata, p. 411, pl. 4, fig. 4.
Phyllodoce, sp., pp. 43, 45, 411.
Pyllodoce Grœnlandica, pp. 411, 502.
Phyllodoce catenula V., pp. 39, 43, 45, 131, pl. 4, fig. 5.
Eulalia pistacia, pp. 43, 45, 131, pl. 5, fig. 2.
Eteone pusilla, p. 45.
Eteone depressa, p. 411.
Nereis pelagica, pp. 39, 43, 45, 411, 502.
Nereis, sp., p. 411.
Nereis virens, pp. 131, 135.
Autolytus cornutus, pp. 43, 132.
Autolytus, sp., p. 43.
Stephanosyllis picta (nom. preoc.), sp. nov., pp. 43, 414, pl. 4, fig. 6, =Stephanosyllis ornata V.
Stephanosyllis ornata, sp. nov., p. 132.
Proceræa gracilis, sp. nov., p. 43, 132, pl. 5, fig. 1.
Gattiola, sp., pp. 439, 39, 411. =[Pterosyllis cincinnata V.]
Leodice vivida, p. 411, 499, 502.
Nothria opalina V., pp. 408, 411, 502, (pl. 4, fig. 1).
Nothria conchilega, pp. 39, 43, 411, 499, 502 (conchylega Sars).
Ninoë nigripes V., pp. 439, 408, 411 (pl. 4, fig. 3), 39, 45, 502.
Lumbriconereis fragilis, pp. 408, 411 (pl. 4, fig. 2), 39, 45, 502. (Lumbrinereis.)
Goniada maculata, pp. 45, 411, 502.
Rhynchobolus albus, pp. 45, 411, 502.
Rhynchobolus dibranchiatus, pp. 132, 135. =[Euglycera dibranchiata V.]
Eumenia crassa, p. 411.
Scalibregma inflatum, pp. 439, 411.
Travisia, sp., p. 411.
Brada, sp., p. 411.
Brada sublævis, p. 502.
Tecturella flaccida, pp. 43, 411, 502. =Flabelligera affinis Sars.
Trophonia aspera, pp. 43, 45, 411, 499, 502.
Ophelia, sp., pp. 45, 411.
Ammotrypane fimbriata, pp. 439, 45, 411, pl. 7, fig. 5.
Sternaspis fossor, pp. 439, 43, 45, 411.
Scolecolepis cirrata, p. 411 (Scololepis).
Polydora, sp., (in shell), pp. 43, 45. =Polydora concharum V.

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Anthostoma acutum V., pp. 439, 39, 45, 411. =[Scoloplos acutus V.]
Cirratulus cirratus, p. 43, 132.
Chætozone setosa, pp. 439, 411.
Dodecaceria concharum, p. 411.
Maldane Sarsii, pp. 439, 43, 45, 411 (Sarsi Mgn.).
Praxilla gracilis, p. 411. =[Praxillella gracilis V.]
Praxilla prætermissa, p. 411. =[Praxillella prætermissa V.]
Praxilla zonalis V., sp. nov., p. 505, pl. 6, fig. 2.
=[Praxillella zonalis V.]
Clymenella torquata, p. 132, pl. 7, fig. 6.
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Notomastus latericeus, p. 411. (latericius.)
Ancistria acuta V., sp. nov., p. 505, pl. 6, fig. 3.
=Notomastus acutus V., 1879.
Arenia, sp. in capillary tubes, sp. nov., pp. 439, 411.
=[Notomastus capillaris V., 1879.]
Cistenides granulatus, pp. 39, 43, 45, 411, 502.
=[Pectinaria granulata.]
Cistenides Gouldii, p. 45. =[Pectinaria Gouldii V.]
Ampharete gracilis, pp. 439, 39, 45, 411.
Ampharete Finmarchica, p. 411.
Amphicteis Gunneri, pp. 45, 411.
Amage auricula, p. 411.
Samytha sexcirrata, p. 411, 502.
Samythella elongata V., p. 411.
Melinna cristata, pp. 439, 39, 45, 411.
Terebellides Stroëmi, pp. 439, 411, 502. (Strömii Sars.)
Pista cristata, pp. 439, 408, 411, 502 (pl. 5, fig. 3).
Scione lobata, pp. 39, 43.
Grymæa spiralis V., sp. nov., p. 407 (cut. 1, descr.) 411, pl. 5, fig. 4.
Thelepus cincinnatus, pp. 39, 43, 411, 499, 502.
Amphitrite cirrata, pp. 439, 41, 43, 45, 411, 503, pl. 6, fig. 5.
Amphitrite Johnstoni, pp. 439, 411. =Amphitrite brunnea Stimp.
Amphitrite Grœnlandica, pp. 439, 39, 411.
Amphitrite intermedia, p. 503.
Amphitrite brunnea V., pp. 45, 132.
Polycirrus, sp., vii, pp. 43, 45, 411.=[Polycirrus phosphoreus V., 1879.]
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Sabella, sp., pp. 39, 503.
Chone, sp., p. 39, 43, 45, 412, 503.
Euchone, sp., p. 412.
Euchone elegans, p. 45, pl. 7, fig. 4.
Fabricia Leidyi, pp. 132, 135. =Fabricia stellaris Bv.
Myxicola Steenstrupii, pp. 43, 412, 503.
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Spirorbis quadrangularis, pp. 41, 43, 503.
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Phascolosoma boreale (?), p. 412, 499 (descr.), 503.
=[Phascolosoma eremita Sars.]
Phascolosoma cæmentarium, pp. 439, 43, 45, 412, 503.
=[Phascolion strombi Theel.]
Phascolosoma tubicola, pp. 439, 412. =[Phascolion tubicola V.]
Thalassema, sp., p. 412. =Thalassema viridis V.
Priapulus, sp., p. 412. =Priapulus pygmæus V.
Chætoderma nitidulum, pp. 45, 408, 412.
Chætoderma lucida, p. 439 (see errata, vol. vii, p. 46).
=Chætoderma nitidulum.

1874.—W. C. M'Intosh. *The Annelids of the Gulf of St. Lawrence, Canada.* <*Annals and Magazine of Natural History, April*, 1874. [*pp.* 261–270, pl. ix, x, setæ.]

The species included in this paper were collected by Mr. Whiteaves, mostly in the deeper parts of the Gulf.

List of Species.

Aphrodita aculeata, p. 261.
Lætmatonice filicornis Kinb., p. 261.
Lepidonotus squamatus, p. 261.
Nychia cirrosa, p. 262.
Nychia Amondseni Mgn., p. 262.
Eunoa Œrstedi Mgn., p. 262. (Oerstedi Mgn.)
Lagisca rarispina, var. occidentalis, var. nov., p. 262, pl. ix, figs. 1–4.
Malmgrenia Whiteavesii, sp. nov., p. 263, pl. ix, figs. 5–7.
Antinoë Sarsi, p. 263.
Eupolynoë occidentalis, sp. nov., p. 264, pl. ix, figs. 8–13.
?=Eucranta villosa Mgn.
Eupolynoë anticostiensis, sp. nov., p. 265, pl. x, figs. 1–4.

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Nemidia (?) *canadensis*, sp. nov., p. 265, pl. x, figs. 5–8.
Nemidia (?) *Lawrencii*, sp. nov., p. 266, pl. x, figs. 9–11.
Polynoë gaspéensis, sp. nov., p. 267, pl. ix, figs. 14, 15, pl. x, figs. 12, 13.
Sthenelais limicola Ehl., p. 268.
Leanira tetragona (Œrst.), p. 268.
Leanira Yhleni (?) Mgn., p. 268, pl. x, fig. 14.
Pholoë minuta, p. 269.

1874.—J. F. Whiteaves. *Report on Deep-Sea Dredging Operations in the Gulf of St. Lawrence* [1873]. *Montreal*, 1874; *pp.* 1–29.
In this, the third Report of this series, Mr. Whiteaves records a few additional species, on p. 14.

Priapulus caudatus Lam. Off Gaspé, etc.
Priapulus, sp. Off Port Hood, N. B.
Phascolosoma cæmentarium Qtf. Northumberland Straits, etc.
=[Phascolion strombi].

1874.—A. E. Verrill. *Explorations of Casco Bay by the U. S. Fish Commission, in* 1873. < *Proceedings of the American Association for the Advancement of Science. Portland Meeting, August*, 1873, [pp. 340–395; pl. 1–6]. Salem, June, 1874.

List of Species.

Cryptonota citrina, pp. 362. =Spinther citrinus V.
Euphrosyne borealis, pp. 358.
Aphrodita aculeata, pp. 351, 367.
Lætmonice filicornis, pp. 351. =Lætmatonice armata V.
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Eunoa nodosa, pp. 351.
Harmothoë imbricata, pp. 351, 355, 357, 362, 367, 370.
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=Nephthys incisa Mgn.
Nephthys ciliata, pp. 351, 367, pl. 5, fig. 7.
Nephthys cæca, p. 370.
Phyllodoce catenula, pp. 355, 358, 361, 362, 367, 370, 380 (descr.), pl. 3, fig. 1.
Phyllodoce, sp., pp. 351, 367.

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Phyllodoce Grœnlandica, p. 351.
Eulalia pistacia, pp. 361, 362, 367, 370, 380 (descr.), pl. 4, fig. 2.
Eteone pusilla, p. 367.
Eteone depressa, p. 351.
Nereis pelagica, pp. 351, 355, 362, 367, 370, pl. 3, figs. 3, 4.
Nereis limbata, p. 373.
Nereis virens, pp. 370, 373, pl. 5, figs. 1, 2.
Autolytus, sp., p. 362.
Autolytus cornutus, pp. 362, 370, 373.
Stephanosyllis picta (typ. error), pp. 361, 362.
=Stephanosyllis ornata V.
Stephanosyllis ornata V., pp. 370, 378 (descr.), pl. 4, fig. 1.
Proceræa gracilis, pp. 361, 362, 370, 379 (descr.), pl. 3, fig. 2.
Gattiola, sp., pp. 351, 355. =[Pterosyllis cincinnata V.]
Gattiola cincinnata V., sp. nov., p. 394, pl. 2, fig. 1.*
=[Pterosyllis cincinnata V.]
Leodice vivida, p. 351.
Nothria opalina, pp. 351, 381 (descr.), pl. 4, fig. 4.
Nothria conchylega, pp. 351, 355, 362.
Ninoë nigripes, pp. 351, 355, 367, 382 (descr.), pl. 3, fig. 5.
Lumbriconereis fragilis, pp. 351, 355, 367, 370, pl. 5, fig. 6. (Lumbrinereis).
Lumbriconereis obtusa, pp. 367, 370, 383 (descr.).
=Lumbrinereis hebes V.
Goniada maculata, pp. 351, 367.
Rhynchobolus albus, pp. 351, 367.
Rhynchobolus dibranchiatus, pp. 370, 373.
=[Euglycera dibranchiata V.]
Scalibregma inflatum, p. 351.
Travisia, sp., p. 351. =Travisia Forbesi.
Brada, sp., p. 351.
Brada granosa, p. 362.
Tecturella flaccida, pp. 351, 362. =Flabelligera affinis Sars.
Trophonia aspera, pp. 351, 362, 367.

* Although this figure was correctly drawn by Mr. Emerton, several errors were introduced by the wood-engraver. The occipital 'epaulets,' which are made to look like eyes, are really small, but prominent, flattened processes, having a very distinct dark border. The first dorsal cirrus on the left side, is wrongly made to appear to arise from the buccal segment, and the bases of the tentacular cirri are not well represented. In this species the ventral cirri of the first two segments are short, thick, flattened, ovate, and brownish in color. I have, therefore, reproduced this drawing, more correctly, on one of the accompanying plates.

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Ophelia sp., p. 351.
Ammotrypane fimbriata, pp. 351, 367, pl. 2, fig. 3.
Sternaspis fossor, pp. 351, 362, 367.
Scolecolepis cirrata, pp. 351, 367, (Scololepis).
Polydora, sp., pp. 362, 367.
Anthostoma fragilis, p. 370. =[Scoloplos fragilis V.]
Anthostoma acutum V., pp. 351, 355, 367, 384 (descr.).
=[Scoloplos acutus V.]
Anthostoma, sp., p. 351. =[Scoloplos, sp.]
Cirratulus cirratus, pp. 361, 362, 367, 370.
Chætozone setosa, pp. 351, 367.
Dodecaceria concharum, pp. 351, 362.
Maldane Sarsii, pp. 351, 367.
Rhodine Loveni, pp. 351, 367.
Axiothea catenula (typ. error), p. 351. =Axiothea catenata Mgn.
Praxilla gracilis, pp. 351, 367. =[Praxillella gracilis V.]
Praxilla prætermissa, p. 351. =[Praxillella prætermissa V.]
Praxilla zonalis, p. 384 (descr.), pl. 5, fig. 4. =[Praxillella zonalis V.]
Praxilla zonata (typ. error), pp. 357, 361, 362, 367.
=[Praxillella zonalis V.]
Clymenella torquata, pp. 367, 370.
Nicomache lumbricalis, pp. 351, 355, 362, 367.
Ammochares, sp., p. 351.
Notomastus latericeus (Sars, typ. error ?), p. 351. =Notomastus latericius.
Notomastus luridus, p. 370.
Areniella filiformis V., sp. nov., pp. 367, 386 (descr.).
Ancistria capillaris V., sp. nov., pp. 351, 355, 367, 385 (descr.).
=[Notomastus capillaris V., 1879.]
Ancistria acuta, pp. 367, 386 (descr.). =[Notomastus acutus V., 1879.]
Cistenides Gouldii, p. 367. =[Pectinaria Gouldii V.]
Cistenides granulatus, pp. 351, 355, 362, 367, 370.
=[Pectinaria granulata.]
Ampharete gracilis, pp. 351, 355, 367.
Ampharete Finmarchica, pp. 351, 367.
Amphicteis Gunneri, pp. 351, 367.
Amage auricula, p. 351.
Samytha sexcirrata, p. 351.
Melinna cristata, pp. 351, 355, 367.
Terebellides Stroemi, pp. 351, 357.
Pista cristata, pp. 351, 357.

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Scione lobata, pp. 355, 358, 361, 362, 367.
Grymæa spiralis V., pp. 351, 387 (descr.), pl. 5, fig. 5.
Thelepus cincinnatus, pp. 351, 355, 358, 362.
Amphitrite cirrata, pp. 351, 358, 362, 367.
Amphitrite Johnstoni, p. 351. =Amphitrite brunnea Stimp.
Amphitrite brunnea, pp. 367, 370.
Amphitrite Grœnlandica, pp. 351, 355.
Amphitrite intermedia, pp. 351, 362, 367.
Polycirrus, sp., p. 351. =[Polycirrus phosphoreus V., 1879.]
Potamilla oculifera, pp. 351, 355, 358, 362, 370.
=Potamilla reniformis Mgn.
Sabella zonalis, pp. 351, 362.
Chone, sp., pp. 351, 355, 362, 367.
Euchone elegans V., pp. 351, 362, 367.
Fabricia Leidyi, pp. 370, 373. =Fabricia stellaris Blainv.
Myxicola Steenstrupii, pp. 351, 362, 370.
Protula media, p. 351.
Filigrana implexa, p. 362.
Vermilia serrula, pp. 351, 355, 358, 361, 362, pl. 4, fig. 3.
Spirorbis lucidus, pp. 351, 355, 358, 362.
Spirorbis quadrangularis, pp. 358, 362.
Spirorbis nautiloides (?), p. 362. =Spirorbis Stimpsoni V.
Spirorbis borealis, pp. 370, 373.
Clitellio irrorata, p. 370.
Halodrillus littoralis, p. 370.
Phascolosoma boreale (?), pp. 351, 367.
=[Phascolosoma eremita Sars.]
Phascolosoma cæmentarium, pp. 351, 355, 362, 387 (descr.).
=[Phascolion strombi Theel.]
Phascolosoma tubicola V.,* pp. 351, 355, 388 (descr.).
=[Phascolion tubicola V.]
Priapulus, sp., p. 351. =Priapulus pygmæus V.
Chætoderma nitidulum, pp. 351, 367, pl. 6, fig. 6.

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* This species appears to be identical with the *Phascolion Spetsbergense* Theel, 1875. (K. Svenska Vet. Akad. Handlingar, iii, 6, p. 16, [sep.], pl. 1, figs. 2, 3.) The latter is regarded as a variety of *P. strombi*, by Danielssen and Koren.

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1874.—S. I. Smith and Oscar Harger. *Report on the Dredgings in the region of George's Banks, in* 1872.<*Trans. Conn. Acad., iii,* [*pp.* 1–57, *pl.* 1–8]. *New Haven, July–Aug.*, 1874.

The identifications and descriptions of the annelids in this paper were by A. E. Verrill. The list includes not only the species from the banks, but also many from the deep water of the Gulf of Maine, and some from the deep water south of the banks, and from Nova Scotia.

List of Species.

Aphrodite aculeata Linné, pp. 4, 5.
Lætmatonice filicornis Kinb., p. 22. =Lætmatonice armata V.
Nychia cirrosa Malmg., p. 12.
Eunoa nodosa Malmg., p. 12.
Lagisca propinqua, p. 20.
Lagisca rarispina Malmg., p. 14.
Harmothoë imbricata Malmg., pp. 5, 8, 12, 20, 22.
Lænilla (?) *mollis* G. O. Sars, pp. 16, 35 (descr.).
Antinoë Sarsi Kinberg, pp. 18, 37 (descr.).
Antinoë angusta V., sp. nov., pp. 22, 36.
Eucrante villosa Malmg., pp. 22, 37 (descr.).
Pholoë minuta Œrsted, p. 16.
Nephthys ciliata Rathke, p. 16, pl. 5, fig. 1.
Nephthys ingens Stimp., pp. 16, 18, 22, 39 (descr.).
=Nephthys incisa Mgn.
Nephthys circinata V., sp. nov., pp. 11, 18, 38.
Phyllodoce catenula V., pp. 5, 39, pl. 4, fig. 3.
Phyllodoce, sp., p. 16.
Eteone depressa Malmg., p. 16.
Eusyllis phosphorea V., sp. nov., pp. 20, 39, pl. 7, fig. 2.
Nereis pelagica Linné, pp. 4, 6, 12, 16, 20.
Leodice vivida (Stimp.) V., pp. 11, 16, 20, 41, pl. 5, fig. 5.
Nothria conchylega Malmg., pp. 10, 12, 16, 18, 20, 22, 41, pl. 7, fig. 3.
Nothria opalina V., pp. 16, 22, 41, pl. 7, fig. 4.
Ninoë nigripes V., pp. 16, 40, pl. 5, fig. 3.
Lumbriconereis fragilis Aud. and Edw., pp. 11, 16, 18, 22, pl. 5, fig. 2, (Lumbrinereis).
Goniada maculata Œrsted, pp. 16, 22, 42.
Rhynchobolus capitatus V., pp. 8, 11, 16, 17, 43.
Ammotrypane fimbriata V., pp. 16, 18.
Eumenia crassa Œrsted, pp. 16, 18.

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Scalibregma inflatum Rathke, p. 22.
Trophonia aspera (Stimp.) V., pp. 16, 18.
Brada, sp., p. 15.
Sternaspis fossor Stimp., pp. 16, 18.
Scolecolepis cirrata Malmg., pp. 16, 22, (Scololepis).
Spiochætopterus (? tubes), pp. 12, 22. Doubtful.
Dodecaceria concharum Œrsted, p. 21.
Chætozone setosa Malmg., p. 18.
Notomastus latericius p. 16, (latericeus Sars, typ. error?)
Ancistria capillaris V., pp. 16, 19, 22.
=[Notomastus capillaris V., 1879.]
Maldane Sarsii Malmg., pp. 16, 19, 22.
Axiothea catenata Malmg., p. 16.
Rhodine Loveni Malmg., pp. 16, 19.
Nicomache lumbricalis Malmg., p. 16.
Praxilla prætermissa Malmg., pp. 16, 19.
=[Praxillella prætermissa V.]
Praxilla gracilis Malmg., p. 16. =[Praxillella gracilis V.]
Praxilla, sp., p. 16.
Clymenella torquata V., p. 4.
Ammochares assimilis Sars, pp. 16, 19, 20, pl. 5, fig. 4.
Cistenides granulata Malmg., pp. 3, 6, 12, 23, (granulatus).
=[Pectinaria granulata.]
Ampharete gracilis Malmg., p. 17.
Ampharete Finmarchica Malmg., p. 17.
Ampharete arctica Malmg., pp. 16, 19.
Amphicteis Gunneri Malmg., pp. 17, 23.
Amphicteis Sundevalli Malmg., p. 19.
Samytha sexcirrata Malmg., pp. 17, 43.
Samythella elongata V., pp. 17, 22 (descr.).
Amage auricula Malmg., p. 23.
Melinna cristata Malmg., pp. 17, 23.
Amphitrite cirrata Müller, p. 17.
Amphitrite Grönlandica Malmg., p. 20.
Pista cristata Malmg., pp. 17, 23, pl. 4, fig. 2.
Thelepus cincinnatus Malmg., pp. 6, 8, 12, 20.
Grymæa spiralis V., pp. 17, 23, 44 (descr.), pl. 4, fig. 1.
Terebellides Stroemi Sars, pp. 17, 19, 23.
Polycirrus, sp., pp. 17, 19. =[Polycirrus phosphoreus V., 1879.]
Potamilla oculifera, pp. 6, 12. =Potamilla reniformis.
Potamilla neglecta, pp. 6, 12, 17, 44.

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Sabella pavonia (?), p. 17.
Chone infundibuliformis Kröyer, p. 20.
Protula media Stimp., pp. 17, 46, pl. 6, fig. 1.
Protula borealis (?), pp. 17, 23, 46. ?=Protula media, var.
Spirorbis valida V., sp. nov., pp. 12, 44, (validus).
=Spirorbis validus V.
Spirorbis nautiloides (?) Lamk., pp. 6, 8, 20, 45 (descr.), pl. 4, fig. 4.
=Spirorbis Stimpsoni V.
Spirorbis lucidus Mörch, p. 12.
Spirorbis quadrangularis Stimp., p. 21.
Phascolosoma boreale Kef., pp. 17, 23, 47.
=[Phascolosoma eremita Sars.]
Phascolosoma cæmentarium Quatr., pp. 6, 12, 17, 19, 23, 46.
=[Phascolion strombi Theel.]
Phascolosoma tubicola V., pp. 11, 17, 23, 46.
=[Phascolion tubicola V.]
Chætoderma nitidulum Lov., p. 17, pl. 8, figs. 3, 4.

1874.—A. E. Verrill. *Notice of some Dredgings made near Salem by Dr. A. S. Packard, Jr., and C. Cooke, in* 1873.<*Sixth Annual Report of the Peabody Academy of Science, Salem, Mass.*

List of Species.

Lepidonotus squamatus, p. 60.
Harmothoë imbricata, p. 60.
Nephthys ingens, p. 60. =Nephthys incisa Mgn.
Amphitrite brunnea, p. 60.
Cistenides Gouldii, p. 60. =[Pectinaria Gouldii V.]
Spirorbis lucidus, p. 60.
Phascolosoma cæmentarium, p. 59. =[Phascolion strombi Theel.]

1875.—A. E. Verrill. *Brief Contributions to Zoology from the Museum of Yale College. No. XXXII.—Results of Dredging Expeditions off the New England Coast, in* 1874.<*American Journal of Science and Arts, vol. ix,* [*pp.* 411–415], *June,* 1875.

This contains an account of the dredgings made in the Gulf of Maine, on Jeffrey's Bank, Jeffrey's Ledge, Cashe's Ledge, etc., by Messrs. A. S. Packard, Jr., Richard Rathbun and C. Cooke, on the Coast Survey steamer "Bache," Capt. Platt, commander, for the U. S. Fish Commission. The list included only species additional to those previously dredged in the same region.

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List of Species.

Euphrosyne borealis, p. 415.
Eunoa nodosa, p. 415.
Lagisca rarispina, p. 414.
Nephthys circinata V., p. 414.
Ancistria capillaris, p. 415. =[Notomastus capillaris V., 1879.]
Amphitrite Grayi, p. 415.
Amphitrite intermedia, p. 415.
Grymæa spiralis V., p. 414.
Sabella neglecta (?), p. 414. =Potamilla neglecta Mgn.
Spirorbis valida V., p. 414 (validus).

1875.—A. E. Verrill. *Brief Contributions to Zoology from the Museum of Yale College. No. XXXIII.—Results of Dredging Expeditions off the New England Coast, in* 1874. [Nos. 2, 3].<*American Journal of Science and Arts, vol. x*, [*pp.* 36–43, *July*, 1875; *pp.* 196–202, *pl. iii, iv, Sept.*, 1875].

These articles contain an account of the dredgings made in the eastern part of Long Island Sound, Fisher's Island Sound, near Block Island, off Stonington, Conn., etc., with tables of localities and temperatures, and lists of species added to the fauna.

List of Species.

Sthenelais, sp. nov., p. 39. =Sthenelais Emertoni V.
Pholoë minuta, p. 39.
Nephthys cæca, p. 39.
Phyllodoce Grœnlandica, p. 39.
Proceræa gracilis, p. 39.
Eusyllis lucifera V., sp. nov., p. 39. =[Odontosyllis lucifera V., 1879.]
Syllis pallida V., sp. nov., p. 39, pl. 3, fig. 6.
Syllis, sp., p. 39.
Lumbriconereis obtusa V., sp. nov., p. 39. (nom. preoc.) =[Lumbrinereis hebes V., 1879.]
Lumbriconereis acuta V., sp. nov., p. 39, pl. 3, fig. 5. =Lumbrinereis acuta V.
Ophelia denticulata V., sp. nov., p. 39.
Arenicola marina, p. 39.
Trophonia aspera, p. 39.
Brada, sp., p. 39.
Polydora, sp., p. 39. =Dipolydora concharum V., 1879.
Polydora, sp., p. 39. =[Polydora gracilis V., 1879.]

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Praxilla, sp., p. 39.
Ancistria capillaris, p. 39. =[Notomastus capillaris V., 1879.]
Cistenides granulatus, p. 39. =[Pectinaria granulata.]
Thelepus cincinnatus, p. 39.
Polycirrus, sp., p. 39. =[Polycirrus phosphoreus V., 1879.]
Pista cristata, p. 40.
Chone, sp., p. 40.
Filigrana implexa, p. 40.
Spirorbis nautiloides? Lam., p. 40.
=Spirorbis Stimpsoni V., 1879.
Spirorbis, sp., p. 40.

1876.—F. M. Trumbull. *On the Anatomy and Habits of Nereis virens.* <*Trans. Conn. Acad.*, *iii*, [*pp.* 265–280, *pl.* 42–44]. *August*, 1876.

This paper includes only the species named in the title. The sexual differences, in this species, are here first described and illustrated. The anatomical details relate chiefly to the external organs and to the circulatory and digestive systems.

1876.—W. C. M'Intosh, *in Jeffreys and Carpenter. The Valorous Expedition. Preliminary Report of the Biological Results of a Cruise in H. M. S. 'Valorous' to Davis Strait in* 1875.<*Proceedings of the Royal Society*, *xxv*, [*pp.* 177–237]. *London*, 1876. [*Annelids*, *pp.* 215–222.]

Lists of the species identified by Mr. M'Intosh, from the collection made in the Gulf of St. Lawrence by Mr. Whiteaves, are given on p. 222, and compared with those dredged by the Valorous Expedition, off the coast of Greenland. No special depths or localities are given for the Gulf of St. Lawrence species. Part of the species here enumerated from the Gulf of St. Lawrence are not included in the lists previously published. [See pp. 298, 306, 307, above.] On the other hand, some of those in the former lists were omitted from this.

List of Species not included in former reports.

Leanira hystricis. ?=L. Yhleni?, see p. 307.
Nephthys incisa.
Phyllodoce Grönlandica.
Nereis pelagica.
Onuphis sicula.

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Glycera capitata. =Rhynchobolus capitatus.
Scoloplos armiger.
Ophelia limacina.
Scalibregma inflatum.
Eumenia crassa.
Scolecolepis cirrata.
Prionospio Steenstrupi.
Capitella capitata. (Doubtful).
Maldane Sarsi.
Nicomache lumbricalis.
? Axiothea catenata.
Owenia filiformis.
Cystenides hyperborea (Cistenides Mgn.) =Pectinaria hyperborea.
Artacama proboscidea Mgn.
Sabella saxicava ?
Chone infundibuliformis.
Sternaspis.

The following species, enumerated in this paper, are included in the lists published previously (see pp. 298, 306, 307): Aphrodita aculeata, Lætmonice filicornis, Nychia cirrosa, N. Amondseni, Ennoa Oerstedi, Eupolynoë occidentalis, Lepidonotus squamatus, Lagisca rarispina, var., Malmgrenia Whiteavesii, Antinoë Sarsi, Nemidia canadensis, N. Lawrencii, Polynoë Gaspéensis, Sthenelais limicola, Leanira tetragona, Pholoë minuta, Nothria conchylega, Lumbriconereis fragilis, Goniada maculata, Ephesia gracilis, Ammotrypane aulogaster, Trophonia plumosa, Praxilla gracilis, Thelepus circinnatus (cincinnatus), Terebellides Strömii, Sabella pavonia.

1879.—A. E. Verrill, *in Kumlin. Contributions to the Natural History of Arctic America, made in connection with the Howgate Polar Expedition*, 1877–78. *Annelids, pp.* 141–143. < *Bulletin U. S. National Museum, No.* 15, 1879.

The species included in this paper were all from Cumberland Gulf and Cumberland Sound. They were mostly collected on the shores, between tides.

List of Species.

Harmothoë imbricata Mgn., p. 141.
Nereis pelagica Linné, p. 141.
Phyllodoce Grönlandica Œrsted ?, p. 141.

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Syllis, sp.,
Cistenides granulata Mgn., p. 141. =[Pectinaria granulata].
Thelepus cincinnatus (Fabr.) V., p. 141.
Spirorbis lucidus Mörch, p. 142.
Spirorbis quadrangularis Stimp., p. 142.
Phascolosoma margaritaceum (Sars) Kor. & Dan. (?), p. 142.

1879.—H. E. Webster. *On the Annelida Chætopoda of the Virginian Coast* [*pp.* 1–69, *plates* 1–11].<*Transactions of the Albany Institute, vol. ix, pp.* 202–269, *Jan.*, 1879.

The species enumerated were from Northampton Co., Virginia, (eastern shore), and are all shallow water and shore species.

List of Species belonging to the New England Fauna.

Lepidonotus squamatus Kinbg., p. 204 [4], pl. 1, figs. 1–5.
Lepidonotus squamatus, var. angustus, p. 205 [5].
Lepidametria commensalis, n. sp., p. 210 [10], pl. 3, figs. 23–31.
Sthenelais picta Verrill, p. 213.
Nephthys ingens Stimpson, p. 213. =Nephthys incisa Mgn.
Nephthys picta Ehlers, p. 214.
Podarke obscura Verrill, p. 216.
Syllis fragilis, sp. nov., p. 217, pl. 4, figs. 42, 43.
Sphærosyllis fortuita, sp. nov., p. 221, pl. 4, figs. 44–48.
Pædophylax dispar, sp. nov., p. 223, pl. 4, fig. 49, pl. 5, figs. 50–55.
Nereis virens Sars, p. 235.
Nereis limbata Ehlers, p. 235, pl. 6, figs. 70–75.
Diopatra cuprea Clpd., p. 236.
Marphysa sanguinea Quatr., p. 236, pl. 6, figs. 76–80, pl. 7, figs. 81–83.
Drilonereis longa, sp. nov., p. 240, pl. 7, figs. 84–88.
Lumbriconereis tenuis V., p. 241. (Lumbrinereis.)
Arabella opalina Verrill, p. 242.
Staurocephalus pallidus Verrill, p. 242 (descr.)*
Rhynchobolus Americanus Verrill, p. 245.
Spiochætopterus oculatus, sp. nov., p. 47, pl. 8, figs. 98–102.
Anthostoma robustum V., p. 258. =[Scoloplos robustus V.]
Anthostoma fragile V., p. 258. =[Scoloplos fragilis V.]

* The supposed error in the original measurement of this species, referred to by Professor Webster, is only apparent, owing to the fact that the specimens of this species contract excessively when put into alcohol. My original measurements were from the living specimens, and are correct. Still larger specimens have since been obtained.

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Cirratulus grandis V., p. 258.
Cymenella torquata V., p. 258.
Maldane elongata V., p. 259.
Sabellaria varians, sp. nov., p. 259. =Sabellaria vulgaris V
Amphitrite ornata V., p. 262.
Scionopsis palmata V., p. 262.
Pista cristata Mgn., p. 263. (Doubtful.)
Lepræa rubra V., p. 263.
Polycirrus eximius V., p. 263.
Enoplobranchus sanguineus Verrill.
Sabella microphthalma V., p. 265.
Hydroides dianthus Verrill, p. 266.

The following additional species, which were described, have not yet been observed on the New England coast:

Lepidonotus variabilis, sp. nov.; Antinoë parasitica, sp. nov.; Phyllodice fragilis (Phyllodoce), sp. nov.; Eumida maculosa, sp. nov.; Syllis gracilis Grube; Odontosyllis? fulgurans Clapd.; Autolytus hesperidum Clapd.; Proceræa tardigrada, sp. nov.; Proceræa? cœrulea, sp. nov.; Nereis irritabilis, sp. nov.; Nereis Dumerillii Aud. and Edw.; Staurocephalus sociabilis, sp. nov.; Trophonia arenosa, sp. nov.; Nerine heteropoda, sp. nov.; Polydora hamata, sp. nov.; Polydora cæca, sp. nov.; Aricia rubra, sp. nov.; Aricidea fragilis, sp. nov.; Ancistria minima Quatr.; Pectinaria (Lagis) dubia, sp. nov.; Melinna maculata, sp. nov.; Lysilla alba, sp. nov.; Potamilla tortuosa, sp. nov.; Cabira incerta, gen. et sp. nov.; Phronia tardigrada, gen. et sp. nov.

1879.—A. E. Verrill. *Preliminary Check-list of the Marine Invertebrata of the Atlantic Coast, from Cape Cod to the Gulf of St. Lawrence.* [*Prepared for the United States Commission of Fish and Fisheries.*] *Author's Edition, published at New Haven, June,* 1879. [*Annelida, pp.* 7–11, 32, *printed August,* 1878, *to April,* 1879.]

This list includes a few species not previously recorded from the New England coast. The rest are contained in the lists already given, and are, therefore, omitted. The total number of polychætous Annelids included in this check-list is 195; of Gephyræa 10. Several new species, included in the check-list, were described in the paper next to be noticed. Although the latter was in type before the printing of the check-list, it was published some months later.

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List of described Species not previously recorded.

Evarne impar, p. 7.
Nephthys longisetosa Œrs. (*non* Mgn.), p. 7.
Nephthys paradoxa Malm., p. 7.
Autolytus, sp., p. 8. =[Autolytus Alexandri Mgn.]
Eusyllis monilicornis Mgn., p. 8.
Nereis zonata Mgn., p. 8.
Scoloplos armiger Blainv., p. 9.
Ophelia limacina Mgn., p. 9.
Lysippe lobata Mgn., p. 10.
Leœna abranchiata Mgn., p. 10.
Lanassa Nordenskiœldi Mgn., p. 11.
Artacama proboscidea Mgn., p. 10.
Euchone tuberculosa Mgn., p. 10.
Chone Duneri Mgn., p. 10.

List of New Species.

Sigalion, sp., p. 7. =Sigalion arenicola V., p. 32.
Sthenelais, sp., p. 7. =Sthenelais gracilis V., p. 32.
Lysilla, sp., p. 10.
Polycirrus, sp., p. 10. =[Polycirrus phosphoreus V.]
Lætmatonice armata V., sp. nov., p. 11.
Eunoa spinulosa V., p. 11.
Sthenelais Emertoni V., p. 11.
Heterocirrus fimbriatus V., 11.
Polydora concharum V., p. 32. =[Dipolydora concharum V.]
Thalassema viridis V., p. 11.
Priapulus pygmœus V., p. 11.

The following changes in nomenclature were introduced in the Check-List:

Arabella opalina V., p. 8. =Lumbriconereis opalina V., 1873.
Enoplobranchus sanguineus V., p. 10.
=Chætobranchus sanguineus V., 1873.
Hydroides dianthus V., p. 11. =Serpula dianthus V., 1873.

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1879.—A. E. Verrill. *Notice of Recent Additions to the Marine Invertebrata of the Northeastern Coast of America, with Descriptions of new Genera and Species, and Critical Remarks on others. Part I.—Annelida, Gephyræa, Nemertina, etc.* <*Proceedings of United States National Museum, vol. ii* [*pp.* 165–205. *Nov.*, 1879].

List of Species.

Sthenelais gracilis V., sp. nov., p. 166.
Sthenelais Emertoni V., sp. nov., p. 166.
Sthenelais picta V., p. 167 (descr.)
Sigalion arenicola V., sp. nov., p. 167.
Lætmatonice armata V., sp. nov., p. 168.
Eunoa spinulosa V., sp. nov., p. 169.
Autolytus ornatus V., sp. nov., p. 170 (name preoccupied).
=[Autolytus varians V., nom. nov.]
Odontosyllis lucifera V., p. 170 (Eusyllis lucifera V., 1875).
Pædophylax longiceps V., sp. nov., p. 170.
Nereis alacris V., sp. nov., p. 171.
=[Nereis megalops V., nereis-form.]
Nereis megalops V., p. 172 (Nectonereis megalops V., 1873.)
Ceratocephale Websteri V., sp. nov., p. 172.
Lumbrinereis hebes V., p. 174 (L. obtusa V., 1875, name preoccupied).
Goniada gracilis V., p. 174 (Eone gracilis V., 1873).
Polydora gracilis V., sp. nov., p. 174.
Polydora concharum V., sp. nov., p. 174.
=[Dipolydora* concharum V.]
Spio limicola V., sp. nov., p. 176.
Spiophanes tenuis V., sp. nov., p. 176. =[Prionospio tenuis V.]
Heterocirrus fimbriatus V., sp. nov., p. 177.
Dodecaceria concharum, p. 178 (descr.)
Praxillura ornata V., gen. et sp. nov., p. 179.
Maldane filifera V., sp. nov., p. 179.
Notomastus gracilis V., sp. nov., p. 180.
Polycirrus phosphoreus V., sp. nov., p. 181.

* The genus *Dipolydora* is established for this species. It differs from typical *Polydora*, especially in having four groups of setæ on each side of the large, specialized, fifth segment, showing plainly that this is composed of at least two united segments. The caudal segment bears four flattened lobes, instead of a sucker. The body is very long, and consists of an unusually large number of segments. The structure, in general, is modified to suit the habit of living in galleries excavated in shells or limestone.

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Trichobranchus glacialis, p. 181.
Spirorbis Stimpsoni V., sp. nov., p. 181.
Tomopteris Smithii V., sp. nov., p. 182.
Priapulus pygmæus V., sp. nov., p. 182.
Thalassema viridis V., sp. nov., p. 183.

1880.—H. E. Webster. *Annelida Chætopoda of New Jersey*, [*pp.* 1–28, *plates not issued*].<*Thirty-second Annual Report on the New York State Museum of Natural History*, [*dated* 1879].

Although put in type in 1879, this paper was not actually published until 1880, and the plates that were prepared for it have not yet been published. To the author I am indebted for an advance copy, received in the autumn of 1880.

The specimens were mostly from Great Egg Harbor, N. J., but some from South Norwalk, Conn., were also included.

The new species, not in italics, have not yet been detected on the New England coast.

List of the Species.

Lepidonotus squamatus Kinbg., p. 1.
Lagisca impatiens, sp. nov., p. 2.
Lepidametria commensalis Webs., p. 3.
Sthenelais picta Verrill, p. 3.
Nephthys incisa Malmg., p. 4.
Nephthys picta Ehlers, p. 4.
Anaïtis speciosa, sp. nov., p. 4, [pl. 1, figs. 8, 9].
Phyllodoce arenæ, sp. nov., p. 5, [pl. 2, figs. 10–12].
Eumida maculosa Webs., p. 6.
Eulalia? annulata Verrill, p. 6.
Eteone alba, sp. nov., p. 6, [pl. 2, figs. 13–16].
Eteone limicola Verrill, p. 7.
Podarke obscura Verrill, p. 7, [pl. 2, figs. 17, 18].
Podarke luteola, sp. nov., p. 7, [pl. 2, figs. 19, 20].
Syllis gracilis Grube, p. 9.
Odontosyllis? fulgurans Clpd., p. 9.
Grubea tenuicirrata Clpd., p. 9.
Pædophylax dispar Webs., p. 10.
Autolytus hesperidum Clpd., p. 10.
Nereis limbata Ehlers, p. 11, [pl. 3, figs. 21, 22].

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Nereis Culveri, sp. nov., p. 11, [pl. 3, figs. 23–30; pl. 4, figs. 31, 32].
Nereis tridentata, sp. nov., p. 13, [pl. 4, figs. 33–40].
Diopatra cuprea Clpd., p. 15.
Marphysa sanguinea Quatr., p. 15.
Drilonereis longa Webs., p. 16.
Lumbriconereis tenuis Verrill, p. 16. (Lumbrinereis.)
Arabella opalina Verrill, p. 16.
Staurocephalus pallidus Verrill, p. 16.
Rhynchobolus Americanus Verrill, p. 16.
Rhynchobolus dibranchiatus Verrill, p. 17.
Goniada solitaria, sp. nov., p. 17, [pl. 4, figs. 41, 42; pl. 5, figs. 43, 44.]
Arenicola? cristata Stimpson, p. 17.
Trophonia affinis Verrill, p. 18.
Spiochætopterus oculatus Webs., p. 18.
Nerine agilis Verrill, p. 18.
Scolecolepis viridis Verrill, p. 18 (descr.).
Scolecolepis tenuis Verrill, p. 18.
Spio setosa Verrill, p. 19.
Polydora hamata Webs., p. 19.
Polydora ligni, sp. nov., p. 19, [pl. 5, figs. 45–47].
Streblospio Benedicti, gen. et sp. nov., p. 20, [pl. 5, figs. 48–50].
Anthostoma fragile Verrill, p. 21. =[Scoloplos fragilis V.]
Cirratulus grandis Verrill, p. 22.
Cirrhinereis fragilis Qtrfg., p. 22.
Notomastus filiformis Verrill, p. 23, [pl. 5, figs. 51–54].
Notomastus luridus Verrill, p. 23.
Clymenella torquata Verrill, p. 23.
Maldane elongata Verrill, p. 24.
Praxilla elongata, sp. nov., p. 24 (? Praxillella).
Praxilla elongata, var. Benedicti Webs., p. 25.
Paraxiothea latens, gen., et sp. nov., p. 25, [pl. 7, figs. 62–66].
Sabellaria varians Webs., p. 26. =Sabellaria vulgaris V.
Cistenides Gouldii Verrill, p. 27. =[Pectinaria Gouldii V.]
Sabellides oculata, sp. nov., p. 27, [pl. 7, figs. 67–69].
Amphitrite ornata Verrill, p. 27.
Scionopsis palmata Verrill, p. 28.
Polycirrus eximius Verrill, p. 28.
Sabella microphthalma Verrill, p. 28.
Hydroides dianthus Verrill, p. 28.

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1880.—J. H. Emerton. *Life on the Seashore, or Animals of our Coasts and Bays.* [138 *pp.*, 12*mo.*, 151 *cuts.*] *Geo. A. Bates, Salem, Mass.*, 1880.

This work contains brief popular accounts of several annelids, with notes on their habits. Many of the figures are the same as those in the Report on the Invertebrata of Vineyard Sound, etc.

List of Species.

Nereis virens, pp. 23, 122, figs. 24–26, (after Turnbull.)
Clymenella torquata, pp. 24, 121, fig, 27.
Cistenides Gouldii, pp. 24, 122, fig. 28. =[Pectinaria Gouldii.]
Polycirrus eximius, p. 25, fig. 29.
Lepidonotus squamatus, p. 26, fig. 30 (original).
Autolytus, sp., p. 86, fig. 106 (original). =Autolytus cornutus, male.
Trophonia affinis, pp. 116, 121, fig. 142.
Diopatra cuprea, pp. 117, 122, fig. 143.
Amphitrite ornata, pp. 118, 123, fig. 134.
Nephthys ingens, p. 122. =Nephthis incisa.
Nephthys cœca, p. 122.
Sternaspis fossor, p. 135, fig. 156.
Aphrodite aculeata, p. 135.

1880.—A. E. Verrill. *Notice of Recent Additions to the Marine Invertebrata of the Northeastern Coast of America, with Descriptions of New Genera and Species and Critical Remarks on others. Part II.—Mollusca, with notes on Annelida, Echinodermata, etc., collected by the United States Fish Commission.* <*Proceedings of United States National Museum, iii, pp.* 356–405, *Dec.*, 1880.

This paper is devoted almost entirely to the mollusca, but in the introductory remarks some of the remarkable annelids recently discovered in the deep water (100–500 fathoms), off the southern coast of New England, are mentioned, in connection with their peculiar tubes, which occurred in vast numbers. Two of the new species are briefly described.

List of Species.

Hyalinœcia artifex, sp. nov., p. 357.
Nothria, sp., p. 358.
Leodice polybranchia, sp. nov., p. 358.

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The notices of the following papers were omitted from their proper places. The first should have been inserted on page 291; the second, on page 296.

1860.—J. W. Dawson. *On the Tubicolous Marine Worms of the Gulf of St. Lawrence.* < *Canadian Naturalist and Geologist, v,* [*pp.* 24–30, *with cuts*] *Feb.*, 1860.

List of Species.

Spirorbis spirillum, p. 25. =Spirorbis borealis.
Spirorbis sinistrorsa, p. 25. ?=Spirorbis lucidus, juv.
Spirorbis nautiloides, p. 25. ?=Spirorbis Stimpsoni V.
Spirorbis carinata, p. 26.
Spirorbis vitrea, p. 26.
Spirorbis cancellata, p. 27, fig. 1, *a–e.*
Spirorbis granulata, p. 27.
Spirorbis porrecta, p. 28. =Spirorbis lucidus.
Serpula (Vermilia) serrula (Stimp.), p. 29 fig. 2, *a–c*, lateral chambers of shell.
Pectinaria Grœnlandica? p. 29. ?=[Pectinaria granulata.]

1868.—A. S. Packard, Jr. *A few Sea Worms.* < *Amer. Naturalist, vol. ii,* [*pp.* 267–275,] *July*, 1868.

This is a popular account of the habits and structure of a few forms, native and foreign. The New England species mentioned are as follows:

No name, p. 270, fig. 1. =Cirratulus cirratus.
Hæmatorrhæa, p. 270, fig. 2. =Polycirrus eximius, young.
Amphitrite cirrata, p. 271, fig. 3, (after Malmgren.)
Pectinaria hyperborea (non Mgn.), p. 274, fig. 4. =Pectinaria granulata, in part.
Nereis denticulata, p. 274. =Nereis pelagica.
Nereis grandis, p. 275. =Nereis virens.

[Printed, October 17, 1881.]

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EXPLANATION OF PLATES.

[VOLUME IV, ARTICLE VIII.]

My thanks are due to Professor S. F. Baird for the use of most of the drawings for the following plates. They were made for the use of the U. S. Fish Commission by Messrs. J. H. Emerton and J. H. Blake, under my direction.

PLATE III.

Figure 1.—*Aphrodite aculeata* Linné. From a small, living specimen, enlarged two diameters. By J. H. Emerton.

PLATE IV.

Figure 1.—*Lepidonotus squamatus* Leach. Head and anterior part of the body, with the proboscis protruded; one of the anterior scales has been removed to partially expose the head. Much enlarged.

Figure 1*a*.—The same. End view of the proboscis.

Figure 2.—*Lepidonotus sublevis* V. Dorsal view, enlarged nearly two diameters.

Figure 3.—*Marphysa Leidyi* Quatr. Head and anterior part of body, dorsal view. Much enlarged.

Figure 4.—*Arabella opalina* V. Anterior part of the body, enlarged five diameters.

Figure 4*a*—The same. One of the parapodia, much enlarged.

Figure 5.—*Diopatra cuprea* (Bosc.). Head and anterior part of body; side view, showing the gills partially expanded. Enlarged two diameters.

Figure 5*a*.—The same. Ventral view, showing the pharynx everted so as to expose the jaws. Enlarged about two diameters.

Drawn from preserved specimens, by J. H. Emerton.

PLATE V.

Figure 1.—*Nereis megalops* V. (Nereis-form = *N. alacris* V.). Dorsal view of the head and anterior part of body, and of the caudal segments and cirri (1*a*). Enlarged about three diameters, from a living specimen, Vineyard Sound, 1875.

Figure 2.—The same. (Female, Heteronereis-form.) Dorsal view of head and anterior segments; the posterior dorsal tentacular cirri are broken off. Enlarged about five diameters, from a living specimen.

Figure 3.—*Nereis limbata* Ehl. Dorsal view of the head and anterior segments, enlarged about four diameters, from a living specimen, Vineyard Sound, 1875.

Figure 3*a*.—The same. Caudal segments and cirri.

Figure 4.—*Phyllodoce catenula* V. Dorsal view of the entire worm, except the caudal segments, which are broken off. Enlarged about three diameters, from life.

Figure 4*a*.—The same. Head and protruded proboscis.

Figure 5.—*Phyllodoce Grönlandica* Œrsted. Head and anterior segments, dorsal view. Enlarged about four diameters, from a living example, Eastport, Me.

Figure 6.—*Eulalia pistacia* V. Head and anterior segments, dorsal view. Enlarged about eight diameters, from life.

Figure 6*a*.—The same. Caudal segments and cirri.

Figures 1 and 3 are by J. H. Blake; the rest by J. H. Emerton.

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PLATE VI.

Figure 1.—*Harmothoë imbricata.* Dorsal view, natural size.

Figure 1*a*.—The same. Portion of a scale, enlarged twelve diameters.

Figure 1*b*.—The same. A small part of a scale, enlarged 190 diameters.

Figure 1*c*.—The same. One of the parapodia bearing dorsal cirri, enlarged ten diameters: *a′* upper, *a″* lower acicula; *dc* dorsal cirrus; *vc* ventral cirrus; *ds* dorsal setæ; *vs*, *vs′*, *vs‴* ventral setæ.

Figure 1*d*.—The same. Setæ, enlarged 200 diameters: *d*, dorsal seta; *v*, *v′*, ventral setæ, side views; *v″*, the same, front view.

Figure 2*a*.—*Lepidonotus squamatus.* A scale, enlarged fourteen diameters.

Figure 2*b*.—The same. Portion of a scale bearing the smaller tubercles, more enlarged.

Figure 3*a*.—*Lepidonotus sublevis* V. A scale, enlarged twelve diameters.

Figure 3*b*.—The same. Portion of a scale, enlarged sixty diameters.

Figure 3*c*.—The same. One of the parapodia, much enlarged.

Figure 3*d*.—The same. Setæ, much enlarged: *d*, dorsal seta; *v*, *v′*, ventral setæ, side and front views.

Figure 4.—*Lepidonotus squamatus ?*. (Young). Head and anterior part of body, with scales removed, much enlarged: *h*, cephalic lobe or head, with eyes; *a*, median antenna; *a′*, lateral antennæ; *c*, *c′*, upper and lower tentacular cirri; *p*, *p*, palpi; *dc*, dorsal cirri; *vc*, ventral cirri; *s*, dorsal and *s′*, ventral fascicles of setæ; *e*, *e*, tubercles to which the scales (elytra) were attached.

Figure 4*a*.—The same. A scale, enlarged eighteen diameters.

Figure 4*b*.—The same. One of the parapodia, enlarged fifty-six diameters; lettering as in 1*e*.

Figure 4*c*.—The same. Setæ, much enlarged; *d*, dorsal seta; *v*, ventral seta.

Figure 5.—*Polynoë* (*Eunoa*) *Acanellæ* Verrill. Head and anterior part of body, enlarged.

Figure 5*a*.—The same. Part of a scale, enlarged sixty diameters.

Figure 6.—*Lætmatonice armata* V. Head and anterior part of body, enlarged four diameters, with some of the anterior scales removed to show the head. Lettering as in fig. 4: *e″*, *e‴*, *e⁗*, elytra.

Figure 6*a*.—The same. Ventral parapodium, showing (*vs*) four of the peculiar setæ, enlarged ten diameters.

Figure 6*b*.—The same. Barbed setæ of the dorsal fascile; *d*, side view; the same, seen edgewise.

Figure 7.—*Sthenelais picta* V. Setæ of ventral fascicle, enlarged 200 diameters: *a*, semi-whorled spinulose seta from upper part of ventral fascicle; *b*, *b′*, two of the compound bidentate setæ with spinose shafts; *c*, *c′*, stouter bidentate setæ, without spinules.

The drawings are by J. H. Emerton, mostly from preserved specimens.

PLATE VII.

Figure 1.—*Sthenelais Emertoni* V. Dorsal view of head, much enlarged. Specimen (figs. 1–1*s*) from Narragansett Bay, station 770, 1880. From a living specimen.

Figure 1*a*.—The same. Ventral view of the head and anterior appendages.

Figure 1*b*.—The same. Side view of head and anterior appendages.

Figure 1*c*.—The same. Side view of middle region of body.

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Figure 1*d*.—The same. Posterior segments and caudal cirri, enlarged eight diameters.

Figure 1*e*.—The same. Transverse section of a segment.

Figure 1*g*.—The same. Setæ from the ventral fascicle, much enlarged: *v*, one with jointed and hooked tip; *v'*, one with jointed tip, acute at the end.

Figure 1*s*.—The same. A scale, enlarged.

Figure 2.—The same. Several forms of setæ from the ventral fascicle: *a*, *a'*, simple acute setæ, with partial whorls of spinules; *b*, one with short, hooked terminal piece, bidentate at tip and not distinctly jointed; *c*, *c'*, setæ with shorter and longer, jointed, acute tips.

Figure 3.—*Sthenelais picta* V. Dorsal view of head and anterior part of body, enlarged four diameters. From an alcoholic specimen.

Figure 3*a*, 3*b*—The same. Dorsal and ventral views of the posterior segments and caudal cirri of a living specimen, much enlarged. Newport, low water, Aug. 8, 1880.

Figure 3*c*.—The same. A scale, enlarged.

Figure 3*d*.—Portion of the edge of the same scale, more enlarged.

Figure 4.—*Pholoë minuta*. Head and anterior segments, much enlarged. Newport, 1880.

Figure 4*a*, 4*b*.—The same. Scales, much enlarged.

Figure 4*c*.—The same. One of the parapodia, much enlarged.

Figure 5.—*Sigalion arenicola* V. Head and anterior segments, dorsal view. The scales of the first pair have been removed; enlarged eight diameters, from an alcoholic specimen.

Figure 5*a*.—The same. One of the scales, enlarged twenty-five diameters.

Figure 5*b*.—The same. One of branched appendages from the outer border of the scale, much enlarged.

Figure 5*c*.—The same. One of the parapodia, enlarged: *ds*, dorsal setæ; *a*, simple spinulated setæ of the ventral fascicle; *b*, *c*, different forms of compound setæ, with jointed, acute tips; *d*, *e*, compound setæ with bidentate hooked tips; *e'*, compound setæ with jointed tips, hooked at end; *v*, *v'*, different forms of capillary setæ in lower part of ventral fascicle, some with joints distally. Original specimen from Nantucket Sound, 1875.

Figure 5*d*.—The same. Some of the setæ more enlarged, lettered as in fig. 5*c*, enlarged 250 diameters.

Figure 6.—*Polynoë* (*Eunoa*) *spinulosa* V. Portion of the edge of one of the scales of the original specimen, from off Sable I., N. S., enlarged seventy-five diameters.

The drawings are mostly from preserved specimens, by J. H. Emerton. The appendages and setæ are camera-drawings from specimens mounted in glycerine-jelly.

Plate VIII.

Figure 1.—*Travisia carnea* V. Dorsal view, enlarged nine diameters.

Figure 1*a*.—The same. Side view of one of the middle segments, more enlarged.

Figure 2.—*Arenicola marina* L. Side view, natural size.

Figure 3.—*Clitellio irrorata* V. Enlarged six diameters.

Figure 3*a*.—The same. One of the fascicles of setæ, much enlarged.

Drawn from life: figures 2–3*a*, by J. H. Emerton; 1, 1*a*, by J. H. Blake.

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PLATE IX.

Figure 1.—*Maldane elongata* V. Posterior segments and caudal appendage, side view, enlarged two diameters.

Figure 1*a*.—The same. Caudal appendage, end view.

Figure 2.—*Rhodine attenuata* V. Head and anterior segments, side view, enlarged six diameters.

Figure 3.—*Ophelia denticulata* V. Side view, enlarged one and one-half diameters.

Figure 4.—*Brada setosa* V. Dorsal view, enlarged three diameters.

Figure 5.—*Scalibregma brevicauda* V. Dorsal view, enlarged four diameters.

Figure 5*a*.—The same. Head and anterior segments, enlarged eight diameters.

Figure 6.—*Cirratulus cirratus*. Dorsal view, enlarged five diameters.

Figure 6*a*.—The same. Side view of head.

Drawn from living examples, by J. H. Emerton.

PLATE X.

Figure 1.—*Nicolea simplex* V. Side view; the posterior segments are omitted, enlarged three diameters.

Figure 2.—*Amphitrite brunnea* Stimp., from Eastport, Me. Side view, enlarged two diameters.

Figure 3.—*Dodecacerea concharum*. Enlarged five diameters.

Drawn from living specimens, by J. H. Emerton.

PLATE XI.

Figure 1.—*Thelepus cincinnatus* V., from Eastport, Me. Side view, slightly enlarged, from life.

Figure 2.—*Pista cristata* Mgn. Side view of head and anterior segments, enlarged four diameters.

Figure 3.—*Scionopsis palmata* V. Side view, enlarged about three diameters, from life.

Figures 4 and 4*a*.—*Enoplobranchus sanguineus* V. Branchial parapodia of two segments, enlarged.

Figure 4*b*.—The same. Tip of one of the branchial divisions, with setæ, much enlarged.

Figures 1 and 2 were drawn from living specimens, by J. H. Emerton; figure 3, from life, by J. H. Blake; figures 4–4*b* are camera-drawings from mounted specimens, by the author.

PLATE XII.

Figure 1.—*Odontosyllis lucifera* V. Dorsal view of the head and anterior part of the body, much enlarged, seen as a transparent object, under a compressor; from a camera-drawing of a living specimen: *p*, pharynx; *g*, stomach or gizzard.

Figure 1*a*.—The same. One of the setæ, greatly enlarged.

Figure 2.—*Pedophylax longiceps* V. Head and front part of body, and (*a*) caudal segments, ventral-view, much enlarged, from a camera-drawing of a living specimen.

Figure 3, 3*a*.—*Autolytus longigula* V., sp. nov. Dorsal view, from life, of a specimen taken at Newport, R. I. Head and anterior segments; 3*a*, caudal segments, enlarged about 20 diameters.

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Figure 3*b*.—The same. Dorsal views, seen as a transparent object, under a compressor, to show the pharynx and stomach. Setæ, etc., are omitted.

Figure 4.—*Autolytus cornutus* A. Ag. Male, from life. Ventral view of the anterior part of body, and of the caudal segments (*a*), much enlarged.

Figure 5.—*Autolytus* (*Proceræa*) *ornatus* V. (?). Male, from Wood's Holl, Mass. Dorsal view, from life, of the anterior and posterior (*a*) portions, more enlarged than fig. 4.

Figure 6.—*Autolytus cornutus*. Female. Ventral view of the head and anterior segments, including the first segment bearing long. capillary, sexual setæ, enlarged 14 diameters.

Figure 7.—*Autolytus varians* V., nom. nov., p. 320. Female. Dorsal view of the anterior segments, including three of those bearing sexual setæ, enlarged.

Figure 8.—*Autolytus Alexandri* Malmgren. Male. Dorsal view, enlarged 14 diameters. Specimen from Salem, Mass., in alcohol, taken in early spring: *a*, median, odd antenna; *a'*, pair of small frontal antennæ; *a''*, large forked palpi, or antennæ, characteristic of the male, in this genus; *tc*, upper and lower tentacular cirri, arising from the concealed buccal segment; *dc'*, large dorsal cirri, arising from the first body-segment (1), which, in this species, bears a fascicle of setæ, beneath (see fig. 8*a*); *dc*, dorsal cirri of segments 2 to 15; *s*, short setæ of the 14 anterior segments; *s''*, long sexual setæ, beginning on the 15th segment.

This may be the sexual form of *Stephanosyllis ornata* V.

Figure 8*a*.—The same. Ventral view, including only the head and 14 anterior segments. Lettering the same as in fig. 8.

Figure 8*b*.—The same. Female. Dorsal view, including the 14 anterior segments, enlarged 14 diameters. From Salem, Mass. Lettering as in fig. 8, but with the following additions: *e*, "epaulets" or lobes extending back from the head; *vc*, ventral tentacular cirrus; *a''*, the pair of long simple frontal antennæ.

Figure 8*c*.—The same. One of the short, compound setæ, with bidentate tip, from a short anterior segment. Greatly enlarged.

Figure 9.—*Autolytus Emertoni* V., sp. nov. Male. Dorsal view of the head. Enlarged 14 diameters. From an alcoholic specimen, taken at Salem, Mass., at the surface, in February, by J. H. Emerton. Lettering as in figs. 8, 8*a*.

Figure 9*a*.—The same. Male. Ventral view, showing the very large dorsal tentacular cirrus (*tc*) and the smaller ventral one (*vc*); the six anterior setigerous segments have short and nearly equal dorsal cirri, the first pair being the smallest and crowded beneath the large tentacular cirri.

Figure 9*b*.—The same. Female. Ventral view of a specimen from Salem, Mass., in alcohol, enlarged 14 diameters. Lettering as in 8*b*.

Figure 9*c*.—The same. Peculiar setæ of the short anterior segments.

Figure 10.—*Autolytus longisetosus*. Male. Dorsal view of the anterior part of a large alcoholic specimen from Salem, Mass., taken in February by J. H. Emerton, enlarged 14 diameters. Lettering as in figs. 8 and 8*a*.

Figure 10*a*.—The same. Male. Ventral view of the head and anterior segments. The ends of some of the appendages are omitted.

Figure 10*c*.—The same. One of the compound setæ of the anterior segments, greatly enlarged.

Figures 1 and 5 are by J. H. Blake; 2 and 7 by the author; the rest by J. H. Emerton.

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IX.—The North American Species of Conops. By S. W. Williston, M.A., M.D.

Of the fourteen species of *Conops* recorded in Baron Osten-Sacken's valuable catalogue of North American Diptera, but four are designated by the author as having been recognized, viz: *excisus* Wied., *genualis* Loew, *pictus* Fab., and *sagittarius* Say. As to the last, it is, however, most probable that the species really recognized was *tibialis* Say, our most common one, and which had been confounded by both Wiedemann and Loew, and re-described by both as *nigricornis* Wied., the real *sagittarius* being possibly described anew as *genualis* Loew, which I believe to be its synonym. *C. pictus* is unknown to me; its recognition must depend upon that of its synonym, *Ramondi* Bigot (teste Loew, vide Osten-Sacken's Catalogue), as the original description of Fabricius is wholly insufficient. In the present paper all of the recognizable descriptions, whose habitats are creditably referred to North America, have been determined with sufficient certainty, except *genualis* Lw., and *castanopterus* Lw., both of which I believe to be of doubtful value. *C. æthiops* Walker has been recognized with as much certainty as most of the descriptions of that author will admit of, while none of Macquart's very insufficiently described species have been recognized, nor the South American species identified by him as also pertaining to North America. *? Conops quadrimaculatus* Ashmead, is a syrphid, probably a *Bacha.*

The material herein described has been wholly derived from Mr. Burgess, to whom my sincerest thanks are due, and my own collections. Although by no means so large as I desired, it is, I trust, sufficient to remove many of the obstacles to the future study of our species. The indiscriminate description of new species, without a considerable knowledge of allied species, is here especially to be deprecated; and, owing to the great individual variation of color, and the paucity of plastic characters, large collections will be essential, eventually, to a complete and satisfactory knowledge.

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My knowledge of such variations I have endeavored to supplement by the study of the descriptions of foreign species, and I deem it worth the while to here give a translation of Dr. Loew's very pertinent remarks upon the value of the specific characters in this genus.

"Those specific characters have the greatest weight here, as elsewhere, that are based upon the differences in form; such are not always easy to find, yet the shape of the antennæ and antennal style, the structure of the face, the length of the proboscis, the structure of the abdomen and of the legs, and finally the neuration offers many good characters. Next in value to these characters are those derived from the yellow or white shimmering markings of the dorsum of the thorax, the pleuræ and the metanotum; so, also, from the form and boundaries of the coloration of the wings, when such is present. The presence or absence of yellow spots on the sides of the metanotum, and the color of the scutellum are also good characters. Finally, the color of the front is quite useful, when cautiously employed, as it is rather changeable in many species. Much less dependence is to be placed upon the characters derived from the dust-markings of the abdomen; least of all upon the coloring of the body in those species that are black and reddish brown, as sometimes they may be quite black, sometimes reddish brown, with more or less black, or sometimes indeed quite ferruginous." (Neue Beitr., i, p. 20.)

The following table of the American genera of Conopidæ, adapted from Schiner, with the aid of a considerable number of our own species, will be of service to many:

Third antennal joint with a terminal style; ocelli wanting. *Conops* Linn.
Third antennal joint with a dorsal, or sub-dorsal bristle:
  Proboscis bent only at the base. *Zodion* Lat.
  Proboscis bent at base and near the middle, the anterior part closely folding back:
    Anal cell, short, obtuse:
      Bristle of antennæ dorsal, proboscis not of unusual length. *Dalmannia* Rob. [Des.
      Bristle of antennæ near the end of third joint (sub-dorsal), proboscis very long, abdomen cylindrical. *Stylogaster* Mac.
    Anal cell, extending well toward the border of the wing, acute:
      Face much produced, inferiorly, the cheeks broader than the vertical diameter of the eyes, abdomen depressed, short. *Myopa* Fab.
      Face moderately produced inferiorly, cheeks not broader than the vertical diameter of the eyes; abdomen mostly cylindrical and moderately long. *Oncomyia* [Rob. Des.

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Schiner has divided the genus *Conops*, upon what appears to me insufficient grounds.* I have retained all our American species under the genus in its wider sense.

In the following table I have not attempted to include the species of Macquart, nor his identifications. I have added the original descriptions at the close of the article, although there is very little possibility of their ever being recognized. *C. pictus* F. is also not included in the table, but its description is given:

A.—Small cross-vein nearly opposite or before the tip of the costal vein and near the middle of the discal cell. **a.**

B.—Small cross-vein beyond the tip of costal vein and near the outer third of discal cell; third joint of antennæ much shorter than the second. **e.**

a.—Third joint of antennæ much shorter than the second. Face and cheeks yellow; facial grooves not darkened; front wholly black; wings brownish before, subhyaline behind, picture indistinct. *obscuripennis*, n. sp

—Third joint of antennæ nearly or quite as long as second, brown of anterior portion of wings with distinct outlines. **b.**

b.—Attenuated portion of antennal style very short, head comparatively narrow, proboscis very short, a hyaline crossband before the tip of the wing. Shining black, dust grayish, front blackish or black. *sylvosus*, n. sp.

—Attenuated portion of style quite long; proboscis considerably longer than the head; the brown reaches the whole length of the wing. **c.**

c.—Stripe of dust on upper half of pleura indistinct or without distinct borders. Front black, cheeks brown behind; proboscis about once and a half the length of head. (Dark colored species.) *bulbirostris* Lw.

—Stripe of dust on upper half of pleura as sharply defined as on the under half. **d.**

d.—Proboscis twice the length of head, stripe on upper half of pleura narrow. Dust on the sides of mentanotum much narrowed toward the disk; front mostly red, darkest above base of antennæ. (Reddish species.) *excisus* Wied.

—Proboscis scarcely once and a half the length of head, much swollen at the base; pleural stripe broad above. Dust broadly

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* Schiner's characters are as follows:

Schenkel nicht verdickt, wenn sie aber verdickt sind, dann ist die Verdickerung an der Basis nicht auffallend, sondern von da bis zur Spitze ein regelmässige; zweiter Hinterleibsring nicht auffallend schmäler und länger als die übrigen Ringe.—*Conops* Lin.

Schenkel an der basis plötzlich und in auffallender Weise verdickt, so dass die Verdickerung immer unregelmässig erscheint; zweiter Hinterleibsring auffallend schmäler und länger als die übrigen Ringe. *Physocephala* Sch.

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covering the sides of metanotum; front mostly yellow; face, cheeks, and under part of occiput yellow; facial grooves with a black spot on each side of middle. *xanthopareus*, n. sp.

e.—The brown of the wings fills out the discal cell to beyond the great cross-vein. **f.**

—Outer portion of discal cell from near the small cross-vein distinctly lighter or hyaline. **g.**

f.—Cheeks uniformly black; dust on the humeri indistinct; posterior orbits with a very narrow whitish line. (Black species.) *tibialis* Say.

—"Cheeks brownish yellow; posterior orbit with yellowish brown border; humeri dusted with golden yellow." *genualis* Lw.

—Cheeks with a yellow spot in the middle, brown in front and behind; humeri with distinct yellow dust. { *sagittarius* Say. { *castanopterus* Lw.

g.—Cheeks uniformly black or brown; basal joint of antennæ very short. **h.**

—Cheeks with a yellow spot in the middle; first joint of antennæ but little or not at all shorter than the third. **i.**

h—Front with deep black transverse and median divaricate stripes. (Dark-colored species.) *furcillatus*, n. sp.

—Front with very narrow reddish stripes or wholly yellow; dorsum of thorax with a broad, median, posteriorly abbreviated, black stripe. (Reddish species.) *Burgessi*, n. sp.

i.—Ultimate segment of fourth vein nearly twice as long as the penultimate; dorsum of thorax with a narrow, median, black stripe. (Reddish species.) *texanus*, n. sp.

—Ultimate segment of fourth vein but little or not at all longer than penultimate. **k.**

k.—Facial grooves and frontal stripes black. (Dark-colored species.) *marginatus* Say.

—Facial grooves but little or not at all darkened; frontal stripes nearly or quite obsolete; dorsum of thorax with three black stripes, narrowly separated or confluent, the median one more abbreviated behind. *affinis*, n. sp.

Conops obscuripennis, n. sp.

♂. Face and cheeks yellow, facial grooves not darkened; near the orbits a rather broad, glistening, yellow border, extending as a line to the vertex. Proboscis nearly black, about as long as the antennæ, much swollen at the base. Antennæ nearly black, first and third joints reddish below, of nearly equal length, scarcely more than one-half as long as second joint; second joint of style moderately projecting; third joint gradually tapering, of medium length. Front wholly black; with a coppery luster, when viewed obliquely. Occiput black, with a rather narrow golden yellow border along the orbit. Thorax black; spots on the inner sides of the humeri, the

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upper border of the disk of the metanotum, and large spots in front of the halteres nearly golden yellow, a small spot below each humerus, and one above the root of the wing, less distinct, of the same color. Upper portion of pleural stripe not distinctly defined. Abdomen black, second segment mostly brownish, its tip, and beginning of the third, pure yellow; first and second segments narrowly, or indistinctly, dusted; third, fourth and fifth with moderately broad yellow hind borders; sixth segment covered on the end with yellowish dust. Legs brown or black; base and tips of femora and basal portions of tibiæ, yellow. Coxæ and anterior tibiæ on their outer portions with a more golden satiny luster. Wings darker before the third longitudinal vein, sub-hyaline behind; the darker portion nowhere with distinct outlines, a little clearer nearer the humeral cross-vein; slightly darker, narrow clouds before the fifth and sixth veins. Posterior cross-vein at right angles. Long. corp., 8–10^{mm}. Five specimens, Va. (Davis); S. Ca., Ga. (Morrison); Mass. (Burgess).

Conops sylvosus, n. sp.

♂. Face obscure white, or yellow, with a broad border of silvery dust near the orbit, extending, obliquely narrowed, to near the vertex; facial grooves not darkened. Cheeks like the face, oral border sharp. Front obscurely, or quite black, extending down on the sides of the base of the antennæ. Vertical callosity black, or reddish-black, flattened, extending far forward. Antennæ black, red on the under sides, third joint as long as second, first joint more than half as long. Second joint of style projecting very little, third joint rather short, with a very brief bristle-like end. Proboscis brownish black, scarcely as long as the two last joints of the antennæ. Occiput black; orbits, except on the upper part bordered with yellowish dust. Thorax black, shining; inner sides of humeri, disk and sides of metanotum, faintly dusted with white. Abdomen black, shining; sides of the second segment more or less red. All the segments, except the sixth, with nearly uniform, rather narrow, yellowish white hind borders; fifth, and especially the sixth, dusted with whitish. Legs black; coxæ, particularly the first pair in front, and a small spot near the tip of each of the anterior tibiæ, with satiny white luster. Extreme base and tips of femora, red or yellow; basal three-fifths of anterior, and two-fifths of hind tibiæ, light yellow; distal portions and tarsi, reddish brown. Wings light brown and pure hyaline, costal cell scarcely lighter; the brown is limited by the third longitudinal

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vein as far as the small cross-vein; then by the fourth longitudinal to the great cross-vein, beyond which is a hyaline cross-band reaching the costal margin, excepting sometimes a faint, narrow cloud along the third vein. A brown spot in the end of the submarginal cell that extends a short distance on the posterior side of the third vein; great cross-vein and the anterior side of the fifth vein from near the small cross-vein, with brownish clouds. Great cross-vein not at all oblique. Long. corp., 9–11^{mm}; Long. al., 6–7½^{mm}. Three specimens. On underbrush, Mass., July 1; Conn., June 25. New England (Sanborn); Burgess' col.

The hairs on the vertex, dorsum of thorax and sides of first abdominal segment, are longer and more abundant than in any other species known to me. Its parasitic habits will probably be found to be quite different from any of the following.

Conops excisus Wied.

Aus. Zw. ii, 234. Lw., Neue Beitr., i, 28.

♂. Face yellow, the orbits with a rather broad glistening border of yellowish white, extending as a narrow line nearly to the vertex; facial grooves not darkened; cheeks more reddish. Proboscis about twice the length of the head; reddish brown; at base and tip black. Antennæ red or reddish brown, more or less black near the end of the third joint; first joint more than half as long as second, second and third of equal length; second joint of style with a very inconspicuous process; third joint at the base not very broad, and soon attenuated into a long, bristle-like point. Vertex red or reddish brown; front mostly filled out with dark, reddish brown, blackish above the base of the antennæ. Occiput brown, orbits with a line of yellowish dust, broader above and extending across below the vertex. Thorax black; humeri, scutellum, and the upper parts of the sides of the metanotum, red. The lateral borders of dorsum may be of the same color. Humeri on their inner sides and extending outwards along their hind border; a small spot in front below; another in front of the roots of the wings above; narrow pleural stripes, distinctly margined above; upper margin of the metanotum; and a large spot in front of each halter, bright golden yellow. Smaller spots on the sides of the disk of the metanotum and just back of the root of the wings, more grayish yellow. Abdomen chiefly black; second segment on its sides or almost wholly, red; the three or four following segments with only the hind border, red, or also with large

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spots on the sides. Hind borders of the segments with bright golden yellow bands, broad on the second, narrower on the third and fourth, and broad again on the fifth; sixth segment behind, similarly colored. Anterior parts of second, fifth, and sixth segments more grayish dusted. Legs red; base of tibiæ yellow, anterior pairs on the outside with white luster; tarsi brownish black, first joint reddish black. Wings rather dark brown and hyaline, distinctly clearer before the first longitudinal vein. From the small cross-vein the discal cell, except a dark brown cloud before the fifth vein, is nearly hyaline, limited in front by a very clear streak between the fourth vein and a sort of spurious vein, extending between the anterior ends of the small cross-vein nearly to the posterior; last segment of the fourth vein with a rather narrow clearer space in front. Sixth vein with a dark brown cloud. Long. corp., 13–16^{mm}. Long. al., 10–11^{mm}. Ga., Flor., N. Carolina (Morrison).

The female, according to Loew, has an unusually large process on the underside of the fifth segment. The red color on the sides of the segments is not a sexual marking.

Conops bulbirostris Loew.

Neue Beitr., i, p. 30.

Very closely allied to the foregoing, yet evidently distinct. A single female specimen was described by Loew which agrees nearly perfectly with two male specimens before me, from Carolina and Georgia. (Burgess' collection.)

The second joint of the antennæ is proportionately a little longer, but I can see no difference in the length of the terminal joint of the style; the proboscis, however, is distinctly shorter, and at its base more swollen. The second segment of the male abdomen, in my specimens, at least, is narrower.

♂. Front wholly and quite black, vertex a more brownish black, face yellow, cheeks brown, the brown not reaching far forward. Antennæ black, somewhat reddish on the undersides of the first and third joints. Thorax and abdomen black throughout. The grayish or grayish yellow, and much less distinct dust markings of the head and thorax are quite as in *excisus*, except that the spot on the side of the dorsum of the thorax, before the root of the wing, is less distinct, the dust on the disk of the metanotum broader and less sharply defined, and the pleural stripe is indistinct or diffused on the upper half. The third and fifth segments of the abdomen have rather narrow, the

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fourth very narrow, gray or slightly yellowish hind borders; sixth segment, on the end, thickly dusted with gray, yellowish at the tip. Legs reddish brown (or "brownish black," Loew), marked quite as in *excisus*. Color of wings rather darker than in *excisus*, especially before the first vein; the first and second basal cells near the fourth vein are lighter or with hyaline streaks; otherwise scarcely at all different. "Of the size of medium specimens, and the shape fully as in *excisus*" (13–14^{mm}), Loew. The specimens before me measure 16–17^{mm}.

An additional female specimen from Georgia has the antennæ and proboscis nearly as in *excisus*, but its coloration and markings similar to *bulbirostris*; the process of the fifth abdominal segment, below, is of extraordinary size. It measures but 9^{mm}. I believe it to be distinct.

Conops xanthopareus, n. sp.

♂ ♀. Face and cheeks yellow, with glistening dust near the orbits, extending narrowly upon the sides of the front. Facial grooves with a black spot on each side of the median prominence; proboscis reddish brown, black at the ends; base much swollen, scarcely once and a half the length of the head. Antennæ reddish brown, considerably blackened at the end of the third joint; first joint more than half as long as second; second and third of equal length; process of second joint of style small, but more projecting than in the two preceding; third joint moderately long, the attenuated portion scarcely as long as its basal portion. Front and vertex yellow, brownish above the base of the antennæ. Occiput brown, its under part pure yellow. Posterior orbits with borders of grayish yellow dust. Thorax black, humeri and more or less of the scutellum red. A spot on the inner side of each humerus, a small spot below; broad pleural stripes, distinctly limited above; upper border of disk of metanotum, extending broadly on its sides, golden or grayish yellow. Abdomen mostly black, sometimes quite reddish upon the sides, especially of the second segment; the four anterior segments rather broadly bordered with golden or grayish yellow, nearly the whole upper surface of the fifth and sixth segments thickly dusted with the same; process of the fifth segment of the female rather small below. Legs red, basal half of tibiæ yellow. Coxæ and outer distal part of anterior tibiæ with silvery luster; tarsi brownish black, last four joints, quite black. Wings brown in front, subhyaline be-

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hind, a stripe in the first basal cell, and the base of the anal cell, pure hyaline. The subhyaline fills out a larger part of the discal and first posterior cells; the stripes along the fifth and sixth veins not so well marked as in *excisus*, otherwise similar. Long. corp., 11–12^{mm}, long. al., 10^{mm}. Four specimens, Texas (Belfrage); Mass., Conn.

Two additional male specimens from Connecticut, differ in their much deeper black, the legs in part, the antennæ almost wholly so, the spot above the base of the antennæ larger, the humeri and scutellum scarcely reddened, and the dust markings throughout are more gray.

Conops tibialis Say.

Journal Acad. Phil., vi, 171.
C. nigricornis Wied., Aus. Zw. Ins., ii, 236, 4. Loew, Neue Beitr., i, p. 31.

♂ ♀. Face waxy yellow; cheeks black, with a narrow silvery line near the orbit; facial grooves deep black. Proboscis black, considerably swollen at the base. Antennæ black; the under sides of the first and third joints somewhat reddish brown; first and third joints of nearly equal length, scarcely half as long as the second; process of second joint of style only moderately projecting; third joint conic, rather short. Vertical callosity nearly black, rather shining, bordered in front by a black, opaque, transverse stripe, continued as a median stripe to near the base of the antennæ, where it divaricates Λ-like on to the sides of the face. Occiput black, dusted with whitish, above posterior orbits with a narrow white line. Thorax deep black, with whitish bloom when viewed obliquely; humeri usually concolorous, sometimes slightly tinged with red, faintly or not at all dusted with white above; sides of metanotum inconspicuously dusted with white. Abdomen black, first and second segments with broader, remaining segments with narrower, posterior yellowish white borders, fourth, fifth, and sixth with white dust above, becoming most abundant on the sixth. Legs black, coxæ with satiny white luster; trochanters, base and extreme tips of femora, yellowish red; basal halves of tibiæ yellow, distal portions reddish brown; anterior pairs, on their outer distal portions with satiny white luster. Tarsi: anterior pairs reddish, becoming black at tips, hind pair mostly black. Anterior half of the wings dark brown, before the first longitudinal vein not much clearer; the brown accompanies the fifth longitudinal vein nearly to the border of the wing, then follows

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parallel to the great cross-vein till it reaches the last segment of the fourth, which it accompanies to near the tip, and then borders the last segment of the third longitudinal in the second posterior cell; sixth longitudinal vein with a dark brown stripe in front. Posterior cross-vein but slightly oblique; third and fourth longitudinal veins distinctly angulated at the tips of the small cross-vein. Long. corp., 11–13^{mm}. Long. al., 9–10^{mm}. Thirty specimens. Indiana (Say); Mass., Conn., D. C., Va., N. C.

Our most common eastern species, and very uniform in its markings. Of the synonymy, there can be no doubt. Loew's description of the first and third joints of the antennæ being about *one-third* as long as the second, I can attribute only to an error, as in neither this nor *sagittarius* is the discrepancy nearly so great. The differences between *tibialis* and *sagittarius* are such that Wiedemann's supposition of the synonymy, drawn from Say's brief description, is quite excusable and fully accounts for his doubts in giving a new name. The description of *tibialis* by Say appeared but a short time previous to Wiedemann's second volume of his Zweiflügelige Insecten, and was also apparently overlooked by Loew. The two species are most readily separated by the color of the cheeks, a character which I have found of excellent value in all the specimens of the genus examined by me.

Two female specimens from Georgia (Morrison), and Texas (Belfrage), in Mr. Burgess' collection, differ in their much larger size (16^{mm}), and deeper black color throughout, yet they are evidently the same.

Conops sagittarius Say (*non* Wied., Loew).

Journ. Acad. Phil., iii, 83, 2.

? *C. genualis* Loew, Neue Beitr., i, p. 32.

? *C. castanopterus* Loew, loc. cit.

♂ ♀. Face waxy yellow; cheeks behind and a transverse spot in front, reddish brown, or brownish black, enclosing a quadrate yellow space; a white line near the orbits, not extending higher than the antennæ. Facial grooves black, lighter in the median line. Proboscis reddish brown, black at the end, considerably swollen at the base. Antennæ red, or reddish brown, deeper on the upper sides of the joints, first and third joints of nearly equal length, scarcely half as long as the second; process of second joint of style a little more prominent, and third joint more acuminate than in *tibialis*. Vertex varying from red to brownish black, bordered in

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front by a black, opaque, transverse stripe, continued as a median stripe to its divarication, near the base of the antennæ. Occiput black, dusted with white, posterior orbit with a moderately broad yellowish white line. Thorax black, with whitish bloom, scutellum wholly or in part red; humeri red or reddish yellow, with a very conspicuous spot of golden yellow dust above, continued on their inner sides, and less conspicuously behind; the narrow, and on the upper half indistinctly limited oblique stripes of the pleuræ, upper border of the disk of the metanotum and large spots on the sides with yellowish gray dust. Abdomen black, more or less red near the tip of second segment; first segment with white, end of second with yellowish dust; hind borders of third, fourth, and fifth segments conspicuously golden yellow, broadest on the fourth; fourth, fifth and sixth segments conspicuously dusted with white or yellowish. Legs red or reddish brown, often quite black on basal half of femora, behind; basal halves of tibiæ yellow, anterior pairs in their distal portions with a satiny white luster, like the coxæ; tarsi black near the tips. Anterior half of wings brown, the markings quite as in *tibialis*, a small subhyaline space more frequent in the first posterior cell. Long. corp., 11–13^{mm}. Long. al., 8–10^{mm}. Seven specimens, Penn. (Say). Mass. Conn.; N. Carolina (Morrison).

Loew's description of *genualis* agrees very well, throughout, except that the cheeks are *brownish yellow*. *C. castanopterus* agrees in some respects even better. I give here careful translations of both descriptions, as they are rather inaccessible.

Conops genualis Loew.

Neue Beitr., etc., i, p. 32.

♂ ♀ "Face yellow; facial grooves black, the cheeks brownish yellow, without luster except on the under half of the orbit, where there is a white line. Front on the sides yellow with a narrow median black line, bifarious above the antennæ and passing as a black line upon each side of the face. Vertical callosity yellowish brown, bordered in front by a narrow black transverse band. Occiput blackish brown, posterior orbit with a border of yellowish brown dust, below yellow. Antennæ dark brown, below nearly ferruginous; first joint about half as long as the second; the third at the end attenuated; second joint of the style moderately large, strongly projecting, the third short. Thorax black, humeri and scutellum blackish brown, the former with nearly golden yellow

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dust above which extends inwards on the front border of the thorax and behind nearly to the roots of the wings; pleuræ with an indistinct white stripe; metanotum above and the two spots on the sides with grayish yellow dust. Abdomen quite black, the first and second segments with rather narrow bands of yellowish dust; on the third segment only a finely yellowish dusted posterior line; the fourth and fifth segments of the male abdomen with a similarly dusted somewhat broader posterior border; the fifth segment with a very narrow posterior line; the last two segments in both sexes with yellow dust. The extension of the fifth segment below in the female as in [*tibialis*], but the last segment somewhat shorter. The coloration of the wing and the neuration wholly as in my specimen of [*tibialis*]. Length of body as in [*tibialis*]."

C. castanopterus Loew.

Neue Beitr., i, 33.

"A third species further removed from [*tibialis*] but more nearly related to *genualis*. It is most readily distinguished by the more blackish brown color of the anterior part of the wings, which is distinctly clearer before the first longitudinal vein, by the ferruginous color of the humeri, the lateral margins of the thorax, the scutellum, and a larger or smaller, sometimes a very large part of the abdominal segments and the legs. The third joint of the antennæ is at the end yet more attenuated; on the cheeks a large yellow spot lies between the brown. In all else quite as in *genualis*. Georgia, Carolina."

Conops furcillatus, n. sp.

? *C. æthiops* Wlk., List., etc., iii, 671.

♂ ♀. Face waxy yellow, facial grooves not darkened, cheeks black. Proboscis black, not much swollen at the base. Vertical callosity black, bordered in front with opaque black, extending as a median stripe to the antennæ and there divaricating and passing as a black line upon each side of the face. Antennæ brownish black, first and third joints more reddish below, first joint very short, scarcely a third as long as second, third joint about half as long; third joint of style short, conical. Occiput black, broadly dusted with whitish below the vertex. Dorsum of thorax black, more or less reddened near the borders; humeri with a spot of yellowish white dust on their

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inner sides. Scutellum more or less reddish, the usual spots on the metanotum indistinctly dusted. Pleuræ chiefly reddish, especially along the sutures, the disks of the segments sometimes mostly blackish. Abdomen black, the second segment, on its sides, and sometimes the beginning of the third, reddish brown; first segment with distinct, second with broader, especially on the sides, third and fourth with moderately broad, fifth with narrower borders of grayish yellow dust; posterior part of fourth, with the fifth and sixth distinctly whitened above. Legs reddish or reddish brown; fore and hind coxæ darker, with satiny luster; femora sometimes darker toward the base, basal halves of all the tibiæ yellow, tips of all the tarsi dark brown or black. Wings brown on the anterior half, lighter before the first longitudinal vein; from a little before the small cross-vein the discal cell is hyaline, excepting a narrow dark cloud before the fourth longitudinal vein, reaching nearly or quite to the great cross-vein; on the anterior side of the fourth vein an oval longitudinal space in the first posterior cell, reaching nearly the whole length of the segment of the vein, a lighter spot in the outer part of the submarginal cell, near the margin. Petiole of the first posterior cell about as long as the posterior cross-vein. Long. corp., 10–12^{mm}, long. al., 7–9^{mm}. White Mts. (Patton, Morrison). Three specimens.

The very short first joint of the antennæ renders it probable that this is Walker's *æthiops*, but his very indefinite description of the front and wings prevents me from feeling at all sure of it.

Conops Burgessi, n. sp.

♂ ♀. Face and front yellow, vertex concolorous or reddish, when the latter, with a more or less faint narrow median stripe, scarcely reaching the base of the antennæ. Facial grooves not distinctly darkened, cheeks red. Proboscis brownish red, black at tip. Antennæ red; first joint very short, scarcely a third the length of second; third joint a little more than half as long as second, rather symmetrically attenuated. Style black, process of second joint small, not conspicuous, third joint not thickened, moderately long, conical. Occiput red, not distinctly dusted. Thorax red; a broad, median, posteriorly abbreviated stripe, spots in front of the humeri and disk of metanotum, except the sides and upper edge, black. No, unless very indistinct, spots of dust near the humeri, nor elsewhere on the thorax. Abdomen red, sometimes somewhat blackish on the sides of the segments, especially of the third; faint spots of dust on the first and near the tip of

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second segment, posterior segments very faintly dusted on their upper sides. Process of fifth segment below, in the female, not large. Legs red, anterior coxæ blackened in front, without distinct luster. Coloration of wings as in the preceding species, the cloud along the fifth vein in the outer part of discal cell, often quite obsolete, scarcely any along the sixth. Long. corp., 10–11^{mm}, Colorado (Prof. Snow); Mendocino, Cal. (O. T. Baron). 16 specimens.

Conops texanus, n. sp.

♂. Face and front yellow, anterior border of the vertical callosity a little darker, a narrow median stripe, divaricate at base of antennæ, brown; facial grooves not darkened. Cheeks with a transverse brown spot in front, continued as a narrow line along the oral border to the brown of the posterior part. Antennæ red, a little darker at the tip of third joint, third joint short, hardly as long as first, distinctly less than half the length of second; second joint of style projecting strongly, nearly as far as third, third short, conic. Proboscis red, base and tip blackish. Occiput brownish below, posterior orbit with a yellowish white border, broader above, continued as a narrow line behind the eye, and broader on the sides of the face. Dorsum of thorax, pleuræ, and scutellum red, humeri lighter, a narrow median dorsal stripe and triangular spot on the disk of metanotum, black; humeri above, a spot on the inner side and behind, with yellow dust, oblique stripe of the pleuræ (diffused on the upper half), upper border of disk and sides of metanotum with fainter, less yellow dust. Abdomen brownish red, with more or less black on upper parts of the segments, first segment distinctly, the others more faintly, dusted. Coxæ usually black, satiny luster distinct, but not conspicuous. Legs red, basal halves of tibiæ yellow, Wings brown and hyaline; costal cell much lighter, nearly hyaline. Discal cell from before the small cross-vein, except a cloud along the fifth vein, and an oval spot in first posterior cell, nearly hyaline. The penultimate segment of the fourth vein is scarcely three-fifths the length of the ultimate. Long. corp., 16^{mm}, long, al., 10^{mm}. One specimen. Waco, Texas (Belfrage, Burgess' col).

Conops marginatus Say.

Journ. Acad. Phil., iii, 82, 1. Comp. W., ii, 73.

♂. Face and front waxy yellow, vertex rather brownish black, with a broad opaque black anterior border, extended as a narrow

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median stripe to the base of the antennæ, there divaricating and passing to the sides of the face; facial grooves black, cheeks with a yellow space between two black ones. Proboscis black. Antennæ black, first and third joints, below, red, of nearly equal length, scarcely half the length of second, third joint quite unsymmetrically attenuated, about half as broad at base as long, second joint of style with a considerably projecting, rather slender process, third joint moderately long, conical. Occiput black, posterior orbits with a narrow silvery line. Thorax black, scutellum red. Humeri above and on the inner sides with silvery white dust, sides of the metanotum and the oblique stripe of the pleuræ (indistinct in its upper part) with white dust. Abdomen black, second segment reddened on the sides near the tip, hind borders of all the segments with yellowish dust, rather broadest on the fourth, fifth and sixth lightly dusted with gray above. Legs brownish red, probably often quite blackish, especially near the base of the femur; coxæ black with silvery luster. Wings brown on anterior half. Costal cell lighter, outer part of discal cell hyaline, except a narrow cloud along the fifth longitudinal vein; a more or less large spot in first posterior, hyaline, outer part of marginal cell less deeply colored. Sixth longitudinal vein without brown cloud. Long. corp., 10–11^{mm}. Long. al., 6–7^{mm}. Two specimens. White Mts.

Conops affinis, n. sp.

♂ ♀. Front either wholly yellow, or with the vertex rather more reddish, and with a brownish anterior border extending as a narrow median stripe to the base of the antennæ and there divaricating. Face yellow, grooves usually quite the same, sometimes a little reddish near the middle. Cheeks with a yellow spot between the brown. Proboscis red, tip black. Antennæ yellowish red, third joint more or less infuscated above. The relation of the joints very nearly as in *marginatus;* the process of the second joint of style is, however, broader, and the third joint shorter and thicker. Occiput brown, posterior orbits with a narrow yellowish line. Thorax red, dorsum with a broad, median, opaque black stripe beginning at the neck and extending beyond the transverse suture, either very narrowly separated or confluent on each with a similar one beginning back of the humerus and extending to near the scutellum. Disk of the metanotum, except the edges, black. Humeri above, a spot on the inner side, a faint one behind, with golden yellow, or in lighter colored specimens grayish dust. Oblique stripes of the pleuræ and sides of the metanotum very

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faintly dusted. Abdomen red, with more or less black, on the dorsum of the third, fourth and anterior part of fifth segments; posterior part of fourth, the fifth and sixth segments, in well preserved specimens, quite conspicuously dusted with golden yellow. Legs red, coxæ mostly, or at least the middle ones in front, black; basal half of tibiæ yellowish. Wings light brown on the anterior half, coloration very nearly as in the preceding, the first posterior cell, sometimes scarcely at all, sometimes for the larger part, nearly hyaline. Long. corp., 10–11^{mm}. Twelve specimens. Kansas Plains; California (Baron); Washington Ter. (Morrison).

This species is intermediate between *marginatus* and *texanus*, holding the same relation to the former as *Burgessi* does to *furcillatus*. From *texanus* it may be distinguished by its much smaller size and broad thoracic stripes, from *marginatus* by the shape of its antennal style and the strong difference in coloration.

Conops pictus Fab.

According to Loew, who apparently has recognized this species, (Vide O. Sacken's Catalogue, Note 258), *pictus* F. is not *pictus* Wied. but *Ramondi* Bigot *is* a synonym of the former. Not having seen the species I give here Fabricius' original description, together with Bigot's:

Fab. Ent. Syst. IV, 391, 3. Caput ferrugineum, rostro nigro orbitaque oculorum aurea. Thorax niger antice utrinque puncto calloso, marginali ferrugineo. Lineolae duae parvae marginis antici, linea lateralis, scutellum lineaque punctaque duo sub scutello flavis. Abdomen hamosum, ferrugineum segmenti primo et secunda nigris margine flavo. Alae hyalinae, costa late flava. Pedes ferruginei, apice nigri.

C. Ramondi Bigot, Ramon de la Sagra, 808.

"Niger, capite ferrugineo; oculis brunneis; antennis ferrugineo-brunneis, basi pallidis; haustello ferrugineo, acumine brunneo; facie ferruginea, aureo-nitente; fronte ferrugineo, linea media brunnea; orbitis, postice, flavidis; thorace nigro-piceo, antice, linea transversali sinuata, interrupta, flava; metathorace postice subtus linea lata transversali, flava; pleuris externe auratis; alis hyalinis, antice margina ferruginea; margine magna apicali fusca; abdomine petiolato, segmentis duobus primis nigris, incisuris pallide flavis; omnibus segmentis posterioribus rubescentibus; incisuris superne obscure brun-

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neis; pedibus, antice, ferrugineis, postice, pallide auratis; tarsis nigricantibus; pulvillis ferrugineis.—Longit. 23 millim.

"Esta bella especie parece casi idéntica al *C. picta* de Fabr. (*Syst. Nat.*), aunque la descripcion que ha dado este autor sea demasiado incompleta para servir á resolver la duda. Por otra parte, nos parece diferir bastante del *C. picta* de Wiedmann (*Aus. Zweiflug., Ins.*), para autorisarnos á proponerla como nueva. Tal vez asi no ofrezca mas que una variedad occidental."

Conops brachyrhynchus Mac., Dipt. Exot., ii, 3, 15, 13.

Long. 5l. ♂. Trompe assez courte, ne dépassant pas le premier article des antennes. Face et front jaunes. Ce dernier brunâtre au milieu. Antennes peu divergentes; premier article testace; deuxième et troisième noirs, d'egale longuer. Thorax et abdomen noirs; deuxieme segment de ce dernier a bord postérieur testacé. Pieds testaces; jambes a base jaune et anneau brun au milieu; tarses bruns. Balancers testaces. Ailes brunés, bord interieur brunâtre; premiere cellule posterieure, discoidale et anale assez claires, nervure terminale de la discoidale oblique. Amerique Septentrionale. M. Bastard.

Conops flaviceps Mac., loc. cit., 15, 14.

Long. 3½ l. ♀. Noir. Face et front jaunes; vertex fauve. Antennes; noirs, massue testacee en-dessous; troisieme article court. Thorax á epaules et tache contigue en arriere testaces; ecusson et metathorax noirs. Abdomen á petiole tres-menu; premier segment noir; deuxieme et partie anterieure du troisieme fauves; le reste de ce segment, quatrieme et cinqueme noirs en-dessus, sixieme fauve, á duvet d'un gris jaunatre pale, bord postericure des troisieme, quatrieme et cinquieme á duvet semblable. Pieds fauves; hanches noires, jambes á partu anterieure jaune, les quarte derniers articles des tarses noirâtres. Ailes á large bord exterieure brun, termine carrement, une petite tache brune á l'extremite. De l'Amerique Septentrionale.

Conops fulvipennis Mac., loc. cit., 13, 10.

Long. 7 l. ♀. Face jaune; partie superieure de la carene noire, parte inferieur des joues noires. Front fauve á borde des yeux jaune et borde intermediare noire. Antennes testaces. Thorax noir, epaulete brunes, deux petites taches de duvet jaunâtre en-dedons de ce derniers. Abdomen noir, segmens bordes posterieurment de duvet

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jaunatre. Pieds fauves; base des cuisses noires. Ailes: cellule costale d'un jaune pale, un large bord exterieur d'un fauve brunâtre, laissant l'extremité de la cellule sous-marginale de la premiere posterieure et de la discoidale, et le bord interieur hyalins; nervure terminale de la cellule discoidale perpendiculaire à sa base.

Conops analis Fab., Syst. Entem, 175, 3.

Antennæ totæ atræ. Caput nigrum, ore late flavo, haustello atro, orbita oculorum postica flavo micante. Thorax niger, utrinque lobo antico linea alba, micante impressa. Abdominus petiolus elongatus ater; facia ante apicem flava. Ultimum segmentum macula magna dorsali flava. Alæ albo-hyalinæ: costa lata nigra. Pedes nigri.

Wied., Aus. Zwei. Ins., ii, p. 237, 5.

Scheitel und Stirn bis zur Fühlerwurzel tief schwartz, wodurch diese Art sich von *costatus* Fabr. leicht unterscheidet, auch ist der Körper überall tiefer schwartz, der obere Rand des Hinterrückens hat nichts Vergoldetes und schimmert kaum an den Seiten ein wenig weisslich messinggelb. An allem Einschnitten des Hinterleibes ist mehr weniger Messinggelbes. Auch die Beine schimmern in gewisser Richtung zum Theil messinggelb. Länge 5 Linien. Aus Südamerika.

C. costatus Fab. Wied., Aus. Zwei. Ins., ii, 238, 6.

Fühlerwurzel goldgelblich, drittes Glied unten ocherbraun; Augenhöhlenränder messinggelb. Untergesicht und Stirn honiggelb; Scheitel mit schwartzer Binde, von deren Mitte eine schwarze Strieme zur Fühlerwurzel absteigt und hier gespalten diese Wurzel umfasst. Rückenschild bräunlichschwartz; Schultern vergoldet und neben ihnen nach innen ein vergoldeter Punkt; Seitenränder des Rückenschildes und eine zu den mittleren Hüftgliedern absteigende Strieme vergoldet, auch an dem hintern Rande und den Seiten des Hinterrückens ist etwas Vergoldetes. Das Gelbe an den Rändern der ersten Hinterleibsabschnitte reibt sich entweder leicht ab, oder verschwindet bei schnellem Zusammentrocknen, zuweilen scheint es am zweiten zu fehlen, zuwielen auch sogar am vierten vorhanden; der sechste Abschnitt ist in gewisser Richtung überall gelb. Flügel der Länge nach halb braun, Schwinger gelb, mit schwarzen oder schwarzbraunen Kopfe, Schenkel mehr weniger braun. Länge 5 Linien. Aus Südamerika.

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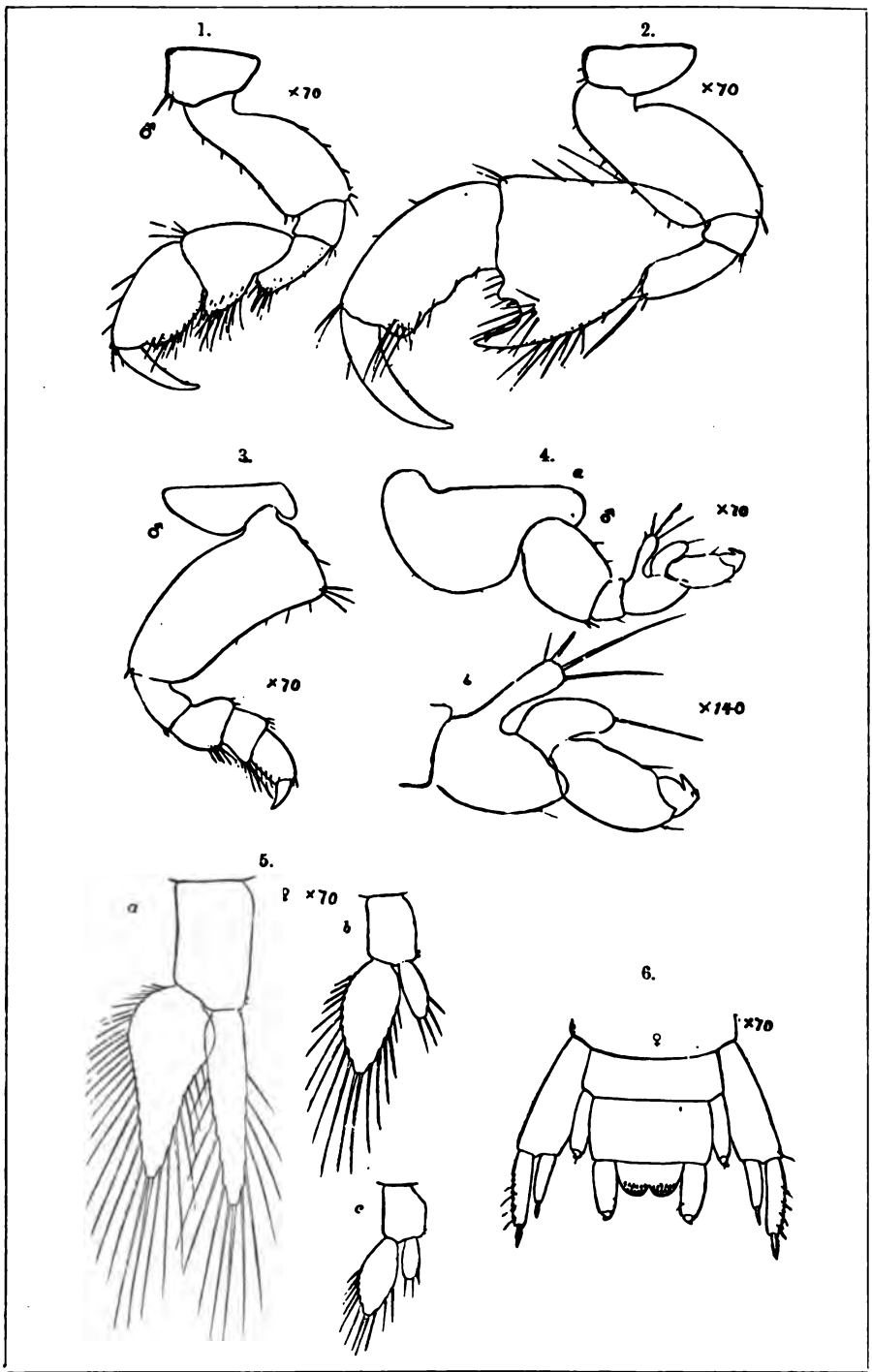

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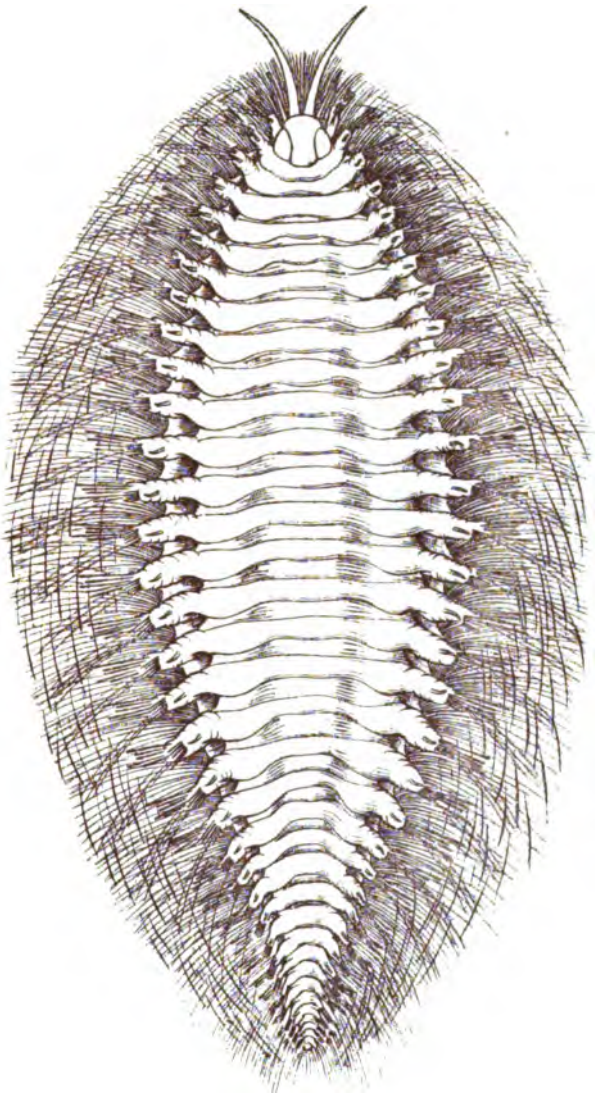

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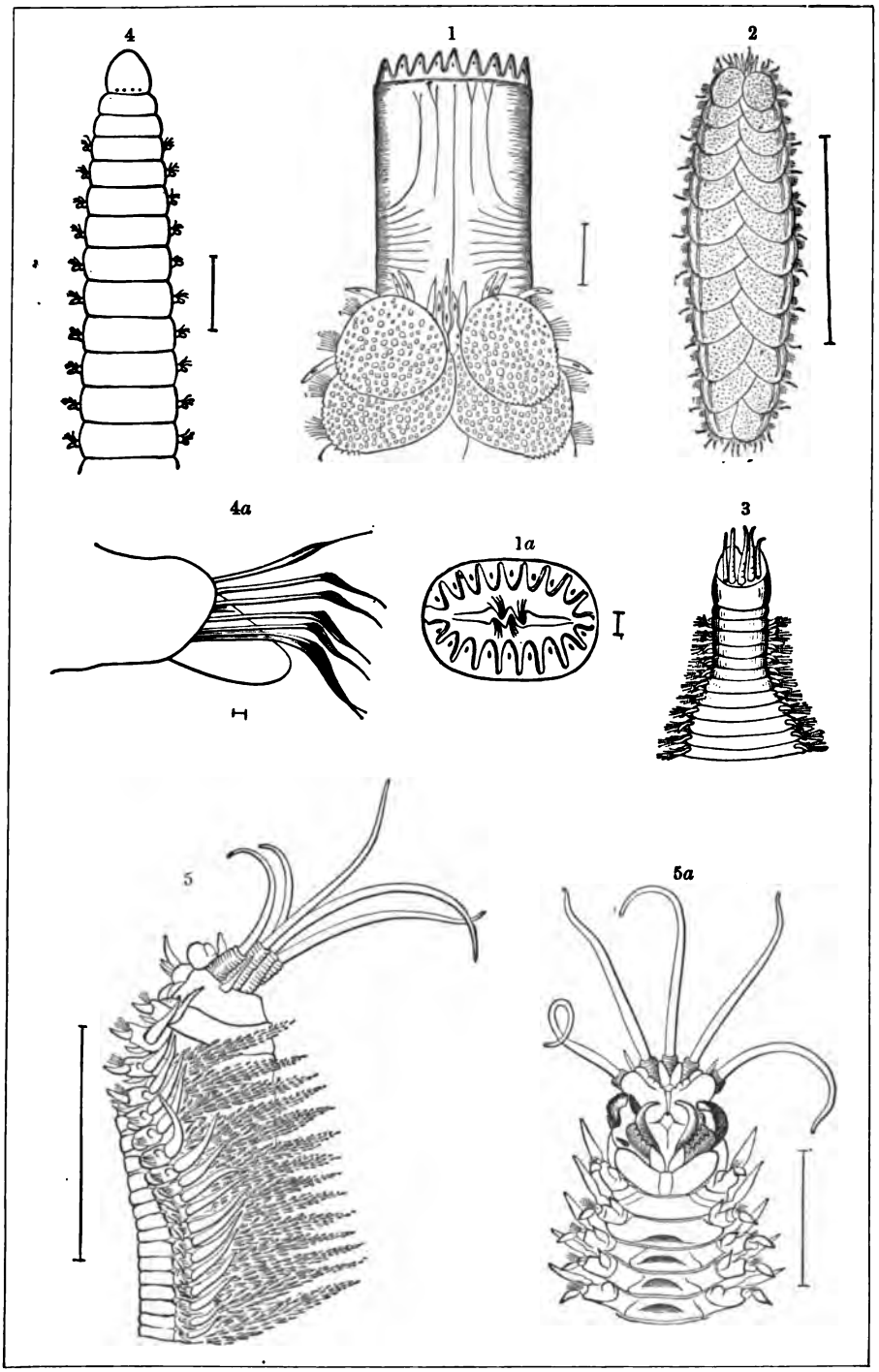

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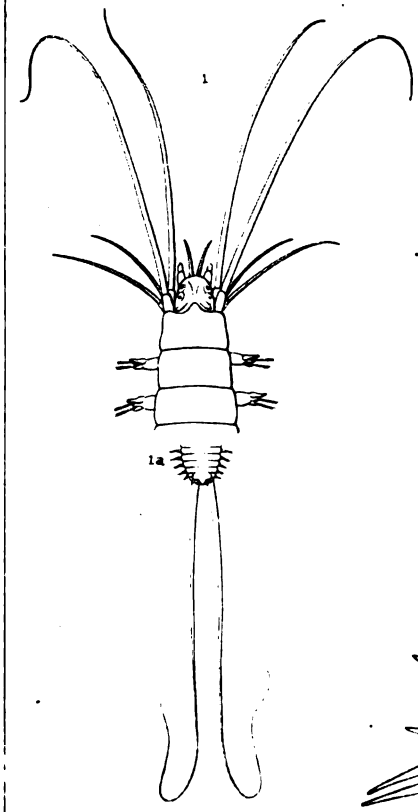

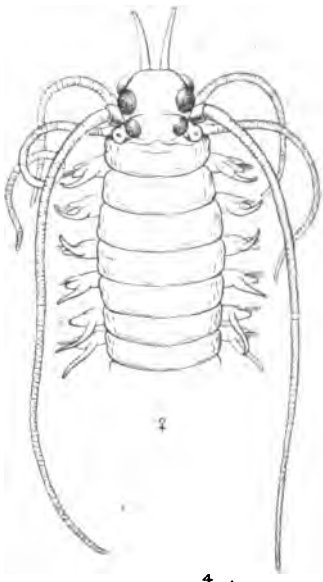

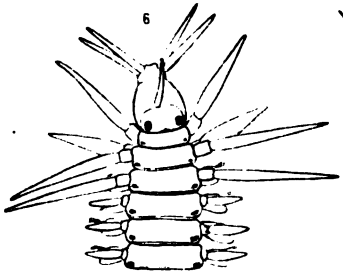

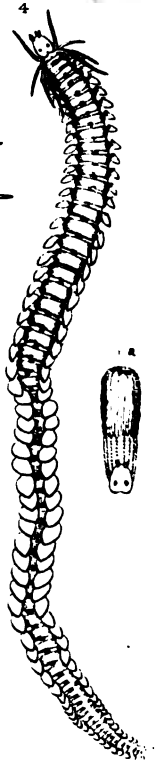

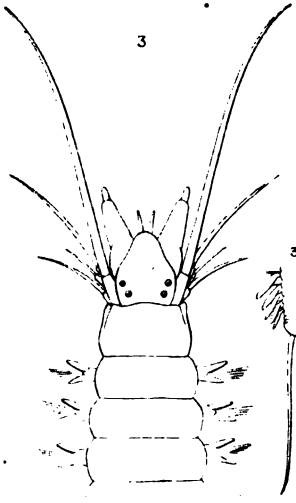

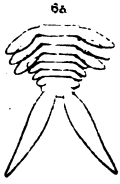

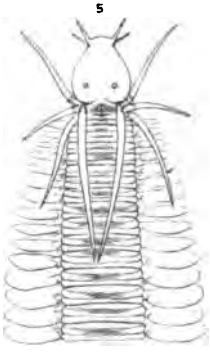

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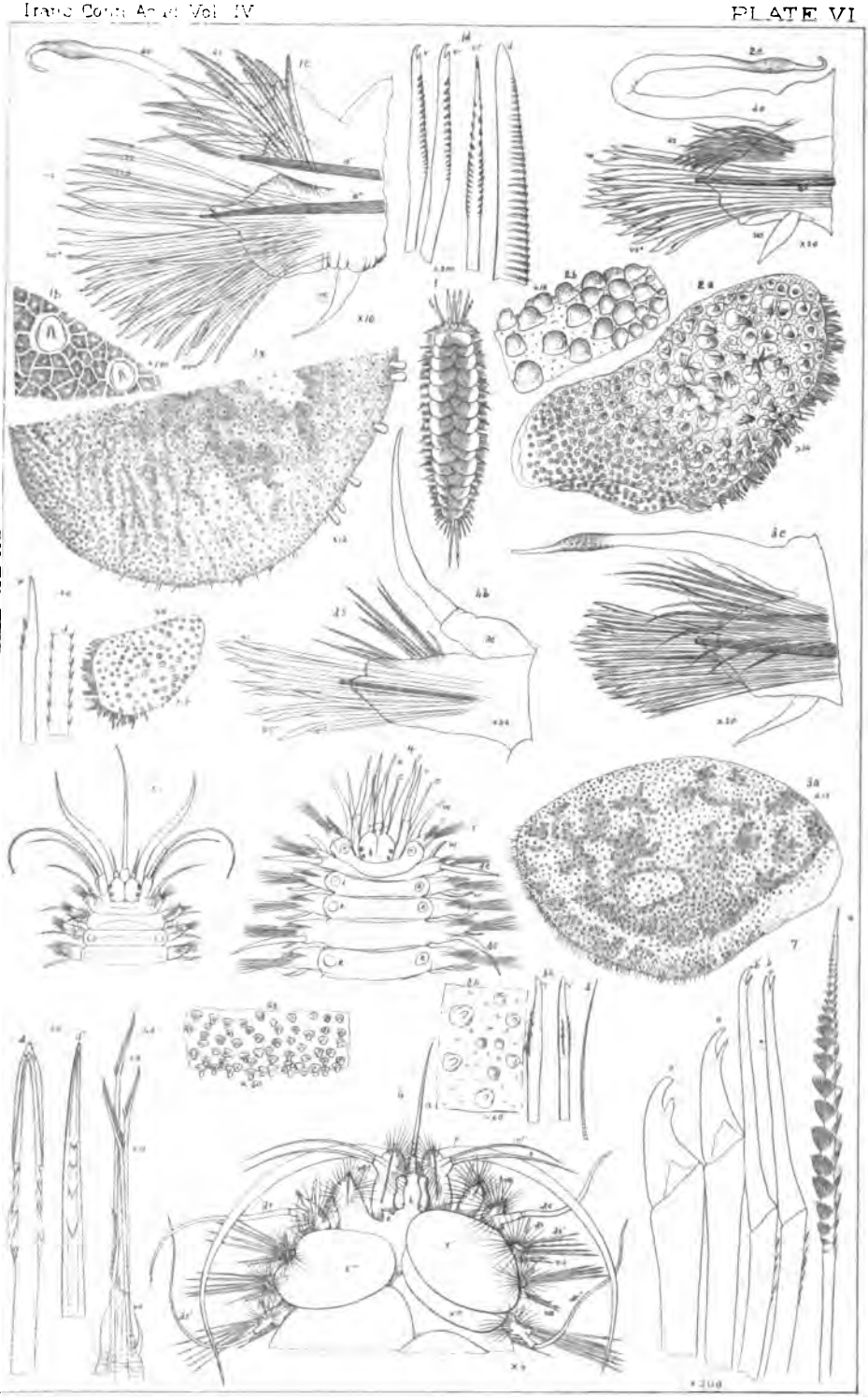

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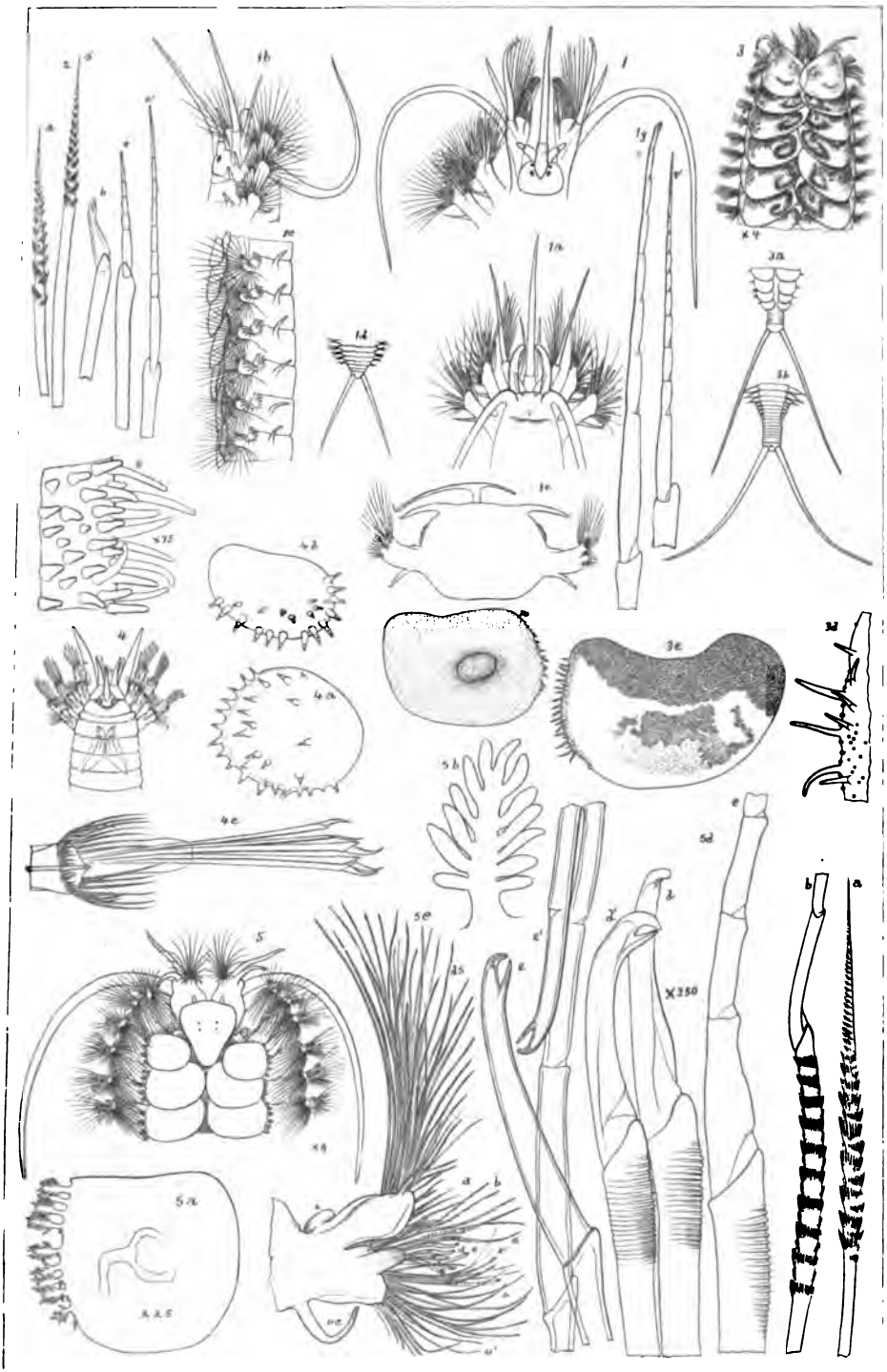

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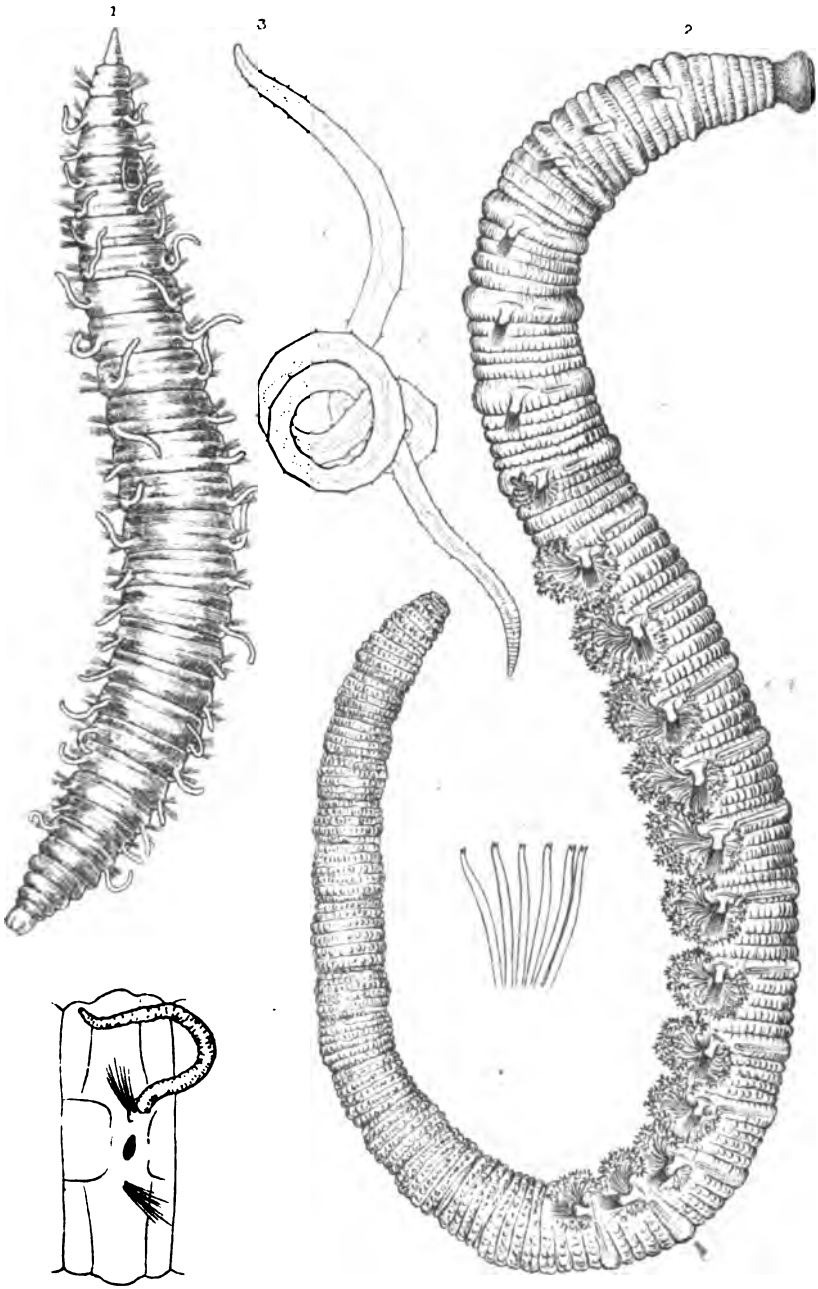

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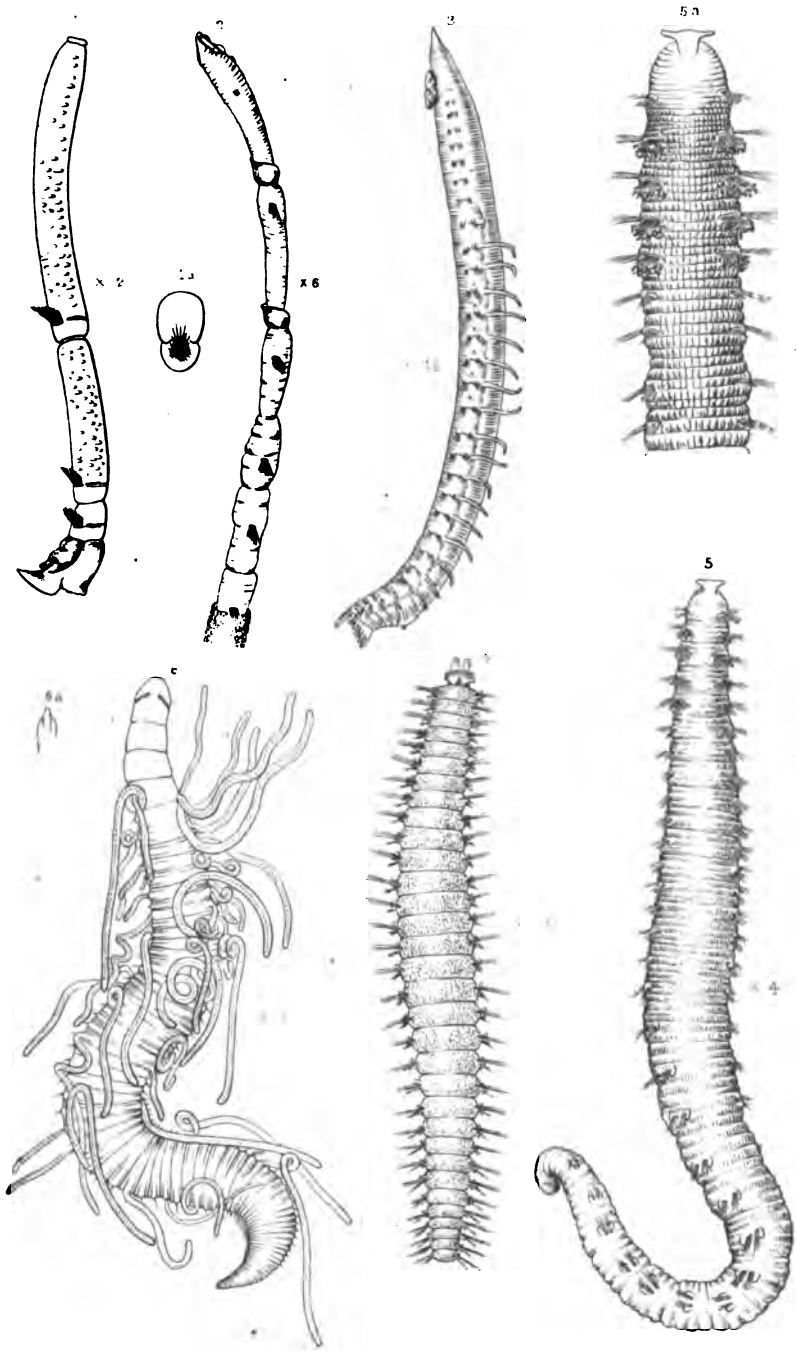

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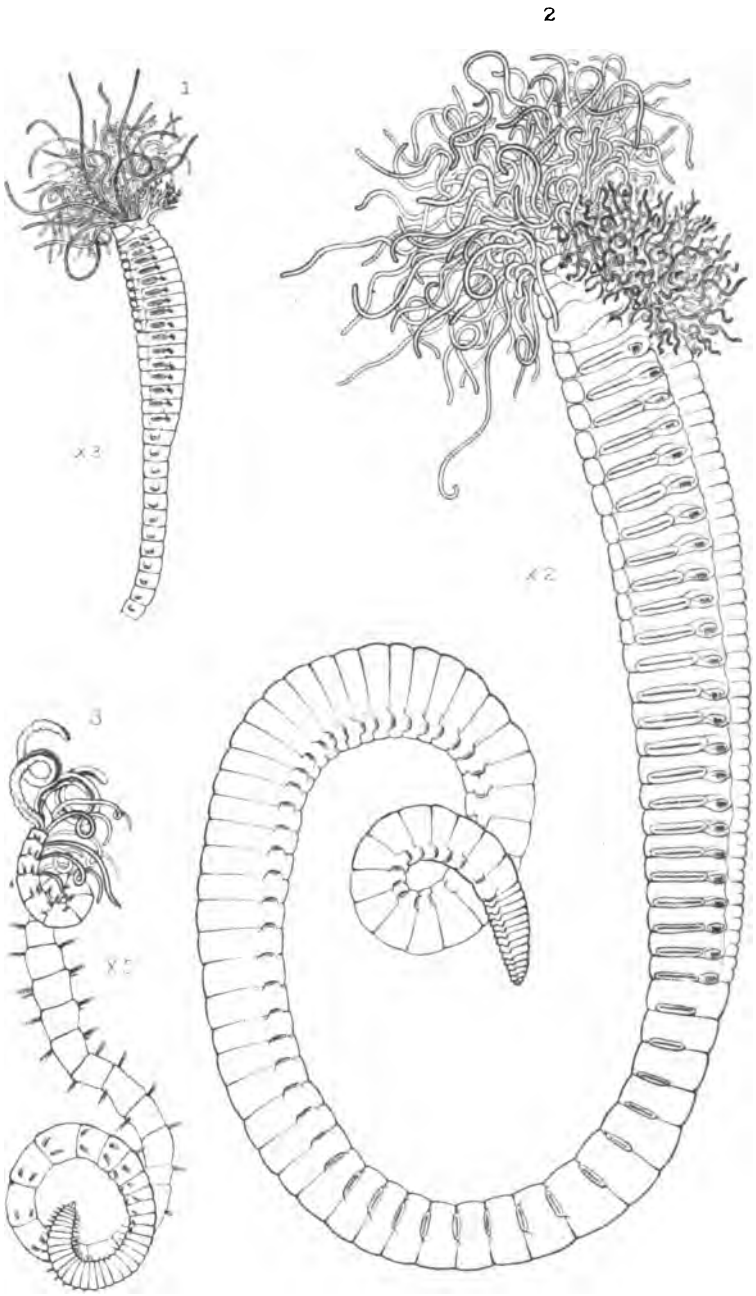

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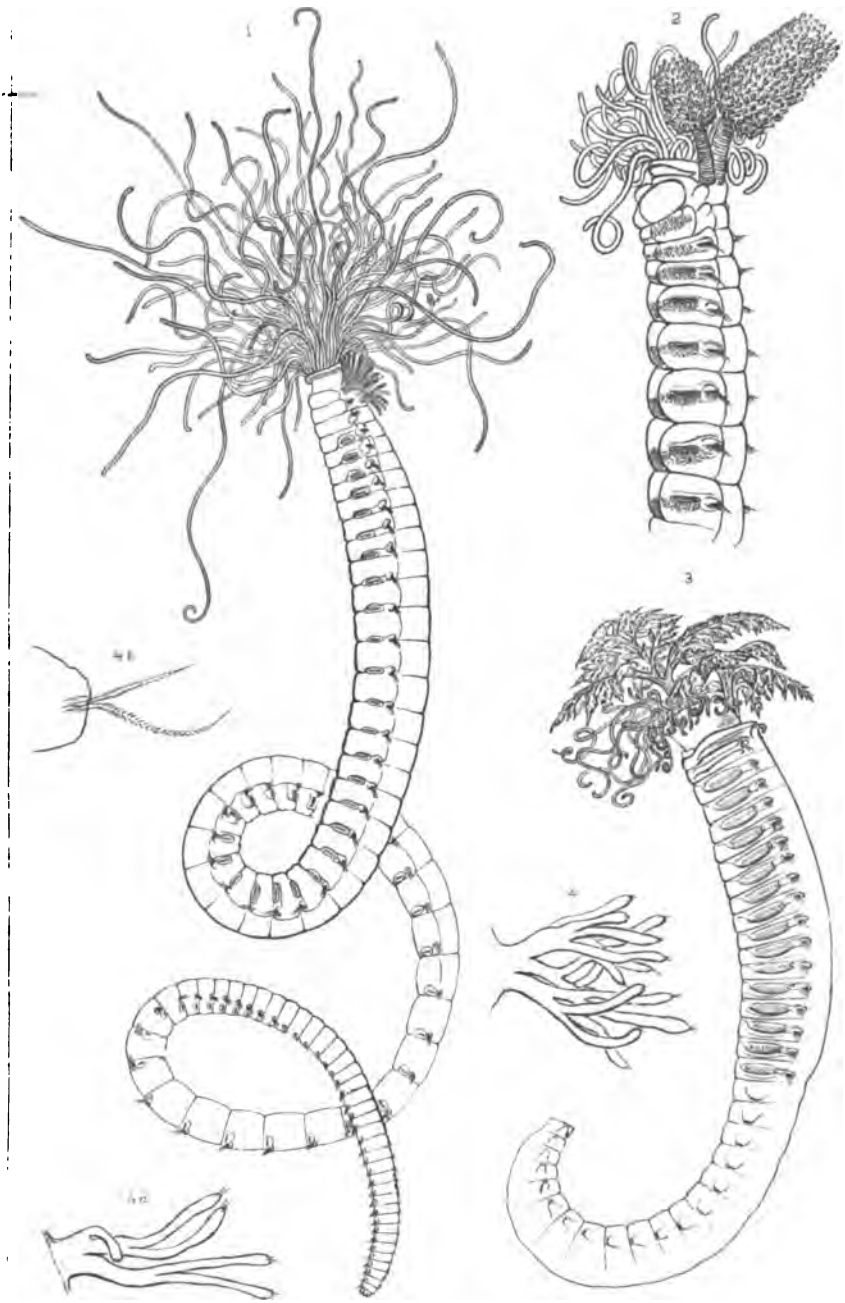

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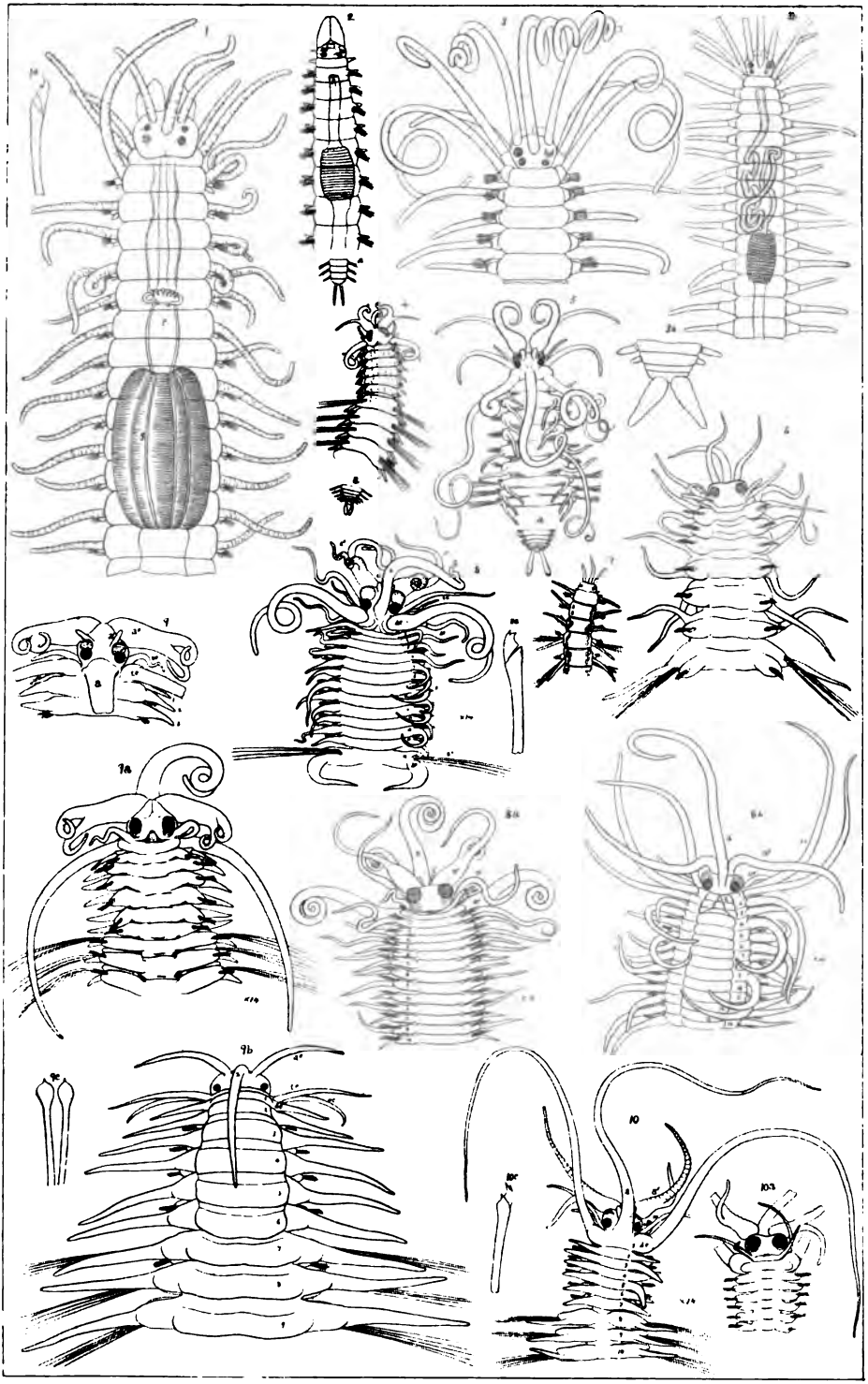

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